STATISTICAL QUALITY CONTROL

(DMSTT21)

(MSC - STATISTICS)

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Unit – I
Lesson – 1

Statistical Basis of Shewhart Control Charts

1.0 Objective :

After going through this lesson, you should be able to:

- Understand the need for Statistical Application in Industry.
- The concepts of Quality and its control, random and assignable causes of variation.
- Shewhart Control charts and their applications in controlling the quality.
- Statistical Basis of Shewhart control charts.
- Role of Normal distribution in the development of control charts and control limits.

Structure

1.1. Introduction to Statistical Quality Control

Earlier, we have studied many Statistical Methods and techniques to make **Statistically valid decisions**. Basically Statistical tools and techniques are useful for making **Optimal Decisions** in many fields like Medicine, Meteorology, Biometry, Industry, Economics, Biology, Biotechnology, Information Technology, Business and so on. Application of statistical tools and techniques to industry, to control the quality of an Industrial output is popularly known as "**Statistical Quality Control (SQC**)". Here the term **'Quality'** play an important role and is to be understood clearly first and then to concentrate on '**How to control the same?'**

In brief, the term **Quality** is defined as "**Fitness for use"**, where as Quality Control means different things to different people. To a Statistician, it may mean the application and solution of Statistical formulae. To the inspector, it can mean a set of sampling tables he is to use cookbook style. To the machine operator, sometimes it has meant an inconsistent putting-up and tearing –down of control charts often coordinated with visits from the buyers or users of the product. Quality Control is as old as industry itself. From the time man began to

manufacture, there has been an interest in quality of industrial output. Thus quality control has a long history.

On the other hand, Statistical Quality Control is a recent one, whose development goes back to **world war - II**. The outbreak of the conflict in 1939 set the United States to think of national defence. U.S. Government has to supply war material, Medicine and food to the armed persons fighting in the war field. Thus armed services enter into the Public Market as large consumers of American output and as such, had an increasing influence on the quality standards of the products purchased. Many Statistical Quality Control Techniques evolved at these juncture to ensure the quality if the industrial output, like, war material, medicine and food which are to be supplied to the military personnel fighting in the war field. SQC techniques basically have three important qualities. Namely they are **Quick, Reliable and Scientific.**

The first to apply the new Statistical methods to solve the problem of Quality Control was Dr. Walter A. Shewhart of the Bell Telephone Laboratories, USA. On May 16, 1924, Shewhart made the first attempt to propose Modern "**Control Chart"**. In 1931, Shewhart published a book on Statistical Quality Control entitled **"Economic Control of Manufactured Product". This book forms the basis for Statistical Quality Control.** Two other Bell System Engineers H.F. Dodge and H.G. Romig took leadership in developing the application of Statistical Theory to SQC particularly in Sampling Inspection to control the quality of industrial output.

1.2. General theory of Control Charts

A **"Control Chart**" is a Statistical device basically, used to study and control of Repetitive processes. Dr. Walter A. Shewhart, its originator suggests that Control Chart may be used as follows:

1. To define goal or standard for a process that the management might strive to attain.
2. To be used as an instrument for attaining that goal by the management and
3. To be used as means for judging whether the goal has been reached by the management or not?

It is important to note that **No Two Units** are a like in this world. However careful the workers are, however precise the machines are, however quality the materials are, we cannot produce two identical items in this world. Some variation or the other will present in the products when the products are produced repetitively. The variation is inevitable in repetitive processes. For instance, if you put your signature 20 times, variation from signature to signature is inevitable. If the variation is within certain limit, we ignore the variation as **'negligible'** and consider all the signatures are same. Suppose, let us think that first 10 signatures are done by your left hand and other 10 are done by your right hand. Then difference between fist set and second set of signatures are large and one cannot consider all the twenty are same. Because the difference is not ignorable and further one can find the cause for that variation. That is signatures are done by left hand and right hand.

First type of variation i.e. ignorable variation is known as **'Random Variation' or 'Chance Variation'** and the second type of variation i.e. large variation or considerable variation is known as **'Assignable Causes of Variation'.** Basic difference between Random Causes of variation and Assignable Causes of variation is that in the former one, variation is very

little (Ignorable) and causes for such variation are large in number. Where as in the latter case, Variation is large in size (Considerable) and causes for such variation are few in number and can be identifiable. From any production process we can identify assignable causes of variation and through proper actions; one can eliminate such causes from the process. But it is impossible to eliminate Random causes or chance causes of variation. If only random causes are alone are present in the production process, we say that the process is "**Under Control**" otherwise, i.e. if assignable causes of variation are also present along with random causes of variation, we say that the process is "**Out of Control**". Control Charts are mainly used to determine statistically whether the given production process is "**within Control**" or "**Out-of Control**". Usually, in industry we consider the following assignable causes of variation. They are:

1. Difference among machines
2. Difference among workers
3. Difference among raw materials
4. Differences in each of these factors over time and
5. Differences in their relationships over one another.

1.2a. Definition of a Control Chart

A control Chart is a graphical representation of Control limits represented as three parallel lines to X-axis known as Upper Control Limit (UCL), Central Line (CL) and Lower Control Limit (LCL). Y-axis consists of the Statistic computed from each sample at different time points. Sample number or sub-grouping of the samples is considered on x-axis.

1.3. Statistical Basis of control charts

Usually, quality of a product is measured through its closely related variable or an attribute. For example, quality of an electric bulb can be measured by determining its life time measured in hours, or Strength of the filament or breaking strength of the shell and so on. Similarly, quality of the cloth can be measured by breaking strength of the yarn or smoothness of the cloth or colour and design of the cloth and so on. Quality of the cracker can be measured through the produced sound measured in decibels. Thus Every Product will have one more "**Quality Characteristics**" through which quality of the product can be analysed statistically. If chance causes are ordered in time or possibly on some other basis they will behave in a random manner, with respect to the **"Quality Characteristic"** collected or measured. They will show no cycles or runs or any defined pattern. Further, no specific variation to come can be predicted from the knowledge of past variation. Variation produced from chance causes follow Statistical laws. For example, if 10 coins are tossed randomly 'n' times, the relative frequencies with which 0,1,2,3,...10 heads occur follow binomial distribution for large 'n'. On similar lines, in a random sample of 'n' units, from a process that is affected only by chance causes, the probabilities of getting 0,1,2,.,.,n defective units follow binomial distribution. On similar lines, one can use Poisson distribution for large values of 'n' or Normal distribution when the quality characteristic is a continuous variable.

Knowledge of the behaviour of chance variation is the foundation on which control chart analysis is based on. If we consider a group of data and is found that the variation conforms to a statistical pattern that might reasonably be produced by chance causes, then we say that the process from which the data is collected is '**Under Control'**. On the other hand, if the variations in the data do not conform to a pattern that might reasonably be produced by chance causes, then it is concluded that one or more assignable causes are at work. In this case, we conclude that the process from which the data is collected is **'Out-of Control'**.

Let us assume that samples of a given size 'n' are taken from a production process of items say electric bulb. Let us assume that these samples are taken from the process more or less at regular intervals. Let the random variable X represents some variable measured from each sample unit. Let X* represents some statistic computed from each sample. That is X* is the sample fraction defective, or the sample mean or the sample variance or sample range or sample Standard Deviation. Then naturally X* will be subjected to sampling fluctuations. If no assignable causes are present, these sampling fluctuations in X* will be distributed in a definite Statistical Pattern as shown in the figure (1.3.1) whose Probability density function is given in equation(1.3.1). If sufficient samples are taken, it is possible to estimate the Process mean and certain extreme points of this distribution. Further we know that, Using Normal Theory approximation, if X* is the sample mean, then it would be possible to estimate the mean of the distribution of X* and the within-sample variation, the standard deviation of X* and using these results one can determine 0.001 probability points. If the distribution is considered on vertical scale and horizontal scale is marked with respect to the time or some other basis for ordering X* and if Horizontal lines are drawn through the estimated mean of X* and through an extreme value on the upper and lower tail of the distribution, of X* as shown in figure (1.3.3) we obtain the Control Chart for X*.

If X* is any 'Statistic' its sampling distribution can be obtained and for large samples this Sampling distribution can be approximated to Normal distribution with mean μ and variance σ^2. If $\mu = 0$ and variance $\sigma^2 = 1$ we obtain Standard Normal Distribution. This is the result can be obtained by Law of large numbers. Definition of Normal and Standard Normal distributions along with their graphs and distribution of area under Normal curve are given as follows for your ready reference.

1.3a. Definition of Normal Distribution

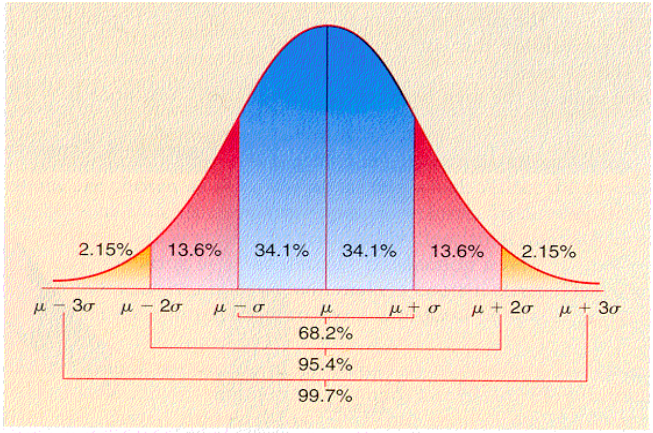
A continuous random variable X is said to have *normal distribution* with mean μ and standard deviation σ if the probability density function is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}(x-\mu)^2/\sigma^2} \qquad -\infty < x < \infty \qquad (1.3.1)$$

where e = 2.7183 and $\pi = 22/7$.

The graph of Normal distribution is given in the following figure (1.3.1).

Fig. (1.3.1) Distribution Area of Normal Curve with mean μ and s.d. σ .



In equation (1.3.1) if μ = 0 and $\sigma^2 = 1$ then normal distribution is known as "Standard Normal Distribution" and is denoted by the random variable Z. It is important to note that whatever may be the distribution of the random variable X, the new variable Z = (x – Mean) / S.D. or (X - μ)/σ follows Normal distribution with mean 0 and variance 1. That is Z follows Standard Normal Distribution.

1.3b. Definition of Standard Normal distribution

A random variable z which has mean 0 and variance 1 is said to have a *Standard Normal Distribution* if its probability density function is given by:

$$f(z) = \frac{1}{\sqrt{2\pi}}\, e^{-z2/2} \qquad -\infty < z < \infty \qquad (1.3.2)$$

where π = 22/7. Z is called Standard Normal Variable (S.N.V.)

The Graph of Probability density function of the Standard Normal Variable is given as follows in the fig.(1.3.2):

Fig. (1.3.2): Graph of standard Normal Distribution mean=0 and S.D. =1

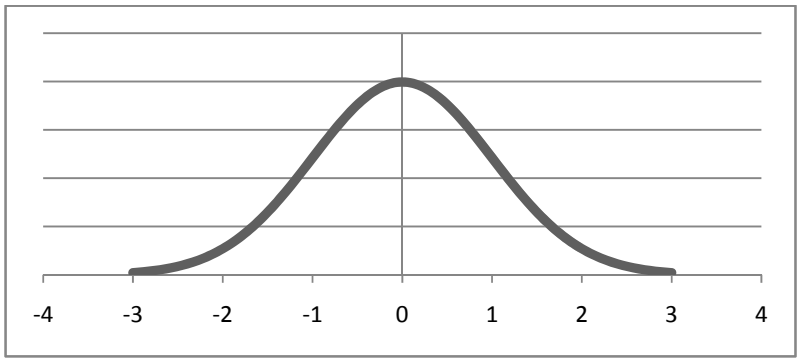
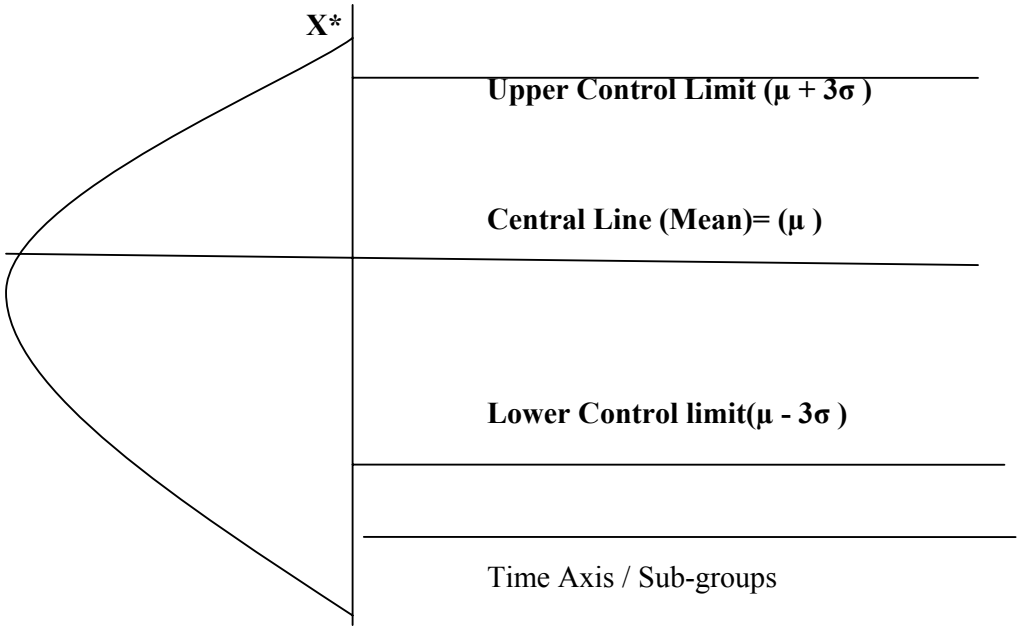


Fig (1.3.3) Control Chart for X*



If the sample values X* are plotted for a significant range of output and time and if all these values fall within the control limits and show no cycles or runs above or below average or runs up or down, then it can be said that the process is in the state of **'Statistical Control'** or 'in control' with respect to the quality variable X measured. Otherwise, we say that the process is **'Out of Control'** and can be assumed that the variation is not because of chance causes but some assignable causes are in force. Hence there is need for searching for assignable causes. After identifying the cause, one can take appropriate action to eliminate the same from the Process. Thus Control Charts are designed to determine whether the given process in in-control or out-of control. In other words, Control Charts are useful to sort out Assignable Causes of variation from Chance Causes of variation present in the process.

1.4 CONTROL LIMITS, PROBABILITY LIMITS AND WARNING LIMITS

The limits on the control chart in fig.(1.3.3) are **0.001 probability limits**. They were Determined, so that, if chance causes alone are at work, the probability of point's falling above the upper Control limit (UCL) is one out of a thousand and the point falling below the Lower Control Limit (LCL) is one out of thousand. If the system of chance causes produces a variation in X*, that follows the normal curve, the 0.001 probability limits are practically equivalent to **3σ limits**. This is because of the fact that under a normal curve, the probability that a deviation from the mean will exceed 3σ in one direction is 0.00135. or in both direction is 0.0027. Thus for normal variation 0.001 probability limits are practically equivalent to 3σ Control limits.

In USA, it is common to use 3σ control limits whether the chance variation in X* follow normal curve or not. It is important to note that, 3σ Control limits are also known as **'Action Limits'** because some action is taken when a point fall outside these limits. British engineers suggest that use of **inner or warning limits** in addition to action limits. Usually 2σ limits or 0.025 probability limits are known as '**warning Limits**' or **'Inner Limits'.** If a point falls

outside warning limits give signal or hint that if the process is continued in the same manner, may lead to out-of control state soon. Thus this gives warning for out-of control state and hence are known as **Warning Limits.** In brief, Action Limits or 3σ limits = $\mu \pm 3\sigma$ and Warning limits or inner Limits = $\mu \pm 2\sigma$, Where μ is the Mean and σ is the standard Deviation of the process calculated from the sample data collected. Sometimes variation of the process σ^2 is known from the past experience, and then we can substitute the known value otherwise it can be estimated from the sample data.

1.5 SUMMARY

However careful we are, we cannot produce to identical items in this world. Presence of some variation or the other is inevitable form item to item in the industrial output. If the variation is within some permissible limits in the quality of the products, we consider all the items are of equal quality. Otherwise we consider them as low quality products. Thus variations are classified into two categories namely (1) Random Causes of Variation and (2) Assignable causes of variation. We use Shewhart control charts to separate random causes of variation and assignable causes of variation present in the production process. Normal Distribution is the basis for the development of control charts and control limits namely UCL. CL and LCL. If all the sample points fall inside these control limits implies that only assignable causes are alone present in the process and its is impossible to remove such random causes of variation and hence, we consider the process is "**Under-control**". If one or more points are falling outside these control limits, we consider that the process of "**Out-of control**" and conclude that along with random causes of variation, some Assignable causes of variation is also present and necessary to detect the cause and remove the same from the process.

1.6 SELF ASSESSMENT QUESTIONS

1. Explain the need of Statistical Quality Control.
2. Explain historical development of SQC.
3. Define SQC and Control Charts.
4. Explain the Statistical Basis for the control Charts.
5. Distinguish Chance cause and assignable cause variation.
6. Explain Assignable causes of variation. Will they follow any Statistical law?
7. Explain the role of Normal distribution in SQC.
8. Distinguish between 0.001 and 3σ control limits.
9. Distinguish between Action limits and Warning limits.
10. What are control limits? When do you say that the process is out-of Control?

1.7. FURTHER READINGS

1. Montgomery, D.C., " Introduction to Statistical Quality Control", John Wiley (Asia) 2001.
2. Biswas.S., "Statistical Quality Control-Sampling Inspection and Reliability", New Central Book Agency Pvt. Ltd. 2003.
3.Wardsworth "Modren Methods for Quality Improvement", John Wiley and Sons 2005.
4.A.J. Duncan "Quality Control and Industrial Statistics", R.J.Taraporevala Sons & Co., 1985.

Unit – I

Lesson – 2

CONTROL CHARTS FOR VARIABLES – X-BAR AND R- CHARTS

2.0. OBJECTIVE :

After going through this lesson, you should be able to:

- Understand the concept of Variable control charts.
- Construction and use of X-bar and R-charts.
- Application of these charts simultaneously, to control the process average and variation
- Operating Characteristic (O.C) curve and Average Run Length (ARL) and their interpretation.

STRUCTURE

2.1. INTRODUCTION

In the last lesson, we have discussed the need of statistical tools in controlling the quality of industrial output. Further, we have understood that the quality of any product can be measured one or more related characteristics of the intended job of any product. Based on the characteristics measured, using the control charts, one can decide whether the produced products have specified characteristics at desired level. That is if all the sample points are falling within the control limits we consider that all the products are **'within the control'**. If one or more points are falling outside these control limits, (that is, above UCL or Below the LCL) we conclude that the products produced are **'Out of Control'** and search for the cause for such variation. This type of analysis, which controls the product's quality is known as **'Product**

Control'. If all such Assignable Causes are identified and eliminated from the production process then the production process from which the products produced will come under Control. This type of Analysis is called **'Process Control'.**

2.2. TYPES OF CONTROL CHARTS

One can use 'control chart' technique for both 'Product control' as well as for 'Process control'. Based on the characteristics measured from the product, Control Charts can be classified into two categories. Namely, (a) Variable Control Charts and (b) Attribute Control Charts.

(a) **Variable Control Charts**: Many quality characteristics can be expressed in terms of numerical measurement. That is, If the characteristic measured from the product is a variable, like, weight or length or Volume or diameter or life time of the product, charts used to analyse such data are called **'Variable Control Charts'**. Examples of Variable Control Charts are:

- **(i)** X – Bar Control Chart or Mean Control Chart .
- **(ii)** Median Control Chart.
- **(iii)** Range or R – Control Chart.
- **(iv)** Midrange Control Chart.
- **(v)** Standard Deviation or σ (Sigma) Control Chart.
- **(vi)** S – Control Chart.
- **(vii)** Coefficient of Variation (C.V.) – Control Chart and so on.

(b) **Attribute Control Charts:** If the characteristic measured from the product is an attribute, like, Working or Not working, Good or Bad, Smooth or rough, Defective or Non- defective, Number of defects in a unit, charts used to analyse such data are called **'Attribute Control Charts'**. Examples of Variable Control Charts are:

- **(i)** P – Control Chart.
- **(ii)** np – Control Chart.
- **(iii)** C – control chart.
- **(iv)** U – control chart and so on.

In the remaining lectures of Unit – I, we introduce Variable Control charts and related topics and in Unit – II, we concentrate topics of Attribute Control charts.

2.3. CONTROL CHARTS FOR VARIABLES – X-BAR AND R-CHARTS

The construction and application of X-bar and R-charts are discussed elaborately at degree level with many problems. At Post Graduate level we concentrate other aspects of these charts elaborately. Before discussing these concepts we revise briefly about the Construction of these charts, for a ready reference. Before going to explain the construction of these charts, first let us discuss the need for the application of both X-bar and R-charts simultaneously.

2.3.1. Need for controlling the mean as well as variance in the Process.

When the quality characteristic is a variable, it is usually necessary to monitor both the mean value of the quality characteristic and also the variability present in the variable. Both the mean and the variability must be in control. Then only we can say that the process or the product is under control. There exist many situations, where the products or the process under control with respect to the mean but may be out of control with respect to the variance. Similarly, the product or the process may be in control with respect to the variance but out of control with respect to the mean. To bring the process or the product under control, both the mean and the variance should in control. To control the mean, we use X-bar chart and to control the variance we use Range chart or Mid-range chart or Sigma or σ chart or S- chart.

Thus we must use these charts in pairs like:

(i) X-bar and R-Charts.
(ii) X-bar and Mid-range chart.
(iii) X-bar and σ – chart.
(iv) X – bar and S-chart and so on.

Hence it is necessary to apply both mean and variance control charts simultaneously.

This is because of the fact that control of any process, means control over the mean and the process variability. Usually process mean is denoted by μ and the process standard deviation by σ and σ^2 is the process variability.

2.3.2. Construction of X-bar and R-charts.

Suppose that a quality characteristic is normally distributed with men μ and standard deviation σ where both μ and σ are known from the past experience. Let $X_{i1},X_{i2},.,.,X_{1n}$ is the i^{th} sample of a characteristic of a product collected from n items. i = 1,2,.,.,m. From the given data calculate sample means $\bar{X}_i$ and sample Ranges R_i i = 1,2,.,.,m from the sample data collected. Where $R_i = X_{max} - X_{min}$. Here, X_{max} = maximum value in the i^{th} sample and X_{min} = Minimum value in the i^{th} sample. Then the best estimator of the process mean μ the grand average of all these m sample means denoted by $\bar{\bar{X}}$ which is calculated as follows:

$$\bar{\bar{X}} = [\bar{X}_1 + \bar{X}_2 + . + . + . + \bar{X}_m] / m \qquad (2.3.1)$$

Let $R_1, R_2.,.,.,R_m$ be are sample ranges calculated from m samples. The average of these ranges is denoted by $\bar{R}$ which is calculated as follows:

$$\bar{R} = [R_1 + R_2 + .,.,. + R_m] / m \qquad (2.3.2)$$

Then, Control limits for $\bar{X}$ or X – bar chart are given as follows:

Control Limits for X – bar Chart:

UCL (Upper Control Limit) $= \bar{\bar{X}} + A_2 \bar{R}$ (2.3.3)

CL (Central Line) = $\bar{\bar{X}}$ (2.3.4)

LCL (Lower Control Limit) = $\bar{\bar{X}}$ - $A_2\ \bar{R}$ (2.3.5)

Similarly Control charts for Range chart or R-chart which is to control process variability is given as follows:

Control Limits for R – Chart:

UCL (Upper Control Limit) = $D_4\ \bar{R}$ (2.3.6)

CL (Central Line) = $\bar{R}$ (2.3.7)

LCL (Lower Control Limit) = $D_3\ \bar{R}$ (2.3.8)

In the above equations, A_2 , D_3 and D_4 are tabulated values for various Sample sizes given in Table A–1 in Appendix. After plotting above control limits on the charts along with sample means $\bar{X}_i$ and sample Ranges R_i i = 1,2,.,.,m we obtain X-bar and R-charts. If all the sample means are falling in between UCL and LCL in X- bar chart and all Sample Ranges are falling in between UCL and LCL of R-chart, we conclude that the process or the product with the given characteristic is in-control. Otherwise, that is, if one or more points are falling outside control limits in X-bar or in R-chart, we conclude that the process or the product is out-of control with respect to the measured characteristic. Thus X-bar chart control the means and R-chart control the process variability. Usually, process variability is measured through the Range if the sample is small. Range is not an efficient measure of variability, if sample size is large. That is as sample size n increases, we should not use the range as a measure of variability.

Above explained procedure is explained with an example, by constructing X-bar and R-charts.

Example (2.3.1):

An injection manufacturing process produces produce injection syringes through an injection moulding process, which are subjected to a comprehensive Strength test. A Sample of five syringes from each day production is collected for 20 days and strength if each syringe is measured in psi. The recorded data is as follows:

Sample Number	X_1	X_2	X_3	X_4	X_5
1	83.4	78.4	82.6	78.2	78.9
2	83.0	81.2	78.7	75.7	77.0
3	78.8	79.6	80.2	79.1	80.8
4	84.5	73.1	78.6	78.7	80.6
5	79.0	77.8	81.2	84.4	81.6

6	75.3	79.9	87.3	89.7	81.8
7	88.6	78.3	78.8	71.0	84.2
8	80.8	74.4	82.5	74.1	75.7
9	84.5	76.9	83.5	81.2	79.2
10	85.5	82.1	82.8	73.4	71.7
11	85.7	75.8	84.3	75.2	81.0
12	79.2	74.9	78.6	77.7	75.3
13	74.5	70.0	80.8	73.4	79.7
14	82.7	81.3	79.1	82.0	79.5
15	78.2	84.4	81.5	86.0	74.5
16	80.5	86.2	76.2	64.1	80.2
17	80.8	81.5	78.4	73.8	78.1
18	80.6	81.2	79.3	73.8	81.7
19	82.1	78.2	75.5	78.2	82.1
20	75.7	75.2	71.1	82.1	74.3

Establish $\bar{\bar{X}}$ and R charts for the Comprehensive strength and determine whether the syringe production process is in control or out of control?

Solution: From the given data calculate sample means $\bar{X}_i$ and sample Ranges R_i i = 1,2,.,.,20 where $\bar{X}_i = [X_1 + X_2 + X_3 + X_4 + X_5] / 5$ sample Ranges $R_i = X_{max} - X_{min}$.

These values are calculated and presented in the following table.

Sample Number	X_1	X_2	X_3	X_4	X_5	$\bar{X}_i$	R_i
1	83.4	78.4	82.6	78.2	78.9	80.3	5.2
2	83.0	81.2	78.7	75.7	77.0	79.1	7.3
3	78.8	79.6	80.2	79.1	80.8	79.7	2.0
4	84.5	73.1	78.6	78.7	80.6	79.1	11.4
5	79.0	77.8	81.2	84.4	81.6	80.8	6.6
6	75.3	79.9	87.3	89.7	81.8	82.8	14.5
7	88.6	78.3	78.8	71.0	84.2	80.2	17.6

8	80.8	74.4	82.5	74.1	75.7	77.5	8.4
9	84.5	76.9	83.5	81.2	79.2	81.1	7.6
10	85.5	82.1	82.8	73.4	71.7	79.1	13.8
11	85.7	75.8	84.3	75.2	81.0	80.4	10.4
12	79.2	74.9	78.6	77.7	75.3	77.1	4.3
13	74.5	70.0	80.8	73.4	79.7	77.3	7.4
14	82.7	81.3	79.1	82.0	79.5	80.9	3.6
15	78.2	84.4	81.5	86.0	74.5	81.1	11.4
16	80.5	86.2	76.2	64.1	80.2	81.1	9.9
17	80.8	81.5	78.4	73.8	78.1	78.4	7.7
18	80.6	81.2	79.3	73.8	81.7	79.4	8.0
19	82.1	78.2	75.5	78.2	82.1	79.2	6.6
20	75.7	75.2	71.1	82.1	74.3	75.5	10.9
					Totals	1590.1	174.6

$\bar{\bar{X}}$ = $[\bar{X}_1 + \bar{X}_2 + . + . + . + \bar{X}_{20}]$ / 20 = 1590.1 / 20 = 79.505

And $\bar{R}$ = $[R_1 + R_2 + ,.,., + R_{20}]$ / 20 = 174.6 / 20 = 8.73.

Further, we can read from table A-1 in Appendix for a sample of size n = 5 we have

A_2 = 0.577 , D_3 = 0 D_4 = 2.115 . Using these values in equations (2.3.3) to (2.3.8), we Control limits for X-bar chart are:

UCL = 79.505 + 0.577 (8.73) = **84.54.**

CL = **79.505.**

LCL = 79.505 – 0.577 (8.73) = **74.47.**

Similarly, control limits for R-chart are:

UCL = 2.115 (8.73) = **18.46.**

CL = **8.73.**

LCL = 0 (8.73) = **0.**

By plotting control limits and $\bar{X}_i$'s (in column no. 7) on the graph, we conclude that all sample means are in between UCL= 84.54 and LCL = 74.47. Similarly, plotting control limits and R_i ,s (in column no. 8) on the graph we can observe that all sample ranges are between UCL

= 18.46 and LCL = 0. Hence we conclude that the production process of injection syringes are in control with respect to the mean and Variance (or Range). X-Bar and R- charts are shown in the following figures (2.3.1) and (2.3.2).

Fig. (2.3.1); X – Bar control chart.

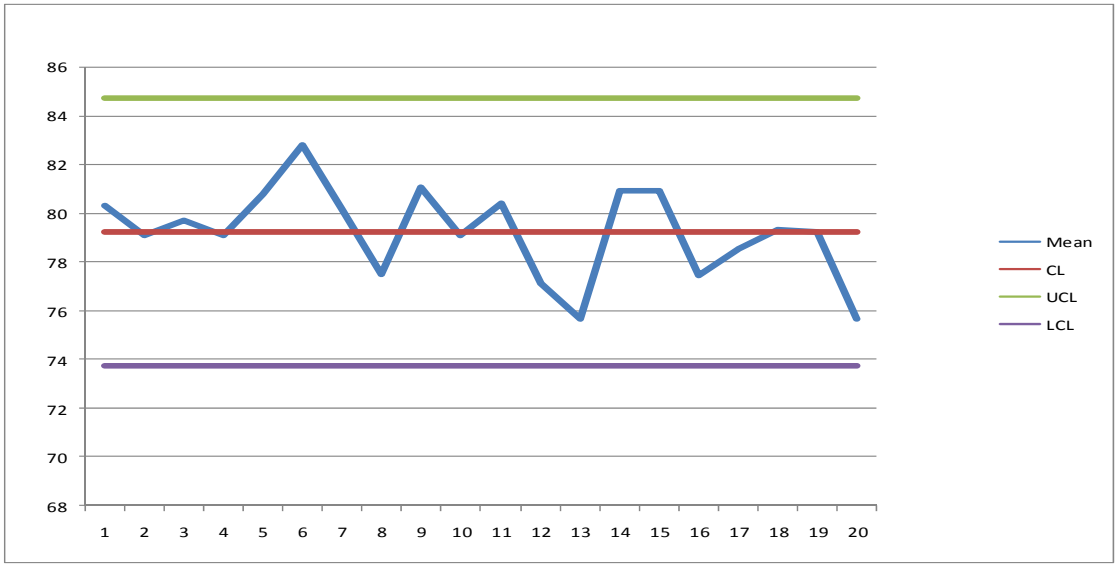
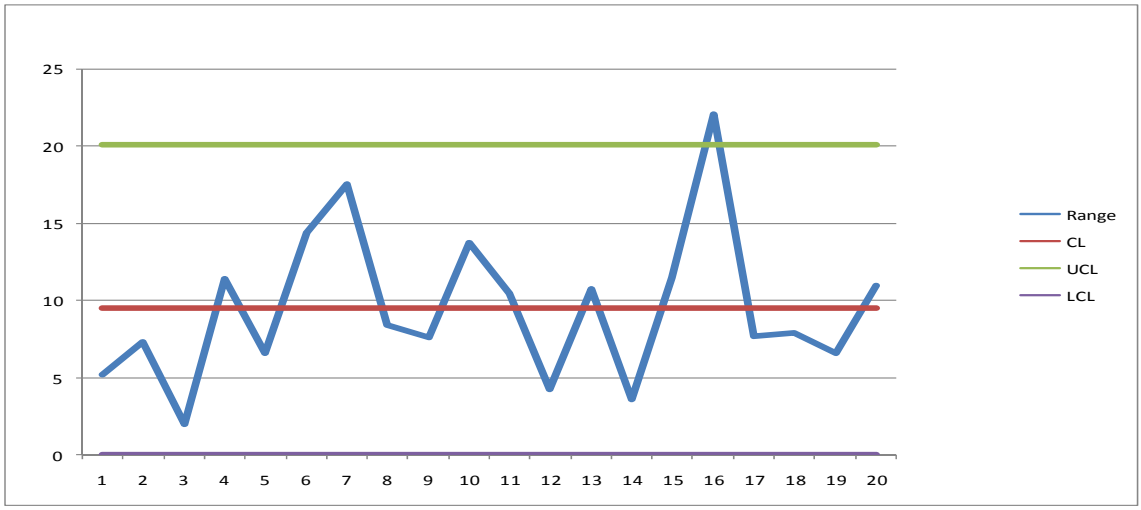


Fig. (2.3.2): Control Chart for ranges.



*2.4. REVISED CONTROL LIMITS AND SUB-GROUPING

Shewhart Control chart is a device to determine whether the given product is within the control or not? If all the sample means fall within the UCL and LCL we decide that the product under consideration is '**in control'**. That is only random causes of variation are present in the production process. There is no need to adjust the process because all the sample means are inside the control limits. If one or more sample means are falling outside the control limits, we conclude that the process is '**out-of Control'**. When the process is out-of control, then these is a need for '**Further Statistical Analysis'**. Such analysis improves the production process. Such analysis is known as '**Process Control'**. Concepts of (a) Revised Control Limits and (b) Sub-grouping explaining below are necessarily used in Process Control.

2.4a. Revised Control Limits.

The set of Control limits obtained above using the data are to be considered as **'Trial limits'** subjected to subsequent revision. Usually, the effective use of any control chart will require Periodic revision of the control limits and centre lines. Some standard practitioners establish regular periods for review or revision of control chart limits, such as every week, every month, or every 50 or 100 samples.

* Foot Note: Concepts of Revision of Control limits and Sub-grouping are necessary for Process Control. Hence are discussed here.

These control limits are useful for controlling the future production. If the production is within control, that is, if all the collected sample means are within the control limits, then, same limits can be considered as revised control limits and can be used as control limits to be used for future production. On the other hand, if one or more sample means are falling out-of control limits then we determine that the process is out-of control. Then there is a need for further analysis. Simply saying the process is out-of control is not the end of the job. We have to identify the cause for the variation and suggest to the management the required action is to be taken, to bring the process within control. For this, we use a special technique known as **'Sub-grouping'**, which we are going to explain in 2.4b. After identifying the assignable cause of variation and management will take appropriate action to eliminate the identified assignable cause for variation from the production process. Now, next question arises is that the remaining process is within control or not? To do this, **remove all sample data from the analysis** which fell out of control and calculate revised control limits by using revised $\bar{\bar{X}}$ and $\bar{R}$ and sample size n. If all these sample points are falling between these revised control limits, then consider these revised control limits, for future production. Otherwise, that is, again if some points are falling outside the control limits repeat the process explained above. That is remove, those points falling outside control limits and calculate revised control limits again by using revised $\bar{\bar{X}}$ and $\bar{R}$ and sample size n. Repeat this procedure until we obtain all the points within control limits. In the process of elimination, if more than one third of sample size n is removed, then it is suggested to go for fresh data from the process and to revise the control limits.

The concept of revision of control limits useful in **'Process Control'** and hence discussed here even thought this concept is not included in the present syllabus.

2.4b. Sub-grouping.

Sub-grouping is the most important tool to identify assignable causes present in the production process. This is the technique to be used when the process is out-of control. When the process is out-of control, then it is the clear evidence that one or more assignable causes are present in the production process. It is already listed in Lesson-1 various assignable causes and one has to identify these assignable causes present in the production process. One can suspect machines may the cause if more machines are producing these products. Let Machine-1, Machine-2,machine-3 and machine-4 are used to produce the product. Then Sub-group those Products produced from machine-1, Machine-2 machine,3 and Machine-4 and calculate these sub-group sample means. Construct Control Charts X-bar and R-charts by considering Sub-groups on X-axis and sample means on Y-axis. If any sample mean is falling outside control limits, then the corresponding machine, producing those products is the cause for causing the variation. Similarly, if workers are the cause for variation, we can sub-group samples according to workers and construct control charts by considering workers on X-axis and identify the worker whose production is out of control. Similarly, if we feel raw material is the cause for variation and then sub-group the samples according to various raw materials, if the suspected cause is power supply and its voltage, one has to subgroup the samples over different power supply modes and so on. Thus sub-grouping is the basic tool with which we can identify the specific cause for assignable variation and suggest to the management to remove such identified cause from the production process so that the process can be brought under control. Identifying the cause is the job of the analyst and steps to remove such identified cause is the management job, to take appropriate action.

In order to apply above charts and take decision, we have used the data collected from five sample units collected on each day production for 20 days. In measuring the quality of the product, some characteristics like length, diameter and weight are to be measured through instruments and these procedures of measuring the quality characteristics does not harm to the product. Hence these procedures are known as '**non- destructive procedures or tests**'. In these tests, the unit put for the test is further usable. But, some quality characteristics like Strength of the shell, breaking strength of a brick, or life time of the product or sound produced by a cracker and so on must be tested based on a '**destructive test procedures'**. That is the unit put for the test is destroyed and cannot be further used. It will be a waste product. More wastage, results increase in the production cost of the product. Hence the sample size 'n' in destructive tests plays a crucial role which determines the wastage. Hence, an optimum sample size is to be determined and based on the sample size, production cost of the product is obtained. This task of determining optimum sample is determined based on some important concepts like **Operating Characteristic** (OC) curve, **Average Sample Number** (ASN) **Curve**, **Average Run Length** (ARL) **Curve** and so on. These concepts are discussed in the following section.

2.5. O.C. AND ARL CURVES FOR X-BAR CHART AND R-CHARTS

The efficiency of X-bar and R-charts to detect shifts in the process quality is explained by their Operating Characteristic (OC) curves. First we discuss the method of construction of

OC curve for X-bar chart and then for R-chart. OC curves for charts used for on-line control of a process.

2.5a. Construct ion of OC curve for X-bar chart

Let us consider the process whose standard deviation σ is known and is a constant and discuss the method of construction of OC curve for X-bar chart. Let μ_0 represent process mean when the process is in-control. Let the mean shift from μ_0 to another mean value $\mu_1 = \mu_0 + k\sigma$, the probability of not detecting this shift on the first subsequent sample or the β-risk is given by:

$$\beta = \Phi\left[\frac{UCL - (\mu 0 + K\sigma)}{\sigma/\sqrt{n}}\right] - \Phi\left[\frac{LCL - (\mu 0 + K\sigma)}{\sigma/\sqrt{n}}\right] \qquad (2.5.1)$$

Where $\mathbf{UCL = \mu_0 + L\sigma / \sqrt{n}}$; $\mathbf{LCL = \mu_0 - L\sigma / \sqrt{n}}$ (2.5.2)

And **Φ** denotes the standard Normal cumulative distribution function with $\mu_0 = 0$ and $\sigma = 1$.

This reduces (2.5.1) as follows:

$$\beta = \Phi(L - K\sqrt{n}) - \Phi(-L - K\sqrt{n}) \qquad (2.5.3)$$

Construction of OC curve for X-bar chart is explained as follows with an example**.**

Example (2.5.1): Construct OC curves for n = 5, K = 2 and L =3 (usual 3 –sigma limits) .

Solution: Here we wish to determine the probability of detecting a shift to $\mu_1 = \mu_0 + 2\sigma$ on the first sample following the shift. Thus we have L = 3, K = 2 and n = 5. Substituting these values in (2.5.3) we have:

$$\beta = \Phi(3 - 2\sqrt{5}) - \Phi(-3 - 2\sqrt{5}) = \beta = \Phi(-1.47) - \Phi(-7.37) \approx 0.0708.$$

This is the β – risk, or the probability of not detecting such a shift. Thus the probability of detecting such shift is $\alpha = 1 - \beta = 1 - 0.0708 = 0.9292$. Thus using equation (2.5.3) we can calculate $\alpha = 1 - \beta$ for various sample sizes. By considering different k values on X- axis and β values on Y – axis, if we draw the curve, we obtain OC curve for X-bar chart.

Following figure represents, OC curves for various values of sample sizes n, for an X-bar chart.

Fig. (2.5.1): Construction of OC curve:

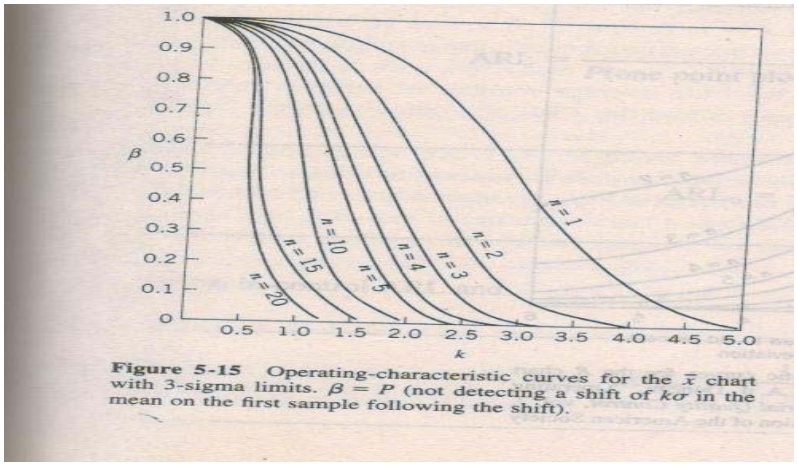


Figure 5-15 Operating-characteristic curves for the $\bar{x}$ chart with 3-sigma limits. $\beta = P$ (not detecting a shift of $k\sigma$ in the mean on the first sample following the shift).

2.5b. Average Run Length (ARL) for X-bar chart.

In any Shewhart Control chart, the Average Run Length (ARL) curve explains the expected number of samples taken before the shift in the process mean is detected. Thus we have:

ARL = 1/ P(one point plots out of control) (2.5.4)

In control ARL = ARL_0 = 1 / α (2.5.5)

Out-of control ARL = ARL_1 = 1/ (1 – β) . (2.5.6)

Thus for example, if β = 0.75 then α = 0.25, ARL = 1 / 1 – 0.75 = 1/0.25 = 4. In other words, the expected number of samples required to detect a shift in the process mean required a sample of size n = 4 or 5.

These results are actually intuitive. If the observations plotted on the control chart are independent, then numbers of points that must be plotted until the first point exceeds a control limit follow a geometric distribution with parameter p. We know the mean of the geometric distribution is 1/p, which represents Average Run Length.

We have observed that getting expression for β is relatively is easy using equation (2.5.1) for construction of ARL curve for X – bar chart. Then, we can draw easily, a set of ARL curves for X- bar chart for different sample sizes n.

2.5c. Construction of OC curve for R-chart.

To construct OC curve for R – chart, the distribution of Sample Range is to be used. The distribution of sample range $W = R / \sigma$ is to be employed. Suppose that the in-control value of the standard deviation is denoted by σ_0 for the given process. Then the OC curve plots the probability of not detecting a shift to a new value of σ_1 (say). Let $\sigma_1 > \sigma_0$ on the first sample following the shift. Figure (2.5.2) represents OC curve in which β is plotted against $\lambda = \sigma_1 / \sigma_0$ (the ratio new to old process standard deviation) for various values of sample sizes n. Critically comparing the OC curves for R- chart, we can observe that R-chart is not very efficient in detecting process shifts in variances based on small sample sizes less than 5. In order to detect these shifts, we must have sample sizes more than 10 or 12. In such situations, we have to use S – chart in the place of R – chart. Thus R – chart is not an efficient one to measure the variability present in the process. This is to be done through S – charts.

Fig. (2.5.2): OC curve in which β is plotted against $\lambda = \sigma_1 / \sigma_0$.

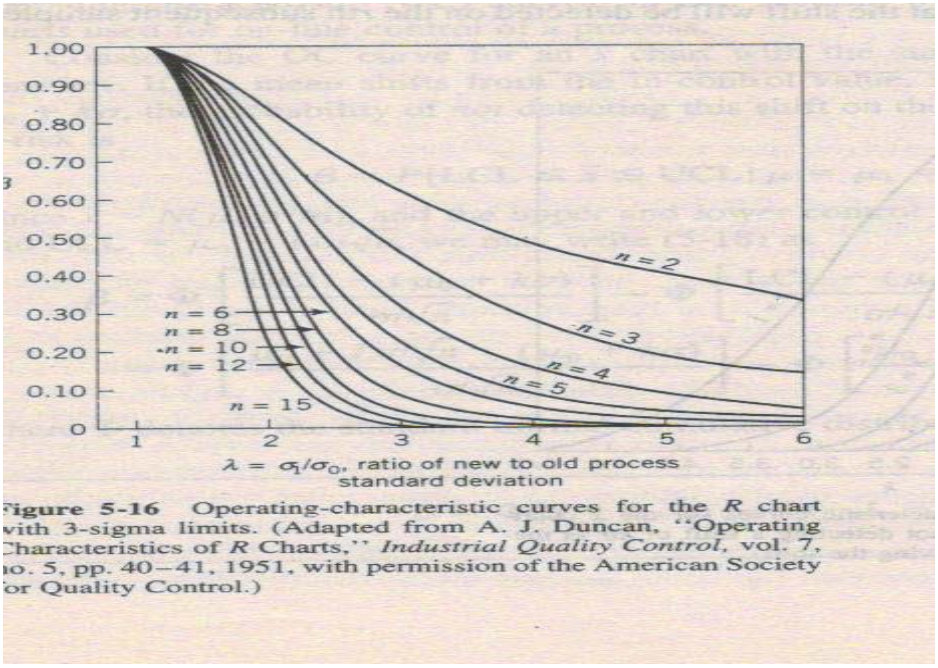


Figure 5-16 Operating-characteristic curves for the *R* chart with 3-sigma limits. (Adapted from A. J. Duncan, "Operating Characteristics of *R* Charts," *Industrial Quality Control*, vol. 7, no. 5, pp. 40–41, 1951, with permission of the American Society for Quality Control.)

OC curves given in Figures (2.5.1) and (2.5.2) are useful for on-line process control; but, is occasionally used to analyse past data that is for product control.

2.6. SUMMARY

In this lecture, we have discussed the method of construction of variable control charts, namely X-bar and R-charts. We have under stood that X-bar charts for controlling or monitoring process average and R-chart for controlling or monitoring the process variability. They should be applied in pairs simultaneously. Role of OC and ARL curves are discussed and method of construction of these curves for X- bar and R – charts are also discussed.

2.7. SELF ASSESSMENT QUESTIONS

1. Discuss various types of control charts.
2. What are control limits and discuss the role of revised control limits.
3. Explain the method of construction of Revised control limits.
4. Explain the method of construction of X – bar and R – charts.
5. Explain the role of OC and ARL curves.
6. Explain the method of construction of OC curves for X – bar and R – charts.
7. Discuss the role of sub-grouping in SQC.
8. Construct X – bar and R – charts for the following data on weights measured in oz of a dry bleach product which is to be monitored with respect to means and Ranges.

Sample No.	X_1	X_2	X_3	X_4	X_5
1	13.4	18.4	12.3	18.2	18.9
2	13.0	11.2	18.7	15.7	17.8
3	18.8	19.6	10.2	19.1	10.8
4	14.5	13.1	18.6	18.7	18.6
5	19.0	17.8	11.2	14.4	11.6
6	15.3	19.9	17.3	19.7	11.8
7	18.6	18.3	18.8	11.0	14.2
8	10.8	14.4	12.5	14.1	15.7
9	14.5	16.9	13.5	11.2	19.2
10	15.5	12.1	12.8	13.4	11.5
11	15.7	15.9	14.3	15.2	11.6
12	19.2	14.4	18.6	17.7	15.3
13	14.5	10.0	10.8	13.4	19.7
14	12.7	11.3	19.7	12.0	19.5
15	18.2	14.8	11.5	16.0	14.7
16	10.5	16.2	16.2	14.1	10.6
17	10.8	11.5	18.4	13.8	18.1
18	10.6	11.2	19.3	13.8	11.7
19	12.1	18.2	15.5	18.2	12.1
20	15.7	15.2	11.1	12.1	14.8

9. The volume of soft drink beverage bottles is an important quality characteristic. The volume is measured (approximately) by placing a gauge over the crown and comparing the height of the liquid in the neck of the bottle against a coded scale. Construct X – bar and R – Charts and draw your conclusions.

Sample No.	X_1	X_2	X_3	X_4	X_5
1	8.4	7.8	8.2	7.8	7.8
2	8.0	8.2	7.7	7.5	7.7
3	7.8	7.6	8.2	7.1	8.8
4	8.4	7.3	8.6	8.7	8.6
5	7.9	7.7	8.2	8.4	8.6
6	7.5	7.9	8.7	8.9	8.8
7	8.8	7.8	7.8	7.1	8.4
8	8.8	7.4	8.5	7.1	7.7
9	8.5	7.9	8.5	8.2	7.2
10	8.5	8.1	8.8	7.4	7.7
11	8.7	7.8	8.3	7.2	8.0
12	7.2	7.9	7.6	7.7	7.3
13	7.5	7.0	8.8	7.4	7.9
14	8.7	8.3	7.1	8.0	7.5
15	7.2	8.4	8.5	8.0	7.5
16	8.5	8.2	7.2	6.1	8.2
17	8.8	8.5	7.8	7.8	7.1
18	8.6	8.2	7.3	7.8	8.7
19	8.1	7.8	7.5	7.2	8.1
20	7.7	7.2	7.1	8.1	7.3

1. Following data represents inner side diameters measured in cms. of Piston rings of Auto mobile manufacturing company in Bombay. Construct Appropriate control charts and draw your conclusions on the process.

Sample Number	X_1	X_2	X_3	X_4
1	283.4	278.4	282.6	278.2

2	283.0	281.2	278.7	275.7
3	278.8	279.6	280.2	279.1
4	284.5	273.1	278.6	280.6
5	279.0	277.8	281.2	284.4
6	275.3	279.9	287.3	289.7
7	288.6	278.3	278.8	271.0
8	279.2	274.9	278.6	277.7
9	274.5	270.0	280.8	273.4
10	282.7	281.3	279.1	282.0
11	278.2	284.4	281.5	286.0
12	280. 8	281.5	278.4	273.8
13	280.6	281.2	279.3	273.8
14	278.5	285.2	278.4	280.3
15	284.9	292.8	279.8	294.8

2.8. FURTHER READINGS

1. Montgomery, D.C., " Introduction to Statistical Quality Control", John Wiley (Asia) 2001.

2. Biswas.S., "Statistical Quality Control-Sampling Inspection and Reliability", New Central Book Agency Pvt. Ltd. 2003.

3.Wardsworth "Modern Methods for Quality Improvement", John Wiley and sons 2005.

4.A.J. Duncan "Quality Control and Industrial Statistics", R.J.Taraporevala Sons & Co., 1985.

Unit – I

LESSON – 3

Median and Mid-Range Control Charts

3.0. OBJECTIVE

After going through this lesson, you should be able to:

- Understand the concept of Variable control charts based on other statistics.
- Construction and use of Median and Mid – Range Control charts.
- Application of these charts to control the process average and variation.
- Interpretation of Median and Mid-range Control charts.

STRUCTURE

3.1. Introduction

One of the basic objections to use X – bar and R – charts are the number of Calculations in the construction of these charts. Further, Arithmetic mean has the primary drawback that it is much affected by the extreme values. The range is also calculated based on extreme values only. Thus, extreme values play crucial roles in calculations of both mean and Ranges. Hence, X-bar and R-charts are much affected by extreme values present in the data collected from a production process. Hence, there is a need to introduce Control charts based on another Statistic where Extreme Values have no influence al all.

Such an average is **'The Median'**. Control charts where control limits depending on Median are (i) median Chart for controlling the Process Average and (ii) Mid-range or Median for

Ranges to control the process variation. Basic advantage of using Median instead of Arithmetic mean is that lesser number of calculations involved in the calculation of Median than the Arithmetic Mean. Some definition, Properties and preliminary applications of median are discussed in the following section.

3.2 DEFINITION AND PROPERTIES OF MEDIAN

By definition, the Median is the halfway point in a distribution of observations; that is, number of data items below or equal to Median Value must be equal to number of data items above or equal to the median value . In other words, the median lies exactly in the middle most position in the arrangement of data items in ascending or descending order of magnitude, when the given data is a raw data or un-classified data. In classified data Median represents that value such that one half of frequencies smaller than or equal to the median value must be equal to another half of frequencies larger than or equal to the median value. Thus the formula for calculation of Median is given as follows.

3.2a. Calculation of Median

Calculation of Median is depending on the type of data. If the data is a raw data, that is, Un-classified data, and number of items 'n' are even in number, then the method of calculation of median as follows:

Calculation of Median (for raw data): Let x_1 , x_2 ,.,.,x_n are the given data items.

n is Odd:

Step – 1: Arrange the given data in an ascending order $X_{(1)}$,$X_{(2)}$,.,.,.,$X_{(n)}$ are arranged in an order. That is $X_{(i)} \leq X_{(i+1)}$ for all values of i = 1,2,.,.,n.

Step – 2: median = $[(n+1)/2]^{th}$ item in the arrangement. (3.2a.1)

Example (3.2a.1): Calculate the median for the data represents the life time of 11 electronic batteries measured in weeks. As follows: 56,89,79,45,35,86,96,75,88,99 and 54.

Solution: Here n = 10 and n is an even number.

Step – 1: Arrange the data in the ascending order of magnitude:

35,45,54,56,75,79,86.88,89,96and 99.

Step – 2: Median = [11 +1 /2] 12/2 = 6^{th} item in the arrangement that is 79 weeks.

n is odd:

When n is even, calculation of Median is as follows:

Step – 1: Arrange the given data in an ascending order $X_{(1)}$,$X_{(2)}$,.,.,.,$X_{(n)}$ are arranged in an order. That is $X_{(i)} \leq X_{(i+1)}$ for all values of i = 1,2,.,.,n.

Step – 2: median = {$(n/2)^{th}$ item in the arrangement+ $[(n/2) + 1]^{th}$ item in the arrangement} / 2 .

(3.2a.2)

Example (3.2a.2): Calculate the median for the data represents the life time of 10 electric bulbs measured in hrs. As follows: 516,889,789,485,395,846,986,785,838, and 594.

Solution: Here n is even.

Step – 1: Arrange the data in an increasing series as follows.

395,485,516,594,785,789,838,846,889, and 986.

Step – 2: $(N/2)^{th}$ item in the arrangement = 10/2 = 5th item in the arrangement = 785.

Similarly, (n/2 + 1)th item in the arrangement (5+1)th = 6 th item = 789.

Median = (785 + 789) / 2 = 1574/2 = 787 hrs.

Calculation of Median (for classified data):

For the given frequency distribution, calculation of Median is as follows:

Step – 1: Calculate Less than Cumulative Frequencies (LTCF) for the given frequency distribution.

Step – 2: Calculate N / 2 and determine Median Class (MC).

Step – 3: Median = $\mathbf{L} + \left[\frac{\frac{n}{2} - C.F.}{f}\right] \mathbf{X} \mathbf{C}$. (3.2a.3)

Where L = Lower limit of the median Class

n = frequency total.

C.F. = Cumulative Frequency up-to the Median Class.

f = Frequency of the Median Class.

C = Class Interval of the Median Class.

Example (3.2a.3): The following data represents the frequency distribution of outside diameter measurements in cms. of 100 machined parts.

Class Interval	5 – 10	10 – 15	15 – 20	20 - 25	25 - 40	40 - 50	50 - 55	55 - 60	60 – 70
Frequency	1	2	5	12	15	35	15	8	7

Solution: Calculate Less than Cumulative Frequencies and n/2.

Class Interval	Frequency	Less Than Cumulative Frequency (LTCF)
5 - 10	1	1
10 -15	2	3
15 – 20	5	8
20 – 25	12	20
25 – 40	15	35
40 – 50	35	70
50 – 55	15	85
55 – 60	8	93
60 – 70	7	100

Here n = 100 and n/2 = 50. Hence Median class is 40 – 50, Class Interval of the Median Class is 10 and C.F. = 35 and f = 35. Using Equation (3.2a.3), we have:

Median = 40 + [(50 – 35) / 35] X 10 = 40 + 4.2 = 44.2 cms.

3.2b. Properties of Median

1. Median is independent of extreme Values.
2. Median is a positional average; that is the data item occupying middle most position.
3. Median divides the data into two equal halves.
4. Median is used for Un-equal intervals without any difficulty.
5. Median can be determined through graph also.
6. Median has operational convenience for the determination and fewer calculations involved than in arithmetic mean.

3.3. APPLICATION OF MEDIAN TO CONTROL PROCESS AVERAGE AND VARIATION

Some authors have suggested the use of Median instead of Arithmetic Mean for controlling the process average. Similarly, Mid-range or Median for sample ranges is to be used to control the process variation. Using Median in the place of arithmetic mean reduces calculations to a large extent. Calculations for calculating UCL, CL and LCL for the control charts, reduces significantly, when compared to calculate $\bar{X}$, $\bar{\bar{X}}$ *and* $\bar{R}$ values. Thus Control charts are constructed based on Median Values for controlling both Process average and the process variation. The median has been frequently used instead of $\bar{X}$ as a central line in control charts for controlling the process average or variation. Median can safely be used even when the underlying distribution is a skewed distribution. Similarly, Median of Ranges can be used as central line, on the charts controlling process variability. Such charts are known as Median Charts and Mid-range Charts. In other words, Median charts are used to control the process average and Mid-range charts are to be used to control the Process Variability.

3.4. CONSTRUCTION OF MEDIAN AND MID-RANGE CONTROL CHARTS

For the construction of Median Control Charts, the sample or sub-group data is to be collected from the process in a usual manner as done in the construction of X-bar and R-charts.

3.4a. Construction of Median Control chart.

Step – 1: Calculate sample or sub-group Medians ($\tilde{X}_i$) i = 1,2,.,.,n for the collected data. Step-2: Similarly calculate Ranges ($\tilde{R}_i$) i = 1,2,.,.,.,n for each sample or sub-group.

Step – 3: Arrange these medians and ranges in an increasing or decreasing order.

Step – 4: Calculate the Grand Median ($\tilde{\bar{X}}$) and Median for Ranges ($\tilde{R}$) Calculated in steps - 1 and 2.

Step – 5: Calculation of control limits for Median chart.

Control Limits for Median Chart:

(a) For controlling the Process average.

UCL (Upper Control Limit) = $\tilde{\bar{X}} + \tilde{A}_2 (\tilde{R})$ (3.4.1)

CL (Central Line) = **Grand Median** = $\tilde{\bar{X}}$ (3.4.2)

LCL (Lower Control Limit) = $\tilde{\bar{X}} - \tilde{A}_2 (\tilde{R})$ (3.4.3)

Similarly Control charts for Range chart or R-chart which is to control process variability is given as follows:

(b) For controlling the process Variation.

UCL (Upper Control Limit) = $\tilde{D}_4$ ($\tilde{R}$) (3.4.4)

CL (Central Line) = $\tilde{R}$ (3.4.5)

LCL (Lower Control Limit) = $\tilde{D}_3$ ($\tilde{R}$) (3.4.6)

Where $\tilde{A}_2$, $\tilde{D}_3$ and $\tilde{D}_4$ are constants noted from the table given in table A-2 in Appendix.

3.4b. Construction of Mid-range Control chart.

In the mid-range control chart, extreme readings are plotted along with the average of the extremes. Mid-range $\tilde{M}$ is defined as the mean of the two extreme data items. That is (Maximum data item + Minimum data item)/2 Or (Largest + Smallest)/2 in the given data.

For example, the data items are 552,579,514,565, 580 and 546 then the mid-range for the given data $\tilde{M}$ = (Largest + Smallest)/2 = (580 + 514)/2 = 547.

Step – 1: Calculate sample or sub-group Mid-ranges ($\tilde{M}_i$) i = 1,2,.,.,n for the collected data.
Step- 2: Similarly calculate Ranges ($\tilde{R}_i$) i = 1,2,.,.,.,n for each sample or sub-group.

Step – 3: Arrange these medians and ranges in an increasing or decreasing order.

Step – 4: Calculate the Grand Median ($\tilde{\tilde{M}}$) and Median for Ranges ($\tilde{R}$) Calculated in steps - 1 and 2.

Step – 5: Calculation of control limits for Mid-range chart.

Control Limits for Mid-range Chart:

(a) For controlling the Process average.

UCL (Upper Control Limit) = $\tilde{\tilde{M}}$ + $\tilde{\tilde{A}}_2$ ($\tilde{R}$) (3.4.7)

CL (Central Line) = **Grand Median** = $\tilde{\tilde{M}}$ (3.4.8)

LCL (Lower Control Limit) = $\tilde{\tilde{M}}$ - $\tilde{\tilde{A}}_2$ ($\tilde{R}$) (3.4.9)

Similarly Control charts for Range chart or $\tilde{R}$-chart which is to control process variability is given as follows:

(b) For controlling the process Variation.

UCL (Upper Control Limit) = $\tilde{D}_4\ (\tilde{R}\ )$ (3.4.10)

CL (Central Line) = $\tilde{R}$ (3.4.11)

LCL (Lower Control Limit) = $\tilde{D}_3\ (\tilde{R}\ )$ (3.4.12)

Where $\tilde{A}_2$, $\tilde{D}_3$ and $\tilde{D}_4$ are constants noted from the table given in Table A-2 in Appendix.

Example (3.4.1): . The volume of soft drink beverage bottles is an important quality characteristic. The volume is measured (approximately) by placing a gauge over the crown and comparing the height of the liquid in the neck of the bottle against a coded scale. Construct Median and Mid-range control Charts and draw your conclusions.

Sample No.	X_1	X_2	X_3	X_4	X_5
1	8.4	7.8	8.2	7.8	7.8
2	8.0	8.2	7.7	7.5	7.7
3	7.8	7.6	8.2	7.1	8.8
4	8.4	7.3	8.6	8.7	8.6
5	7.9	7.7	8.2	8.4	8.6
6	7.5	7.9	8.7	8.9	8.8
7	8.8	7.8	7.8	7.1	8.4
8	8.8	7.4	8.5	7.1	7.7
9	8.5	7.9	8.5	8.2	7.2
10	8.5	8.1	8.8	7.4	7.7
11	8.7	7.8	8.3	7.2	8.0
12	7.2	7.9	7.6	7.7	7.3
13	7.5	7.0	8.8	7.4	7.9
14	8.7	8.3	7.1	8.0	7.5
15	7.2	8.4	8.5	8.0	7.5
16	8.5	8.2	7.2	6.1	8.2
17	8.8	8.5	7.8	7.8	7.1
18	8.6	8.2	7.3	7.8	8.7
19	8.1	7.8	7.5	7.2	8.1
20	7.7	7.2	7.1	8.1	7.3

Solution: Construction of Median Control chart:

First Calculate sample Medians and Ranges from the given data as follows:

Sample No.	X_1	X_2	X_3	X_4	X_5	Medians ($\tilde{X}_i$)	Ranges (R_i)
1	8.4	7.8	8.2	7.8	7.8	7.8	0.6
2	8.0	8.2	7.7	7.5	7.7	7.7	0.7
3	7.8	7.6	8.2	7.1	8.8	7.8	1.7
4	8.4	7.3	8.6	8.7	8.6	8.6	1.4
5	7.9	7.7	8.2	8.4	8.6	8.2	0.9
6	7.5	7.9	8.7	8.9	8.8	8.7	1.4
7	8.8	7.8	7.8	7.1	8.4	7.8	1.7
8	8.8	7.4	8.5	7.1	7.7	7.7	1.7
9	8.5	7.9	8.5	8.2	7.2	8.2	1.3
10	8.5	8.1	8.8	7.4	7.7	8.1	1.4
11	8.7	7.8	8.3	7.2	8.0	8.0	1.3
12	7.2	7.9	7.6	7.7	7.3	7.6	0.7
13	7.5	7.0	8.8	7.4	7.9	7.5	1.8
14	8.7	8.3	7.1	8.0	7.5	8.0	1.6
15	7.2	8.4	8.5	8.0	7.5	8.0	1.3
16	8.5	8.2	7.2	6.1	8.2	8.2	2.4
17	8.8	8.5	7.8	7.8	7.1	7.8	1.7
18	8.6	8.2	7.3	7.8	8.7	8.2	1.4
19	8.1	7.8	7.5	7.2	8.1	7.8	0.9
20	7.7	7.2	7.1	8.1	7.3	7.3	1.0

Grand Median = $\tilde{\tilde{M}}$ = 7.9. Similarly Medan for Ranges = $\tilde{\tilde{R}}$ = 1.4.

Now calculate control limits for Median chart for the process average as follows: Here $\tilde{A}_2$ = 0.712 , $\tilde{D}_3$ = 0 and $\tilde{D}_4$ = 2.179 for n = 5 are constants noted from the table A-2 in Appendix.

(i) For controlling the Process average.

UCL (Upper Control Limit) = $\tilde{\tilde{X}}$ + $\tilde{A}_2$ ($\tilde{R}$) = 7.9 + 0.712(1.4) = **8.8968** (3.4.13)

CL (Central Line) = Grand Median = $\tilde{\bar{X}}$ = **7.9** (3.4.14)

LCL (Lower Control Limit) = $\tilde{\bar{X}}$ - $\tilde{A}_2$ ($\tilde{R}$) =7.9 – 0.712(1.4) = **6.9032** (3.4.15)

Similarly Control charts for Range chart or $\tilde{R}$-chart which is to control process variability is given as follows:

(ii) For controlling the process Variation.

UCL (Upper Control Limit) = $\tilde{D}_4$ ($\tilde{R}$) = 2.179(1.4) = **3.0506**. (3.4.16)

CL (Central Line) = $\tilde{R}$ = **1.4** (3.4.17)

LCL (Lower Control Limit) = $\tilde{D}_3$ ($\tilde{R}$) = **0** (3.4.18)

Median Control chart is given in Figures (3.4.1) and (3.4.2)

Fig(3.4.1) : Median Control Chart for controlling the process average.

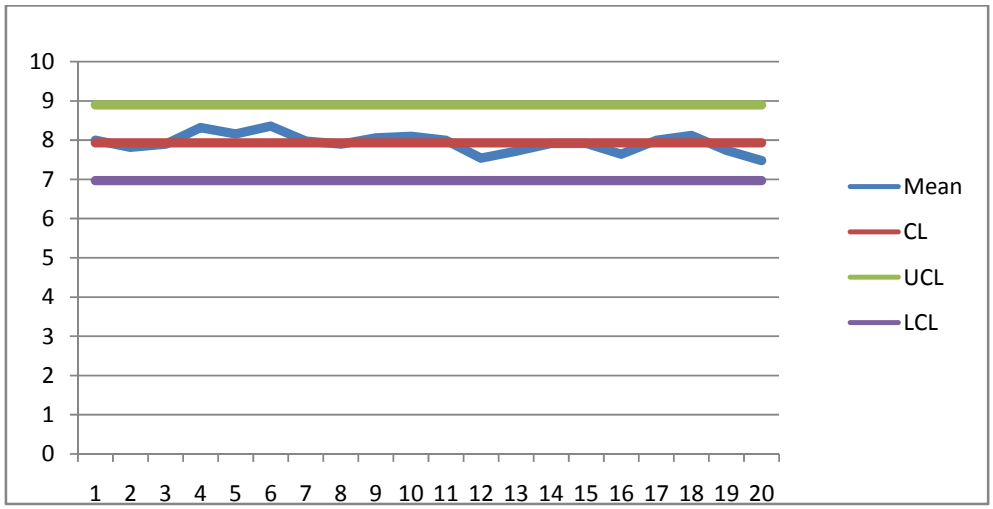
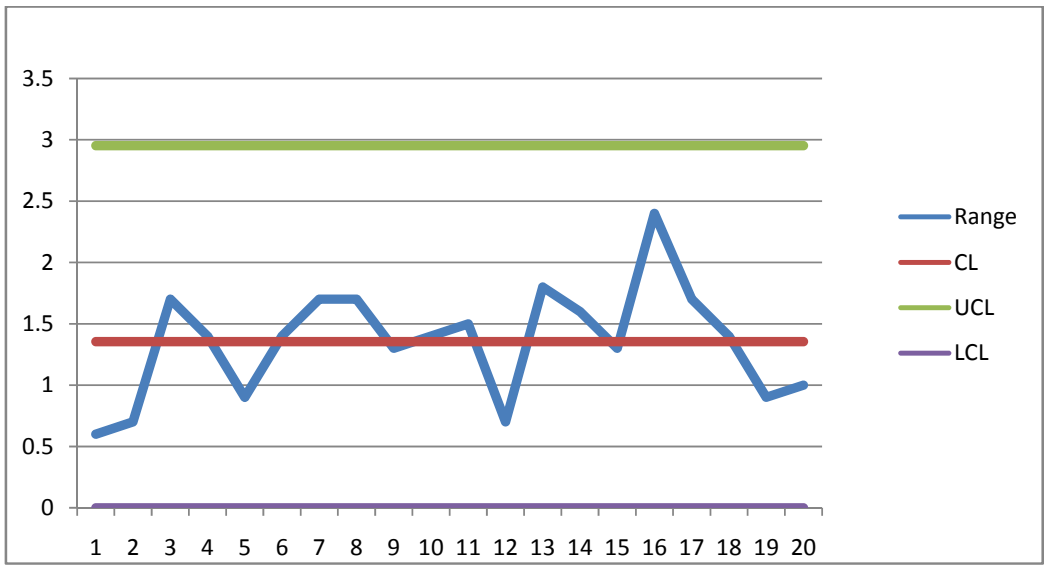


Fig.(3.4.2) :Range Chart for controlling process variation.



Conclusion: Since all the points are within control limits, with respect to the process average and the process variation, we conclude that the process is 'under control'.

That is only random causes of variation are alone present in the volume of soft drink beverage bottles.

Construction of Mid-range control chart:

From the given data calculate sample mid- ranges and ranges as follows:

Sample No.	X_1	X_2	X_3	X_4	X_5	Mid-ranges	Range
1	8.4	7.8	8.2	7.8	7.8	8.1	0.6
2	8	8.2	7.7	7.5	7.7	7.85	0.7
3	7.8	7.6	8.2	7.1	8.8	7.95	1.7
4	8.4	7.3	8.6	8.7	8.6	8	1.4
5	7.9	7.7	8.2	8.4	8.6	8.15	0.9
6	7.5	7.9	8.7	8.9	8.8	8.2	1.4
7	8.8	7.8	7.8	7.1	8.4	7.95	1.7
8	8.8	7.4	8.5	7.1	7.7	7.95	1.7
9	8.5	7.9	8.5	8.2	7.2	7.85	1.3
10	8.5	8.1	8.8	7.4	7.7	8.1	1.4
11	8.7	7.8	8.3	7.2	8	7.95	1.5

12	7.2	7.9	7.6	7.7	7.3	7.55	0.7
13	7.5	7	8.8	7.4	7.9	7.9	1.8
14	8.7	8.3	7.1	8	7.5	7.9	1.6
15	7.2	8.4	8.5	8	7.5	7.85	1.3
16	8.5	8.2	7.2	6.1	8.2	7.3	2.4
17	8.8	8.5	7.8	7.8	7.1	7.95	1.7
18	8.6	8.2	7.3	7.8	8.7	8	1.4
19	8.1	7.8	7.5	7.2	8.1	7.65	0.9
20	7.7	7.2	7.1	8.1	7.3	7.6	1
					median	7.95	1.4

Thus we have: $\tilde{\tilde{M}}$ **= 7.95,** $\tilde{R}$ =1.4, $\tilde{\tilde{A}}_2$ = 0.679, $\tilde{D}_3$ =0 , $\tilde{D}_4$ =2.179 are constants noted from the table A-2 in Appendix, for n = 5 .

Control limits for Mid-range control chart are:

(c) For controlling the Process average.

UCL (Upper Control Limit) = $\tilde{\tilde{M}}$ + $\tilde{\tilde{A}}_2$ ($\tilde{R}$)= 7.95 + 0.679(1.4) = **8.9** (3.4.19)

CL (Central Line) = **Grand Median** = $\tilde{\tilde{M}}$ = **7.95** (3.4.20)

LCL (Lower Control Limit) = $\tilde{\tilde{M}}$ - $\tilde{\tilde{A}}_2$ ($\tilde{R}$)= 7.95 – 0.679(1.4) = **6.99** (3.4.21)

Similarly Control charts for Range chart or $\tilde{R}$-chart which is to control process variability is given as follows:

(d) For controlling the process Variation.

UCL (Upper Control Limit) = $\tilde{D}_4$ ($\tilde{R}$) = 2.179(1.4) = **3.051** (3.4.22)

CL (Central Line) = $\tilde{R}$ = **1.4** (3.4.23)

LCL (Lower Control Limit) = $\tilde{D}_3$ ($\tilde{R}$) = **0** (3.4.24)

Figures (3.4.3) and (3.4.4) represent mid-range control charts for the process average and the process variation respectively.

Fig. (3.4.3): Mid Range Control Chart for controlling the process average.

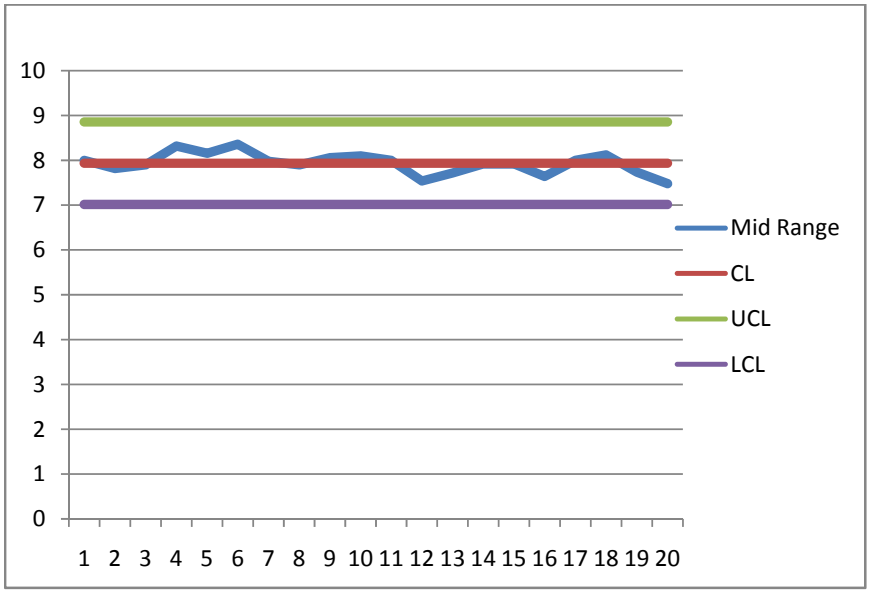
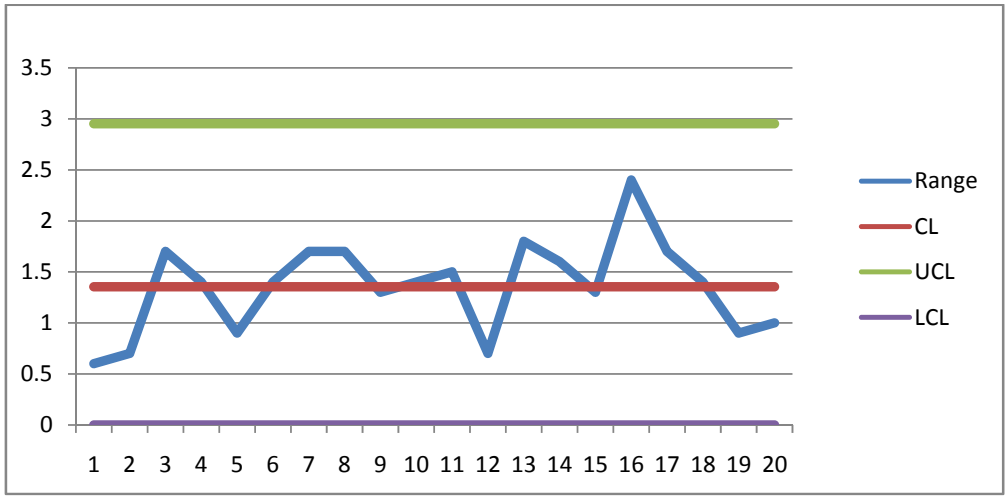


Fig. (3.4.3): Range Control Chart for controlling the process variation.



Conclusion: Since all the sample mid-ranges and ranges are inside the control limits, we conclude that the process under consideration is within control. That is there is no need to search for assignable causes of variation, in the production process with respect to process average and process variability.

3.5. ADVANTAGES OF MEDIAN AND MID-RANGE CONTROL CHARTS

Following are main advantages of **median** control chart.

(1) Median control charts have less arithmetic when compared to X-bar or R-Charts.
(2) Easier to compute control limits.

(3) Calculations of median involve less calculation when compared to the arithmetic Mean calculations.
(4) Comparison between process capability and process achievement are easily made.

The **mid-range** Control charts have all the above advantages as explained above except the fourth one. That is mid-range control chart does not provide the comparison between process capability and process achievement. This is because of the fact that mid-ranges do not provide a very good estimate of central tendency when long tail distributions or Skewed distributions are involved. Similarly, when sample size n is larger than 5 or 6, range charts are suitable charts to control the variability present in the process. The charts useful for large values of n are introduced in the next lesson.

3.6. SUMMARY

Application of Median in the place of arithmetic is discussed in this lesson. In arithmetic mean, calculations are more. These calculations are fewer, in determining the median. To reduce mathematical calculations, Median or mid-range control charts are used when the sample size n is small. When sample sizes are more, we have to use Sigma charts or S-charts to control process average.

3.7. SELF ASSESSMENT QUESTIONS

1. Discuss basic advantages of Median control charts over X-bar charts.

2. What are Median and Mid-range control charts? Discuss their applications.

3. Explain the method of determining control limits of Median control chart.

4. Discuss advantages of Median and mid-range control charts.

5. The following data represents the frequency distribution of inside diameter measurements in cms. of 150 piston rings. Calculate Median for the following distribution.

Class Interval	5 – 10	10 – 20	20 – 30	30 - 35	35 - 40	40 - 50	50 - 55	55 - 60	60 - 65
Frequency	3	5	10	12	25	55	15	18	7

6. Following data represents inner side diameters measured in mms. of Piston rings of Auto mobile manufacturing company in Bombay. Construct Median and mid-range control charts and draw your conclusions on the process.

Sample Number	X_1	X_2	X_3	X_4
1	283.4	278.4	282.6	278.2
2	283.0	281.2	278.7	275.7
3	278.8	279.6	280.2	279.1
4	284.5	273.1	278.6	280.6
5	279.0	277.8	281.2	284.4
6	275.3	279.9	287.3	289.7
7	288.6	278.3	278.8	271.0
8	279.2	274.9	278.6	277.7
9	274.5	270.0	280.8	273.4
10	282.7	281.3	279.1	282.0
11	278.2	284.4	281.5	286.0
12	280. 8	281.5	278.4	273.8
13	280.6	281.2	279.3	273.8
14	278.5	285.2	278.4	280.3
15	284.9	292.8	279.8	294.8

7. Following data represents outside diameters measured in cms. of Axial rods of Auto mobile manufacturing company in Calcutta. Construct Median and mid-range control charts and draw your conclusions on the process.

Sample Number	X_1	X_2	X_3	X_4
1	285	278	286	278
2	283	282	278	275
3	288	296	282	279
4	285	273	286	286
5	279	278	282	284
6	275	279	283	287
7	286	283	278	271
8	279	279	278	277
9	275	270	288	274
10	287	283	279	282

11	278	284	285	286
12	28 8	285	278	278
13	286	282	279	278
14	278	285	278	283
15	299	298	298	298
16	276	276	288	295
17	288	280	286	292
18	278	276	298	297
19	274	299	283	287
20	285	272	289	274

8. Explain the method of construction of Mid-range control chart with an example.

9. Explain the method of construction of Median control charts.

10. critically compare Median control charts with X-bar control charts.

3.8. FURTHER READINGS

1. Montgomery, D.C., " Introduction to Statistical Quality Control", John Wiley (Asia) 2001.

2. Biswas.S., "Statistical Quality Control-Sampling Inspection and Reliability", New Central Book Agency Pvt. Ltd. 2003.

3.Wardsworth "Modern Methods for Quality Improvement", John Wiley and sons 2005.

4.Bertrand L. Hansen "Quality control – Theory and Applications" Prentice-Hall of India-2005.

Unit – I

LESSON – 4

SIGMA, STANDARD DEVIATION AND C.V. CONTROL CHARTS

4.0. OBJECTIVE

After going through this lesson, you should be able to understand:

- The need for Sigma and Standard deviation Control Charts.
- The need for CV and control charts using CV.
- Other Variable control charts based on some simple statistics.
- Properties and applications of above charts.

Structure:

4.1. Introduction.

4.2. Construction of X – bar and S- Charts.

4.3. Introduction to Coefficient of Variation (CV).

4.4. Construction of CV Control charts.

4.5. Other Variable control charts properties and applications..

4.6. Summary.

4.7. Self Assessment Questions.

4.8. Further readings.

4.1. Introduction

In the previous lessons we have introduced variable control charts line X-bar, Median charts for controlling process average and R-chart, Mid-range charts for controlling the variability present in the process. In all these charts, small sample sizes less than five or six are considered. As sample size increases, range lose its efficiency in estimating the variability present in the process. To estimate the variability present in any process, we require calculating the Standard Deviation σ or sample standard deviations are to be used.

Hence, there is a need to introduce control charts based on σ or S. These charts are known as Sigma charts or σ – charts or S – charts. Construction of these control charts are discussed in the following section.

4.2. CONSTRUCTION OF X – BAR AND S- CHARTS.

Usually X-bar and R- charts are widely used, it is occasionally desirable to estimate the process Standard deviation directly instead of indirectly through the use of range R.

This leads to control chars for X-bar and S-charts where, S represents the sample standard deviation calculated from sample data using the formula:

$$S = \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} \frac{(xij - \bar{X})^2}{mn-1}} \qquad (4.2.1)$$

Some authors prefer to call s-chart as σ- chart. Usually if σ^2 is unknown variance of a probability distribution, then an unbiased estimate of σ^2 is the sample variance S^2 .

If x_1 , x_2 ,.,.,x_n are data items, then:

$$S^2 = \sum_{i=1}^{n}(xi - \bar{x})2 \ / \ n-1 \text{ and } S = \sqrt{S^2} \qquad (4.2.2)$$

However, the sample standard deviation S is not an unbiased estimator of σ. Further, we observe that S actually estimate $c_4 \sigma$, where c_4 is a constant that depends upon the sample size n. Further we know that the standard deviation of S is $\sigma\sqrt{(1-(c_4)^2}$. This information is used to construct control limits for X – bar and S – charts as follows:

First calculate sample means $\bar{x}_i$ and sample standard deviations S_i i = 1,2,.,.,m . Then by using formulae calculate grand means $\bar{\bar{x}}$ and $\bar{S}$ from the given data. Here

$$\bar{\bar{x}} = [\bar{x}_1 + \bar{x}_2 + ,.,.,., + \bar{x}_m \ ] \ / \ m. \qquad (4.2.3)$$

$$\bar{S} = [S_1 + S_2 + .,.,., + S_m \ ] \ / \ m. \qquad (4.2.4)$$

Then control limits for S – Chart If σ value is given then we have:

$$UCL = c_4 \sigma + 3\sigma\sqrt{(1-(c_4)^2} \qquad (4.2.5)$$

$$CL = c_4 \sigma \qquad (4.2.6)$$

$$LCL = c_4 \sigma - 3\sigma\sqrt{(1-(c_4)^2} \qquad (4.2.7)$$

Let, $B_5 = c_4 - 3\sqrt{(1-(c_4)^2}$ and $B_6 = c_4 + 3\sqrt{(1-(c_4)^2}$

Then we have:

$$UCL = B_6 \sigma \qquad (4.2.8)$$

$$CL = c_4 \sigma \qquad (4.2.9)$$

$LCL = B_5 \sigma$ (4.2.10)

If σ value is not known or given, then we have to estimate this σ by using the past data by using the formulae (4.2.2) and (4.2.4). Then $\bar{S}/c_4$ is considered as unbiased estimate for σ . Therefore the parameters of S – chart are:

$UCL = \bar{S} + 3\,\bar{S}/c_4\,\sqrt{(1-(c_4)^2}$ (4.2.11)

$CL = \bar{S}$ (4.2.12)

$LCL = \bar{S} - 3\,\bar{S}/c_4\,\sqrt{(1-(c_4)^2}$ (4.2.13)

Let, $B_3 = 1 - 3/c_4\,\sqrt{(1-(c_4)^2}$ and $B_6 = 1 + 3/c_4\,\sqrt{(1-(c_4)^2}$

Then, Control limits for S- chart are:

$UCL = B_4\,\bar{S}$ (4.2.14)

$CL = \bar{S}$ (4.2.15)

$LCL = B_3\,\bar{S}$ (4.2.16)

When $\bar{S}/c_4$ is used as an estimate of σ, then the Control Limits for $\bar{x}$ chart are:

$UCL = \bar{\bar{x}} + 3\,\bar{S}/c_4\,\sqrt{n}$ (4.2.17)

$CL = \bar{\bar{x}}$ (4.2.18)

$LCL = \bar{\bar{x}} - 3\,\bar{S}/c_4\,\sqrt{n}$ (4.2.19)

Let $A_3 = 3/(c_4\sqrt{n})$, then control limits for $\bar{x}$ chart are:

$UCL = \bar{\bar{x}} + A_3\,\bar{S}$ (4.2.20)

$CL = \bar{\bar{x}}$ (4.2.21)

$LCL = \bar{\bar{x}} - A_3\,\bar{S}$ (4.2.22)

Where, A_3 , B_3 and B_4 are constants to be noted from table given A-1 in Appendix, for various sample sizes n. Here $\bar{S}$ and S are calculated using equations (4.2.2) and (4.2.4).

Example (4.2.1): The following data represent the inside diameter measurements (mm) of automobile Piston Rings manufactured by an automobile spare parts production parts at Chennai. Construct $\bar{x}$ and S – charts and draw your conclusions.

Sample No.	X_1	X_2	X_3	X_4	X_5
1	73.982	73.984	73.995	74.017	74.013
2	73.992	74.007	73.015	73.989	74.014
3	73.998	74.000	73.990	74.007	73.995
4	74.012	74.014	73.998	73.999	74.007
5	74.000	74.010	74.013	74.020	74.003
6	73.995	73.992	74.001	74.011	74.004
7	74.002	73.996	73.993	74.015	74.009
8	74.009	73.994	73.997	73.985	73.993
9	73.985	74.003	73.993	74.015	73.988
10	74.004	74.000	74.007	74.000	73.996
11	74.006	73.967	73.994	74.000	73.984
12	74.000	73.984	74.005	73.998	73.996
13	74.006	74.010	74.018	74.003	74.000
14	74.004	73.999	73.990	74.006	74.009
15	74.015	74.008	73.993	74.000	74.010
16	74.030	74.002	74.019	73.992	74.008
17	73.988	74.024	74.021	74.005	74.002
18	73.995	74.006	73.994	74.000	74.005
19	74.008	73.995	74.009	74.005	74.004
20	73.994	73.998	73.994	73.995	73.990
21	73.983	74.002	73.998	73.997	74.012
22	73.994	74.012	73.986	74.005	74.007
23	73.984	74.002	74.003	74.005	73.997
24	74.982	74.001	74.015	74.005	73.996
25	74.010	73.989	73.990	74.009	74.014

Solution: For the given data, first calculate the sample mans and Standard deviations using formulae (4.2.2) and then calculate Grand mean $\bar{\bar{x}}$ and $\bar{S}$ Using (4.2.3) and (4.2.4) as follows:

Sample Number	X_1	X_2	X_3	X_4	X_5	$\bar{x}_i$	S_i
1	73.982	73.984	73.995	74.017	74.013	73.998	0.0162
2	73.992	74.007	73.015	73.989	74.014	74.003	0.0122
3	73.998	74.000	73.990	74.007	73.995	73.998	0.0063
4	74.012	74.014	73.998	73.999	74.007	74.006	0.0073
5	74.000	74.010	74.013	74.020	74.003	74.009	0.0080
6	73.995	73.992	74.001	74.011	74.004	74.001	0.0075
7	74.002	73.996	73.993	74.015	74.009	73.003	0.0091
8	74.009	73.994	73.997	73.985	73.993	73.996	0.0087
9	73.985	74.003	73.993	74.015	73.988	73.997	0.0123
10	74.004	74.000	74.007	74.000	73.996	74.001	0.0042
11	74.006	73.967	73.994	74.000	73.984	73.990	0.0153
12	74.000	73.984	74.005	73.998	73.996	73.997	0.0078
13	74.006	74.010	74.018	74.003	74.000	74.007	0.0070
14	74.004	73.999	73.990	74.006	74.009	74.002	0.0074
15	74.015	74.008	73.993	74.000	74.010	74.005	0.0087
16	74.030	74.002	74.019	73.992	74.008	74.010	0.0148
17	73.988	74.024	74.021	74.005	74.002	74.008	0.0147
18	73.995	74.006	73.994	74.000	74.005	74.000	0.0055
19	74.008	73.995	74.009	74.005	74.004	74.004	0.0055
20	73.994	73.998	73.994	73.995	73.990	73.994	0.0029
21	73.983	74.002	73.998	73.997	74.012	73.998	0.0105
22	73.994	74.012	73.986	74.005	74.007	74.001	0.0106
23	73.984	74.002	74.003	74.005	73.997	73.998	0.0085
24	74.982	74.001	74.015	74.005	73.996	74.000	0.0122
25	74.010	73.989	73.990	74.009	74.014	74.002	0.0119
					Totals	1850.028	0.2350

Then, $\bar{\bar{x}}$ = 1850.028/25 = 74.001 and $\bar{S}$ = 0.2350 / 25 = 0.0094 . Further the constants

$A_3 = 1.427$, $B_3 = 0$ and $B_4 = 2.089$ from the table A-1 in appendix. Then, the control limits for $\bar{x}$ charts are as follows:

UCL $= \bar{\bar{x}} + A_3 \bar{S} = 74.001 + (1.427)(0.0094) =$ **74.014.** (4.2.23)

CL $= \bar{\bar{x}} =$ **74.001** (4.2.24)

LCL $= \bar{\bar{x}} - A_3 \bar{S} = 74.001 - (1.427)(0.0094) =$ **73.988** (4.2.25)

Further, control limits for S – charts are :

UCL $= B_4 \bar{S} = (2.089)(0.0094) =$ **0.0196** (4.2.26)

CL $= \bar{S} =$ **0.0094** (4.2.27)

LCL $= B_3 \bar{S} (0)(0.0094) =$ **0** (4.2.28)

Control charts along with the control limits for both charts are drawn in the following figures (4.2.1) and (4.2.2).

Fig.(4.2.1): Mean Chart for controlling process average.

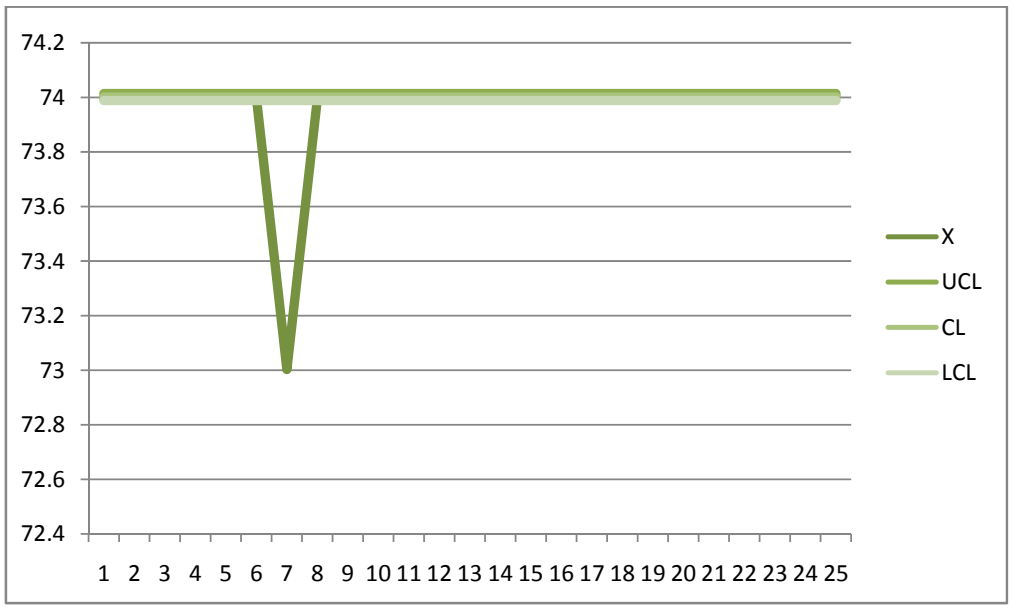
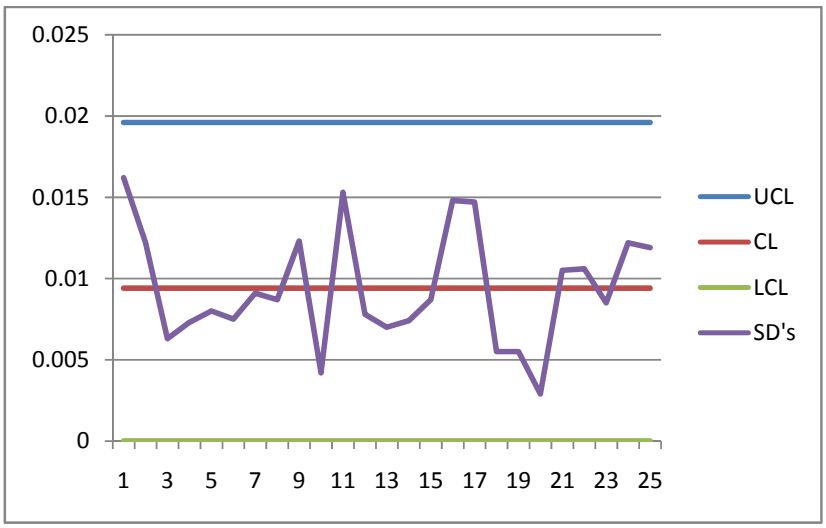


Fig.(4.2.2): Sigma Chart for controlling process variation.



Conclusions: Since all the sample means are within the control limits in $\bar{x}$ chart, we conclude that the production of piston rings with respect to the inside diameter is within control with respect to the process average. Similarly, all the sample standard deviations are within control limits in S – chart, we conclude that the products are within control with respect to the inside diameter with respect to the variance. Since, both charts are within control, we conclude that the production process of piston rings is producing the products as per our requirement and hence are considered as quality products with respect to the inside diameter. Further calculation of revised control limits for this process is not required because, the process is within the control. Hence, same control limits for both $\bar{x}$ chart and S – chart can be sued to control future production.

4.3. INTRODUCTION TO COEFFICIENT OF VARIATION (CV)

So far, we have discussed Variable control charts based on sample means, Median, Ranges, Standard Deviation σ or sample standard deviation 's'. All these statistics are depending on the measuring units in which the sample data are measured. For example, if the data is in mm all these measures are also in mm. If the data is in grams, all these are also should be expressed in grams and so on. For example, if the quality characteristic is the inside diameter of the piston ring it is to be measured in mm or cms. If the quality characteristic is the weight of the piston, it is to be measured in grams or kgs. If the quality characteristic is lifetime, then it is to be measured in hours (hrs). Some quality characteristics are to be measured in oz or psb and so on. Comparison between different measuring units will become difficult particularly, when conversion is not possible. Hence we need a measure which is free from these measuring units. Such a statistical measure is known as **'Coefficient of variation (CV)'** denoted by 'V' and is defined as:

Coefficient of Variation = CV = V = [(Standard Deviation) / Mean] x 100

$$= (\sigma / \bar{x}) \times 100 \qquad (4.3.1)$$

After calculating Sample means and Standard deviations, we can calculate V using (4.3.1).

To illustrate its use, consider two players A and B are measured for their efficiency. Player A is a Hockey player and Player B is a Cricket Player. Their mean $\bar{x}$ and Standard deviation σ are as follows:

	Player – A	Player - B
Mean $\bar{x}$	20 Goals	200 Runs
Standard deviation σ	2 Goals	40 Runs

Among these two players, one has to be selected for a Sports Prize. There no possibility of converting hockey goal to cricket runs. Hence we calculate the Coefficient of Variation CV.

CV of Player – A $= V_A = (2/20)100 = 10$ and C.V. of player B $= V_B = (40 / 200)100 = 20$.

Here, $V_A < V_B$. Hence , V_A is considered for the prize because C.V. of player A is smaller than the CV of player B. The data which has smaller variance is to be preferable than the larger one. Hence Player A is preferred to player B. Thus in quality control also CV for various quality characters can be measured to compare variation present in the data which are in different measuring units. Control charts based on coefficient of Variation CV are known as **CV – Control Charts or % CV control charts**.

4.4. CONSTRUCTION OF % CV CONTROL CHARTS

The central line and control limits on the %CV chart are calculated based off the SD chart. For specified limits the calculations are:

$$\%\text{CV Centerline} = \frac{\bar{s}}{\bar{\bar{X}}} \times 100 \qquad (4.4.1)$$

Where $\bar{s}$ is the specified central line of the data displayed on the SD chart and $\bar{\bar{X}}$ is the specified mean as defined in the control limit record or the grand mean calculated from the given data.

$$\%\text{CV UCL} = \frac{UCL_{SD}}{\bar{\bar{X}}} \times 100 \qquad (4.4.2)$$

$$\%\text{CV LCL} = \frac{LCL_{SD}}{\bar{\bar{X}}} \times 100 \qquad (4.4.3)$$

Where UCL_{SD} and LCL_{SD} are the specified control limits from the SD chart and $\bar{\bar{X}}$ is the process mean as defined in the control limit record or calculated from the data.

Calculated Control Limits and Central line

The central line and control limits on the %CV chart are calculated based off the SD chart. For calculated limits the calculations are:

$$\%\text{CV Calculated Centerline} = \frac{\bar{s}_c}{\bar{\bar{X}}_c} \times 100 \qquad (4.4.4)$$

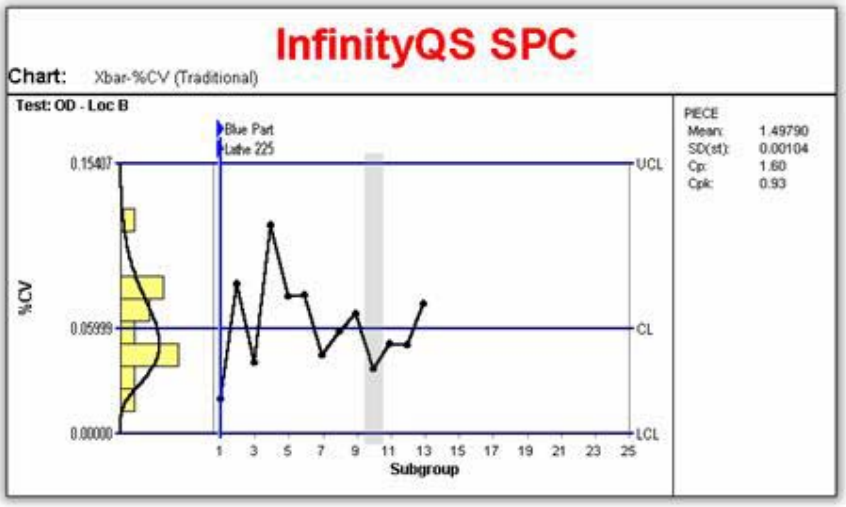
Where $\bar{s}_c$ is the calculated central line of the data displayed on the SD chart and $\bar{\bar{X}}_c$ is the calculated mean as defined in the control limit record.

$$\%\text{CV UCL}_c = \frac{UCL_{SD_c}}{\bar{\bar{X}}_c} \times 100 \qquad (4.4.5)$$

$$\%\text{CV LCL}_c = \frac{LCL_{SD_c}}{\bar{\bar{X}}_c} \times 100 \qquad (4.4.6)$$

Where UCL_{SDc} and LCL_{SDc} are the calculated control limits from the SD chart and $\bar{\bar{X}}_c$ is the calculated central line on the $\bar{X}$ chart.

Fig. (4.4.1): Example of % CV Control Chart.



Applications of %CV Charts.

One example where %CV charts are used is in the textile industry. The variation among tensile strength measurements from thin thread is significantly smaller than measurements taken from heavy thread. This is due to the inherent physical properties of fibre. Another use of the %CV statistic is to compare items that have different means, but share a common relative variation. Comparing multiple solutions with different chemical concentrations compounded in the same mixing line may be best analysed using the %CV control chart.

4.5. OTHER VARIABLE CONTROL CHARTS PROPERTIES AND APPLICATIONS

There other variable control charts like Moving Average Control charts MA Charts, Exponentially weighted Moving Control charts (EWMA), Control charts based on sums, Multivariate control Charts, Hoteling- T control chars, CUSUM charts and so on. These charts have special applications and construction methods also different. These special type of control charts are discussed in coming lectures in other Units. At present we are closing this Unit -1 with the charts discussed earlier.

4.6. SUMMARY

In this lecture, we have discussed the need of other control charts to measure the Variation present in the production process. Usually, variation is measured through the Range. But range is not an efficient measure of variability when the sample size is large say N = 10 or 12 or more. Hence there is a need to use the Standard Deviation (SD) or σ chart or S – Chart (when σ is not known). We have also discussed the construction, application and limitations of these charts. Finally, CV charts are introduced and discussed their construction and applications. Other variable control charts are also listed whose details are studied in the forthcoming lectures.

4.7. SELF ASSESSMENT QUESTIONS

1. Explain the need of Standard Deviation instead of the range to measure the variability.
2. Explain the method of construction of σ chart.
3. Explain the need and construction of S – Chart.
4. Explain the need and construction of % CV chart.
5. Discuss the application of % CV chart with two suitable examples.
6. The net weight (in oz) of dry bleach product is to be monitored with respect to the average weight and variability in weights. Construct $\bar{x}$ and s-charts and draw your conclusions.

Sample No.	X_1	X_2	X_3	X_4	X_5
1	16.3	16.2	16.4	16.3	16.5
2	16.1	16.1	16.4	16.5	16.0

3.	16.2	16.1	16.2	16.1	16.5
4	15.9	16.6	16.7	16.2	16.5
5	16.6	16.3	16.4	16.1	16.5
6	16.4	16.2	16.4	16.3	16.2
7	16.4	16.1	16.3	16.2	16.2
8	16.1	15.8	16.7	16.6	16.4
9	15.8	16.3	16.2	16.1	16.6
10	16.4	16.4	16.5	16.0	15.8
11	16.4	16.1	16.6	16.4	16.1
12	16.4	16.0	16.3	16.4	16.4
13	16.3	15.9	15.9	16.2	16.4
14	16.3	16.2	15.9	16.4	16.2
15	16.1	16.3	16.5	16.1	16.5
16	16.2	16.4	15.9	16.3	16.4
17	16.0	16.2	16.4	16.5	16.1
18	16.0	16.2	16.3	16.3	16.2
19	16.5	16.3	16.2	16.3	16.4
20	16.1	16.2	16.5	16.4	16.3

7. Construct % CV chart for the data given in question no. 6.

8. Explain the use of Coefficient of Variation with a suitable example.

9. The fill volume of soft drink beverage bottles is an important quality characteristic. The Volume is measured (approximately) by placing a gauge over the crown and Comparing the liquid in the neck of the bottle against a coded scale. On this scale, a reading of 0.0 corresponds to the correct fill height. Fifteen samples of size n=10 have been collected. Construct $\bar{x}$ and s-charts and draw your conclusions.

Sample No.	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
1	0.0	0.0	0.5	1.0	1.5	1.0	-1.0	1.0	1.5	-1.0
2	0.0	0.5	-2.0	0.0	-1.0	1.5	-1.5	0.0	-2.0	-1.5
3	1.0	-0.5	0.0	0.0	0.0	0.5	-1.0	1.0	-2.0	1.0
4	0.0	-1.5	-0.5	1.5	0.0	0.0	0.0	-1.0	0.5	-0.5
5	-0.5	3.5	0.0	-1.0	-1.5	-1.5	-1.0	-1.0	1.0	0.5
6	0.0	-2.0	-0.5	0.0	-0.5	2.0	1.5	0.0	0.5	-1.0

7	0.5	1.0	-1.0	-0.5	-2.0	-1.0	-1.5	0.0	1.5	1.5
8	2.5	0.5	2.0	1.0	1.0	-1.0	0.5	1.5	0.5	-1.5
9	1.5	1.0	1.0	-1.0	0.0	-1.5	-1.0	-1.0	1.0	-1.0
10	0.0	0.0	0.0	-0.5	0.5	1.0	-0.5	-0.5	0.0	0.0
11	1.0	-1.0	-1.0	-1.0	0.0	1.5	0.0	1.0	0.0	0.0
12	-2.0	-1.5	1.5	1.5	0.0	0.0	0.5	1.0	0.0	1.0
13	0.0	1.5	0.0	0.0	2.0	-1.5	0.5	-0.5	2.0	-1.0
14	-1.0	-0.5	-0.5	-1.0	0.0	0.5	0.5	-1.5	-1.0	-1.0
15	1.0	0.0	1.5	1.5	1.0	-1.0	0.0	1.0	-2.0	-1.5

10. For the data in question 9 construct % CV chart and draw your conclusions.

4.8. FURTHER READINGS

1. Montgomery, D.C., " Introduction to Statistical Quality Control", John Wiley (Asia) 2001.

2. Biswas.S., "Statistical Quality Control-Sampling Inspection and Reliability", New Central Book Agency Pvt. Ltd. 2003.

3.Wardsworth "Modern Methods for Quality Improvement", John Wiley and sons 2005.

4.Bertrand L. Hansen "Quality control – Theory and Applications" Prentice-Hall of India-2005.

UNIT-II
LESSON – 5

CONTROL CHARTS FOR ATTRIBUTES

5.0. OBJECTIVE

After going through this lesson, the student should be able to understand:

- The need for Control charts for attributes.
- Comparison between variable and attribute control charts.
- Various types of Attribute Control charts.
- Construction and applications of p and np – control charts.

STRUCTURE:

5.1. INTRODUCTION

In the Unit – 1, we have discussed the applications of Variable Control charts where the Quality Characteristic of a product is a variable like, Inside or outside diameters of Piston rings, or Volume of soft drink in a bottle, or weight or length or life time of products produced, and so on. **Many times, the quality characteristic will be an attribute**, like The product is of good quality or Bad quality; Smooth finishing or rough finishing; handy or not; defective or non-defective; attractive or not; and so on. For example, Pen can be classified based on smooth writing or not. If the **pen** is smooth writing, it is considered as good product otherwise, it is considered as bad product. When the quality characteristic is an attribute, Control charts to be used to analyze such data are known as **'Attribute Control Charts'**. Even if the quality characteristic is a Variable, we can convert the data into an attribute data. For instance, if the life time of an **electric bulb** is more than 1000 hours, then the bulb is considered as a good one other wise a defective one. Similarly, if the inner diameter of the piston ring is in the interval [73.05mm, 75.05mm] then the piston ring is a good one otherwise a defective one. Like this,

we can convert the variable data into attribute data. Dealing attribute data is much easier in data collection and have easier calculations in the analysis. Hence, **'Control charts for attributes'** are more preferable than '**Control charts for Variables**'.

5.2. COMPARISON BETWEEN VARIABLE AND ATTRIBUTE CONTROL CHARTS

Control charts for Attributes have advantages as well as some dis-advantages, when compared to Control charts for variables. They are explained as follows:

Advantages:

1. Attribute control charts are much easier to construct when compared to Variable control charts.
2. Attribute control charts have much easier calculations when compared to variable Control charts.
3. One can always use an attribute control chart in the place of Variable control chart by converting variable data to an attribute data.
4. Many quality characteristics cannot be conveniently represented numerically, but it is easy to classify the product as defective or non defective. Recently, '**Conforming**' or **'non-confirming'** is popularly used instead of **'non-defective'** and **'defective'** respectively.
5. This type of attribute data is widely used in the semi-conductor industry, Electronic products manufacture industry, Textile industry and so on.
6. Attribute control charts are more popularly used and have important applications than Variable control charts.
7. Attribute control charts are economical when compared to Variable control charts.
8. Attribute control charts require less time to construct when compared to Variable control charts.
9. Attribute control charts are easier to construct than Variable control charts.
10. Many quality characteristics can be considered at a time to determine the product is confirming the quality or not confirming.

Dis-advantages:

1. Attribute control charts are generally not as informative as Variable control charts.
2. Attribute control charts are generally not as accurate as Variable control charts.
3. Variable data or numeric data is more informative than attribute data and hence, Variable control charts require smaller sample size 'n' than sample size in Attribute control charts.
4. Variable control charts are preferable in destructive test since they require less number of sample units. This reduces the testing units and testing time and hence cost of the product.
5. Wastage can be controlled through variable control charts, when compared to attribute control charts.

Well, above discussed are some important points to be remembered before deciding to use a variable Control chart or an attribute control chart. Comparison can be made critically

between Attribute and variable control charts before taking the decision, which chart is to be used in a particular production process.

5.3 VARIOUS TYPES OF ATTRIBUTE CONTROL CHARTS

When the data collected is in the form of attributes, that is, the product produced is classified as 'good' or 'bad'; 'defective' or 'non-defective'; 'conforming' or non-conforming', we generally calculate the following statistics:

1. The fraction of non-conforming units in the collected sample denoted by the random variable 'p'. Control charts based on such fraction defectives p are known as p – charts. That is, when we count number of defectives, we use p- chart. If the variable is number of non conformities in the sample of size 'n' instead of 'p', we call such charts as np – charts.
2. Sometimes it is more convenient to deal with the number defects per sample, denoted by the random variable 'c'. That 'c' represents number of defects counted in the sampled units, then, Control chart based on number of defects per unit are known as c –chart.
3. When it is preferable to count number of defects per unit, denoted by 'u', then control chart used to monitor the variable 'u' is known as u – chart.

Before explaining the construction of these charts, it is important to distinguish the difference between '**defect'** and a **'defective'**. These two are always confusing and needs clarity and are to be distinguished clearly. If any quality characteristic is not having the required specifications, then it is called as a **'defect'** and the product having such defect is called as '**defective'**. A defective item may have one or more defects, but, the item is a defective one. When the unit is small or the unit is a single part which cannot be further divisible, like electric bulb, or pen or pencil or piston ring, or a cracker, we classify such products a 'defective' or 'non-defective'; 'non-conforming' or 'conforming' unit. The unit may have one or more defects with respect to one or more quality characteristics. In such situations, we use fraction defectives 'p' and p-chart or number of defectives per sample 'np' and use np – chart. Sometimes it is convenient to count number of defects per sample, then we use c – chart. If it is convenient to count number of defects per unit, we use u – chart. Construction and application of these charts are discussed in the following lessons.

5.4. Construction of p – Chart and np – chart (for fixed sample size n)

When the product under consideration is a single unit or the major quality character of the product is one on which the product is decided as confirming and non-conforming or good or defective product then we calculate the **fraction of non-conforming** which is denoted by the random variable 'p' and is defined as the ratio of the number of non-conforming items in the population to the total number of items in that population. The items may have several quality characteristics that are examined simultaneously by the inspector and determine the product is conforming to the standards or not? That is the product is classified as **'conforming'** or '**non-**

conforming', based on one or more quality characteristics. If the population consists of 'n' units and let 'D' units are non-conforming units out of 'n' items in the population. Then
P = D / n represents the fraction or probability of non-conforming units. The statistical principle to develop Control chart for 'D' is Binomial distribution whose probability mass function is given By:

$P[D = x] = \binom{n}{x} P^x q^{n-x}$ x = 0,1,2,.,.,.,n. (5.4.1)

Where q = 1- P . Here P is estimated as sample fraction non-conforming which is denoted by $\hat{p}$ = D / n . Here D represents number of non-conforming units in the sample of size n.
Using the properties of Binomial distribution we know that the mean and the variance of $\hat{p}$ is given as follows:
Mean of $(\widehat{p})$ = P and Var $(\hat{p})$ = $\sigma(\widehat{p})$ = [P(1 – P)] / n . (5.4.2)

5.4a. Construction of p – chart

Then, the control limits for p – chart are given as follows:
UCL = P + 3 $\sqrt{\mathbf{P(1 - P)/n}}$ (5.4.3)
Center Line = CL = P
LCL = P - 3 $\sqrt{\mathbf{P(1 - P)/n}}$ (5.4.4)

Above control limits can be used when the population mean P is known or the standard Value is specified by the management. If the value of P is not known, this is to be estimated from the sample data or observed data collected. Let the sample data collected has m samples each of size n. Then the sample fraction of non-conforming from i^{th} sample is denoted by $\hat{p}_I$, where,
$\hat{p}_I$ = D_i / n i=1,2,.,.,m. (5.4.5)

Then the average of these individual sample fractions non-conforming denoted by $\bar{p}$ is an unbiased estimate of population fraction of non-conforming P. Where $\bar{p}$ is defined as:
$\bar{p} = [\sum_{i=1}^{m} Di\ ] /mn = [\sum_{i=1}^{m} \hat{p}i\ ] / m$. (5.4.6)
Using $\bar{p}$ as an unbiased estimate of P, we have the Control limits of p – chart is given as follows:

UCL = $\bar{p}$ + 3$\sqrt{\bar{p}\ (1 - \bar{p}\ ) /n}$ (5.4.7)
CL = $\bar{p}$ (5.4.8)
LCL = $\bar{p}$ + 3$\sqrt{\bar{p}\ (1 - \bar{p}\ ) /n}$ (5.4.9)
Now, the method construction of p – chart is explained with the following example.

Example (5.4.1): Apple juice manufacturing company uses cardboard cans of size 12 oz. These cans are formed on a machine, by spinning them from cardboard stock and attaching a metal bottom panel. By inspection of a can, we may determine, when filled with juice, it could possibly leak either on the side seam or the bottom panel. Anywhere, leak is possible, the can is considered as a defective one or non-conforming can. Otherwise it is considered as conforming can. For this a sample of size n = 100 cans are randomly selected for 30 days production of cans. Number of non-conforming cans is recorded from each sample as follows:

Day:	1	2	3	4	5	6	7	8	9	10
No. of Non-Conforming cans:	20	10	26	22	40	36	48	30	18	24
Day:	11	12	13	14	15	16	17	18	19	20
No. of Non-Conforming cans:	24	30	16	20	8	14	32	18	28	20
Day:	21	22	23	24	25	26	27	28	29	30
No. of Non-Conforming cans:	12	14	26	18	10	12	34	24	44	16

Construct a suitable control chart and draw your conclusions.

Solution: From the given data, calculate fraction of non-conforming $\hat{p}_I$ from each sample using (5.4.5) and calculate Grand sample mean $\bar{p}$ using (5.4.6). Then we have:

Day:	1	2	3	4	5	6	7	8	9	10
No. of Non-Conforming cans:	20	10	26	22	**40**	**36**	**48**	30	18	24
$\hat{p}_I$:	0.2	0.1	0.26	0.22	**0.4**	**0.36**	**0.48**	0.3	0.18	0.24
Day:	11	12	13	14	15	16	17	18	19	20
No. of Non-Conforming cans:	24	30	16	20	8	14	32	18	28	20
$\hat{p}_I$:	0.24	0.3	0.16	0.2	0.08	0.14	0.32	0.18	0.28	0.2
Day:	21	22	23	24	25	26	27	28	29	30
No. of Non-Conforming cans:	12	14	26	18	10	12	34	24	**44**	16
$\hat{p}_I$:	0.12	0.14	0.26	0.18	0.1	0.12	0.34	0.24	**0.44**	0.16

Grand total of Non-conforming units = 694.

Hence $\bar{p}$ = 694 / (30)(100) = 0.2313. Therefore the Control limits of p – chart are given as follows: (Points falling outside the UCL are bolded in the above table).

UCL = $\bar{p}$ + 3$\sqrt{\bar{p}(1-\bar{p})/n}$ = 0.2313+3$\sqrt{(0.2313)(0.7687)/100}$

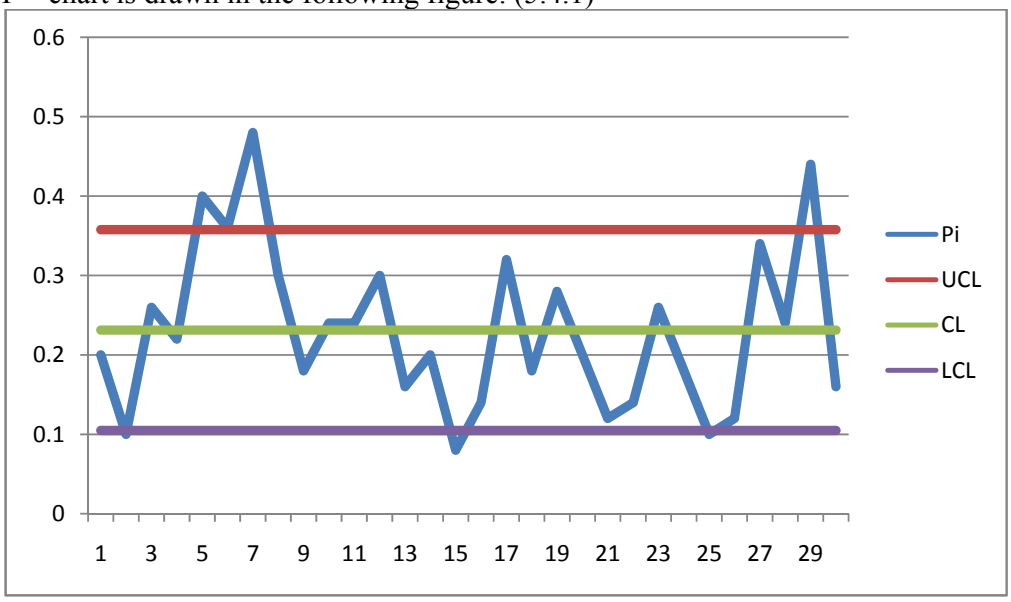
= 0.2313 +3 (0.04217) = 0.2313 + 0.1265 = 0.3578 (5.4.10)

CL = $\bar{p}$ = 0.2313 (5.4.11)

LCL = $\bar{p}$ + 3$\sqrt{\bar{p}(1-\bar{p})/n}$ = 0.2313 -$\sqrt{(0.2313(0.7687)/100}$

=0.2313 – 3(0.04217) = 0.2313 – 0.1265 = 0.1048 (5.4.12)

P – chart is drawn in the following figure: (5.4.1)



Conclusions: Since four p_i are falling outside the UCL we conclude that the production of cardboard cans is out of control. There is a need to identify the cause for variation. There are some assignable causes are present. Hence there is a need to identify the cause and to eliminate them from the production process of cardboard cans.

5.4b. Construction of np – chart

Sometimes, we are interested to plot number of non-conforming units instead of $\hat{p}_I$ values. Then we have to use np instead of $\bar{p}$ in the control limits. Such charts are known as np – control charts. The control limits for np – control chart are as follows:

$$\mathbf{UCL} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} \quad (5.4.13)$$

$$\mathbf{CL} = n\bar{p} \quad (5.4.14)$$

$$\mathbf{LCL} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} \quad (5.4.15)$$

Example (5.4.2): Construct np – control chart for number of non-conforming apple juice cans given in example (5.4.1) and draw your conclusions.

Solution: we have calculated the grand mean $\bar{p}$ for the given data is calculated as $\bar{p} = 0.2313$ **and n = 100.** Then, Control limits for np – chart is calculated as follows;

$$\mathbf{UCL} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 23.13+3\sqrt{23.13(0.7687)}$$
$$= 23.13 + 3(4.21664) = 23.13+12.65 = \mathbf{35.78} \quad (5.4.16)$$

$$\mathbf{CL} = n\bar{p} = 100\,(0.2313) = 23.13 \quad (5.4.17)$$

$$\mathbf{LCL} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} \quad 23.13 - 3\sqrt{23.13(0.7687)}$$
$$= 23.13 - 3\,(4.21664) = 23.13 - 12.65 = \mathbf{10.48}. \quad (5.4.18)$$

Conclusions: Since same sample points are falling outside the UCL, conclusions drawn in example (5.4.1) holds good here also.

5.5. Construction of p – chart with varying sample size n

Sometimes, in some applications of the control chart, for fraction non-conforming, there is a need to go for 100% inspection of the process output. In such situations, the output in any process per unit of time is not constant because output depend on the raw material available, machine times in use and workers present (some workers may go on leave) and so on and hence the output varies from time to time per unit of time or per day or per month. Hence the units to be inspected, that is the quantity 'n' varies from time to time. In such situations, sample size 'n' is a variable but not constant. It is observed in previous calculations that while calculating the control limits, the sample size 'n' is involved in the denominator. If 'n' is a constant value, then we are getting control limits as straight lines. If 'n' changes from time to time, control limits are not straight lines but we get control lines with different widths. For instance, the Control limits of p – chart with varying 'n' are given as follows:

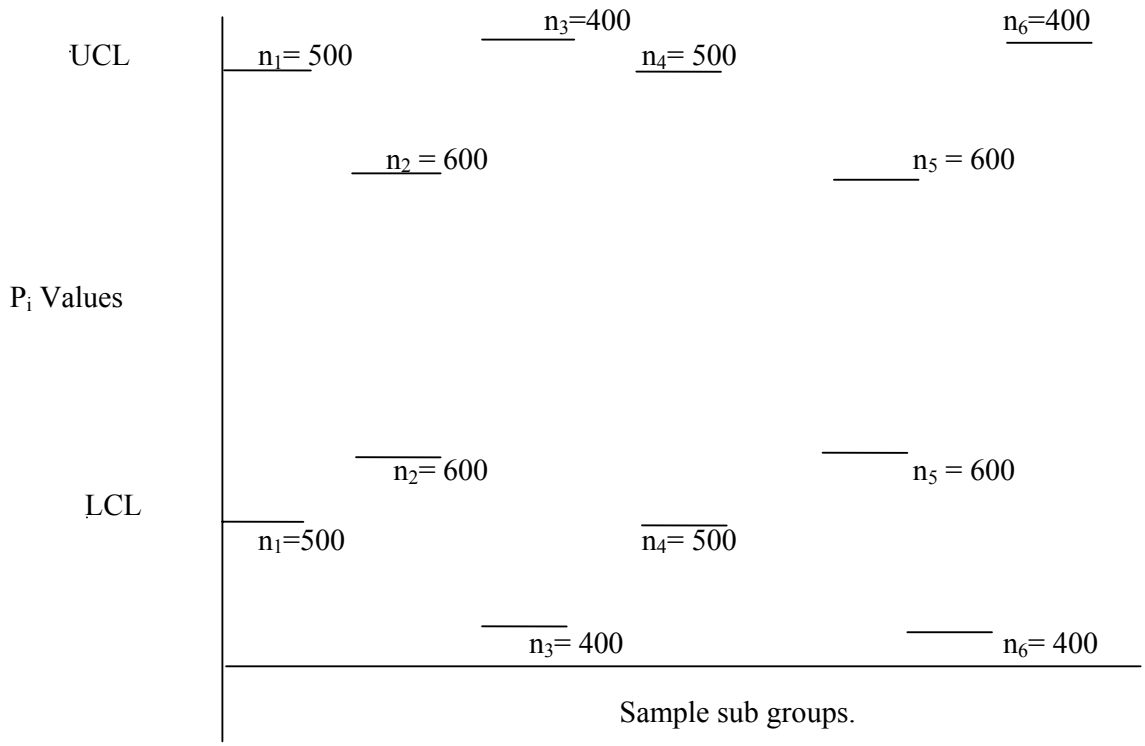
$$\mathbf{UCL} = \bar{p} + 3\sqrt{\bar{p}\,(1-\bar{p})/n_i} \qquad (5.5.19)$$

$$\mathbf{CL} = \bar{p} \qquad (5.5.20)$$

$$\mathbf{LCL} = \bar{p} + 3\sqrt{\bar{p}\,(1-\bar{p})/n_i} \qquad (5.5.21)$$

In the above equations, as n_i values changes widths of control limits also will change as shown in the following figure:

Fig: (5.5.1) Control limits of p – chart with varying sample size.



There are three approaches / methods to construct and operating a control chart with a variable sample size n.

5.5a: Method – 1: Variable width Control limits:

The first and most preferable method of constructing control limits when the sample size is a variable is that draw the control limits as it is with varying widths for different sample sizes as explained above in fig (5.5.1). It is important to note that in this method, the widths are inversely proportional to the square root of sample size 'n_i ' (as shown in Fig. (5.5.1).
This is the most useful method than other two because decision making is done properly, without re-calculating the control limits, when a need comes. This method is explained with the following example.

Example (5.5a.1): Construct a p – chart for the following data where the sample size is a variable as shown below:

Day:	1	2	3	4	5	6	7	8	9	10
No. of Non-Conforming cans:	8	10	8	12	10	20	10	10	15	16
Sample size n_i:	120	80	80	110	110	110	80	100	120	100

Solution: For the above problem, calculate $\bar{p}$ and $\mathbf{p_i}$ for the given data and then calculate the control limits as shown in the following table.

Sample Number I	number of non-conforming Units D	Sample size n_i	Sample fraction of non-conforming p_i	S.D $\sigma\hat{p}$	Control Limits. LCL	UCL
1.	8	120	0.067	0.089	0.031	0.209
2.	10	80	0.125	0.109	0.011	0.229
3.	8	80	0.100	0.109	0.011	0.229
4.	12	110	0.109	0.093	0.027	0.213
5.	11	100	0.110	0.097	0.023	0.217
6.	20	110	0.182	0.093	0.027	0.213
7.	10	80	0.125	0.109	0.011	0.229
8.	10	100	0.100	0.097	0.023	0.217
9.	15	120	0.182	0.089	0.031	0.209
10.	16	100	0.160	0.097	0.023	0.217

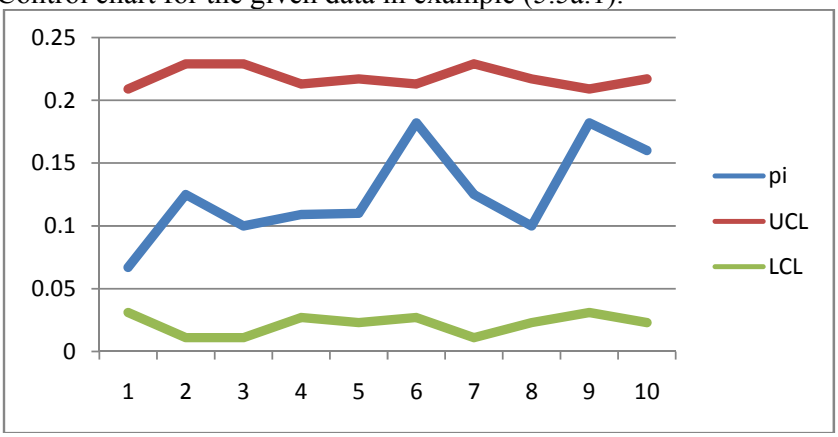
Here, m = 10, $\Sigma D_i = 120$ and $\Sigma n_i = 1000$ and $\sigma\hat{p} = 3\sqrt{\bar{p}(1 - \bar{p})} / n_i$

$\bar{p} = \Sigma D_i / \Sigma n_i = 120 /1000 = 0.12$. hence we have $\bar{p}(1 - \bar{p}) = (0.12)(0.88) = 0.1056$.

Calculate UCL and LCL using (5.5.19) and (5.5.21) as shown in last two columns in the table.

Comparing p_i with UCL and LCL, we conclude that the process is under control. If we draw the control chart, we obtain similar figure as shown in fig (5.5.1).

Fig. (5.5a.1): Control chart for the given data in example (5.5a.1).



Even though calculating control limits involve more calculations, this method – 1 is preferable to other methods which we are going to explain below.

5.5b. Method – 2: Control limits based on average sample size

The second approach is to construct the control limits based on the average sample size. Since the average sample size $\bar{n} = \Sigma n_i / m$ is constant, we obtain UCL and LCL as straight lines as in previous control charts with constant sample size n. basically, there is a difficulty in determining a sample fraction of non conforming p_i is inside or out-side these average UCL or LCL. If p_i is closer to these limits, then there is a necessity to recalculate the actual control limits with actual sample size n_i and take appropriate decision. The sample fraction of non-conforming is inside or outside control limits. If all the sample fractions of non-conforming are inside control limits, we decide that the process is 'under Control' otherwise, determine that the process is 'out of control'. The procedure is explained with the following example.

Example (5.5b.1): Construct Control chart for the data given in example (5.5a.1) using average sample size.

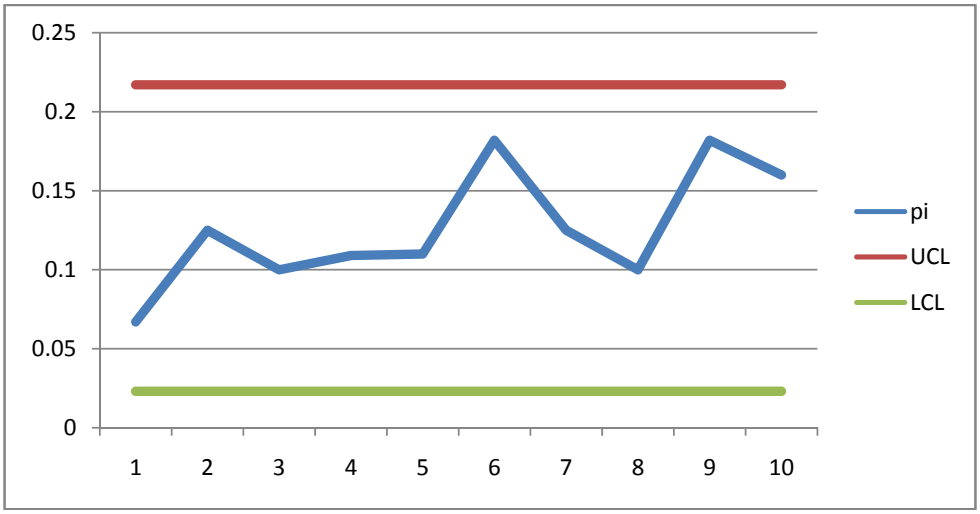
Solution: Here Average sample size $\bar{n} = \Sigma n_i / m = 1000 / 10 = 100$.
The control limits for p – chart by using average sample number is given in the following table.

Sample Number I	number of non-conforming Units D	Sample size n_i	Sample fraction of non-conforming p_i	S.D $\sigma\hat{p}$	Control Limits. LCL	UCL
1.	8	120	0.067	0.097	0.023	0.217
2.	10	80	0.125	0.097	0.023	0.217
3.	8	80	0.100	0.097	0.023	0.217
4.	12	110	0.109	0.097	0.023	0.217
5.	11	100	0.110	0.097	0.023	0.217
6.	20	110	0.182	0.097	0.023	0.217
7.	10	80	0.125	0.097	0.023	0.217
8.	10	100	0.100	0.097	0.023	0.217

9.	15	120	0.182	0.097	0.023	0.217
10.	16	100	0.160	0.097	0.023	0.217

We can observe that, same conclusion taken earlier, that is the process is under control can be taken here also by using method – 2. Suppose the calculated value of Sample faction non-conforming p_i = 0.210 and sample size n_i = 120, in this method we conclude that the sample fraction of non-conforming p_i is inside the UCL. Where as if we calculate actual UCL for n_i = 120, we have, UCL = 0.209 (from Previous example (5.5a.1) for n = 120). Thus p_i is out of control. Like this, when a doubt arises, one can calculate, correct UCL or LCL and take appropriate decision. Otherwise this method reduces calculations of Control limits.

Fig. (5.5b.1): Control chart is given in example (5.5b.1).



5.5c. Method – 3: Standardized Control Chart

Using the central limit theorem, we can convert any variable X to standard normal variable Z by using the conversion as:

$$Z = (X - \text{mean of } X) / \text{Standard Deviation} = (X - \bar{X}) / \sigma_x \quad (5.5c.1)$$

Hence, the third approach is based on the Standardized variable Z. Using the relation (5.5c.1) we can convert all sample fractions for non-conforming units p_i 'sto Z_i 's as follows:

$$\mathbf{Z_i = (p_i - \bar{p}) / \sigma\hat{p}} \quad (5.5c.2)$$

Where, $$\mathbf{\sigma\hat{p} = 3\sqrt{\bar{p}(1 - \bar{p}) / n_i}} \quad (5.5c.3)$$

Then the control limits will become UCL =+ 3 ; CL = 0 and LCL = -3. (using the properties of Standard Normal Distribution with mean zero and variance 1). Thus decision making is easy, if the calculated values of Z_i are in the interval (-3 , +3) we conclude that the process is within control. Otherwise, we decide that the process is 'Out – of control'. Application of this method is explained with the following example.

Example (5.5c.1): Construct the control chart for the data given in example (5.5a.1) using method – 3.

Solution: For the given data, we know that:

$m = 10$, $\Sigma D_i = 120$ and $\Sigma n_i = 1000$ and $\sigma\hat{p} = 3\sqrt{\bar{p}(1 - \bar{p}) / n_i}$

$\bar{p} = \Sigma D_i / \Sigma n_i = 120/1000 = 0.12$. hence we have $\bar{p}(1 - \bar{p}) = (0.12)(0.88) = 0.1056$.

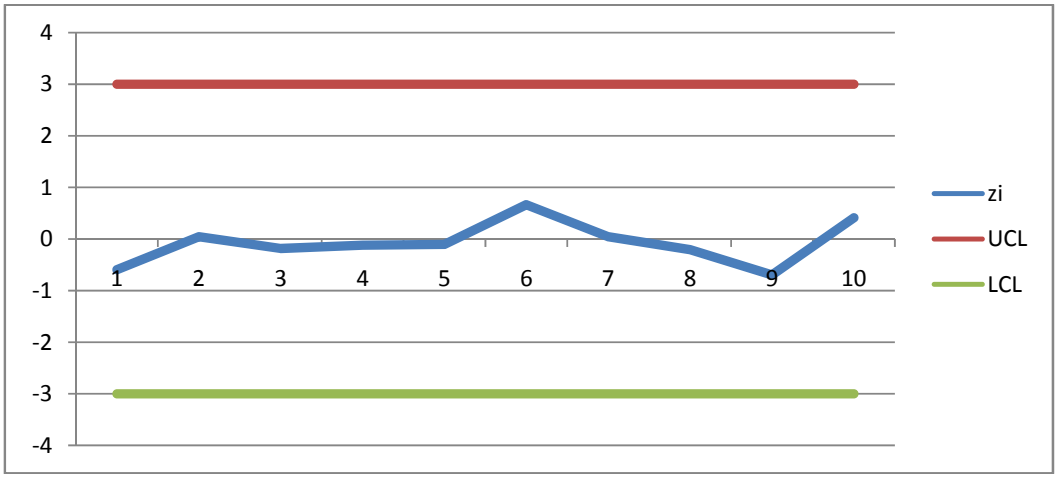
Calculations of Z_i 's are done in the following table.

Sample Number I	number of non-conforming Units D	Sample size n_i	Sample fraction of non-conforming p_i	S.D $\sigma\hat{p}$	Standard Normal Variable Z_i
1.	8	120	0.067	0.089	-0.59551
2.	10	80	0.125	0.109	0.04587
3.	8	80	0.100	0.109	-0.18349
4.	12	110	0.109	0.093	-0.11828
5.	11	100	0.110	0.097	-0.10309
6.	20	110	0.182	0.093	0.66667
7.	10	80	0.125	0.109	0.04587
8.	10	100	0.100	0.097	-0.20619
9.	15	120	0.182	0.089	- 0.69663
10.	16	100	0.160	0.097	0.41237

Conclusions: Since all Z_i values are in the range (-3,+3) we conclude that the process is in-control. Decision is similar to the earlier methods.

Here also control limits are Straight lines UCL at +3 and LCL at -3.

Fig.(5.5c.1): Control chart is given in example (5.5c.1).



When the sample sizes are not equal, we can use any one of the methods explained above and conclusions can be drawn accordingly.

5.6. SUMMARY

In this lecture, we have explained the need for attribute control charts for analyzing the attribute data. Critical comparison between Variable control charts and attribute control charts is made and discussed their relative advantages and disadvantages. Further, types of Attribute control charts are discussed and explained the method of construction of p – charts with fixed sample size n. When sample sizes are varying we have explained three methods available in literature. The method of construction of these charts is explained with suitable examples. We have also discussed the method of construction of np – chart explained the method with suitable example.

It is important to remember that p – charts are to be used when the products are classified as good or bad; defective or non-defective and so on based on one or many quality characteristics. Here we use Binomial Distribution and its properties to determine control limits.

5.7. SELF ASSESSMENT QUESTIONS

1. Explain the need of Attribute control charts.
2. Critically compare Attribute control charts with Variable control charts.
3. Explain how? Variable control charts reduce wastage when compared to Attribute control charts.
4. When the testing procedures involve destructive methods, which charts you prefer?
5. Explain the method of construction of p- chart with fixed sample size n.
6. Explain the problem of Varying sample size.
7. Explain any two methods of construction of p – chart with varying sample size.
8. Explain the method of construction of np – chart.
9. A process that produces titanium forgings for auto mobile turbocharger wheels is to be controlled through use of p – chart. A sample of 150 is taken each day for 20 days and recorded number of non-conforming units as follows:

Day:	1	2	3	4	5	6	7	8	9	10
Non-conforming Units :	5	0	2	1	2	5	2	4	2	3

Day:	11	12	13	14	15	16	17	18	19	20
Non-conforming Units :	2	4	1	3	6	0	1	2	4	2.

Draw Control chart and determine whether the process is in-control or out of control.

10. Using the above data in question 9, construct np – chart and draw your conclusions.

11. Based on the following data construct p and np – charts. Compare your conclusions.

Day:	1	2	3	4	5	6	7	8	9	10
Non-conforming Units :	2	6	12	5	3	4	3	2	4	8

12. A process produces rubber belts in lots of size 2500. Inspection of these belts reveals the following data recorded for 20 days.

Day:	1	2	3	4	5	6	7	8	9	10
Non-conforming Units :	285	327	230	435	346	311	327	308	346	221
Day:	11	12	13	14	15	16	17	18	19	20
Non-conforming Units :	407	414	198	394	456	385	460	225	420	450

13. Plastic parts are produced through an injection molding process produce and found the following non conforming units.

Day:	1	2	3	4	5	6	7	8	9	10
Non-conforming Units :	28	37	23	45	36	11	27	38	36	22
Sample size :	500	200	100	400	500	100	200	300	400	200

Construct suitable control chart using any two methods and compare your conclusions.

14. Explain the method of construction of standardized control chart.
15. A textile mill wishes to check the flow of towels packed 100 in each bundle. Number of non conformities is recorded as follows. Construct Standardized Control Chart.

Day:	1	2	3	4	5	6	7	8	9	10
Packets inspected	20	40	20	30	20	40	40	20	30	20
Non-conformities:	235	240	150	260	240	380	220	320	180	250

5.8. FURTHER READINGS

1. Montgomery, D.C., " Introduction to Statistical Quality Control", John Wiley (Asia) 2001.

2. Biswas.S., "Statistical Quality Control-Sampling Inspection and Reliability", New Central Book Agency Pvt. Ltd. 2003.

3.Wardsworth "Modern Methods for Quality Improvement", John Wiley and sons 2005.

4.Bertrand L. Hansen "Quality control – Theory and Applications" Prentice-Hall of India-2005.

UNIT –II

Lesson – 6

Construction of C – chart with fixed and varying sample sizes

6.0. Objective

After going through this lesson, the student should be able to understand:

- The need for Control charts for number defects.
- Construction and application of C – charts .
- Construction and application of u - charts.
- Construction u – Chart with Variable sample size n.

Structure:

6.1. Introduction.

Control charts discussed in the last lesson is useful when the product or the unit is simple with smaller number of sub-units like pen, piston ring, or electric bulb, pencil, or electronic circuit and so on. In such situations, we can classify the unit into defective or non-defective easily. On the other hand, If the unit is a complicated one consisting of many sub units like T.V., Computer, Railway engine, Ship or Generator and so on, it is not advisable to simply classify the unit as defective or not. Because, the nature of failure will also plays a very important role. Some are serious failures and some are not so serious. Further, it is convenient to count number of defects in a unit rather than simply classifying it as a defective unit. Hence, sometimes, when the unit is large in size consisting of many sub parts, we count number of defects per unit. Further a defective unit or non-conforming unit may contain one or more defects. For example, consider a unit with 5 defects and another unit with 25 defects. Both units are classified as Non-conforming or defective units. Hence, it is necessary to count number of defects rather than simply considering them as defective items. In such situations we use C – charts. Here C is a random variable representing total number of defects / non-

conformities identified in a unit. It is possible to develop control charts for either **total number of non-conformities in a unit or average number of non-conformities per unit.**

6.2. Construction and application of C - charts.

Let the random variable 'C' represent number non-conformities or defects identified in an inspection unit of product. Usually an inspection unit will be a single unit of product, like CPU or mother board of a computer, Engine in a auto mobile, a T.V. or a washing machine. It is always not necessary that an inspection unit must contain only a single unit as explained above. The inspection unit is simply, an entity for which it is convenient to keep records. Sometimes it may be a group of 5 or 10 units produced in a batch or through a machine. For example, an inspection unit may be 20 mother boards produced in a batch of production. Here we assume that number of defects in an inspection unit represented by 'C' follow Poisson distribution, with probability mass function as follows:

$$P(x) = e^{-c}c^{x} / x!\ , x = 0,1,2,.,., \infty. \qquad (6.2.1)$$

Here x represents number of non-conformities and c > 0 is the mean or parameter of the Poisson distribution. Further we know that in Poisson distribution Mean = Variance = c.

Then the control limits for C – chart are as follows:

UCL = c + 3√ c (6.2.2)

Centre Line = CL = c (6.2.3)

LCL = c - 3√c (6.2.4)

Here we also assume that the parameter 'c' is known that is, the parameter is available from the past history of a production process under consideration. If c value is not known, this is to be estimated from the sample data by using the sample mean $\bar{c} = \Sigma C_i / n$, where, c_i represent number of defects / non-conformities in the i^{th} inspection unit. Where i = 1,2,.,.,.n. and n represents total number of units inspected. Then control limits for c – chart are:

UCL $= \bar{c} + 3\sqrt{\bar{c}}$ (6.2.5)

CL $= \bar{c}$ (6.2.6)

LCL $= \bar{c} - 3\sqrt{\bar{c}}$ (6.2.7)

Now, we illustrate the construction of c – chart with an example.

Example (6.2.1): Following data represents number of non conformities / defects observed 500 printed circuit Mother boards of a computer manufacturing company. Data is collected for 20 days and is given below. Construct c – chart and draw your conclusions.

Day:	1	2	3	4	5	6	7	8	9	10
Nonconformities:	28	32	23	35	34	31	37	35	34	22

Day:	11	12	13	14	15	16	17	18	19	20
Non-conformities:	19	24	18	34	**50**	35	35	22	42	**50**.

Solution: From the given data we have $\Sigma C_i = 640$ and n = 20 hence, we have:

$\bar{c} = \Sigma C_i / n = 640/20 = 32.$

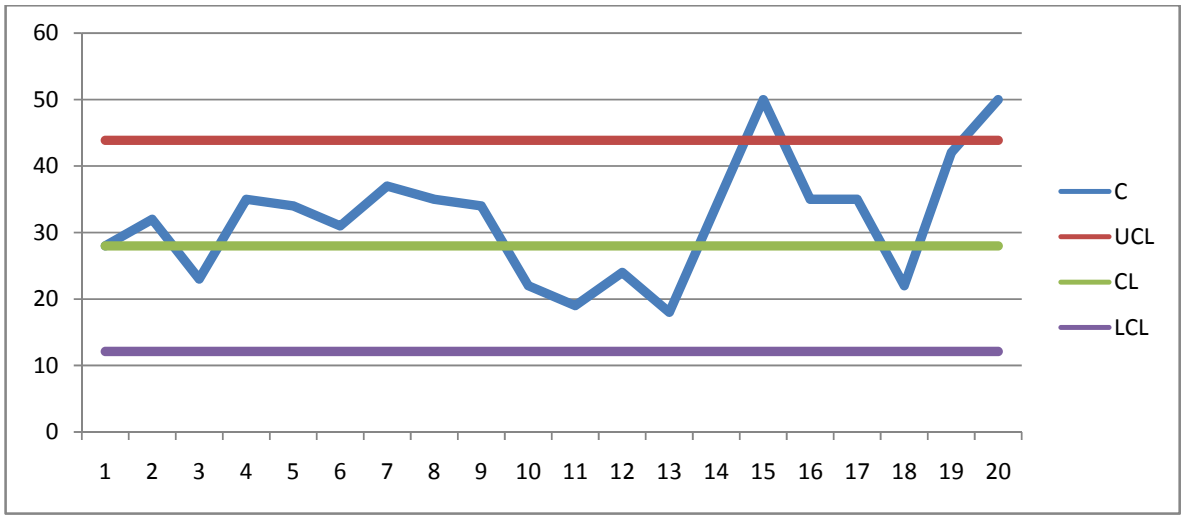
Therefore control limits for c – chart are:

$UCL = \bar{c} + 3\sqrt{\bar{c}} = 32 + 3\sqrt{32} = 32 + 16.971 = 48.971$ (6.2.8)

$CL = \bar{c} = 32$ (6.2.9)

$LCL = \bar{c} - 3\sqrt{\bar{c}} = 32 - 3\sqrt{32} = 32 - 16.971 = 15.029$ (6.2.10)

Fig.(6.2.1): C-chart with fixed sample size.



Conclusions: Since two points on 15 th day and 20th day are falling out of UCL = 48.971,

We conclude that the process is out of control and one has to search for the cause of producing more defective mother boards that is 50 boards out of 500 mother boards are defective. Hence there is a need to search for assignable cause responsible for producing more defective boards on 15th and 20th day.

To suggest the control limits for future control, we have to calculate revised control limits by eliminating the two samples from the analysis. Thus we have: n = 18 and

$\Sigma C_i = 540$. Therefore revised $\bar{c} = \Sigma C_i / n = 540/18 = 30$. Therefore the **revised control limits** for **modified c- chart** are:

$UCL = \bar{c} + 3\sqrt{\bar{c}} = 30 + 3\sqrt{30} = 30 + 16.43 = 46.43$ (6.2.11)

$CL = \bar{c} = 30$ (6.2.12)

$LCL = \bar{c} - 3\sqrt{\bar{c}} = 30 - 3\sqrt{30} = 30 - 16.43 = 13.57$ (6.2.13)

Since remaining 18 samples are having non-conforming mother boards within the control limits, that is in the interval (13.57, 46.43), we conclude that the assignable causes occurred on 15th and 20th day are removed from the process, the remaining production of mother boards are within the control. Hence, control limits given in (6.2.11) to (6.2.13) are considered as modified control limits useful for future control.

Similar type of exercise is to be done, when ever, the process is out of control. Simply, saying the process is 'out of control' will not help the management.

Suggesting the source for assignable causes is necessary and most required action. This procedure of revised control limits is mainly used to control future production, which is required procedure in **'Process Control'**.

6.3. Choice of sample size n

In the earlier method of construction of c – chart, we have considered the inspection unit is exactly equal to one, where, one inspection unit is equal to 500 mother boards. The inspection unit is determined based on the operational or data collection convenience. However, there is no need to inspect one unit only per day. If the inspection is 100% total output, some days the output may be 1500 mother boards (that is 3 inspection units) or 750 mother boards on a power cut-day / machine repair day / strike day (that is, 1.5 inspection units) or some times, the production is 1750 mother boards (that is 3.5 inspection units) and so on. Similarly, sample size is determined based on statistical principles (using results discussed in sampling methods to determine sample size n). Further, economic considerations are also to the considered (particularly in destructive test procedures) in determining the sample size 'n'. In such situations, the inspection units changes from day to day depending on the production on that day or cost of the test or error to be controlled. Thus inspected units 'n' are not a constant as considered in the example (6.2.1). Thus there exists a need to develop control charts, when the sample size is a variable. That is total number of units inspected in a day need not be an integer but can take values 2.0 or 2.5 or 1.5 or 3.0 inspection units. For example the inspection unit n = 2.5. That total number of mother boards to be inspected are 2.5(500) = 1250 mother boards. Similarly, if n = 1.5, then mother boards to be inspected are 1.5(500) = 750 and so on.

Construction of c – chart for variable inspection unit's size can be dealt with two methods which are explained as follows:

6.3a: Method – 1: **Direct method**.

In this method, it is simply consider redefining an inspection unit that is equal to n times the old inspection unit. For example, if old inspection unit is 500 mother boards and the sample mean is $\bar{c}$ and the present inspection unit size is 2.5, then the new revised sample mean or central Line (CL) = $\bar{c}$ = n ($\bar{c}$). In the example (6.2.1) $\bar{c}$ = ΣC_i / n =640/20 = 32, then the revised $\bar{c}$ when n = 2.5 is given by $n\bar{c}$ = 2.5(32) = 80 mother boards. The control limits for the revised c – chart are calculated this revised sample mean or central line $n\bar{c}$. thus the control limits for the revised c – chart are:

Revised UCL = $n\bar{c}$ + 3 $\sqrt{n\,\bar{c}}$ (6.3.1)

Revised CL = $n\,\bar{c}$ (6.3.2)

Revised LCL = $n\,\bar{c}$ - 3 $\sqrt{n\,\bar{c}}$ (6.3.3)

Example (6.3.1): For the data given in the problem (6.2.1), construct revised control limits when (I) n = 2.5 and (ii) n = 1.2.

Solution: (i) Revised control limits when n = 2.5 are calculated as follows:

Revised UCL = $n\bar{c}$ + 3 $\sqrt{}$ $n\bar{c}$ = 2.5(32) + 3 $\sqrt{}$2.5(32)

$$= 80 + 26.8328 = 106.8328 \quad (6.3.4)$$

Revised CL $= n\bar{c} = 2.5\,(32) = 80$ (6.3.5)

Revised LCL $= n\bar{c} - 3\sqrt{n\bar{c}} = 2.5(32) - 3\sqrt{2.5(32)}$

$$= 80 - 26.8328 = 53.1672 \quad (6.3.6)$$

(ii) When n = 1.2.

Revised UCL $= n\bar{c} + 3\sqrt{n\bar{c}} = 1.2(32) + 3\sqrt{1.2(32)}$

$$= 38.4 + 18.59 = 56.99 \quad (6.3.7)$$

Revised CL $= n\bar{c} = 1.2\,(32) = 38.4$ (6.3.8)

Revised LCL $= n\bar{c} - 3\sqrt{n\bar{c}} = 1.2(32) - 3\sqrt{1.2(32)}$

$$= 38.4 - 18.59 = 19.81 \quad (6.3.9)$$

As explained above, one has to revise the control limits, as and when the inspection unit n changes. To avoid, this revision of control limits every time, we use another method of constructing control chart known as u – chart, which is explained as follows:

6.3.b. Method – 2: The u-Chart.

The second approach to tackle the problem of varying inspection unit size involves setting up of control chart based on the average number of non-conformities per inspection unit. This procedure is popularly known as 'u – charts' whose construction procedure and their applications are explained in the following section.

6.4. Construction of u – Chart with constant sample size n.

The construction if u-chart is mainly based on the average number of non-conformities per inspection unit. Let the total number of non-conformities is denoted by the random variable X, in a sample of 'n' inspection units. Then, the average number of non-conformities per inspection unit is denoted by 'u' and is defined as :

$$u = X / n \quad (6.4.1)$$

Further, it is assumed that the random variable X follows Poisson distribution. Hence, control limits of u – chart or control chart for average number of non-conformities per unit are as follows:

UCL $= \bar{u} + 3\sqrt{(\bar{u}/n)}$ (6.4.2)

CL $= \bar{u}$ (6.4.3)

LCL $= \bar{u} - 3\sqrt{(\bar{u}/n)}$ (6.4.4)

Where, $\bar{u} = \sum_{i=1}^{m} ui \,/\, m$, . (6.4.5)

Where, m represent number of samples collected each of size n. Here $\bar{u}$ represents the observed average number of non-conformities per inspection unit, calculated from the data collected. The method of construction of u – chart is explained with the following example.

Example (6.4.1): A computer manufacturer wishes to establish a control chart for non-conformities per unit. An inspection unit consists of single computer. A sample of five

computer are selected from each day production and collected the data for 20 days. The data collected is given as follows. Construct an u – chart and draw your conclusions.

Day:	1	2	3	4	5	6	7	8	9	10
Defects ($X_{i)}$:	5	7	10	8	6	12	11	7	5	9
Day:	11	12	13	14	15	16	17	18	19	20
Defects (X_i):	10	12	8	14	10	16	11	7	10	15.

Solution: First construct Average number of non-conformities per unit $\mathbf{u_i} = X_i / n$ for the given data. Here n = 5 and m = 20.

Day:	1	2	3	4	5	6	7	8	9	10
Defects ($X_{i)}$:	5	7	10	8	6	12	11	9	5	9
$\mathbf{u_i} = X_i / n$ **:**	1.0	1.4	2.0	1.6	1.2	2.4	2.2	1.8	1.0	1.8
Day:	11	12	13	14	15	16	17	18	19	20
Defects (X_i):	10	12	10	12	10	**16**	11	12	10	**15**.
$\mathbf{u_i} = X_i / n$ **:**	2.0	2.4	2.0	2.4	2.0	**3.2**	2.2	2.4	2.0	**3.0**.

$\bar{u} = \sum_{i=1}^{m} ui$ **/ m = 40 / 20 = 2.0.**

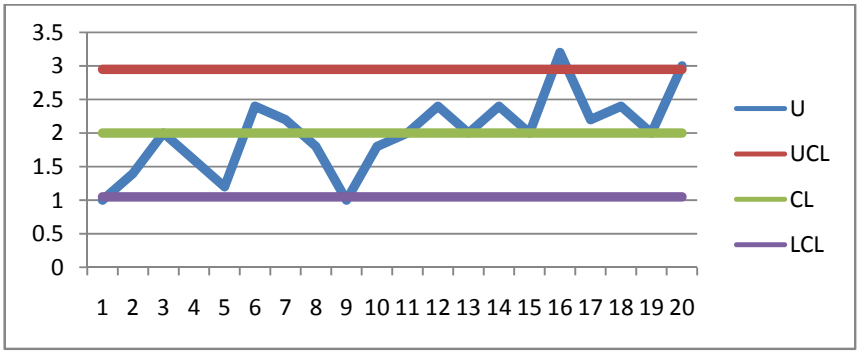
UCL $= \bar{u} + 3\sqrt{(\bar{u}/n)} = 2.0 + 3\sqrt{2.0/20} = 2.0 + 0.949 = 2.949$ (6.4.6)

CL $= \bar{u} = 2.0$ (6.4.7)

LCL $= \bar{u} - 3\sqrt{(\bar{u}/n)} = 2.0 - 3\sqrt{2.0/20} = 2.0 - 0.949 = 1.05$ (6.4.8)

Since two points that is production on 16th and 20th day are falling outside UCL, we conclude that the production process of computers is out of control.

Fig.(6.4.1): U-chart.



In order to control future production, since the process is out of control, we have to revise control limits for u - chart. Remove those samples, fell out of control and construct revised $\bar{u}$ and UCL, CL and LCL as follows:

Revised $\bar{u}$ **= 33.8 / 18 = 1.878 and**

Revised UCL $= \bar{u} + 3\sqrt{(\bar{u}/n)} = 1.878 + 3\sqrt{1.878/18} = 1.878 + 0.969 = 2.847$ (6.4.9)

Revised CL $= \bar{u} = 1.878$ (6.4.10)

Revised LCL $= \bar{u} - 3\sqrt{(\bar{u}/n)} = 1.878 - 3\sqrt{1.878/18} = 1.868 - 0.969 = 0.899$ (6.4.11)

Since all the remaining 18 samples are within the control, we can suggest above control limits for future control.

6.5. Construction u – Chart with variable sample size n.

Control charts for non-conformities are occasionally formed using 100% inspection of the product. When this method of sampling is used, the number of inspection units in a sample will usually will not be a constant but a variable. This problem is already discussed in the construction of p – chart elaborately, in section 5.5. Similar situation arises, here also in the construction of u – chart. For example, in textile industry, inspection of rolls of cloth or in paper manufacturing industry, inspection of paper rolls often, leads to a situation, in which sample size is a variable, because, not all rolls are exactly the same length or width. If a control chart for non-conformities (c – chart) is used in this situation, both central line or control limits varies along with the sample size 'n'. In such control charts, where control limits are varying with the sample size, decision making will be difficult and interpretations are erroneous. To avoid this situation, it is strongly advisable to use u – chart with variable sample size n. In this chart, Central Line is a constant and Control limits vary inversely with the square root of the sample size n. As we have discussed in the construction of p – chart with varying sample sizes. The method of construction of u – chart with variable sample size is explained as follows, using method – 2 and method- 3 discussed in 5.5:

6.5a. Construction of u – chart based on an average sample size $\bar{n}$.

For constructing control chart with variable sample size, actually there are three methods available as discussed in section 5.5. But, in method – 1 the control limits and CL are changing as sample changes and creates confusion in taking the decision, it is not advisable. Hence, we

use those methods where we get constant control limits. Among these Method – 2 is the one where control limits are calculated based on Average sample size $\bar{n}$, where

$$\bar{n} = \sum_{i=1}^{m} ni \ / \ m \qquad (6.5a.1)$$

Then calculate $\bar{u} = \sum_{i=1}^{m} ui \ / \ m \qquad (6.5a.2)$

And $u_i = X_i / n_i \qquad (6.5a.3)$

The control limits for u – chart are:

UCL = $\bar{u} + 3\sqrt{(\bar{u}/n_i)} \qquad (6.5a.4)$

CL = $\bar{u} \qquad (6.5a.5)$

LCL = $\bar{u} - 3\sqrt{(\bar{u}/n_i)} \qquad (6.5a.6)$

It can be observed that, the CL is not involving the term n_i but is based on sample average $\bar{n}$. Hence, CL is a straight line as shown in the graph. But, UCL and LCL are involving the term n_i and hence, are not straight lines but varies from sample to sample, depending its size n. This method is explained with the following example.

Example (6.5a.1): In a textile industry, dyed cloth is inspected for the occurrence of defects per every 50 square meters. The data on 10 rolls of cloth are given below:

Roll Number	:	1	2	3	4	5	6	7	8	9	10
Size of the Roll	:	625	600	500	400	650	500	475	500	600	525
Defects	:	23	19	14	12	20	11	7	10	21	16

Construct u – chart based on average sample size and determine whether the process in control or not?

Solution: In the given problem, the inspection unit is 50 sq.mts. First calculate the sample sizes n_I using the roll sizes and number of non-conformities per inspection unit 'u_i ' , UCL and LCL are calculated as follows. Here $\bar{u} = \sum_{i=1}^{m} ui \ / \ m = 153 / 107.5 = 1.42.$

Roll No.	Size of the Roll	Defects (X_i)	Sample size (n_i)	$u_i = X_i / n_i$	LCL	UCL
1.	625	23	12.5	1.84	0.41	2.43
2	600	19	12.0	1.58	0.39	2.45
3	500	14	10.0	1.40	0.29	2.55
4	400	12	8.0	1.50	0.16	2.68
5	650	20	13.0	1.54	0.43	2.41
6	500	11	10.0	1.10	0.29	2.55
7	475	07	9.5	0.74	0.26	2.58
8	500	10	10.0	1.00	0.29	2.55
9	600	21	12.0	1.75	0.39	2.45
10	525	16	10.5	1.52	0.32	2.52
	Total	153	107.5			

Conclusion: Since, all u_i 's are within LCL and UCL, we conclude that the process is within control. This implies that, only random causes of variation are present in the process and there is no need for searching for assignable causes. It is un-economical to search for Assignable Causes for variation.

6.5b. Construction of u – chart using standardized control chart.

Construction of Standardized control charts are based on central limit theorem, through which we can convert any random variable in to standard normal variate.

This procedure is already explained under method – 3 in section 5.5 elaborately. Using the procedure, we can convert the variable u_i to Z_i as follows:

$$Z_i = (u_i - \bar{u}) / (\sqrt{\bar{u}} / n_i). \qquad (6.5b.1)$$

Then the limits for standardized control chart are UCL = +3, CL=0 and LCL = -3 which are all straight lines. Calculation of Z_i 's are given in the following table. Here $\bar{u} = 1.42$

Roll No.	Size of the Roll	Defects (X_i)	Sample size (n_i)	$u_i = X_i / n_i$	Standard Variable $(\sqrt{\bar{u}} / n_i)$.	Z_i
1.	625	23	12.5	1.84	0.337	1.426
2	600	19	12.0	1.58	0.344	0.465
3	500	14	10.0	1.40	0.377	-0.053
4	400	12	8.0	1.50	0.421	0.190
5	650	20	13.0	1.54	0.330	0.364
6	500	11	10.0	1.10	0.377	-0.849
7	475	07	9.5	0.74	0.387	-1.757
8	500	10	10.0	1.00	0.377	-1.114
9	600	21	12.0	1.75	0.344	0.959
10	525	16	10.5	1.52	0.368	0.272

Since, all Z_i are in the interval (-3,+3), we conclude that the process is under control and there is no need for searching assignable causes.

6.6. Summary

In this lesson, we learnt that in some situations, there is a need for calculating average number of non-conformities per inspection unit. Distinction between Inspection Unit and sample unit are different. They need not be same in many situations. Inspection unit is mainly determined on the maintaining accounts purpose. C – Chart is to be constructed to represent average number of non-conformities per sample unit. Whereas, u – chart is to be used to represent average number of non-conformities per inspection unit. U- Chart can also be constructed for fixed sample size and variable sample size. U – Chart for variable sample size can be constructed using (i) average sample size and (ii) standardized normal variable.

6.7. Self Assessment Questions

1. Distinguish between a defect and defective with suitable examples.
2. Explain the method of construction of C – chart.
3. Explain the difference between c – chart and U – Chart with a suitable example.
4. Explain the method of construction of u – chart for fixed sample size.
5. Explain both methods of construction of u – chart with variable sample size.
6. The following data represents the number of non-conforming bearings and seal assemblies in a sample of size 100. If any points plot outside control limits, assume that the assignable causes are found and eliminated; determine revised control limits, for future control. Also draw control charts.

Sample No.:	1	2	3	4	5	6	7	8	9	10
Defects (X_i):	15	0	9	5	5	6	1	4	5	7
Sample no.:	11	12	13	14	15	16	17	18	19	20
Defects (X_i):	10	8	6	3	1	4	7	5	2	7.

7. A process that produces titanium forgings, for automobile turbocharger wheels is to be controlled through use of fraction of non-conforming control chart. A sample of size 150 is taken for 20 days and the results are shown below. Construct a suitable control chart and if necessary, construct revised control limits. Also draw control charts.

Day:	1	2	3	4	5	6	7	8	9	10
Defects (X_i):	3	2	4	2	6	2	1	2	0	5
Day:	11	12	13	14	15	16	17	18	19	20
Defects (X_i):	2	4	1	3	6	0	1	2	3	2.

8. A paper mill uses a control chart to monitor the imperfection in finished rolls of paper. Production output is inspected for 20 days and the results are tabulated below. Construct a u – chart using (i) Average sample size and (ii) using standardized normal variable. Also draw control charts.

Day:	1	2	3	4	5	6	7	8	9	10
Number of rolls:	18	22	18	18	24	22	21	22	24	24
Defects (X_i):	18	18	12	14	20	18	17	20	18	16
Day:	11	12	13	14	15	16	17	18	19	20

Number of rolls:	20	20	18	18	20	18	20	22	24	20
Defects (X_i):	14	10	9	14	10	20	16	12	20	18

9. The following data represents the number of non-conformities per 1000 meters in telephone cable. Analyze the data with the suitable control chart.

Sample number:	1	2	3	4	5	6	7	8	9	10	11
Defects (X_i):	20	6	9	11	15	8	3	6	7	4	9
Sample number:	12	13	14	15	16	17	18	19	20	21	22
Defects (X_i):	24	19	0	13	5	10	8	7	3	1	1

10. A process produces rubber belts where inspection unit is 2500 belts. Number of non-conforming belts are as follows:

Day:	1	2	3	4	5	6	7	8	9	10
Defects in belts (X_i):	308	342	311	285	327	230	346	221	435	230
Day:	11	12	13	14	15	16	17	18	19	20
Defects in belts (X_i):	407	221	269	131	414	198	331	285	394	456.

6.8. FURTHER READINGS

1. Montgomery, D.C., " Introduction to Statistical Quality Control", John Wiley (Asia) 2001.

2. Biswas.S., "Statistical Quality Control-Sampling Inspection and Reliability", New Central Book Agency Pvt. Ltd. 2003.

3.Wardsworth "Modern Methods for Quality Improvement", John Wiley and sons 2005.

4.Bertrand L. Hansen "Quality control – Theory and Applications" Prentice-Hall of India-2005.

UNIT –II

Lesson – 7

Various types of control limits – Natural tolerance and Specification limits

7.0. Objective

After going through this lesson, the student should be able to understand:

- Various types of control limits.
- Relation between various types of control limits.
- Applications of 3σ limits, warning limits, probability limits.
- Distinction between Natural tolerance and specification limits.

Structure:

7.1. Introduction

Control limits used in all the previous lessons are popularly known as **3- Sigma** Control limits because they are away 3σ distance on either sides of the centre Line. That is UCL and LCL are calculated as **Mean $\pm$ 3 σ**, where σ is the process Standard Deviation. These limits are calculated using the area property of the Normal distribution and it is a well known fact that the area covered under Normal Curve Between **Mean $\pm$ 3 σ** is **99.73%** of the total area under the curve. This implies that the probability of a sample point falling outside UCL and LCL is 0.0027. Which implies that the probability of a sample point falling above UCL or below LCL is (1 – 0.9973) / 2 = 0.0027/2 = **0.00135**. Using the symmetric property of Normal distribution, we can obtain above calculations. Thus a sample point falling above UCL or below LCL is given by 0.00135 $\cong$ 0.001. Hence these limits are approximately considered as **0.001 probability limits**.

Thus **3-sigma limits** are approximately equated to **0.001 probability limits**. This lesson is devoted to discuss various types of control limits and their relation with each other. Now we proceed to discuss the concept of viewing control chart as the problem of testing of Hypothesis.

7.2. Control charts VS testing of hypothesis

Control chart theory we have discussed so far is nothing but the usual testing of hypothesis studies in the statistical inference. In any statistical testing of hypothesis problem, a test is to be developed to test whether to accept Null Hypothesis H_0 or the Alternative Hypothesis H_1 based on the statistic calculated using sample observations. Usually, there are two types of errors in testing of hypothesis. Namely, Type – I error and its probability is denoted by **α** and Type – II error and its probability is denoted by **β** which are defined as:

Probability of Type – I error = Pr[rejecting H_0 when it is true] = α = Producer's Risk and

Probability of Type – II error = Pr[accepting H_0 when it is wrong] = β =Consumer's Risk..

In testing of hypothesis, the size of type – I error **α** is known as **'Size of the test'** and **(1 – β)** is called as the **'power of the test'**. In Testing of hypothesis is the test is suggested such that, for a given size α at 0.05 or 0.01 we obtain the most powerful test. That is the test for which (1 – β) is maximum this in turn implies that β is minimum.

In statistical quality control, **α** = probability of committing type – I error is known as **'Producer's Risk'** and **β** = probability of committing type – II error is known as **'Consumer's Risk'**. We have to suggest control charts, where, for a fixed producer's risk α, not committing type – II error that is (1 – β) is maximum. That is consumer's risk is as minimum as possible.

Using control limits theory, if all the sample points are between UCL and LCL, we accept the Null Hypothesis H_0 otherwise we accept H_1 , where:

Null hypothesis H_0 = The Process is Under Control or there are no assignable causes of variation are present in the production process and random causes of variations are alone present.

Alternative Hypothesis H_1 =The process is out of control or there are assignable causes of variation in the process of production.

If no sample statistic on the chart falling outside UCL or LCL, we accept the Null Hypothesis H_0. That is the process is **'Under Control'**. Otherwise, we Accept H_1 and conclude that the process is **'Out of Control'**.

Thus control chart technique is nothing but the problem of testing of hypothesis discussed in Statistical Inference. In testing of hypothesis, by using Neymann -Pearson Lemma, we determine the **'Best Critical Region (BCR)**' for a given size. The test based on such a BCR is called **'Most Powerful Test (MPT)'**. Control limits are also determined based on such principle and hence, are called Most powerful procedure to determine whether the process is in control or out of control. In Statistical quality control, '**critical region'** is Above UCL and Blow LCL the area in between LCL and UCL is **'acceptance region'**. The gap between LCL and UCL is 6 times the Standard Deviation σ that is the distance between LCL and UCL is 6σ. Thus **Control chart technique is considered as usual testing of hypothesis discussed in statistical Inference.**

7.3. Various types of control limits.

There are many types of control limits used in Statistical Quality Control or in Statistical Process Control. Further it is important to note that the technique of Control Chart is mainly based on the type of **control limits** used in the control chart. Popular control limits used in control chart technique are:

1. Three Sigma or 3σ control limits.
2. Probability limits.
3. Warning limits.
4. Specification limits.
5. Natural tolerance limits and so on.

These limits are to be explained in detail along with their inter relation ships.

7.3a. Three Sigma or 3σ control limits:

Three sigma control limits are defined as:

Upper control Limit =UCL = Mean or expected value of $(\bar{x})$ + 3 S.D $(\bar{x})$ = μ + 3σ /√n (7.2a.1)

Central Line = CL = Mean = expected value of $(\bar{x})$ (7.2a.2)

Lower Control Limit = LCL = Mean or expected value of $(\bar{x})$ + 3 S.D $(\bar{x})$ = μ + 3σ /√n (7.2a.3)

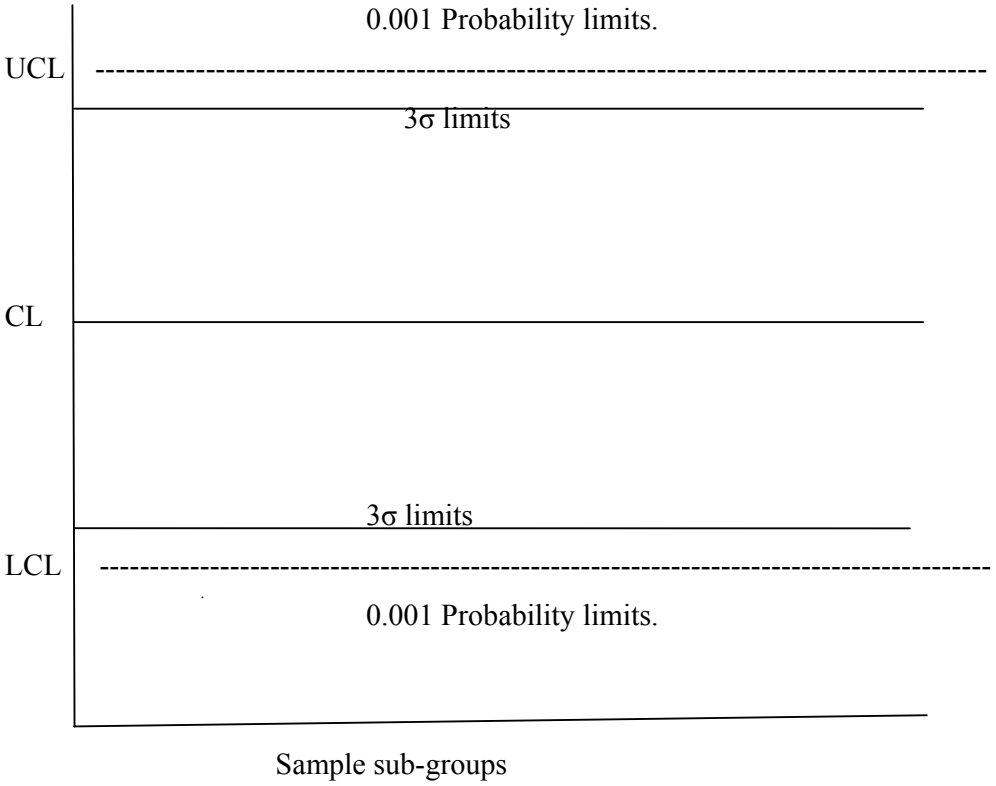
These limits are popularly known as '**Three sigma or 3σ'** Control limits and are to be used when the production process under consideration with regards to the quality variable follow Normal Distribution. All the limits we have calculated in the previous lessons are 3σ limits using the normality assumption. The probability of a sample point falling outside control limits if only Random causes alone are present in the production process is 0.0027. In other words, if no assignable causes of variation are present in the process, a sample point falling above UCL or below LCL is 0.00135. It is important to note that, even if the distribution is slightly deviating from normal distribution, still it is safe to use normal distribution and calculate control limits. That is, not all distributions of quality variable observations in quality control will be normal. Most of the techniques, though the underlying distribution is not normal or moderately skewed, will still be susceptible to Normal law estimation.

7.3b. Probability Limits.

The 3σ control limits are to be used, only when the quality characteristic under consideration follow Normal Distribution. One can also construct control charts using **'0.001 Probability Limits'**. These limits are determined such that, if chance causes alone were at work, the probability of a point falling above the UCL would be one out of 1000. Similarly, if chance causes alone were at work, the probability of a point falling below LCL is one out of 1000. Hence these control limits are called 1 / 1000 = 0.001 probability limits. Since, two points out of thousand (both sides put together) is a very small risk, 0.001 probability limits may be said to give practical assurance that, if a point falls outside, the variation was produced by an assignable cause. Two out of 1000 is purely arbitrary figure; and there is no reason why the probability of exceeding the limits by chance could not be set as one out of a hundred or higher. It is customary to use 0.001 Probability limits that approximate 0.002 standard, but this is not necessary.

Further, one can observe that if the system of chance causes produces a variation in X, that follows the Normal Distribution, the 0.001 probability limits are practically equivalent to 3σ control limits discussed above in 7.3a. It is already discussed in 7.3a that the in 3σ control limits, the probability of a point falling a point above UCL or below LCL is 0.00135 which is approximately equal to 0.001, in **0.001 probability limits**. The Relation between 3σ limits and 0.001 Probability limits are explained in the following figure. In America, it is common to use 3σ limits whether the chance variation in the quality variable X follow the normal curve or not.

Fig. (7.3b.1): relation between 3σ limits and 0.001 Probability limits.



Now proceed to discuss the concept of warning limits and their applications in Quality control in the following section.

7.4. Control limits and warning limits.

Usually 3σ limits or 0.001 Probability limits are also referred as **'Action Limits'**. This is because of the fact that the action of searching for assignable causes of variation is taken when a sample point falls outside these limits. British writers suggest that 2σ limits, that is $\mu \pm 2\sigma$ limits or **Inner Limits** can be used as '**Warning Limits'**. This implies that, if a point fall outside these inner limits will give a hint or warning or caution or signal that if the process is allowed to run on present condition, there is a chance that future sample points may fall out of control. Hence, inner limits are popularly known as Warning limits, because it warns the situation of out of control in near future. On similar lines, British prefer to use Probability

limits to Sigma limits and use 0.001 limits as action limits and 0.025 limits as warning limits instead of 3σ and 2σ limits.

7.5. Specification and Natural tolerance limits.

It is very important to note that the control limits discussed earlier, namely control limits have no relationship or connectivity with 'Specification' limits on the process. The control limits are drawn on the basis of 'Natural Variability' of the process, which is measured by the process standard deviation σ. Which means that Control limits calculations involves the Natural tolerance of the process determined by the **Upper Natural Tolerance Limit (UNTL) and Lower Natural Tolerance Limit (LNTL)**. These Natural tolerance Limits are 3σ times, above and below the process mean μ.

The '**specification limits'**, on the other hand, are determined externally. They may be set by the management, the manufacturing engineers, the customers, or by the product developers / designers. But, it is important to remember that one must have knowledge of inherent process variability σ^2, when setting the 'specification limits'. It is very interesting to state that there **exists no mathematical or Statistical relationship between the 'control limits' and 'specification limits'.**

'Specification limits' are to be considered when we plot individual observations and **'Control limits'** are to be plotted when we consider the sample means, or medians or ranges or standard deviations.

7.6. Summary

In this lesson, various types of limits used in the construction of control charts are discussed. First, the similarities between Control charts and testing of hypothesis are discussed and explained the role of type-I and type-II errors in Statistical Quality Control.

The difference between 3σ limits and 0.001 probability limits are discussed. The concept of inner limits or warning limits and the relation between inner limits and action limits are discussed. Finally, Control limits, Natural Tolerance Limits and Specification limits, their internal relationships are discussed.

7.7. Self Assessment Questions

1. Explain various types of limits used in control charts.
2. Explain Producer's Risk and Consumer's Risk with suitable examples.
3. Explain how control charts are considered as testing of hypothesis problem.
4. Explain Inner control limits and their applications.
5. What are Probability limits? Explain the relation between 3σ control limits and 0.001 probability limits.
6. Distinguish between Action limits and Warning limits.
7. Distinguish between Control limits and Specification limits.
8. Explain Natural tolerance limits and Control limits.
9. Explain 3σ and 2σ limits.
10. Explain 0.001 and 0.025 Probability limits.

7.8. Further Readings

1. Montgomery, D.C., "Introduction to Statistical Quality Control", John Wiley (Asia) 2001.

2. Biswas. S., "Statistical Quality Control-Sampling Inspection and Reliability", New Central Book Agency Pvt. Ltd. 2003.

3.Wardsworth "Modern Methods for Quality Improvement", John Wiley and sons 2005.

4.Bertrand L. Hansen "Quality control – Theory and Applications" Prentice-Hall of India-2005.

UNIT – III
LESSON – 8

CUMULATIVE SUM (CUSUM) CONTROL CHARTS

8.0. OBJECTIVE

After going through this lesson, you should be able to understand:

- The need for CUSUM charts.
- Basic Principles of Cusum charts.
- Construction and application of CUSUM charts.
- Application of V-mask and Cusum Design.

STRUCTURE:

8.1. INTRODUCTION

Control charts discussed in the previous lessons are popularly known as **'Shewhart Control charts'** as they are based on the principles of control charts developed by **Dr. Walter A. Shewhart, an engineer working in Bell Telephone Laboratories, USA, in the year 1920**. In all these control charts, basically, decision making principle is that:

A samples of observations are collected from a production process at time 't' and if all the sample Statistics like, means or medians or Ranges or Standard deviations or proportions of non-conformities or number of non-conformities calculated from the sample collected at time point 't' are within UCL and LCL, we conclude that the process is '**Within control'** otherwise the process is '**out of control'**. That is if any sample statistic is falling above UCL or Below LCL, or both, we conclude that the process is out of control.

In the above procedure, the decision is mainly based on latest samples collected at time point 't'. But, in this procedure, the information collected from previous samples, that is before

time point 't' is completely ignored or not considered at all. This is the major criticism on Shewhert Control charts. On other words, Shewhart control charts concentrates on the sample information collected at time 't' and completely ignores the samples previously collected before the time point 't'. There is a need to consider **previous / past history** of the process also revealed by earlier collected samples. That is one has to use the information **Up to time 't',** but not consider only the information **at time 't'**.

To overcome this Criticism, Prof. E.S. Page, first proposed another type of control charts known as **Cumulative Sum (CUSUM) Control Charts** in the year 1954. Details of these charts like, construction and applications are discussed in remaining sections of this lesson.

8.2. THE NEED AND BASIC PRINCIPLES OF CUSUM CHARTS

A major disadvantage of Shewhert control charts is that it only uses the information about the process provided by the latest plotted sample points and it ignores any information given by the entire sequence of samples collected from the process up to time 't'. This feature makes the Shewhart Control chart relatively insensitive, to **small, sudden and persistent shifts** that took place in the production process. In this point of view Shewhart control charts are less sensitive to **small, sudden and persistent** changes. Hence, there is a need to identify such changes occurred in the process by the control charts. CUSUM charts can quickly identify such changes immediately after their occurrence in any production process. In this respect, CUSUM control charts are more sensitive than Shawhart Control charts. This point is explained elaborately in the coming section with an example.

Thus there is a need to propose another type of charts, which can be used as an alternative to Shewhart control charts, which are more sensitive to identify small, sudden and persistent changes occurring in the production process. Such charts are known as CUSUM control charts in which **information collected from the entire sequence of samples up to time't'** are considered **in making the decision** about the production process.

8.2a. Basic Principles of CUSUM charts

Cumulative Sum (CUSUM) charts directly incorporates all the information provided by all the samples, collected up to time point 't', by plotting Cumulative Sums of the deviations of the sample values (usually Means) $\bar{x}$'s from a target value of the process mean μ_0 .

Let X_1 , X_2 , .,.,.,X_n are the sample observations of size $n \geq 1$, collected from a production process about a quality characteristic X, Say diameter (inner / outer) of a piston ring, or life time of the electric bulb, or breaking strength of the yarn or a brick. Let $\bar{x}_j$ is the average of j^{th} sample. The Cumulative Sum (CUSUM) Control chart is formed by plotting the quantity:

$$C_i = \sum_{j=1}^{i}(\bar{x}j - \mu) \qquad (8.2a.1)$$

Against the sample 'i'. C_i is called the Cumulative Sum up to and including the i^{th} sample. Thus C_i contains all the information from X_1 , to X_i i = I,2,.,.,n. and hence can combine information from several samples collected up to time point 't'. Cumulative Sum Control charts

are more efficient that Shewhart Control charts and the comparison between them is discussed in the following section.

8.3. COMPARISON BETWEEN SHEWHART CONTROL CHARTS AND CUSUM CHARTS

Earlier discussion on Shewhart Control charts and cumulative Sum control charts can combine and one can compare these charts critically as follows:

Table (8.3.1): Critical Comparison between Shewhart Control charts and Cumulative Sum Charts.

S.No.	Shewhart Control charts	Cumulative Sum (CUSUM) Charts.
1.	Shewhart Control chart uses the information provided by the latest collected sample at time 't'.	CUSUM charts uses the information provided by the sequence of samples collected up to time 't'.
2.	These charts are less sensitive to identify small, sudden and persistent changes/Shifts occurred in the process.	These charts are more sensitive to identify the small, sudden and persistent changes / Shifts occurred in the process.
3.	These charts are based on statistics calculated based on the latest collected sample.	These charts are based on Cumulative Sums calculated based on all the samples collected including the latest collected sample.
4.	Decision making is based on the latest plotted sample point.	Decision making is based on all the plotted sample points including the latest one.
5.	Shewhart control charts have control limits on the chart	Cusum Control charts have no control limits on the chart.
6.	Sample statistics are like random walk around the population mean μ.	Cumumative sums are like random walk with mean Zero.

In order to demonstrate the efficiency of Cumulative sums over Shewhart control chart, to detect small sudden and persistent changes, a test data with sudden change in the process mean is changed and analyzed the data using both charts and compared the both charts in identifying this small change occurred in the process mean. For this, consider the following example which contains a small change in the process mean $\mu = 10$ to a process mean $\mu = 11$ with the same process Standard Deviation $\sigma = 1$:

Example (8.3.1): A test data of size 20 is generated first, from a Normal distribution with mean $\mu = 10$ and Standard deviation $\sigma = 1$. Then, another sample of 10 observations are drawn next, from another Normal distribution with mean $\mu = 11$ and the standard deviation $\sigma = 1$. Shewhart control chart and Cumulative sums, were drawn for the above 30 data items and compared the efficiency of these charts in identifying the change in the process mean μ. The data generated is as follows.

Sample Number i:	1	2	3	4	5	6	7	8	9	10
Observation X_i :	9.45	7.99	9.29	11.66	12.16	10.18	8.04	11.46	9.20	10.34
Sample Number i:	11	12	13	14	15	16	17	18	19	20
Observation X_i :	9.03	11.47	10.51	9.40	10.08	9.37	10.62	10.31	8.52	10.84
Sample Number i:	21	22	23	24	25	26	27	28	29	30
Observation X_i :	10.90	9.33	12.29	11.50	10.60	11.08	10.38	11.62	11,31	10.52

Solution: It is given that, in the above data first 20 observations are drawn from N(10,1) and last 10 observations are drawn from N(11,1). Thus the 3 Sigma control limits for $\bar{X}$ chart are:

UCL = 13
CL = 10 and
LCL = 7

For CUSUM chart, first calculate $\mathbf{D_i = (X_i - 10)}$ then Cumulative Sums are calculated as $\mathbf{C_i = D_i + C_{i-1}}$. Calculated values of $\mathbf{D_i}$ **and** $\mathbf{C_i}$ are given in columns 3 and 4 respectively in the following table:

Table (8.3.1): Comparison of Shewhart Control chart and Cumulative Sums

Sample Number i	X_i	$D_i = (X_i - 10)$	$C_i = D_i + C_{i-1}$
1	9.45	-0.55	-0.55
2	7.99	-2.01	-2.56 =(-2.01 – 0.55)
3	9.29	-0.71	-3.27 =(-0.71 – 2.56)
4	11.66	1.66	-1.61 =(1.66 – 3.27)
5	12.16	2.16	0.55 =(2.16 – 1.61)
6	10.18	0.18	0.73 =(0.18 + 0.55)
7	8.04	-1.96	-1.23 = (-1.96 + 0.73)
8	11.46	1.46	0.23 =(1.46 – 1.23)
9	9.20	-0.80	-0.57 =(-0.80 + 0.23)
10	10.34	0.34	-0.23 = (0.34 – 0.57)
11	9.03	-0.97	-1.20 =(-0.97 – 0.23)
12	11.47	1.47	0.27 = (1.47 – 1.20)
13	10.51	0.51	0.78 =(0.51 + 0.27)
14	9.40	-0.60	0.18 =(-0.60 +0.78)
15	10.08	0.08	0.26 =(0.08 + 0.18)
16	9.37	-0.63	-0.37 =(-0.63 + 0.26)
17	10.62	0.62	0.25 =(0.62 – 0.37)
18	10.31	0.31	0.56 =(0.31 + 0.25)
19	8.52	-1.48	-0.92 =(-1.48 +0.56)
20	10.84	0.84	-0.08 =(0.84 – 0.92)
21	**10.90**	**0.90**	**0.82 =(0.90 – 0.08)**
22	**9.33**	**-0.67**	**0.15 =(-0.67 +082)**
23	**12.29**	**2.29**	**2.44 = (2.29 +0.15)**

24	**11.50**	**1.50**	**3.94 = (1.50 +2.44)**
25	**10.60**	**0.60**	**4.54 = (0.60 +3.94)**
26	**11.08**	**1.08**	**5.62 = (1.08 + 4.54)**
27	**10.38**	**0.38**	**6.00 = (0.38 + 5.62)**
28	**11.62**	**1.62**	**7.62 =(1.62 +6.00)**
29	**11.31**	**1.31**	**8.93 = (1.31 +7.62)**
30	**10.52**	**0.52**	**9.45 = (0.52 + 8.93)**

Note: In the above table the second sample of 10 observations taken from N(11,1) and the corresponding calculations are in **bold** figures. The corresponding control charts are given in fig. (8.3.1) and (8.3.2).

Fig. (8.3.1):Shewhart control chart.

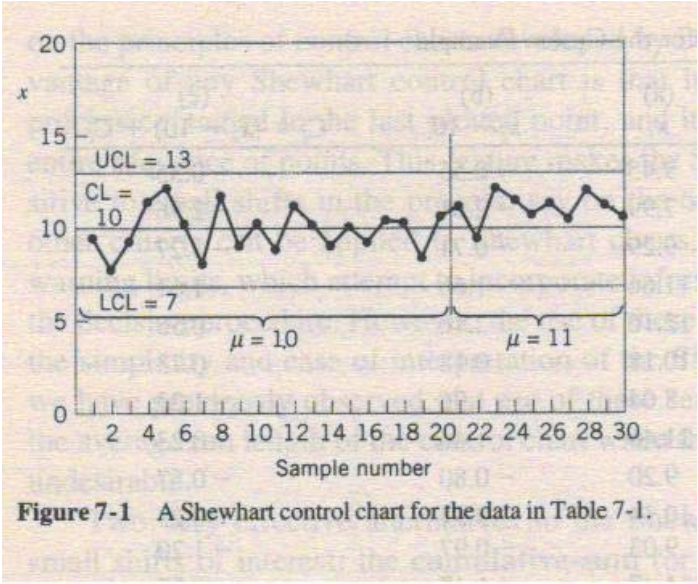


Figure 7-1 A Shewhart control chart for the data in Table 7-1.

Fig.(8.3.2) : Cusum Control Chart for C_i in Table (8.3.1).

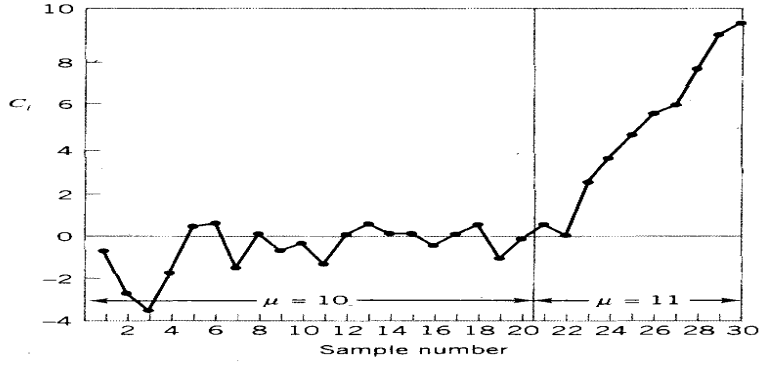


Figure 7-2 Plot of the cumulative sum from column (c) of Table 7-1.

Conclusion: If we plot X_i values and control limits on y – axis and samples on X-axis, we can observe that all X_i's are in the interval (7,13) that is within LCL and UCL and hence we conclude that the process is within control and hence conclude that there is no change occurred in the process mean and can be considered as the process mean $\mu = 10$. This is intact wrong because from sample 21, observations are taken from N(11,1) and this chart cannot identify the change of process mean μ from 10 to 11. Where as if we observe the Values of C in the last column in table (8.3.1), an significant up-word trend can be observed from sample 21 onwards.

This indicates that Cumulative Sums can identify the change, however small the change is. Thus Cumulative Sums are more sensitive and can identify the any small, sudden and persistent change occurred in the process.

8.4. CONSTRUCTION OF CUSUM CHARTS

To construct the CUSUM charts, first calculate C_i values using (8.3a.1). Consider C_i values on Y-axis and sample number i on x-axis. CUSUM plots are not control charts having UCL, CL and LCL, because they lack Statistical Control Limits. There are two ways of representing CUSUM charts, namely, **(a) Tabular or algorithmic CUSUM Method** and **(b) V – mask form of CUSUM Method.** These two methods of representing CUSUM charts are explained as follows:

8.4a.1: The Tabular or algorithmic CUSUM for monitoring the process mean.

Now we proceed to discuss the method of Tabular or Algorithmic CUSUM for monitoring the mean of a given Process. CUSUMs may be constructed both for individual observations and for the averages of rational subgroups or sub samples. In practice, we generally consider the construction of CUSUMs for individual observations. Hence, CUSUMs for individual observations is first and then, extend the method for rational subgroup means.

Let X_i , i= 1,2,.,.,n, be the a set of n observations of the quality characteristic measured from a production process. When the process is in-control and X_i follow Normal distribution with mean $\boldsymbol{\mu_0}$ and the standard deviation σ, where the value of σ is known or can be estimated from the sample data collected, we proceed to monitor the process mean first using CUSUMs and later, we will discuss monitoring the process variability σ^2 using CUSUMs. It can also be considered $\boldsymbol{\mu_0}$ as the process 'Target Value' for the quality characteristic X. This type of situation will occur in chemical industry, where X represents the viscosity and $\boldsymbol{\mu_0}$ represent the target value such as 2000 centistokes at 100^0 C. The objective is to control X at the target value $\boldsymbol{\mu_0}$. If the process drifts or shifts off from this target value, the CUSUM chart will give signal and an adjustment to be made to bring the variable X, that is the catalyst feed rate nearer to the target value $\boldsymbol{\mu_0}$. Sometimes, this may be considered as hint that some assignable causes of variation is present in the process, which is to be identified and to be eliminated from the process. This is similar to the action taken in Shewhart Control chart, when the process is out of control.

The tabular CUSUM works by accumulating deviations from $\boldsymbol{\mu_0}$ that are above the target value with $\mathbf{C^+}$ and accumulating deviations from $\boldsymbol{\mu_0}$ that are below the target value with **C.** These values $\mathbf{C^+}$ and $\mathbf{C^-}$ are called Upper and Lower CUSUMs respectively, which are defined as:

$\mathbf{C^+_i = Max\ [0, X_i - (\mu + K) + C^+_{i-1}\ ]}$ (8.4a.1)
and $\mathbf{C_i^- = Max\ [0, (\mu - K) - X_i + C^-_{i-1}\ ]}$ (8.4a.2)
where, the starting values are $\mathbf{C^+_0 = C^-_i = 0.}$

In the above equations (8.4a.1) and (8.4a.2), **K is known as 'reference value' or 'Slack Value' or 'allowance value'** and is chosen about half way between the target value μ_0 and out of control
Value of the mean, say μ_1, that we are interested to identify the shift quickly.

Thus if the shift is expressed in standard deviation units as $\mu_1 = \mu_0 + \delta\sigma$ or $\delta = |\mu_1 - \mu_0| / \sigma$ then K is one half of the magnitude of the shift. That is $\mathbf{K = (\delta / 2)\ \sigma = |\mu_1 - \mu_0| / 2}$ (8.4a.3)

It is important to note that C^+_i **and** C^-_i are accumulate deviations from the target value μ_0 that are greater then K, with both quantities reset to **ZERO** on becoming negative. If either C^+_i **or** C^-_i **exceeds the decision interval H, the process is considered to be out of control.**

Here the decision interval **H** is usually considered as five times the process Standard Deviation σ. This tabular or Algorithmic CUSUM procedure explained above is illustrated with an example as follows:

Example (8.4a.1): Construct the CUSUM chart using Tabular CUSUM method, for the data considered in example (8.3.1) and draw your conclusions.

Solution: From the data given in the example (8.3.1), we know that, $\mu_0 = 10$, the size each sample is of size n = 1 and $\sigma = 1$. Let us assume that we are interested on the magnitude of the shift in X_i from the process mean $\mu_0 = 10$ which is of magnitude $1.0\sigma = 1.0(1.0) = 1.0$ and hence, the out of control value of the process mean is $\mu_1 = 10 + 1 = 11$. Now we proceed to develop the tabular with K = 1/2 . This is because of the fact that, the shift size is 1.0σ and $\sigma = 1$.

Further, the decision interval H is to be considered usually as five times the Standard deviation and hence, $H = 5\sigma = 5\ (1.0) = 5$. Calculations involved in Tabular or Algorithmic method of CUSUM charts involve the calculations of C^+_i and C^-_i using equations(8.4a.1) and (8.4a.2) for each period. For example, for period – 1, we have:

$\mathbf{C^+_1 = Max\ [0, X_1 - 10.5 + C^+_0\ ]}$ (8.4a.3)
$\mathbf{C^-_1 = Max\ [0, 9.5 - X_1 + C^-_0\ ]}$ (8.4a.4)
Where, $\mathbf{C^+_0 = C^-_0 = 0.}$ (8.4a.5)

Further, K = 0.5 and $\mu_0 = 10$ and $X_1 = 9.45$, we have:
$\mathbf{C^+_1 = Max\ [0, 9.45 - 10.5 + 0\ ] = 0}$ (8.4a.6)
$\mathbf{C^-_1 = Max\ [0, 9.5 - 9.45 + 0\ ] = 0.05}$ (8.4a.7)
Similarly, for period – 2, we have $X_2 = 7.99$ and :
$\mathbf{C^+_2 = Max\ [0, X_2 - 10.5 + C^+_1\ ] = Max[0, 7.99 - 10.5 + 0] = 0}$ (8.4a.8)
$\mathbf{And\ C^-_2 = Max\ [0, 9.5 - X_2 + C^-_1\ ] = Max[0, 9.5 - 7.99 + 0.05] = 1.56}$ (8.4a.9)

Calculations for other periods are calculated on similar lines for each X_i and results are presented in the following table, using above explained steps for $\mathbf{C^+_i}$ **and** $\mathbf{C^-_i}$ **; i=1,2,.,.,.,n**. In the table (8.4a.1) , the quantities N^+ and N^- indicate the number of consecutive periods, in which the cusums $\mathbf{C^+_i}$ **or** $\mathbf{C^-_i}$ have been non-zero.

Table (8.4a.1): The tabular or Algorithmic CUSUMS for the data given in problem (8.3.1).

Period i	X_i	$X_i - 10.5$	$\mathbf{C^+_i}$	$\mathbf{N^+}$	$9.5 - X_i$	$\mathbf{C^-_i}$	$\mathbf{N^-}$
1	9.45	-1.05	0	0	0.05	0.05	1
2	7.99	-2.51	0	0	1.51	1.56	2
3	9.29	-1.21	0	0	0.21	1.77	3
4	11.66	1.16	1.16	1	-2.16	0	0
5	12.16	1.66	2.82	2	-2.66	0	0
6	10.18	-0.32	2.50	3	-0.68	0	0
7	8.04	-2.46	0.04	4	1.46	1.46	1
8	11.46	0.96	1.00	5	-1.96	0	0
9	9.20	-1.3	0	0	0.30	0.30	1
10	10.34	-0.16	0	0	-0.84	0	0
11	9.03	-1.47	0	0	0.47	0.47	1
12	11.47	0.97	0.97	1	-1.97	0	0
13	10.51	0.01	0.98	2	-1.01	0	0
14	9.40	-1.10	0	0	0.10	0.10	1
15	10.08	-0.42	0	0	-0.58	0	0
16	9.37	-1.13	0	0	0.13	0.13	1
17	10.62	0.12	0.12	1	-1.12	0	0
18	10.31	-0.19	0	0	-0.81	0	0
19	8.52	-1.98	0	0	0.98	0.98	1
20	10.84	0.34	0.34	1	-1.34	0	0
21	**10.90**	0.40	0.74	2	-1.40	0	0
22	**9.33**	-1.17	0	0	0.17	0.17	1
23	**12.29**	1.79	1.79	1	-2.79	0	0
24	**11.50**	1.00	2.79	2	-2.00	0	0
25	**10.60**	0.10	2.89	3	-1.10	0	0
26	**11.08**	0.58	3.47	4	-1.58	0	0
27	**10.38**	-0.12	3.35	5	-0.88	0	0
28	**11.62**	1.12	4.47	6	-2.12	0	0
29	**11.31**	0.81	5.28	7	-1.81	0	0
30	**10.52**	0.02	5.30	8	-1.02	0	0

Conclusions: Critically comparing the values of C^+_i in the above table, we conclude that Upper side cusums, that is C^+_i exceeds H = 5, at the period no. 29 onwards. Hence the process is out of control. Using tabular cusums, we can also predict the possible point at which the shift occurred at which point, we conclude that the process is out of control at period 29 and N^+ value at 29^{th} period is 7. Therefore, possible occurrence of the shift might be at 29 – 7 = 22. We can

expect that the shift in process occurred between the periods 22nd and 23rd . The Control chart for the Cusums are given in Fig (8.4a.1).

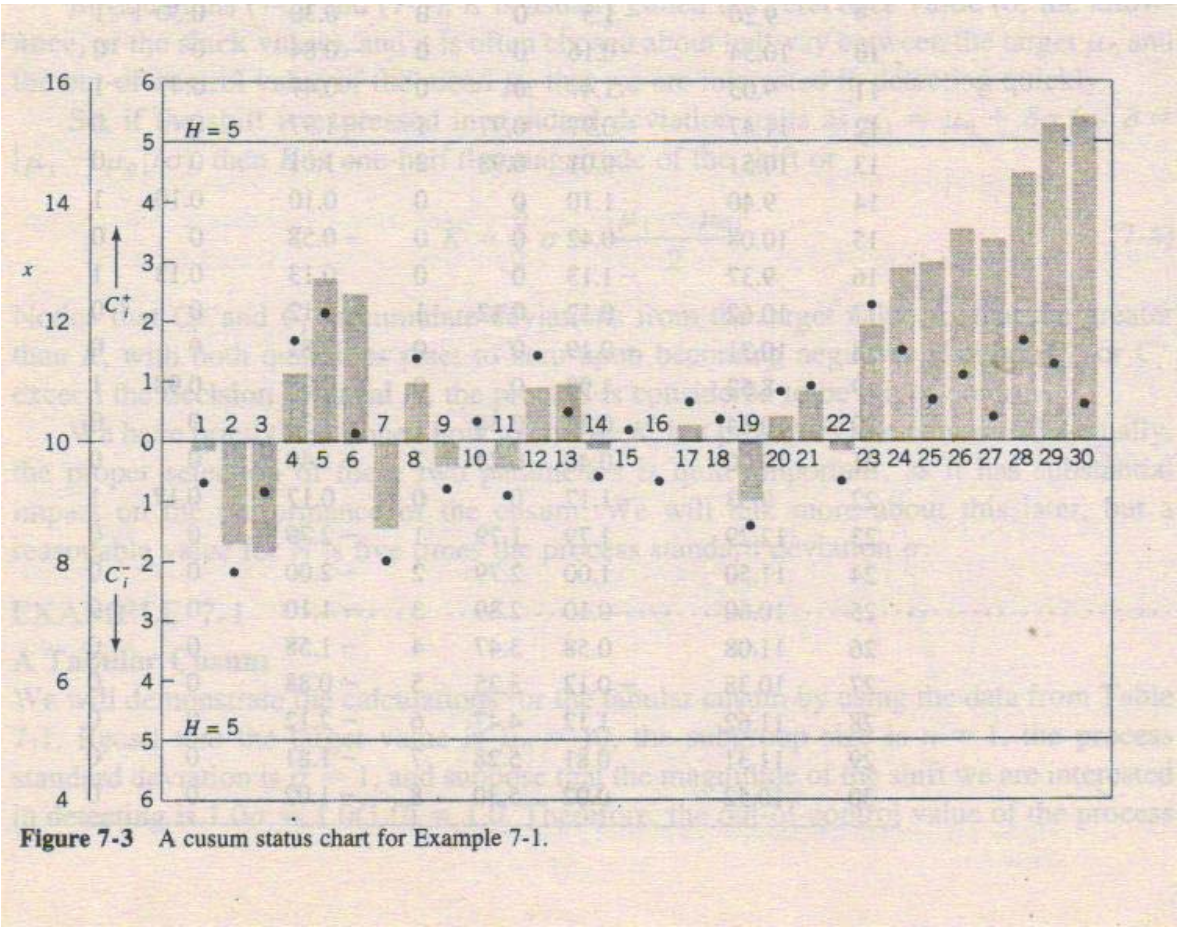


Figure 7-3 A cusum status chart for Example 7-1.

Now we proceed to explain the method of using V – Mask in CUSUM charts in the following section.

8.4b: The V – Mask Procedure

An alternative procedure to the tabular Cusums explained above is the V – mask control scheme proposed by Barnard in the year 1959. The V – mask is applied to successive values of the cusum statistic $\mathbf{C_i} = \sum_{j=1}^{i} \boldsymbol{Yj} = \mathbf{Y_i} + \mathbf{C_{i-1}}$ (8.4b.1)

Where, $\mathbf{Y_i}$ is the standardized observation, that is, $\mathbf{Y_i} = (\mathbf{X_i} - \boldsymbol{\mu_0}) / \boldsymbol{\sigma}$ (8.4b.2)

A typical V – mask, its parameters and its application are discussed as follows:

Fig.(8.4b.1) : A typical V – mask

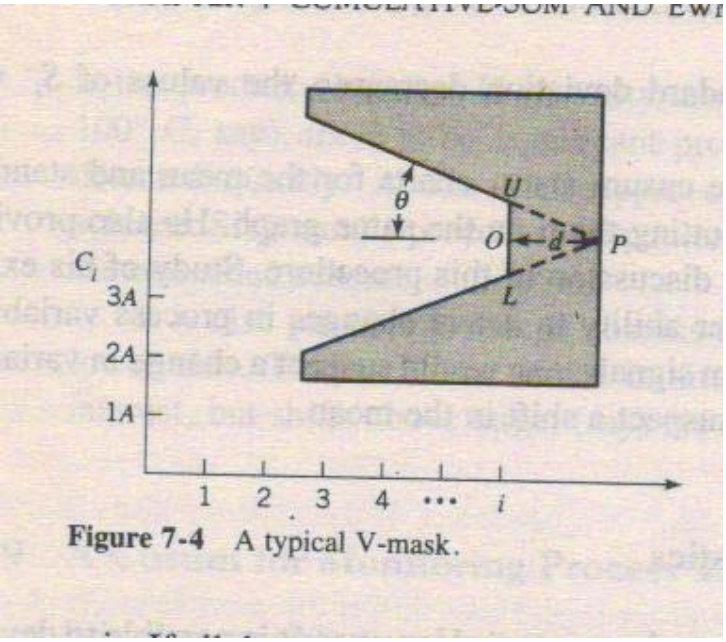


Figure 7-4 A typical V-mask.

Decision procedure with V – mask:

The decision procedure consists of placing the V – mask on the cumulative sum control chart with the point '0' on the last value of C_i and the 'op' line, parallel to the X – axis where, sample number 'i' is considered. If all cumulative sums plotted earlier, that is C_1 , C_2 ,.,.,.,C_i fall within the Angle of the V – mask, we say that the process is 'in control'. Otherwise, the process is out of control. That is if any of the cumulative sums plotted lie out of the angle, that is outside the arms of the V – mask, the process is considered to be out of control. This V – mask **procedure is to be applied immediately**, when a new cumulative sum is plotted on the cusum chart.

The distance from O to P is called **'Lead Distance'** and is denoted by 'd' and half of the angle of V – mask is denoted by θ. These values of 'd' and 'θ' are called the parameters of the V - mask. These values are determined by the concerned design engineer and Different V – masks can be constructed for different values of 'd' and 'θ'. The performance of V – mask, is determined, based on these parameters, d and θ values.

It is important to note that it is strongly recommended to the quality control engineers that, V – Mask procedure should not be used because of the following disadvantages of the V – mask procedure:

1. The V – mask is a two-sided scheme; it is not very useful for one-sided process monitor problems.
2. The head start feature, very useful in practice, cannot be implemented with the V – mask.
3. Sometimes, it is difficult in making interpretation of results by the practitioners.
4. The biggest drawback associated with V – mask is determination of α and β that is producers risk and consumers risk respectively.

Because of above drawback, application of V – mask procedure has limited applications. The Tabular or Algorithmic procedure and the V – mask procedure are equivalent under the following conditions:

If K = A tan (θ) and h = A d tan (θ) = dk. (8.4b.3)

In the above two equations, A is the horizontal distance on the V – mask plot between successive points in terms of unit distance on the y – axis. Thus it is strongly advised that the tabular cusum chart procedure is recommended than, V – mask procedure.

8.5. RECOMMENDATIONS FOR CUSUM DESIGN

It is under stood from the previous section that, cusum designs using Tabular or Algorithmic CUSUM procedure are best used when compared to V – mask procedures. Further, Tabular CUSUM design is mainly depending on the reference value K and the decision interval H. It is usually recommended that these parameters be selected to provide good Average Run Length (ARL) performance. Here, we suggest some general recommendations for selecting H and K values.

Define, $H = h\sigma$ and $K = k\sigma$, where, σ is the process standard deviation estimated by the sample observations used, in forming cusums. Usually, choosing $h = 4$ or $h = 5$ with $k = ½$ will generally provide a cusum that has good ARL properties, against a shift of about 1σ in the process mean. Further, the value of k is to be chosen, based on the relative size of the shift, we want to detect, that is usually, $k = \delta / 2$, where δ is the size of the shift, in standard deviation units. This approach will match Shewhat Control chart with usual 3σ limits.

8.6. SUMMARY

In this lesson, we have discussed the need for studying CUSUM charts as an alternative to Shewhart control charts. Differences between these charts are discussed and Construction of CUSUM charts along with their relative merits and demerits are discussed. There are two methods available for the construction of CUSUM charts namely (i) Tabular or Algorithmic approach and (ii) V – mask approach. Critical comparison is made between these two approaches.

8.7. SELF ASSESSMENT QUESTIONS

1. Explain the need of Cumulative sum charts.
2. Distinguish between Shewhart control charts and CUSUM charts.
3. Discuss chief advantage of CUSUM chart.
4. Explain the method of construction of CUSUM chart using Tabular procedure for controlling the process mean.
5. Bath concentrations are measured hourly, in a chemical process. Data (in PPm) for the last 36 hours are collected and are given as follows:

160	158	150	151	153	154	158	162	186	195	179	184
190	189	185	182	181	180	183	186	197	186	197	192
206	210	216	212	211	202	218	215	214	206	218	220

Determine whether the process is in control or not? Using tabular cusum using the standard values as h = 5 and k = ½ and the process target is μ_0 = 175.

6. Explain the V – mask and explain its parameters.
7. Critically compare Tabular and V – mask cusum procedures.
8. Explain when a Tabular procedure will be equal to a V – mask procedure? in controlling the process mean.
9. Viscosity measurement on a polymer are made every 10 minutes by an on-line viscometer. Thirty six observations are recorded as follows:

3169	3205	3185	3188	3173	3203	3187	3183	3162	3209	3192	3175
3154	3208	3199	3174	3239	3211	3197	3171	3145	3214	3193	3180
3160	3215	3190	3179	3172	3209	3183	3175	3175	3203	3197	3174

Construct a tabular cusum chart for the above data by considering h = 5 and k = ½. The target viscosity for this process is μ_0 = 3200.

10. A machine is used to fill cans with motor oil additive. A single sample can is selected every hour randomly and the weight of the can is recorded as follows. The process standard deviation σ = 0.05 and the process target mean μ = 8.02 oz. Set up A tabular CUSUM for the process using h = 5 and k = ½.

8.00	8.01	8.02	8.01	8.06	8.07	8.01	8.04	8.02	8.01	8.04	8.07
8.05	8.04	8.03	8.05	8.06	8.04	8.05	8.06	8.04	8.02	8.03	8.05

8.8. FURTHER READINGS

1. Montgomery, D.C., " Introduction to Statistical Quality Control", John Wiley (Asia) 2001.
2. Biswas.S., "Statistical Quality Control-Sampling Inspection and Reliability", New Central Book Agency Pvt. Ltd. 2003.
3.Wardsworth "Modern Methods for Quality Improvement", John Wiley and Sons 2005.
4.Bertrand L. Hansen "Quality control – Theory and Applications" Prentice-Hall of India-2005.

UNIT – III
Lesson – 9

EXPONENTIALLY WEIGHTED MOVING AVERAGE CONTROL CHARTS

9.0. OBJECTIVE

After going through this lesson, you should be able to understand:

- The need for EWMA charts.
- EWMA Control charts Monitoring process Mean.
- Construction of EWMA control charts.
- Design and extension of EWMA Control charts.

STRUCTURE

9.1. INTRODUCTION

Cumulative sum control charts discussed in the earlier lecture are used when the sub-groups are of size n =1. The Exponentially Weighted Moving Average (EWMA) Control Charts are also considered as good alternatives to Shewhart Control charts, when we are interested to detect **small, sudden and persistent** shifts present in the production process. When compared to CUSUM charts, EWMA charts are easier to set up and convenient to operate for the samples of size $n \geq 1$. First we discuss the construction of EWMA charts with sample size n = 1 and extend the same when the sample size n > 1. EWMA control charts were first introduced by **Roberts in the year 1959**.

9.2. THE NEED OF EWMA CHARTS

When the process under goes a small and sudden shifts, from the process target value μ_0 , immediately, there is a need to detect the change, on the chart and action is to be taken immediately to bring back the process production to the target value. EWMA charts are very effective to detect small process shifts which are sudden and persistent. Such small shifts occurring at random intervals sometimes may miss on the chart, because on the control chart, points are sample points selected randomly from the production process. Hence, there is a need

to consider all the sample points collected up to time t. As we have discussed in Cusum charts, Cumulative sample information may provide good method of identifying such small and sudden changes occurred in the process mean. The design parameters of the chart are the multiple if sigma used in the control limits of Shewhart Control charts. Construction and applications of EWMA control charts are much easier than CUSUM charts discussed earlier.

EWMA control charts are popularly used in the analysis of Time series data and can be used in time series forecasting methods.

9.3. EWMA CHARTS FOR MONITORING PROCESS MEAN

The exponentially Weighted Moving Average (EWMA) denoted by Z_i is defined as:

$$\mathbf{Z_i = \lambda X_i + (1-\lambda) Z_{i-1}} \qquad (9.3.1)$$

Where, $0 < \lambda \leq 1$ and is a constant and the starting value, that is required with the first sample at i = 1 is to be considered usually, as the process target value μ_0. This implies that, $Z_0 = \mu_0$. Sometimes, the average of the preliminary sample data can be used as the starting value of the EWMA. That is $Z_0 = \bar{X}$. Now we proceed to demonstrate that the EWMA Z_i is a weighted average of all previous sample means as follows:

Substitute, $\mathbf{Z_{i-1}}$ on the right hand side of equation (9.3.1), then we have:

$$\mathbf{Z_i = \lambda X_i + (1-\lambda)[\lambda X_{i-1} + (1-\lambda) Z_{i-2}]} \qquad (9.3.2)$$

Thus we have:

$$\mathbf{Z_i = \lambda X_i + \lambda(1-\lambda) X_{i-1} + (1-\lambda)^2 Z_{i-2}} \qquad (9.3.3)$$

Continuing on similar lines, recursively $\mathbf{Z_{i-j}}$ **, j = 2,3,.,.,i,** we obtain:

$$\mathbf{Z_i = \lambda \sum_{j=0}^{i-1} (1-\lambda)^j X_{i-j} + (1-\lambda)^I Z_0} \qquad (9.3.4)$$

Thus the weights $\lambda(1-\lambda)^j$ decreases geometrically, which are the weights of previous samples, and decreases along with the age of the sample mean.

For example, if $\lambda = 0.2$, then the weight to assign the current sample is 0.2 and the weights to assign to preceding sample means are $(0.2)(1 - 0.2)$, $(0.2)(1-0.2)^2$ $(0.2)(1-0.2)^3$ and so on. That is 0.16, 0.128, 0.1024 and so forth. Therefore the weights decreases along with the age of the sample. These sample weights, decreases Geometrically. Hence, **'EWMA Control charts'** are sometimes, called as **'Geometric Moving Average (GMA) Control Charts'**. Further, we know that, these weights sum to unity. Hence, we have:

$$\mathbf{\lambda \sum_{j=0}^{i-1} (1-\lambda)^j = \lambda\left[\frac{1-(1-\lambda)^j}{1-(1-\lambda)}\right] = 1-(1-\lambda)^j} \qquad (9.3.5)$$

Since, EWMA control charts can be viewed as weighted average of all past and current observations, they are very sensitive to detect the small. Sudden and persistent changes present in any production Process. In EWMA control charts, control limits are not straight lines as in Shewhart Control charts, but they increase in width along with the sample size 'i',
i =1,2,.,.,n.

9.4. CONSTRUCTION OF EWMA CONTROL CHARTS

Let $X_1, X_2, .,.,X_n$ are independent observations collected from a production process representing some quality characteristic of the product. Let X_i are assumed to be independent random variables with variance σ^2. Then the random variable Z_i where:

$$\mathbf{Z_i = \lambda X_i + (1 - \lambda) Z_{i-1}} \quad (9.4.1)$$

Has the variance:

$$\mathbf{\sigma^2 (Z_i) = \sigma^2 [\lambda / (2 - \lambda)] [1 - (1 - \lambda)^{2i}].} \quad (9.4.2)$$

Therefore, the EWMA Control charts can be constructed by plotting Z_i versus the sample number 'i' or time . Then the control limits for EWMA control chart are calculated as follows:

$$\mathbf{UCL = \mu_0 + L\sigma \sqrt{[\lambda / (2 - \lambda)] [1 - (1 - \lambda)^{2i}]}} \quad (9.4.3)$$

$$\mathbf{Central\ Line = CL = \mu_0} \quad (9.4.4)$$

$$\mathbf{LCL = \mu_0 - L\sigma \sqrt{[\lambda / (2 - \lambda)] [1 - (1 - \lambda)^{2i}]}} \quad (9.4.5)$$

In the above equations, the term $\mathbf{[1 - (1 - \lambda)^{2i}]}$ approaches to unity as the value of 'i' increases. Thus for large values of i, EWMA control chart have the control limits as follows;

$$\mathbf{UCL = \mu_0 + L\sigma \sqrt{[\lambda / (2 - \lambda)]}} \quad (9.4.6)$$

$$\mathbf{Central\ Line = CL = \mu_0} \quad (9.4.7)$$

$$\mathbf{LCL = \mu_0 - L\sigma \sqrt{[\lambda / (2 - \lambda)]}} \quad (9.4.8)$$

We prefer to have EWMA charts for smaller values of 'i' and actual control limits given in equations (9.4.3) to (9.4.5) are to be used. This will improve the performance of EWMA control charts in general. We demonstrate the construction of this chart with an example.

Example (9.4.1): A test data of size 20 is generated first, from a Normal distribution with mean $\mu = 10$ and Standard deviation $\sigma = 1$. Then, another sample of 10 observations are drawn next, from another Normal distribution with mean $\mu = 11$ and the standard deviation $\sigma = 1$. Construct EWMA control chart with $\lambda = 0.10$ and $L = 2.7$ and draw your conclusions.

Sample Number i:	1	2	3	4	5	6	7	8	9	10
Observation X_i :	9.45	7.99	9.29	11.66	12.16	10.18	8.04	11.46	9.20	10.34
Sample Number i:	11	12	13	14	15	16	17	18	19	20
Observation X_i :	9.03	11.47	10.51	9.40	10.08	9.37	10.62	10.31	8.52	10.84
Sample Number i:	21	22	23	24	25	26	27	28	29	30
Observation X_i :	10.90	9.33	12.29	11.50	10.60	11.08	10.38	11.62	11,31	10.52

Solution: From the given problem, it is given that the target value of the mean $\mu = 10$ and the standard deviation is $\sigma = 1$. The calculations of EWMA chart are as follows:

Let $Z_0 = 10$, then $Z_1 = \lambda X_1 + (1 - \lambda) Z_0 = 0.1(9.45) + 0.9 (10) = 9.945$.

Similarly, $Z_2 = \lambda X_2 + (1 - \lambda) Z_1 = 0.1(7.99) + 0.9 (9.945) = 9.7495$ and so on proceeding on similar lines, we have for other values of i=3,4,5,.,.,.,.,30 are given in the table (9.4.1). The control limits are calculated as follows:

For period i= 1 we have:

$$\mathbf{UCL = \mu_0 + L\sigma \sqrt{[\lambda / (2 - \lambda)][1 - (1 - \lambda)^{2i}]}}$$

$$\mathbf{= 10 + 2.7(1)\sqrt{[0.1/(2 - 0.1)][1-(1-0.1)^{2(1)}]} = 10.27} \quad (9.4.9)$$

$$\mathbf{Central\ Line = CL = \mu_0 = 10} \quad (9.4.10)$$

$$\mathbf{LCL = \mu_0 - L\sigma \sqrt{[\lambda / (2 - \lambda)][1 - (1 - \lambda)^{2i}]}}$$

$$\mathbf{= 10 - 2.7(1)\sqrt{[0.1/(2 - 0.1)][1-(1-0.1)^{2(1)}]} = 9.73} \quad (9.4.11)$$

Period i	X_i	EWMA Z_i values.
1	9.45	9.945
2	7.99	9.7495
3	9.29	9.70355
4	11.66	9.8992
5	12.16	10.1253
6	10.18	10.1307
7	8.04	9.92167
8	11.46	10.0755
9	9.20	9.98796
10	10.34	10.0232
11	9.03	9.92384
12	11.47	10.0785
13	10.51	10.1216
14	9.40	10.0495
15	10.08	10.0525
16	9.37	9.98426
17	10.62	10.0478
18	10.31	10.074
19	8.52	9.91864
20	10.84	10.0108
21	10.90	10.0997
22	9.33	10.0227
23	12.29	10.2495
24	11.50	10.3745
25	10.60	10.3971
26	11.08	10.4654
27	10.38	10.4568
28	11.62	10.5731
29	11.31	**10.6468**
30	10.52	**10.6341**

In the above table, out of control points are shown in **bold** figures this implies that the process is out of control at period 29. That is the control chart gives the signal at the period 28.

For Period i = 2, we have:

$$UCL = \mu_0 + L\sigma\sqrt{[\lambda/(2-\lambda)][1-(1-\lambda)^{2i}]}$$
$$= 10 + 2.7(1)\sqrt{[0.1/(2-0.1)][1-(1-0.1)^{2(2)}]} = 10.36 \quad (9.4.12)$$
$$\text{Central Line} = CL = \mu_0 = 10 \quad (9.4.13)$$
$$LCL = \mu_0 - L\sigma\sqrt{[\lambda/(2-\lambda)][1-(1-\lambda)^{2i}]}$$
$$= 10 - 2.7(1)\sqrt{[0.1/(2-0.1)][1-(1-0.1)^{2(2)}]} = 9.64 \quad (9.4.14)$$

On similar lines, one has to calculate the control limits for various values i, until they stabilize at steady state values given in equations (9.4.6) to (9.4.8). Thus for large values of i we have:

$$UCL = \mu_0 + L\sigma\sqrt{[\lambda/(2-\lambda)]}$$
$$= 10 + 2.7(1)\sqrt{[0.1/(2-0.1)]} = 10.62 \quad (9.4.15)$$
$$\text{Central Line} = CL = \mu_0 \quad (9.4.16)$$
$$LCL = \mu_0 - L\sigma\sqrt{[\lambda/(2-\lambda)]}$$
$$= 10 - 2.7(1)\sqrt{[0.1/(2-0.1)]} = 9.38 \quad (9.4.17)$$

The EWMA Control chart is shown Fig. (9.4.1) as follows:

Fig.(9.4.1): EWMA Control Chart.

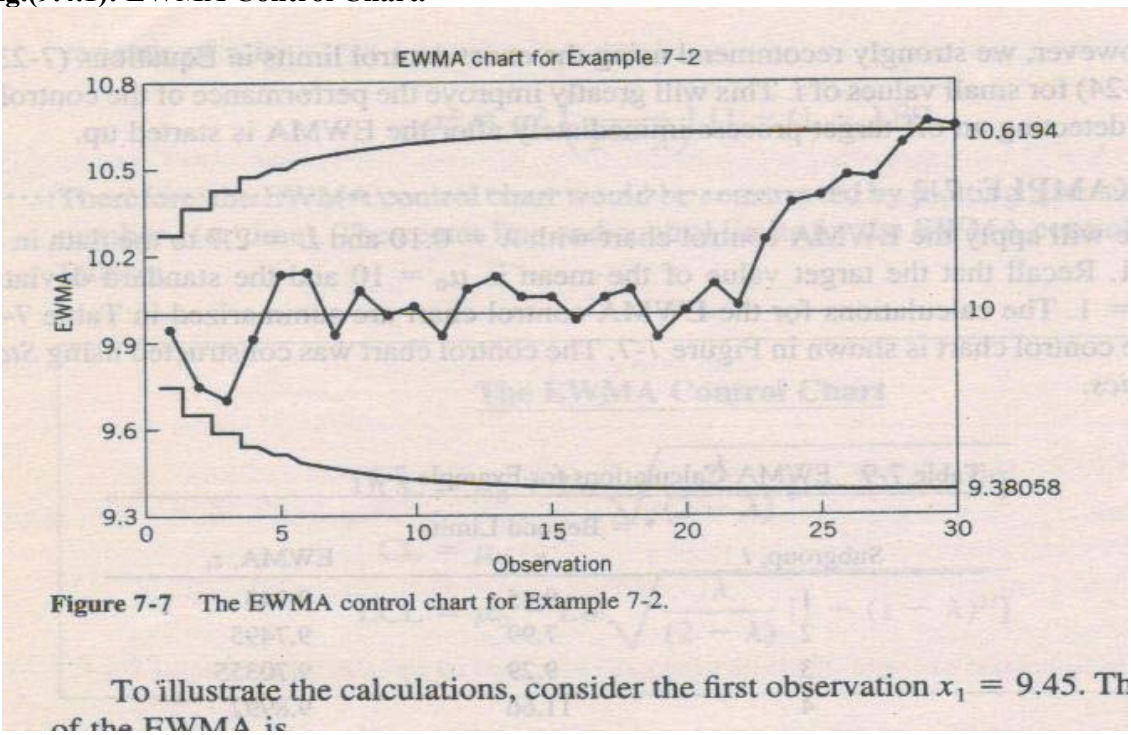


Figure 7-7 The EWMA control chart for Example 7-2.

To illustrate the calculations, consider the first observation $x_1 = 9.45$. Th
of the EWMA is

9.5. DESIGN AND EXTENSION OF EWMA CONTROL CHARTS

The design of EWMA charts is mainly depending on the parameters of the chart. That is the multiplicative constant L and the value of λ in the calculations of control limits of the chart.

These parameters are determined based on the performance of Average Run Length (ARL) curve for the chart. Those parameters, whose ARL which is closely shows performance of the CUSUM control chart for detecting small shifts, are to be chosen for EWMA Control chart.

There have been several theoretical studies available in literature, on the ARL properties of EWMA control charts. These studies provide average run Length tables or graphs for different range of values of L and λ parameters. The following table shows the performance of EWMA control charts for different values of these parameters.

Table (9.5.1): Average Run Length (ARL) for several EWMA control Schemes.
{Copied from Lucas and Sauccucci – 1990}

Shift in mean (multiple of σ)	L = 3.054 and λ =0.40	L = 2.998 and λ =0.25	L = 2.962 and λ =0.20	L = 2.814 and λ =0.10	L = 2.615 and λ =0.05
0	500	500	500	500	500
0.25	224	170	150	106	84.1
0.50	71.2	48.2	41.8	31.3	28.8
0.75	28.4	20.1	18.2	15.9	16.4
1.00	14.3	11.1	10.5	10.3	11.4
1.50	5.9	5.5	5.5	6.1	7.1
2.00	3.5	3.6	3.7	4.4	5.2
2.50	2.5	2.7	2.9	3.4	4.2
3.00	2.0	2.3	2.4	2.9	3.5
4.0	1.4	1.7	1.9	2.2	2.7

Critically comparing the ARL's for various values of the parameters L and λ values, we can observe that:

- As L and λ values are decreasing, ARL's in general are also same or decreasing up to the shift 1.00 and are increasing for shifts larger than 1.00.
- As the shift sizes are increasing, ARL's are decreasing.

In practice, it is found that, if the Values of λ are in the interval $0.05 \leq \lambda \leq 0.25$, then We get good EWMA control charts. The values of λ =0.05, λ =0.10, and λ =0.20 are also good choice, Further, it is observed that, to detect small shifts better to choose smaller λ values. Further, it is observed that L = 3 that is usual three sigma limits, works reasonably well, particularly when values of λ are large.

Like Cusum charts, EWMA charts are also performs well for small, sudden and persistent shifts in the process mean or variance. But they do not react to large shifts as quickly as Shewhart Control charts. However, EWMA charts are often superior to the cusum charts for large shifts particularly, when λ = 0.10. A better way of further improving the problem of identification of large shifts in the production process is to combine Shewhart control charts with EWMA charts. This will improve the sensitivity of the procedure to detect shifts (both small and large) present in the production process.

9.5a. Extension of EWMA Control charts for n > 1.

So far, we have discussed the method of construction of EWMA control charts when the sample size n = 1. However, if the sample sizes n are greater than 1, replace X_i by $\overline{X}i$ i=1,2,.,.,m and σ by σ / √n in equations (9.4.1) to (9.4.8) and construct Z_i 's and EWMA control limits.

9.6. SUMMARY

In this lesson, we learnt the need of EWMA control charts and discussed that these control charts are applied as an alternate to Cusum charts to detect small, sudden and persistent changes. WE can also construct these charts for all sample sizes. That is for n greater than or equal to 1. Control limits depend on two parameters namely, L , λ and are to be selected based on the performance of Average Run Length (ARL) curve, for various values of these parameters. If all the Z_i values are within the control limits, we conclude that there are no shifts present in the process and the production process is within the control otherwise, we determine that the process is out – of control and can search for the causes for such shifts. Eliminating such assignable causes, we can bring back the process under control.

9.7. SELF ASSESSMENT QUESTIONS

1. Explain the need of EWMA control charts.
2. Critically compare Shewhart, Cusum and EWMA control charts.
3. Explain how EWMA control charts are sensitive to Cusum control charts.
4. Derive the control limits of EWMA control charts.
5. What are the parameters of EWMA control charts? Explain how they are to be determined.
6. Following data represents the concentration readings from a chemical process.
   Construct EWMA control chart and draw your conclusions.

197	188	195	189	195	192	196	194	199	197
195	196	199	198	194	193	196	210	211	209
208	209	209	297	206	210	208	207	210	211.

7. Following data represent the temperature measurements on an intermediate chemical product. Construct EWMA control chart and draw your conclusions.

526	533	523	518	510	500	528	533	515	510
535	533	522	517	518	490	495	498	482	491.

8. Explain the method of construction of EWMA control charts for the sample sizes n >1.
9. Explain the role of ARL in determining the parameters of EWMA control charts.
10. Explain why? EWMA control charts are known as GMA control charts?

9.8. FURTHER READINGS

1. Montgomery, D.C., " Introduction to Statistical Quality Control", John Wiley (Asia) 2001.
2. Biswas.S., "Statistical Quality Control-Sampling Inspection and Reliability", New Central Book Agency Pvt. Ltd. 2003.
3.Wardsworth "Modern Methods for Quality Improvement", John Wiley and Sons 2005.
4.Bertrand L. Hansen "Quality control – Theory and Applications" Prentice-Hall of India-2005.

UNIT – III
LESSON – 10

MOVING AVERAGE AND MULTIVARIATE CONTROL CHARTS

10.0. OBJECTIVE

After going through this lesson, you should be able to understand:

- The need for Moving Average (MA) charts.
- Construction of MA control charts.
- Need for studying Multivariate Control charts.
- Various types of Multivariate control charts.

STRUCTURE:

10.1 INTRODUCTION

Exponentially Weighted Moving Average (EWMA) control charts explained in the Previous lesson is mainly used the concept of weighted Average of all the previous sample observations collected from the beginning of the production of a product. Occasionally, there is a need of another type of time-weighted control chart, based on a simple, un-weighted moving average. In many real life problems, present value of a variable is not depend on all the previous values of the variable, but depend on few latest values. That is the variable depends only on few recent past data items but not on all the data items. This concept is introduced in **Moving Average Control Charts.**

10.2 THE NEED FOR MOVING AVERAGE(MA) CHARTS

Usually, collecting data on a quality characteristic of product from a production process can be considered as **'Time Series'**. When time of production of the item play an important role to identify the assignable causes, like, power supply at time t, continuous works hours of the

machine on which the product is produced, comparison of efficiency of workers working in day or night shifts and so on, time of production of the item play an important role. Hence there is a need to collect time series data relating to the quality characteristic of the product.

When we deal with instruments or machines having aging effect, we require time series data in industry. In such type of data, in general efficiency of the working of the instrument having age effect depending on the information like how long the instrument is working? the age / life time of the equipment? and so on. In such situations, present status is depending on the immediate past conditions but not on distant events. Hence, there is a need to go for Moving Averages with period 'K', immediate past data. In such situations, we need to construct Moving Average control charts with period K.

10.3. THE CONSTRUCTION OF MOVING AVERAGE (MA) CHARTS

In order to construct Moving Average control charts, we need not know all the previous history of the process. It is enough if we know only latest few data items, about the process. This concept is introduced through Moving average control charts, by defining a statistic known as **Moving Average M_i** , i = 1,2..,.,n and here i represent the time period. Let X_1 ,X2 ,.,.,.,X_n are data items collected from a process at time points t_1 ,t_2 ,.,.,t_n . Such type of data is popularly known as '**time series'** Data. Then the moving average of Span 'K' at time i denoted by M_i is defined as:

$$\mathbf{M_i = [\ X_1 + X_2 + .,.,.,+ X_j\ ] / j \quad If\ j = 1,2,.,.,K} \qquad (10.3.1)$$

$$\mathbf{M_i = [\ X_i + X_{i-1} + .,.,.,+ X_{i-k+1}\ ] / K,}\ i = K,K+1,.,.,.,n. \qquad (10.3.2)$$

That is, at time period i the oldest observation / data item in the Moving Average set is dropped and the latest data item is added to the set. Further, the variance of the Moving Average M_i is given by:

$$\mathbf{V(M_i) = (1/K^2) \sum_{i-k+1}^{i} V(Xj) = (1/K^2) \sum_{i-k+1}^{i} \sigma = \sigma^2 / K.} \qquad (10.3.3)$$

Let, the process target value is denoted by μ_0 , then the control limits of Moving Average (MA) Control chart are given as follows:

Case 1: If j is less than K.

$$\mathbf{UCL = \mu_0 + 3\sigma / \sqrt{j},} \quad J = 1,2,.,.,K. \qquad (10.3.4)$$

$$\mathbf{CL = \mu_0} \qquad (10.3.5)$$

$$\mathbf{LCL = \mu_0 - 3\sigma / \sqrt{j}\ ,} \quad J = 1,2,.,.,K. \qquad (10.3.6)$$

Case 2: If j is greater than or equal to K:

$$\mathbf{UCL = \mu_0 + 3\sigma / \sqrt{K}} \qquad (10.3.7)$$

$$\mathbf{CL = \mu_0} \qquad (10.3.8)$$

$$\mathbf{LCL = \mu_0 - 3\sigma / \sqrt{K}} \qquad (10.3.9)$$

Where, k is the length of the span, k Under consideration. The control procedure consists of calculating the fresh Moving average M_i as each observation Xi become a available / collected. Plot control limits, M_i values against each 'i' and conclusions can be drawn as usual depending on the nature of spread of points on the chart.

In general, the magnitude of the shift of interest and K are inversely proportional. That is small shifts would be guarded against more effectively by longer-span Moving Averages and vice versa. It is important to note that MA control charts are Particular case of EWMA control charts, where, **equal weights** are given for the latest K data items at time 'i '. In EWMA control charts these **weights decrease geometrically**. In this sense, EWMA control charts more generalized version of Moving Average control charts. Thus MA control chart is more effective than the Shewhart control chart in detecting small process shifts, but are less effective than cusum or EWMA charts in detecting small shifts. MA charts are simpler to construct than Cusum / EWMA charts. Construction of MA control chart is explained with the following example:

Example (10.3.1): Construct Moving average (MA) control chart with period 5 and draw your conclusions for the data given in example (9.4.1).

Solution: First calculate Moving sums and Moving averages M as shown in table (10.3.1). It is given in the problem that $\mu_0 = 10$ and $\sigma = 1.0$. Then, control limits for Moving Average control chart are calculated as follows:

Case 1: j < k. that is J = 1,2,.,.,K. Since, K = 5, we have to calculate control limits for j = 1,2,3,4. Using equations (10.3.4) to (10.3.6) we have:

J = 1: UCL = 10 + 3(1.0)/√1 = 13 and LCL = 10 – 3(1.0)/√1 = 7.0

J =2 : UCL = 10 + 3(1.0)/√2 = 12.1213 and LCL = 10 – 3(1.0)/√2 = 7.8787

J =3: UCL = 10 + 3(1.0)/√3 = 11.73205 and LCL = 10 – 3(1.0)/√3 = 8.26795

J = 4: UCL = 10 + 3(1.0)/√4 = 11.5 and LCL = 10 – 3(1.0)/√4 = 8.5.

Table (10.3.1): Calculation of Control limits with K = 5.

Period i	X_i	Moving Sums	Moving Averages M_i	LCL	UCL
1	9.45	9.45	9.45	7.00	13.00
2	7.99	17.44	8.72	7.8787	12.1213
3	9.29	26.73	8.91	8.26795	11.73205
4	11.66	38.39	9.5975	8.5	11.5
5	12.16	50.55	10.11	8.66	11.34
6	10.18	51.28	10.256	8.66	11.34
7	8.04	51.33	10.266	8.66	11.34
8	11.46	53.50	10.7	8.66	11.34
9	9.20	51.04	10.208	8.66	11.34
10	10.34	49.22	9.844	8.66	11.34
11	9.03	48.07	9.614	8.66	11.34
12	11.47	51.50	10.3	8.66	11.34
13	10.51	50.55	10.11	8.66	11.34
14	9.40	50.75	10.15	8.66	11.34
15	10.08	50.49	10.098	8.66	11.34
16	9.37	50.83	10.116	8.66	11.34
17	10.62	49.98	9.996	8.66	11.34
18	10.31	49.78	9.956	8.66	11.34
19	8.52	48.90	9.78	8.66	11.34

20	10.84	49.66	9.932	8.66	11.34
21	10.90	51.19	10.238	8.66	11.34
22	9.33	49.90	9.98	8.66	11.34
23	12.29	51.88	10.367	8.66	11.34
24	11.50	54.86	10.972	8.66	11.34
25	10.60	54.62	10.924	8.66	11.34
26	11.08	54.80	10.96	8.66	11.34
27	10.38	55.85	11.17	8.66	11.34
28	11.62	55.18	11.036	8.66	11.34
29	11.31	54.99	10.998	8.66	11.34
30	10.52	54.91	10.982	8.66	11.34

Case 2: when j > K. that is j = k,k+1,.,.,n. In the problem j = 5,6,.,.,30 we have Control limits using equations (10.3.7) to (10.3.9) are calculate as follows:

UCL = μ_0 + 3σ / √K = 10 + 3 (1.0) /√5 = 11.34 (10.3.10)

CL = μ_0 = 10 (10.3.11)

LCL = μ_0 - 3σ / √K = 10 – 3(1.0)/√5 = 8.66 (10.3.12)

Calculated control limits are given in columns 5 and 6 of table (10.3.1) the MA control chart is given in Figure (10.3.1).

Fig.(10.3.1): EWMA Control Chart.

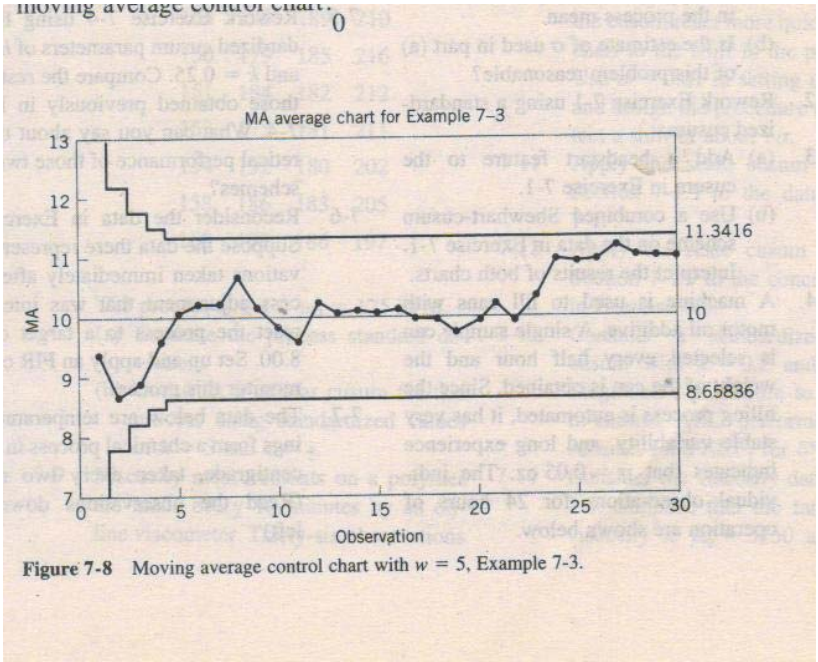


Figure 7-8 Moving average control chart with $w = 5$, Example 7-3.

Conclusion: The moving control chart given in fig. (10.3.1) we can observe that, no points fell out of control limits. Hence, we can conclude that the process is in control.). In the figure, we can observe that the trend of Moving averages M_i show, increasing trend above central line (CL). Which gives hint that the shift occurred at the period I = 22 or 23.

Further, we can observe that in moving control charts, initial control limits for $j < K$ are wider than their final steady-state control limits, which became constant values for $j \geq K$. In the example (10.3.1), K = 5. Thus, calculations of control charts, in Moving Average (MA) Control charts are relatively easier and simpler to implement, when compared to CUSUM / EWMA control charts.

10.4. NEED FOR MULTIVARIATE CONTROL CHARTS

In all the lectures discussed so far, we have considered control charts and their construction based on a Single variable data. That is, we have considered the process monitoring based on univariate data. On other words, the quality characteristic of the product is mainly depend on a single variable like, life time of the unit, or breaking strength of the yarn or brick, or concentration or temperature of the mixture in a chemical industry and so on.

In practice, many real life problems, quality of the product may depend on many variables like, diameter, length and content of potash in a bullet, Length, weight and height and number of beams / rays produced from, and lifetime of a T.V. color tube, Smoothness, % of terrine mixture or wool mixture in the cloth, breaking strength of the yarn produced from a textile industry, and so on. One possible alternative to deal such multivariable problem is to analyze each variable separately and draw conclusions. But this procedure will be lengthy and time consuming and in accurate, because, it ignores the inter relationships between the variables. Hence**, there is need to analyze all the data Simultaneously**. Such Simultaneous Monitoring or Control of two or more related quality characteristics is known as **"Multivariate Methods"** and such control charts used to analyze multivariate data are called **"Multivariate Control charts". Thus there is a need to develop the theory relating to Multivariate Control charts.** Before introducing these control charts, first we need some knowledge on concepts like, Multivariate data, Multivariate Normal Distribution, Mean Vector and variance Covariance Matrix and so on. Thus we concentrate on these concepts along with various types of Multivariate control charts in the next section.

10.5. BASIC CONCEPTS AND VARIOUS TYPES OF MULTIVARIATE CONTROL CHARTS

First, let us consider the problem of monitoring p – variables X_1 , X_2 ,.,.,X_p relating to p – quality characteristics. Arrange these p – variables in a p – component vector $X^T = X' = (X_1, X_2, .,.,X_p)$. Let $\mu^1 = (\mu_1, \mu_2, .,.,\mu_p)$ be the mean vector and Σ is a pxp Variance covariance matrix.

In the matrix Σ diagonal elements are variances and off-diagonal elements are co-variances of $X_1, X_2, .,.,X_p$. Now the squared standardized distance from X to μ in matrix notation is given as $(X - \mu)^1 \Sigma^{-1} (X - \mu)$. Then, the multivariate normal density function f(x) is given by:

$$F(\mathbf{X}) = \{ 1 / [(2\pi)^{p/2} |\Sigma|^{1/2}] \} \; e^{-1/2[(X - \mu)' \Sigma^{-1} (X - \mu)]} \qquad (10.5.1)$$

Where, $-\infty < X_j < +\infty$, j = 1,2,.,.,p.

A multivariate normal distribution for p = 2 is called a "**bivariate normal**" distribution.

Above explained multivariate or bivariate normal distributions are used in the theory of Multivariate Process monitoring problems depending on p is greater than or equal to 2 respectively.

The following are some popularly used Multivariate Control Charts. Namely:

1. Chi Square (X^2) Control Charts.
2. Hotelling T^2 Control Charts.
3. Multivariate EWMA control charts.
4. Latent Structure Control Charts.
5. Principal Component (PCA) Control charts.
6. Partial Component methods of control charts.
7. Partial Least Squares(PLS) methods and so on.

There are many methods of analyzing Multivariate Process Monitoring and we discuss only those, included in the present syllabus in the forth coming lessons. Thus the multivariate process monitoring techniques are applied when we need to control many quality variables at a time to control the quality of the product under consideration.

10.6. SUMMARY

This lesson summarizes the need and application of Moving Average (MA) control charts and discussed that these charts are particular case of EWMA control charts where equal weights are given to the latest K data items.

Further, concentration is diverted to explain the need for monitoring the production process based on several variables, say p simultaneously at a time. When the quality of a product is depending on many variables, and is to be controlled simultaneously, we require using 'Multivariate Control Charts". In this theory Multivariate normal distribution is popularly used along with its Mean Vector and Variance Co-variance Matrix. We have also discussed various types of multivariate quality control methods applicable in Statistical Quality Control.

10.7. SELF ASSESSMENT QUESTIONS

1. Discuss various advantages of Moving Average control charts.
2. Distinguish between EWMA and MA control charts.
3. Explain the method of construction of MA control charts.
4. What are Moving Averages? Discuss their applications.
5. Following data represents the concentration readings from a chemical process. Construct MA control chart for the process mean $\mu_0 = 190$ and K = 5 and draw your conclusions.

197	188	195	189	195	192	196	194	199	197
195	196	199	198	194	193	196	210	211	209
208	209	209	297	206	210	208	207	210	211.

6. Viscosity measurement on a polymer are made every 10 minutes by an on-line Viscometer. Thirty six observations are recorded as follows:

3169	3205	3185	3188	3173	3203	3187	3183	3162	3209	3192	3175
3154	3208	3199	3174	3239	3211	3197	3171	3145	3214	3193	3180
3160	3215	3190	3179	3172	3209	3183	3175	3175	3203	3197	3174

Construct a moving average control chart for the above data by considering k = 3. The target viscosity for this process is $\mu_0 = 3200$.

7. A machine is used to fill cans with motor oil additive. A single sample can is selected every hour randomly and the weight of the can is recorded as follows. The process standard deviation $\sigma = 0.05$ and the process target mean $\mu = 8.02$ oz. Construct M A control chart for the process using k = 4.

8.00	8.01	8.02	8.01	8.06	8.07	8.01	8.04	8.02	8.01	8.04	8.07
8.05	8.04	8.03	8.05	8.06	8.04	8.05	8.06	8.04	8.02	8.03	8.05

8. Explain the need for Multivariate Process monitoring with suitable examples.
9. Explain the role of Multivariate normal distribution in Multivariate control charts.
10. List out various types of multivariate control charts.

10.8. FURTHER READINGS

1. Montgomery, D.C., " Introduction to Statistical Quality Control", John Wiley (Asia) 2001.
2. Biswas.S., "Statistical Quality Control-Sampling Inspection and Reliability", New Central Book Agency Pvt. Ltd. 2003.
3.Wardsworth "Modern Methods for Quality Improvement", John Wiley and Sons 2005.
4. A.J. Duncan "quality control and industrial Statistics" R.J. Taraporevala and D.B. Taraporivala sons & Co. Pvt. Ltd. Secnd Edn. 1970.

UNIT – III
LESSON – 11

CHI-SQUARE AND HOTELLING T^2 CONTROL CHARTS

11.0. OBJECTIVE

After going through this lesson, you should be able to understand:

- The need and application of Chi squire control charts.
- Construction of Hotelling T^2 control charts.
- Applications of chi-square and Hotelling T^2 Control charts.
- Other Multivariate control charts available in literature.

STRUCTURE:

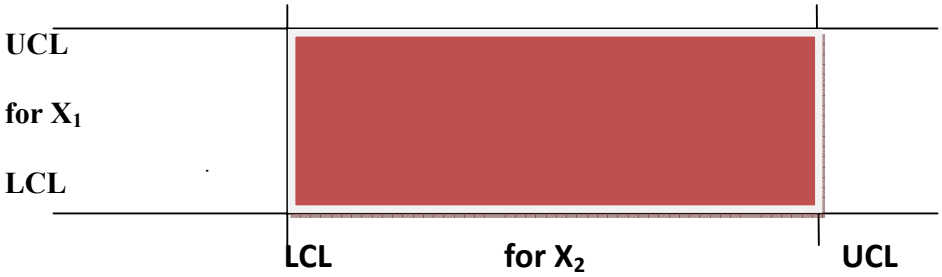
11.1. INTRODUCTION

It is under stood from the previous lesson that, there is need to control many variables at a time and hence, multivariable control charts are to be developed to deal such problems. When we deal many variables at a time, the first and foremost question that arises is that, Are these variables are dependent or independent? To answer such problems, first of all we have to apply Chi-square test and determine the inter relation between the quality variables/ characteristics under consideration. Hence we have to study the application of Chi square test statistic and its applications in solving **multivariate quality control problem** which is explained as follows.

11.2. THE MULTIVARIATE QUALITY CONTROL PROBLEM

It is already mentioned that, there exists number of situations in industry **to control many**

Quality variables, simultaneously, at a time. That is we have to deal two or more quality characteristics of a product to determine whether the product is in conformance to the quality standards or not. For example, consider a soft drink bottles consisting of 200 ml drink. We have to consider many variables like, quantity, taste, ingredients, softness sweetness, thickness, and natural colors mixed and so on. The drink conform the quality, if all these quality characteristics are accordance to their standards. Similarly, in Textile industry, to determine the quality of the cloth, we need to consider many quality characteristics like, breaking strength of the yarn, % mixture of different fiber, smoothness, design, color, durability, latest models, and so on and so forth. Similarly, An automobile ball bearing conforms standards with regards to inner diameter (X_1) and outer diameter (X_2). Both X_1 and X_2 together determine the usefulness of the product. Hence, both X_1 and X_2 are to be measured to determine the quality of the ball bearings. Let X_1 and X_2 have independent normal distributions, then we can apply $\bar{X}$ chart independently on each characteristic. Calculate sample means $\bar{X}_1$ and $\bar{X}_2$ and if sample means fall within the control in their respective control charts, then we can decide that the process is in control. This method is equivalent to the pair of means ($\bar{X}_1$, $\bar{X}_2$) plotting jointly within the common region as shown in the following figure (11.2.1).



Above figure (11.2.1) shaded portion represents common in control region for both ($\bar{X}_1$, $\bar{X}_2$). Like above, constructing two independent control charts for $\bar{X}_1$ separately and $\bar{X}_2$, separately is very misleading and hence both variables are to be considered on a single chart simultaneously. This is explained as follows:

By using the property of normal distribution, we know that, the sample mean exceeds three sigma limits on either directions is 0.0027. If the same control limits are applied for $\bar{X}_1$ separately and $\bar{X}_2$ separately, and the joint probability that both variables exceed their control limits simultaneously, when they are both in control is given by (0.0027)(0.0027) = **0.00000729**, Which is much smaller than 0.0027. Similarly, the probability that $\bar{X}_1$ and $\bar{X}_2$ will simultaneously plot inside the control limits, when the process is really in control is (0.9973)(0.9973) = **0.99460729**. Hence, the use of Separate $\bar{X}$ charts has discarded simultaneous monitoring of both $\bar{X}_1$ and $\bar{X}_2$ is recommended. This is because of the fact that type – I error and point correctly plotting in control are not up to the desired level as desired. This will become more serious when we consider more variables.

In general, if we consider 'p' statistically independent quality characteristics, for a particular product, and if an $\bar{X}$ chart with P[Type – I error] = α, is maintained on each $\bar{X}$ chart, then, the true probability of Type – I error for the joint control procedure is

$$\alpha' = [\, 1 - (1 - \alpha)^p \,] \qquad (11.2.1)$$

and the probability that all p means will simultaneously plot inside their control limits when the process is in control is :

$$P[\text{all p means plot in control}] = (1 - \alpha)^p \qquad (11.2.2)$$

Hence, **'Multivariate control charts'** which deal all the variables simultaneously, at a time are **necessary** to analyze **multivariable process monitoring problems.**

11.3. CONSTRUCTION OF CHI-SQUARE CONTROL CHARTS

First, we consider a simple case of Multivariate process monitoring and control problem with two quality characteristics X_1 and X_2 . Then, we extend the case for 'p' quality characteristics X_1 ,X_2 ,.,.,.,X_p . Let the two quality characteristics X_1 and X_2 are jointly distributed according to the bivariate normal distribution with parameters μ_1 and μ_2 , for the quality characteristics X_1 and X_2 respectively. Let σ_1 and σ_2 be the standard deviations of X_1 and X_2 respectively and σ_{12} denote the covariance between X_1 and X_2 . Further, we assume that the values of σ_1 , σ_2 and σ_{12} are known and μ_1 , μ_2 are estimated from the sample means $\bar{X}_1$ and $\bar{X}_2$ calculated from a sample of size n, then the statistic X_0^2 defined as follows, will have a chi-square distribution with 2 degrees of freedom.

$$\mathbf{X_0^2 = [n / \sigma_1^2 \sigma_2^2 - \sigma_{12}^2]\, X}$$
$$\mathbf{[\sigma_2^2 (\bar{X}_1 - \mu_1)^2 + \sigma_1^2 (\bar{X}_2 - \mu_2)^2 - 2\, \sigma_{12}^2 (\bar{X}_1 - \mu_1)(\bar{X}_1 - \mu_1)]} \qquad (11.3.1)$$

The above equation given in equation (11.3.1) is the basis of a control chart for the process means μ_1 and μ_2. If the process means remain at the values μ_1 and μ_2, then the value of $\mathbf{X_0^2}$ should be less than upper control limit UCL = $\mathbf{X_{\alpha,2}^2}$ where $\mathbf{X_{\alpha,2}^2}$ **is the upper α percentage point of the chi-square (X^2) distribution with 2 degrees of freedom.** If any on of the means shifts to some new (out of control) value, then the probability that the statistic $\mathbf{X_0^2}$ exceed the UCL increases.

Consider the sample number on X- axis and $\mathbf{X_0^2}$ values on y – axis and draw the UCL at $\mathbf{X_{\alpha,2}^2}$**,** as shown in the following figure (11.3.1). The resultant control chart is called **"Chi-square Control Chart".**

11.3a: Advantages of Chi-square control chart:

It is important to note that Chi-square control chart has no lower con troll limits (LCL) and the time sequence of the data is preserved by this control chart. The additional advantage of Chi-square control chart is that, the state of the process is characterized by a single number $\mathbf{X_0^2}$ **which can be compared easily with another single number $\mathbf{X_{\alpha,2}^2}$. This is particularly helpful when there are two or more quality characteristics.** The analogy discussed in Chi-square control chart can easily extendable for more than two quality characteristics. Let us assume that there are 'p' quality characteristics of a product which are to be analyzed jointly, and let the joint probability distribution of these 'p' quality characteristics is a p-variate normal distribution with mean vector μ and Variance covariance matrix Σ. The procedure of extending

the analysis to 'p' variables is simple based on the procedure explained above for two variables. First calculate sample mean for each of the 'p' quality characteristics from the sample data of size 'n' collected from the production process on these 'p' quality characteristics. These set of 'p' sample means be represented by a p X 1 vector $[\bar{X}]' = [\bar{X}_1 \ , \bar{X}_2 \ ,.,.,., \bar{X}_p \ ]$. Then the test statistic **X_0^2 plotted on the Chi-square control chart is defined as:**

$$\mathbf{X_0^2 = n (\bar{X} - \mu)^1 \Sigma^{-1} (\bar{X} - \mu)} \qquad (11.3.2)$$

Where, $\mu' = [\mu_1 , \mu_2 , .,.,\mu_p \ ]$ is the vector of in control means of each quality characteristic and Σ is the variance co-variance matrix of size p X p. The Upper control limit (UCL) for chi-square control chart is given as:

$$\mathbf{UCL = \quad X_{\alpha,2}^2} \qquad (11.3.3)$$

This methodology leads another important control charts known as '**Hotelling T^2 control charts'** which are discussed in the following section.

11.4. CONSTRUCTION OF HOTELLING T^2 CONTROL CHARTS

Above explained procedure of constructing control chart for p =2 can easily be extended to p > 2. That is if we want to monitor more than two quality variables simultaneously, the construction procedure explained for chi-square control chart can be easily extended to any 'p'. Such a control chart useful for more than two quality variables is known as **'Hotelling T^2 control chart'**. Thus Hotelling T^2 control chart is the extension of Chi-square control chart and is most popularly used Multivariate Process monitoring control chart. It is a direct analog of the univariate Shewhart $\bar{X}$ control chart. There are two versions of the Hotelling T^2 control chart. Namely: (1) For sub-group data and (2) For individual observations.

11.4a. Hotelling T^2 for sub-group data.

In practice, it is necessary to estimate the parameters, namely the mean vector μ and the variance covariance matrix Σ using the sample information of size 'n', taken, when the process is assumed in control. The sample means and variances are to be calculated from each sample using the usual procedure and formulae. That is:

$$\mathbf{\bar{X}_{jk} = \frac{1}{n} \sum_{i=1}^{n} X_I \ ; \ j = 1,2,.,.,p \text{ and } k = 1,2,.,.,m.} \qquad (11.4a.1)$$

$$\text{And } \mathbf{S^2_{jk} = \frac{1}{n-1} \sum_{i=1}^{n} (X\,ijk - \bar{X}_{jk})^2 \ ; \ j = 1,2,.,.,p \text{ and } k = 1,2,.,.,m.} \qquad (11.4a.2)$$

Where, X_{ijk} is the ith observation on the jth quality characteristic in the kth sample. The co-variance between quality characteristic j and quality characteristic h in the kth sample is , denoted by $\mathbf{S_{jhk} = \frac{1}{n-1} \sum_{i=1}^{n} (X\,ijk - \bar{X}_{jk})(Xijk - \bar{X}_{hk}) \ ; \ k = 1,2,.,.,m \text{ and } J \neq h.}$ (11.4a.3)

These calculated statistics n amely, $\mathbf{\bar{X}_{jk}}$, $\mathbf{S^2_{jk}}$ and $\mathbf{S_{jhk}}$ are then averaged over all m samples to obtain $\bar{\bar{X}}$,$\bar{\mathbf{S}}^2_j$,$\bar{S}_{jh}$, j ≠ h which are given as follows:

$$\bar{\bar{X}} = \frac{1}{m} \sum_{k=1}^{m} \bar{X}jk \quad \mathbf{J = 1,2.,.,.p.} \qquad (11.4a.4)$$

$$\bar{S}^2_j = \frac{1}{m}\sum_{k=1}^{m} S^2_{jk} \quad j = 1,2,.,.,p. \tag{11.4a.5}$$

$$\text{And } \bar{S}_{jh} = \frac{1}{m}\sum_{k=1}^{m} S_{jhk} \ , \ j \neq h. \tag{11.4a.6}$$

The $\{ \bar{\bar{X}}_j \}$ are the elements of the vector $\bar{\bar{X}}$ and the p x p average covariance matrix S is formed as follows:

$$S = \begin{bmatrix} \bar{S}_{21} & \cdots & \bar{S}_{1p} \\ \vdots & \ddots & \vdots \\ \bar{S}_{1p} & \cdots & \bar{S}_{2p} \end{bmatrix} \tag{11.4a.7}$$

The average of the sample covariance matrix **S** is an unbiased estimate of **Σ**, when the process if in control. Now we proceed to define the Hotelling T^2 statistic using $\bar{\bar{X}}$ and S in the place of μ and Σ in equation (11.3.2). Thus we have the test statistic T^2 for Hotelling T^2 control chartas follows:

$$T^2 = n\,(\bar{X} - \bar{\bar{X}})'\, S^{-1}\, (\bar{X} - \bar{\bar{X}}) \ . \tag{11.4a.8}$$

The above explained procedure is known as Hotelling T^2 Control charts for sub-group data.

Control Limits for Hotelling T^2 Control chart:

In order to construct Hotelling T^2 control chart, First calculate Hotelling T^2 Statistic from the given data, using equation (11.4a.8) and the control limits for the multivariate quality control applications are to be selected carefully as pointed out by Alt (1985). There are two distinct phases of control chart usage and based on the usage of Hotelling T^2 control chart.

In **Phase – 1,** the chart is used to establish the control limits for testing whether process was in control or not? Based on the m preliminary sub-groups / samples were drawn and sample statistics $\bar{\bar{X}}$ and S were computed using (11.4a.4) and (11.4a.7). The basic objective of this phase – 1 is to obtain an in control set of observations, so that control limits for **phase – 2**, where we consider the process monitoring of future production. This procedure is popularly known as **'Retrospective Analysis'**. Thus, in phase – 1 the control limits are calculated as follows:

$$UCL = \frac{p(m-1)(n-1)}{mn-m-p+1} \ F_{\alpha,p,mn-m-p+1} \ . \tag{11.4a.9}$$

$$\text{And } LCL = = 0 \ . \tag{11.4a.10}$$

In phase – 2,when the chart is used for future production, control limits are calculated as follows:

$$UCL = \frac{p(m+1)(n-1)}{mn-m-p+1} \ F_{\alpha,p,mn-m-p+1} \ . \tag{11.4a.11}$$

$$\text{And } LCL = = 0 \ . \tag{11.4a.12}$$

When, μ and Σ are estimated from a large number of preliminary samples, it is customary to use $UCL = X^2_{\alpha,2}$ in the above equations (11.4a.9) and (11.4a.11), both in Phase – 1 and phase – 2. For the Retrospective Analysis of the preliminary samples, to test the statistical control and to establish control limits, also occur in the univariate control chart settings, and is

discussed in the concept of 'Revised Control charts' and 'Revised Control limits' used for future control. The basic difference between Phase – 1 and phase – 2 control limits will be negligible if the number of samples collected m = 20 or 25. In this situation, control limits in phase – 1 and phase – 2 will nearly coincide with each other. Now we explain the construction of Hotelling T^2 control chart for phase – 1, in the following example.

Example (11.4a.1): Consider the Tensile manufacturing company, where The Tensile Strength (X_1) measured in psi and diameter of a textile fiber (X_2) measured (X 10^{-2} inches)are to be controlled jointly. The quality control engineer has decided to use n = 10 fiber specimens in each sample. He has collected 20 preliminary samples are collected and sample means; variances and co-variances are calculated and are given as follows. Construct Hotelling T^2 control chart and draw your conclusions.

Sample	Means		Variances and co-variances		
Number K	$\bar{X}_{1k}$	$\bar{X}_{2k}$	S^2_{1k}	S^2_{2k}	S_{12k}
1.	115.40	1.04	1.29	0.85	0.68
2.	115.72	1.06	1.31	0.89	0.76
3.	115.58	1.05	1.18	0.83	0.79
4.	115.47	1.03	1.20	0.95	0.70
5.	115.63	1.04	1.24	0.91	0.83
6.	115.29	1.11	1.23	0.89	0.82
7.	115.83	1.07	1.17	0.76	0.75
8.	116.01	1.05	1.26	0.55	0.72
9.	114.90	1.06	1.24	0.82	0.81
10.	115.75	0.99	1.45	0.79	0.78
11.	115.92	1.05	1.17	0.86	0.95
12.	116.15	1.09	1.19	0.87	0.83
13.	115.25	1.10	1.40	0.83	0.80
14.	114.98	1.05	1.25	0.78	0.75
15.	115.55	1.06	1.01	0.80	0.76
16.	115.90	1.07	1.16	0.73	0.80
17.	116.21	1.05	1.02	0.85	0.81
18.	115.05	1.09	1.30	0.90	0.82
19.	115.91	1.06	1.26	0.85	0.81
20.	115.25	1.04	1.25	0.87	0.80

Solution: In the problem, it is given that n = 10, m = 20, p = 2, Sample means, Variances and co-variances are calculated and are given as follows:

Sample	Means		Variances and co-variances			Control chart Statistics	
Number K	$\bar{X}_{1k}$	$\bar{X}_{2k}$	S^2_{1k}	S^2_{2k}	S_{12k}	T^2_k	$\lvert S_k \rvert$
1.	115.40	1.04	1.29	0.85	0.68	0.62	0.63
2.	115.72	1.06	1.31	0.89	0.76	0.35	0.59
3.	115.58	1.05	1.18	0.83	0.79	0.00	0.36
4.	115.47	1.03	1.20	0.95	0.70	0.19	0.65
5.	115.63	1.04	1.24	0.91	0.83	0.70	0.19
6.	115.29	1.11	1.23	0.89	0.82	2.56	0.42

7.	115.83	1.07	1.17	0.76	0.75	1.11	0.33
8.	116.01	1.05	1.26	0.55	0.72	3.86	0.17
9.	114.90	1.06	1.24	0.82	0.81	9.96	0.36
10.	115.75	0.99	1.45	0.79	0.78	1.13	0.54
11.	115.92	1.05	1.17	0.86	0.95	2.41	0.10
12.	116.15	1.09	1.19	0.87	0.83	5.92	0.35
13.	115.25	1.10	1.40	0.83	0.80	3.01	0.52
14.	114.98	1.05	1.25	0.78	0.75	7.54	0.41
15.	115.55	1.06	1.01	0.80	0.76	0.03	0.23
16.	115.90	1.07	1.16	0.73	0.80	1.89	0.21
17.	116.21	1.05	1.02	0.85	0.81	8.29	0.21
18.	115.05	1.09	1.30	0.90	0.82	6.77	0.50
19.	115.91	1.06	1.26	0.85	0.81	2.14	0.41
20.	115.25	1.04	1.25	0.87	0.80	2.16	0.45
Means:	**115.59**	**1.06**	**1.23**	**0.83**	**0.79**		

From the given data we can calculate that: $\bar{\bar{X}}_1 = 115.59$ psi, $\bar{\bar{X}}_2 = 1.06 (\times 10^{-2})$ inch $\bar{S}^2_1 = 1.23$ $\bar{S}_2 = 0.83$ $\bar{S}_{12} = 0.79$. Hence, the T^2 statistic is calculated as follows using equation (11.3.1)):

$$\mathbf{T^2 = \frac{10}{(1.23)(0.83) - (0.79)2}\ [0.83\ (\bar{X}_1 - 115.59)^2 + 1.23\ (\bar{X}_2 - 1.06)^2 - 2(0.79)(\bar{X}_1 - 115.59)(\bar{X}_2 - 1.06)]} \qquad (11.4a.13)$$

The control limits for Hotelling T^2 chart are calculated as follows using equation (11.4a.9):

UCL = $[2(19)(9)] / [20(10) - 20 - 2 + 1]$ $\mathbf{F_{0.001,2,(20)(10)-20-2+1}}$

= $[342 / 179]\ F_{0.001,2,179}$ = (1.91) (7.81) = **13.72** (11.4a.14)

Similarly, for **phase – 2 UCL = 15.6 using equation (11.4.11)**.

Instead of F critical values, if we have used the Chi-square critical values, at $\alpha = 0.001$, then the UCL = 13.816 which is close to the UCL for phase – 1.

Conclusions: Based on the Hotelling T^2 control chart, we conclude that the process is in control because no T^2 points are above UCL.

11.4b. Construction of Hotelling T^2 control chart for individual observations:

There are many situations, we have to construct the Hotelling T^2 control chart for n = 1, that is for individual observations. Such situations will occur frequently, in chemical industries, where we have multiple characteristics that must be monitored with sample size n = 1. Such charts are known as 'Multivariate Control Charts for individual observations'. Hence, we discuss here the construction of such charts. Let there are **m** samples each of size **n = 1** and we have to monitor **p** quality characteristics observed in each sample. Let **X** and **S** be the sample mean vector and variance covariance matrix respectively, calculated from the collected data. Then the Hotelling T^2 statistic is calculated as follows:

$$\mathbf{T^2 = (X - \bar{X})' S^{-1} (X - \bar{X})} \qquad (11.4b.1)$$

For phase – 2 control limits are calculate as follows:

$$\mathbf{UCL} = \frac{p(m+1)(m-1)}{(m2-mp)} \; \mathbf{F_{\alpha,p,,m-p}} \; . \qquad (11.4b.2)$$

And $\mathbf{LCL = = 0}$. (11.4b.3)

When the number of preliminary samples m is large, say m > 100, many engineers or practitioners use UCL as:

$$\mathbf{UCL} = \frac{p(m-1)}{(m-p)} \; \mathbf{F_{\alpha,p,,m-p}} \; . \qquad (11.4b.4)$$

And $\mathbf{LCL = = 0}$. (11.4b.5)

Or $\mathbf{UCL = X_{\alpha,2}^2}$ (11.4b.6)

And $\mathbf{LCL = 0.}$ (11.4b.7)

For m > 100, the equation (11.4b.4) is more reasonable than (11.4b.6). The chi-square limit is only approximate in the covariance matriX S is known. This chi-square approximation is more frequently used in practice when the sample size is large. Suppose, if p ≥ 10 and m > 250, then we have minimum 2500 observations and Chi-square approximation will give correct value. In the year 1992, Tracy and Young showed that if n = 1, the phase – 1 limits can be calculated based on β distribution and hence control limits for phase – 1 are given as follows:

$$\mathbf{UCL = \{[(m-1)^2] / m\} \; \beta_{\alpha\,,\,p/2,\,(m-p-1)/2}} \; . \qquad (11.4b.8)$$

And $\mathbf{LCL = 0}$. (11.4b.9)

Where $\beta_{\alpha\,,\,p/2,\,(m-p-1)/2}$ is the upper α percent point of beta distribution with parameter **P / 2 and (m – p – 1) / 2. Phase – 1 control limits based on F – distribution or chi – square distribution may be inaccurate and hence, beta distribution is used, when sample size is large.**

In the multivariable control charts, interpretation of out of control points are to be analyzed carefully, because, we have to identify the quality characteristics, among all considered, with regards to which, the product is out of control. This interpretation is discussed in the following section.

11.5. INTERPRETATION OF OUT OF CONTROL SIGNALS

Usually the major difficulty encountered with any multivariate control chart is a practical interpretation of out-of – control signal. That is to identify, which of the p variables or which sub-set of them is responsible for the out of control signal? This question is always is not easy to answer. This type of problem will not arise in Univeriate case, because, the quality characteristic under the analysis is only one and hence with regards to that quality characteristic the product is out of control. But it is not so easy in multi variable problems. To solve this problem, one direct procedure is to plot X – bar control charts individually for all p variables X_1 ,X_2 ,.,.,X_p. But this procedure is lengthy and should not be used. We have discussed in the last lecture, why this procedure should not be used under the heading 'need for Multivariate control charts' (section 10.4). There are many procedures to deal this situation.

Alt in the year 1985 suggested that using X – bar charts, with Bonferroni type control limits. Thaat is replace $Z_{\alpha/2}$ in the X – bar control limits calculations with $Z_{\alpha/(2p)}$. This

approach reduces number of false alarms associated with using many simultaneous univariate control charts.

Hayter and Tsui in the year 1994 extended the idea given by Alt and suggested a procedure for exact simultaneous confidence intervals. This procedure can also be used when the assumption of normality is not valid.

Jakson in the year 1980 suggested using control charts based on the principal components, which are linear combinations of the original variables. The basic disadvantage of this approach is that the principal components do not always provide a clear interpretation of the situation with respect to the original variables. However, they are often effective in diagnosing an out-of control signal, particularly in cases where the principal components do have an interpretation in terms of the original variables.

Another very useful approach to diagnosis of an out-of-control signal is to decompose T^2 statistic into components that reflect the contribution each individual variable. If T^2 is the current value of the statistic, and $T^2_{(i)}$ is the value of the statistic for all process variables except the ith one. Then Runger, Alt and Montogomery in 1996 showed that: the statistic $\mathbf{D_i = T^2 - T^2_{(i)}}$ is an indicator if the relative contribution of the ith variable to the overall Statistic representing the quality of the product based on 'p' quality variables. Calculate D_i i= 1,2,.,.,.,p and focusing attention on the variable for which D_i are relatively large. This procedure has an additional advantage that calculations can be performed using standard software packages.

11.6. SUMMARY

In this lesson we have discussed two multivariate control charts, namely, Chi-square control charts and Hotelling T^2 control charts. Further Chi-square control charts are popularly used when the quality characteristics are two. If we want to control more than two quality characteristics, we use Hotelling T^2 control chart. Further Hotelling T^2 control chart construction runs on parallel lines of X – bar control chart for single variables and in the construction of control limits of this chart, we use the Multivariate Normal distribution. We have discussed the construction for sub-group and and for individual observations. We have also discussed various procedures, to interpret the out of control signal, in multivariate Control charts.

11.7. SELF ASSESSMENT QUESTIONS

1. What is a Chi-Square control chart. Discuss its application.
2. Explain the method of construction of Chi-square control chart.
3. Explain the multivariate quality control problem with suitable example.
4. Explain the need for multivariate control charts with an example.
5. What is Hotelling T^2 Statistic? Explain its application in Multivariate quality Control.
6. Explain the method of construction of Hotelling T^2 control chart for Sub-group data.
7. Explain the method of construction of Hotelling T^2 control chart for individual observations.
8. Explain the role of β and Chi-square distributions in Multivariate control charts.

9. What is meant by Variance and covariance matrix? Also explain how do you estimate the same from a sample data.
10. Explain various methods relating to the interpretation of out-of-control signal in Multivariate Control charts.

11.8. FURTHER READINGS

1. "Introduction to Statistical Quality control", D.C. Montogomery, John Wiley (Asia) 2001.

2. "Modern methods for quality Improvement" Wordsworth, John Wiley and sons. 2002.

3. "Statistics of Quality control – Sampling Inspection and Reliability" Biswas, S. New central Book agency Pvt. Ltd., - 2003

UNIT – III
LESSON – 12

ANALYSIS OF MEANS (ANOMS)

12.0. OBJECTIVE

After going through this lesson, you should be able to understand:

- The need and application of Analysis of Means (ANOMs).
- Construction of ANOM control charts (For equal and un-equal Sample sizes).
- Construction of ANOMs for $\bar{X}$ charts.
- Construction of ANOMs for 'p' charts.

STRUCTURE:

12.1. INTRODUCTION

The shewhart control chart is a common introduction to statistical analysis of quality characteristics of an industrial output. These charts are to be used when we want to identify any Assignable Causes of variation (non-random variability) is present in the production process or not. If they are present and the remedy is known, then an adjustment of the process is to be made to remove the identified assignable cause. Analysis of Means (ANOMs) is an extension of Shewhart control chart technique and is used as an alternative tool to the Analysis of Variance (ANOVA) to compare more than two variable means at a time. The graphical representation of ANOMs is similar to 'Control Charts' discussed earlier and the interpretation is similar to the interpretation of control charts used in Statistical Quality control. Basic difference between control charts and ANOMs charts is that in the former one, we have control Limits, where as in latter one, we have decision lines, namely:

(1) Upper Decision Line (UDL) (2) Central Line (CL) and (3) Lower Decision Line (LDL)
similar to **UCL,CL and LCL** in control charts. ANOMs technique was first developed by Prof. Ott in the year 1967 for comparing a group of treatment means to see if any one

of the means $\bar{X}_i$ (i = 1,2,.,.,n) differ significantly from the overall mean $\bar{\bar{X}}$. This technique was extended by Tomlison and Lavingna in the year 1983 to Statistical Quality Control Problems, particularly to analyze percent defective data obtained from Silicon crystal growing, which is the first step in the semi-conductor manufacturing process. Parra and Loaiza in the year 2003 applied ANOMs technique to a case study data from chemical and pharmaceutical industries and demonstrated the application and powerful visualization and communication tool. Thus ANOMs are used to analyze multiple quality characteristics means in industry.

12.2. CONSTRUCTION OF ANOM CHARTS FOR $\bar{X}$ (WITH EQUAL SAMPLE SIZE)

The ANOM technique was first developed by Ott in the year 1967 and extended by Shilling in the year 1973, used for the analysis of means treatment effects. It is important to note that analysis of means (ANOM) procedure is an appropriate method in those situations where the factors involving with fixed effects and is inappropriate method for factors involving random effects. For fixed effects, the model assumes factor level means are constant. We can also consider factors with random effects, then the factor level means are random variables and hence, in that case, the aim is to estimate the variance among means rather than mean effects. Now we proceed to explain the method of construction of ANOMs for variable data with fixed sample sizes.

The one way classification model results when an experimenter obtains 'K' independent random samples of size n. Thus we have nxK observations and each of this sample is coming from k different populations. The data consists of quantitative measurements like inner diameter or outer diameter of piston ring, or stencil strength and so on, representing some quality characteristic of units from a production process. This quality characteristic is denoted by X_{ij}, represent the measurement obtained from jth unit in the ith sample. i=1,2,.,.,K and j = 1,2,.,.,n. From the given data, calculate K sample means $\bar{X}_i = \sum_{j=1}^{n} Xij$ /n , i = 1,2,.,.,K. Further, it is assumed that, these K means are from normally distributed populations with common variance denoted by σ^2. Let $\bar{\bar{X}}$ represent the grand mean and s^2 the pooled estimate of the common but unknown variance σ^2 which as defined as follows:

$$\bar{\bar{X}} = [\sum_{i=1}^{K} \bar{X}i\ ] / K = [\sum_{i=1}^{K} \sum_{j=1}^{n} Xij\ ] / Kn. \quad (12.2.1)$$

$$s^2 = [\sum_{i=1}^{K} s_i^2] / K, \quad (12.2.2)$$

$$\text{and } s_i^2 = \sum_{j=1}^{n} [Xij - \bar{X}_i]^2 / (n-1), i = 1,2,.,.,K \quad (12.2.3)$$

Then the Upper Decision Ling (UDL) and Lower Decision Line (LDL) for ANOM Chart are given as follows:

$$UDL = \bar{\bar{X}} + h_{\alpha,k,v}\, s \sqrt{(K-1)/(Kn)} \quad (12.2.4)$$

$$\text{And } LDL = \bar{\bar{X}} - h_{\alpha,k,v}\, s \sqrt{(K-1)/(Kn)} \quad (12.2.5)$$

Here, $h_{\alpha,k,v}$ represent the table value at level of significance α, k represent number of means to be compared and v represent degrees of freedom, where V = (n – 1)K. These values

are obtained from tables (given in table A-3 in appendix), and **s** is the standard deviation obtained by taking square root taken to s^2 given in equation (12.2.2).

Plot the sample means against the decision lines to obtain ANOM control chart for the sample means $\bar{X}$'s. Based on the spread of these means on the chart, we can conclude whether there is any statistically significant difference among the means. That is, if the entire sample means fall between, UDL and LDL, it is interpreted as the production process is in control or only random variability is present in the process. In other words, there is no significant difference between the means or the process is statistically in control. If a point falls outside either above UDL or below LDL, it is considered as an evidence for the presence of non-random cause or assignable cause (with risk α), in the production process.

12.3. CONSTRUCTION OF ANOM CHARTS FOR $\bar{\bar{X}}$ (WITH UNEQUAL SAMPLE SIZE)

We can extend the procedure of construction ANOM control chart with equal sample sizes explained in the previous section to the samples with unequal sample sizes simply by replacing the term 'n' by 'n_i', where n_i represent the size of the ith sample, i = 1,2,.,.,K and instead of $\bar{\bar{X}}$ we have to calculate weighted grand mean $\bar{\bar{X}}_w$ which is defined below. The procedure is explained as follows:

Let X_{ij} j= 1,2,.,.,n_i , and i = 1,2,.,.,K, represent the quality characteristic jth unit in ith sample of size n_i . Then calculate sample means and weighted grand mean as follows:
Sample means $\bar{X}_i = \sum_{j=1}^{ni} Xij / n_i$, **i = 1,2,.,.,K.** Then calculate the weighted grand mean $\bar{\bar{X}}_w$ as follows:

$$\bar{\bar{X}}_w = [\sum_{i=1}^{K} ni\bar{X}i] / N, \text{ where } N = [\sum_{i=1}^{K} ni] \quad (12.3.1)$$

Then calculate s^2, the estimate of the variance of individual observations as follows:

$$S^2 = [\sum_{i=1}^{K}(ni - 1)Si^2] / (N - K) \quad (12.3.2)$$

$$\text{Where } S_i^2 = [\sum_{j=1}^{ni} (Xij - \bar{X}i)^2] / (n_i - 1) \quad (12.3.3)$$

Is the variance of the ith sample variance I = 1,2,.,.,K.
Then calculate the estimate of the standard deviation of individual observations as $s = \sqrt{s^2}$, where s^2 is given in equation (12.3.2). Then decision lines for ANOM chart are given as follows:

$$UDL = \bar{\bar{X}}_w + h_{\alpha,k,v}\, s \sqrt{(N - n_i) / (Nn_i)} \quad (12.3.4)$$

$$\text{And } LDL = \bar{\bar{X}}_w - h_{\alpha,k,v}\, s \sqrt{(N - n_i) / (Nn_i)} \quad (12.3.5)$$

Where, $h_{\alpha,k,v}$ is obtained from tables (given in appendix) at α % level of significance, for comparing k sample means at v degrees of freedom, where v = (N – K).

12.4. COMPARISON BETWEEN SHEWHART CONTROL CHARTS AND ANOM CHARTS

The shewhart control discussed earlier and the ANOM charts discussed now, have similarities in the construction and making decisions. But they differ with each other in the sense that, while constructing control limits we use the properties of Normal distribution and calculated 3 σ control limits as:

$$\bar{\bar{X}} \pm 3\,\hat{\sigma}_x = \bar{\bar{X}} \pm A_2\,\bar{R}. \qquad (12.4.1)$$

Where as in ANOM charts we draw decision lines as:

$$\bar{\bar{X}} \pm H_\alpha\,\hat{\sigma}_x \ . \qquad (12.4.2)$$

$$\text{Where } H_\alpha = \text{Max}\ \{\ [\bar{X}_n - \bar{\bar{X}}\ ]\ /\hat{\sigma}_x\ ,\ [\bar{\bar{X}} - \bar{X}_1\ ]\ /\hat{\sigma}_x\ \}. \qquad (12.4.3)$$

It is important to note that, when K = 2, the ANOM graph is simply a graphical form of Student's t-test. This implies that the relation between ANOM chart and Shewhart control chart is similar to the relation between Student's t distribution and Normal distribution. When k is greater than 2, we have to use Analysis of Variance to test all k sample means. Thus ANOM is a useful alternative to the Analysis of Variance (ANOVA) for comparing k independent means based on multiple significance test given by Halperin et al in 1955. An ANOM chart conceptually similar to shewhart control chart, but portrays decision lines, so that both magnitude differences and Statistical Significance of the treatments may be assed simultaneously. Thus using ANOM chart, we can analyze simultaneously, two quantities, namely (1) magnitude differences between treatment means and (2) Significance difference between treatment means under consideration.

12.5. CONSTRUCTION OF ANOMS FOR 'P' CHARTS.

When the data consists of number of units having a particular attribute, then we can calculate the proportion 'p' of units having the characteristic. We can construct ANOM chart for analyzing such proportions 'p_i's, using ANOM control charts for proportions. Here we use Binomial distribution, which can be considered as approximation to Normal distribution. That is the Normal approximation to the Binomial distribution is generally considered to be adequate if np > 5 and n(1 – p) > 5, where p is the proportion of items in the population having the attribute of interest.

In acceptance sampling plans, we are interested on the proportion of defective items 'p' in the lot submitted for inspection. Now we proceed to explain the method of construction of ANOM chart for proportions.

Let $X_1, X_2, .,.,X_K$ represent number of defective items in K different samples each of size 'n' selected from the production process. Then compute K proportions p_i , where $p_i = X_i / n$ i=1,2.,.,K. Then calculate overall proportion mean $\bar{p} = \sum_{i=1}^{K} pi / K$. Then, the standard error of $\bar{p}$, That is S.E ($\bar{p}$) =S = $\sqrt{\bar{p}\ (1 - \bar{p}\ )} / n$. Then the decision lines on the ANOM chart are given as follows:

$$\mathbf{UDL} = \bar{p} + h_{\alpha,K}\, S \sqrt{(K-1)/K}. \quad (12.5.1)$$

$$\mathbf{LDL} = \bar{p} - h_{\alpha,K}\, S \sqrt{(K-1)/K}. \quad (12.5.2)$$

Where, $h_{\alpha,K}$ is the table (A-3) value at α % level of significance and K represent the number of proportions considered. The degrees of freedom is considered as infinite, because, one can approximate the Binomial distribution with Normal distribution for infinitely large values of n.

Plot the proportions p_i i=1,2,.,.,K along with the decision Lines given in equations (12.5.1) and (12.5.2). if all the proportions, fall between the decision lines, accept the hypothesis that all the K proportions are equal or the process is statistically in control or there is no evidence for assignable causes of variation. That is only random causes are alone acting in the process. If any one of the proportions fall above UDL or below LDL , we conclude that the process is out-of control or there exists significant difference between sample proportions. This implies that, 'Assignable causes of variation' are present in the production process. On similar lines, by using Poisson distribution, we can construct ANOM charts for Count Data, or number of defects per unit and construct ANOM C – chart. Now we proceed to consider one example for proportions.

Example (12.5.1): A metal containers manufacturing company at Hyderabad, studied the effect of copper concentration on the failure rate of these metal containers after storage. The experiment was conducted at three levels of copper namely, 5%, 10% and 15%. The Metal container corrosion data is given as follows:

	Level of Copper			
	5%	10%	15%	20%
Number of containers examined n:	100	100	100	100
Number of failures observed X_i :	16	45	58	62

Analyze the data through ANOM chart for proportions and draw your conclusions.

Solution: In the given example K = 4 and n = 100. Thus Calculate P_i and $\bar{p}$ as follows:

	Level of Copper			
	5%	10%	15%	20%
Number of containers examined n:	100	100	100	100
Number of failures observed X_i :	16	45	58	62
Proportions p_i :	0.16	0.45	0.58	0.62

$\bar{p} = \sum_{i=1}^{K} pi / K = [0.16+0.45+0.58+0.62] / 4 = 1.81/4 = 0.4525.$

S.E $(\bar{p})$ =S = $\sqrt{\bar{p}\,(1-\bar{p})/n} = \sqrt{(0.4525)(0.5475)/100} = \sqrt{0.00248} = 0.04977.$

For risk α = 0.01, K = 4 and degrees of freedom infinity, we have table value $\mathbf{h_{\alpha,K} = 3.01}$

Therefore, decision lines on the ANOM chart for proportions are given as follows:

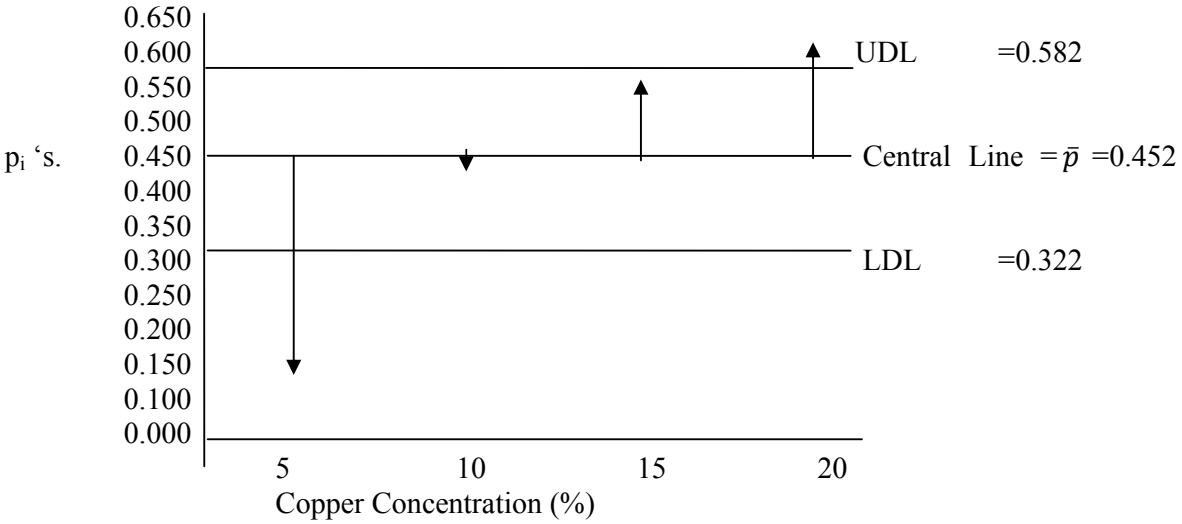
$\mathbf{UDL} = \bar{p} + h_{\alpha,K}\, S \sqrt{(K-1)/K} = 0.4525 + 3.01(0.04977)\sqrt{(3/4)}$

$= 0.4525 + 0.12974 = \mathbf{0.58224}$ (12.5.3)

$\mathbf{LDL} = \bar{p} - h_{\alpha,K}\, S \sqrt{(K-1)/K}. \quad 0.4525 - 3.01(0.04977)\sqrt{(3/4)}$

$= 0.4525 - 0.12974 = \mathbf{0.32276}$ (12.5.4)

Fig. (12.5.1): ANOM chart for proportions.



Conclusion: Critically comparing the graph we conclude that failure rate containers is increasing along with the copper level. Hence, to reduce the failure rate, prefer to fix the copper level at 5% only which can reduce the failure rate significantly in the metal cans production.

12.6. SUMMARY

In this lecture, we have discussed Analysis of Means (ANOM) charts, their construction. Further, a comparison between Shewhart control charts and ANOM charts are discussed. We have also discussed the method of construction of ANOM charts for sample means $\bar{X}$ **'s** with equal and unequal sample sizes. We have also discussed the method of construction of ANOM charts for proportions and given a hint that they can be constructed for number of defects per unit. That is ANOM chart for count Data.

12.7. SELF ASSESSMENT QUESTIONS

1. Explain the concept of ANOM chart.
2. Distinguish between Shewhart control charts and ANOM charts.
3. Explain the method of construction of ANOM chart for sample means of equal sizes.
4. Explain the method of construction of ANOM chart for sample means with unequal sizes.
5. Explain the method of construction of ANOM chart for proportions.

12.8. FURTHER READINGS

1. Ellis R. Ott " Analysis of means – A Graphical Procedure", Journal of quality Technology, Vol. 15, No.1, January 1983, pp 10 – 18.
2. Lloyd S. Nelson "Exact Critical Values for Use with the Analysis of Means", Journal of Quality Technology, Vol. 15, No.1,January 1983, pp 40 – 44.
3. Hiroshi Ohta " A procedure for Pooling Data ny the Analysis of Means", Journal of Quality Technology, Vol. 13, No.2, April, 1981, pp 115 – 119.

UNIT – III
Lesson – 13

ANALYSIS OF MEANS (ANOMS) FOR VARIABLE DATA

13.0. Objective

After going through this lesson, you should be able to understand:

- The comparison between Analysis of Variance and Analysis of Means (ANOMs).
- Construction of ANOM charts – One way classification.
- Construction of ANOM charts – Two way classification.
- Application of ANOM charts.

STRUCTURE

13.1. INTRODUCTION

When the data to be analyzed is variable data, like, life times of Units measured in hours, or outer diameter or inner diameter of piston ring measured in mm, or chemical content in a medicine or amount of copper in an alloy, we get variable data, because the quality characteristic is a variable. When such variable data is to be analyzed with respect to many (say K) sample means, usually, we apply Analysis of Variance (ANOVA). Through ANOVA, we can test the equality of several means. In recent years, the application of Analysis of means (ANOM) is effectively used to solve the economic problems of industry. ANOM can be used for Attribute data, as well as Variable data. In the last lesson, we have discussed the ANOM method applicable for attribute data namely for proportions or for Count Data. In this lesson, we discuss ANOM charts for Variable data.

13.2. ANOM CONTROL CHARTS FOR VARIABLE DATA

Analysis of means (ANOM) is the latest and popular technique developed by Ott in the year 1967 and is extensively used in solving variety of industrial problem. Basically, ANOM is an useful alternative to the analysis of variance, to compare several sample means at a time. As we have two types of ANOVA, namely, one way and two way analyses, ANOM also has two methods of analysis, namely (1) ANOM model for one-way classification or ANOM for one factor model and (2) ANOM model for two-way classification or ANOM for two factors model. ANOM charts are similar to Shewhart control charts to analyze industrial data. Through ANOM charts we can analyze two things simultaneously, namely :(1) magnitude of differences among treatment means and (2) Significance of the treatment effects. Hence, ANOM charts are more appropriate than shewhart control charts to monitor K sample means at a time. Further, it is important to note that ANOM procedure is appropriate for factors having fixed effects on the quality characteristics of the product. To study fixed effects, the model assumes that the factors level means are constant. However, for random effects, the factor level means are random variables and in that case, purpose is to analyze or estimate the variance among the treatment means, rather than studying mean effects. In this lesson, we present step-by-step instructions for performing the Analysis of Means (ANOM), with variable data relating to the quality characteristics of products / Industrial Experiments involving Variable Data. Now we proceed to explain ANOM model with one factor effect, in the following section.

13.3. CONSTRUCTION OF ANOM CHARTS FOR ONE FACTOR MODEL FOR EQUAL SAMPLE SIZES

When we want to analyze the data with one factor effect like effect of different workers in different shifts A,B and C on the production or effect of different machines M-1,M-2,M-3,M-4 from which products are produced or effect of different raw materials on the quality product, we obtain one- way classified data. Hence, one has to consider ANOM charts for one factor model. That is, the one factor model results, when an experimenter obtain K independent random samples of size n_i (i =1,2,.,.,K), further, we assume that, each of these samples came from K different populations. These K populations might be K different shifts of workers, or K different machines or K different treatments, or k different methods of production or k different raw materials used in the production, as explained earlier. Further, the data is variable data, measurable from each sampled unit like inner or outer diameter of the piston ring or abruption of moisture by a concrete mixture, or the effect of cleaning solution with different concentrations of a chemical and so on. Thus we obtain one-way classified data, where, one has to test the Statistical significance between K sample means $\bar{X}_1$, $\bar{X}_2$.,., $\bar{X}_K$ simultaneously. ANOM procedure for one factor model, runs on similar lines explained in the previous lesson for ANOM chart for sample means $\bar{X}$. The step by step procedure to carryout ANOM one-way classified data to analyze K sample means is explained as follows:

Step – 1: Compute Sample means $\bar{X}_I$'s , where $\bar{X}_i = \sum_{j=1}^{n} Xij$ /n , i = 1,2,.,.,K from the data collected.

Step – 2: Compute Grand mean $\bar{\bar{X}}$ using equation (12.2.1).

Step – 3: Compute S, an estimate of the standard deviation of the observations collected. This is the square root of S^2 , where S^2 is calculated by using equations (12.2.2) and (12.2.3).

Step – 4: Obtain the critical value h_α , from the tables at α % level of significance, for testing K means at (n – 1)K degrees of freedom.

Step – 5: Determine the decision lines using equations (12.2.4) and (12.2.5) as follows:

$$\mathbf{UDL} = \bar{\bar{X}} + h_{\alpha,k,v}\, s \sqrt{(K-1)/(Kn)} \qquad (13.3.1)$$

And $$\mathbf{LDL} = \bar{\bar{X}} - h_{\alpha,k,v}\, s \sqrt{(K-1)/(Kn)} \qquad (13.3.2)$$

Step – 6: Plot the means calculated in step- 1 along with decision lines calculated in step – 5 on a graph to obtain ANOM chart for One-way classified data.

Conclusion: If the entire sample means plotted, fall inside decision lines, we conclude tha the process is in-control that is only random causes are present in the production process and there is no evidence of 'Assignable causes of variation'. On the other hand, if any one point fall above UDL or below LDL, we conclude that the process is out of control and conclude that there is an evidence that there is one or more 'Assignable causes of variation' present in the production process and hence there is need for detecting the causes for assignable causes and are to be eliminated from the process to bring the production process under control. Now we proceed to explain the above explained procedure with an example.

Example (13.3.1): The following data represent the results of an experiment which studied on the absorption of moisture in concrete slabs, with different proportions of Iron, namely 2%, 5%, 10% 15 % and 20%. Above five different iron percentages in the concrete aggregates were studied based on six samples each to moisture for 72 hours and the moisture absorption is measured based on the increased weight %. Based on the data, determine whether there is any significant effect of % of iron, in the concrete in the moisture absorption, using suitable ANOM control charts. Draw your conclusions based on the ANOM chart drawn.

	Proportion of Iron				
Sample Number	2%	5%	10%	15%	20%
1	449	517	551	679	677
2	417	633	457	656	643
3	555	583	450	613	573
4	517	508	731	522	511
5	438	580	499	631	615
6	415	595	582	563	639

Solution: From the given data, calculate sample means, Grand mean, sample standard deviations and the estimate the sample grand sample variance as follows:

	Proportion of Iron				
Sample Number	2%	5%	10%	15%	20%
1	449	517	551	679	677
2	417	634	458	656	643
3	555	584	451	613	572
4	516	508	732	522	512
5	438	582	499	632	616

6	415	595	633	564	640
Totals :	2790	3420	3324	3666	3660
Sample Means $\bar{X}_i$:	465	570	554	611	619
Sample variances S^2_i :	3319	2303	12134	3455	3594

The grand mean $\bar{\bar{X}}$ = [465+570+554+611+610] / 5 = 2810/5 = 562.
And the grand standard deviation S^2 = [3319 + 2303 + 12134 +3455+3594]/5 = 24805/5 = 4961.
Hence S = √4961 = 70.4.

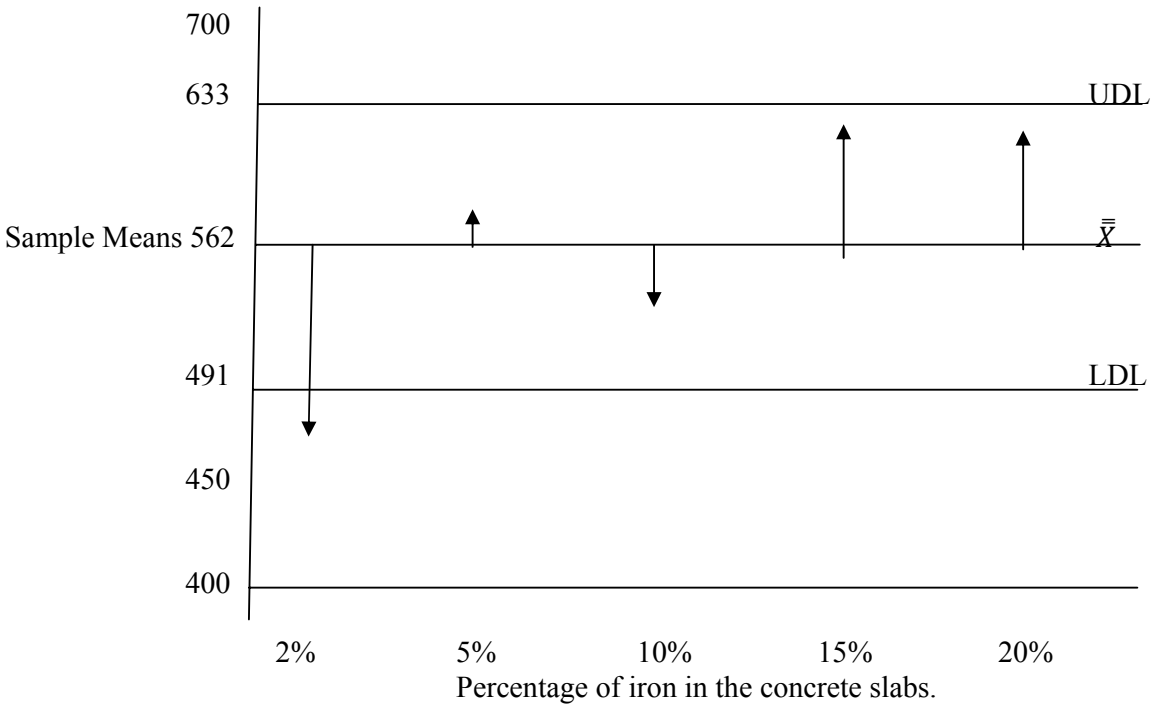
The table value h_α = 2.75 at α = 0.05 ,with k = 5 and k(n-1) 5(6-1) = 25 degrees freedom, (From the tables A-3 in the appendix).

The control limits for ANOM chart for one way classified data with equal sample sizes are:

UDL = $\bar{\bar{X}}$ **+** $h_{\alpha,k,v}$ **s** $\sqrt{(K-1)/(Kn)}$ **= 562+2.75(70.4)√(4/30)= 633** (13.3.3)

And **LDL =** $\bar{\bar{X}}$ **-** $h_{\alpha,k,v}$ **s** $\sqrt{(K-1)/(Kn)}$ **=562-2.75(70.4)√(4/30)= 491.** (13.3.4)

The ANOM control chart for one way classification with equal sample size is given as follows:
Fig. (13.3.1) ANOM control chart for one way classification.



Conclusions: It is observed from the graph that the process if out of control. Sample at 2% is below LDL hence there is evidence that 'Assignable causes of variation' is present in the process. There exists, statistically significant differences between the sample means.

13.4. CONSTRUCTION OF ANOM CHARTS FOR ONE FACTOR MODEL FOR UNEQUAL SAMPLE SIZES

We can extend the procedure of construction ANOM control chart for one way with equal sample sizes explained in the previous section to the samples with unequal sample sizes simply by replacing the term 'n' by 'n_i', where n_i represent the size of the ith sample, I = 1,2,.,.,K and instead of $\bar{\bar{X}}$ we have to calculate weighted grand mean $\bar{\bar{X}}_w$ which is defined below. The procedure is explained as follows:

Let X_{ij} j= 1,2,.,.,n_i , and i = 1,2,.,.,K, represent the quality characteristic jth unit in ith sample of size n_i . Then calculate sample means and weighted grand mean as follows:
Sample means $\bar{X}_i = \sum_{j=1}^{ni} Xij / n_i$, **i = 1,2,.,.,K.** Then calculate the weighted grand mean $\bar{\bar{X}}_w$ as follows:

$$\bar{\bar{X}}_w = [\sum_{i=1}^{K} ni\bar{X}i] / N, \text{ where } N = [\sum_{i=1}^{K} ni] \quad (13.4.1)$$

Then calculate s^2, the estimate of the variance of individual observations as follows:

$$S^2 = [\sum_{i=1}^{K}(ni - 1)Si^2] / (N - K) \quad (13.4.2)$$

Where $S_i^2 = [\sum_{j=1}^{ni} (Xij - \bar{X}i)^2] / (n_i - 1)$ (13.4.3)
Is the variance of the ith sample variance I = 1,2,.,.,K.

Then calculate the estimate of the standard deviation of individual observations as $s = \sqrt{s^2}$, where s^2 is given in equation (12.3.2). Then decision lines for ANOM chartone way classified data with unequal sample sizes are given as follows:

$$UDL = \bar{\bar{X}}_w + h_{\alpha,k,v}\, s \sqrt{(N - n_i) / (Nn_i)} \quad (13.4.4)$$
$$\text{And } LDL = \bar{\bar{X}}_w - h_{\alpha,k,v}\, s \sqrt{(N - n_i) / (Nn_i)} \quad (13.4.5)$$

Where, $h_{\alpha,k,v}$ is obtained from tables (given in appendix) at α % level of significance, for comparing k sample means at v degrees of freedom, where v = (N – K).

ANOM control chart can be drawn on similar lines explained earlier after calculating the sample means, Grand Mean, Weighted Standard Deviation and UDL and LDL. Conclusions can be drawn based on the chart obtained.

13.5. CONSTRUCTION OF ANOM CHART FOR TWO FACTOR MODEL

The analysis explained earlier, that is, ANOM chart for one factor can be extended to an ANOM chart for two factors. When, we deal with two factors say, Factor A and Factor B, we have to consider three effects, namely, (i) Main effect of factor A, (ii) Main effect of factor B and (iii) Interaction effect of factor A and factor B denoted by AXB. To calculate interaction effects, we have to consider Factorial experiments and the corresponding factorial analysis, dealt in detail in 'Design of Experiments' paper, under the topic 'Factorial Experiments'. Thus calculation of factorial effects is not discussed in this lecture, but we will broad outlines of

analyzing the three effects explained above. Further, it is necessary to collect data with both factors A,B in replicates, say 'r' replications. For example Levels of factor A are 5 and Levels of Factor B are 4 and replicates r = 3, then the data consists of 60 observations, as shown below.
Table (13.5.1) : Cross Classification or Two factor Data.

Factor A	Factor B				
	1	2	3	4	5
1	X111	X121			X151
	X112	X122			X152
	X113	X123			X153
	.				.
2	.				.
3	.				.
	.				.
4	X411				X451
	X412……				X452
	X413				X453.

In general, If Factor A has 'M' levels, factor B has 'L' levels and replicates 'R', and Xijk represent the experiment result of data relating ith level of Factor A,jth level of Factor B belonging to kth replicate. I = 1,2,.,.,M,j = 1,2,.,.,L and r = 1,2,.,.,R. Thus we obtain MxLxR observations.

Method of construction of ANOM chart for two factor Model is explained as follows:
Step – 1: First calculate Sample means$\bar{X}$ij and Sample variancesS^2ij, for i = 1,2,.,.,M and j = 1,2,.,., L, over all replicates.. Thus we obtain MxL sample means and variances table known as Means and variances Matrix of size MxL of Factors A and Factor B.

Step – 2: Then calculate Row or Factor – A means $\bar{X}$i. , Column or Factor B means $\bar{X}$.j , and grand mean $\bar{\bar{X}}$, from the table of means and variances calculated in step – 1.

Step – 3: Using equations (13.3.1) and (13.3.2) and table values at respective degrees of freedom, determine UDL and LDL and draw ANOM charts separately (i) for Rows or Factor A , (ii) for Columns or Factor B and (iii) for Interaction Effects Between factor A and factor B. Thus we obtain three sets of Decision lines and three ANOM charts.

If the plotted means on all the three ANOM charts show in control, then we conclude that the process is 'in control'. On any one chart show out of control signal, we conclude that the process is 'out of control'.

Note: Calculation of interaction effects between various factors are studied separately in the Design of Experiments paper under the topic 'Factorial Designs'. Hence, here we have not discussed the method of calculation of interaction effects between factors.

13.6. SUMMARY

In this lesson, we have discussed the method of construction of ANOM charts for Variable data involving one factor or for one way classified data. Since, ANOM charts are considered as alternative method of analysis for Analysis of Means (ANOVA), here also we obtain two types of data. Namely (i) one way classified data or one factor model and (ii) two way classified data or factors involving two factors. In two factor ANOM charts are similar to factorial designs where we can also analyze Interaction effects along with main effects of the factors. We have to construct ANOM charts separately for analyzing (i) Main effect of factor-1, (ii) Main effect of factor – II and (iii) Interaction effects of Factor – I and Factor – II.

13.7. SELF ASSESSMENT QUESTIONS

1 Explain the analogy between ANOVA and ANOM with suitable examples.
2 Explain the method of construction of ANOM chart for one factor model with equal number of observations.
3 The following data represent the effect of cleaning solution in ounces per gallon is Measured. The data is as follows:

Cleaning Solution Concentrations.

0%	10%	15%	20%	25%
15	15	19	25	32
15	14	22	28	35
16	15	21	26	31
14	16	22	27	33
16	15	21	25	32

Analyze the data using ANOM chart for one factor model and draw your conclusions.

1. Explain the method of construction of ANOM chart for one factor model with un-equal number of sample sizes.
2. Briefly outline the method of ANOM chart for two factor model.

13.8. FURTHER READINGS

1. Ellis R. Ott " Analysis of means – A Graphical Procedure", Journal of quality Technology, Vol. 15, No.1, January 1983, pp 10 – 18.
2. Lloyd S. Nelson "Exact Critical Values for Use with the Analysis of Means", Journal of Quality Technology, Vol. 15, No.1,January 1983, pp 40 – 44.
3. Hiroshi Ohta " A procedure for Pooling Data ny the Analysis of Means", Journal of Quality Technology, Vol. 13, No.2, April, 1981, pp 115 – 119.

UNIT – IV
Lesson – 14

Acceptance Sampling Plans

14.0. Objective

After going through this lesson you will be in position to understand:

- The meaning and need for Acceptance Sampling Plans, their assumptions, advantages and disadvantages.
- Analogy between Acceptance Sampling Plans and Usual Testing of Hypotheses.
- Some fundamental concepts like producer's risk and consumer's risk.
- Various types of Acceptance Sampling Plans, and
- Concepts of O.C., A.S.N,A.O.Q, A.Q.L and L.T.P.D.

Structure

14. 1.Introduction

It is important to note that '**Inspection**' is the most important activity to control the quality of any '**industrial output'**. Inspection of Raw material, Semi-finished products, or finished products before sending to the market and after reaching to the market, are essential aspects of Quality assurance by the industry / Producer / Supplier. The basic objective of such inspection is to accept or reject the product inspected. Hence such inspection procedures are popularly known as '**Acceptance Sampling Plans'**. Further, these plans are to be applied on the products supplied in the form of lots, like ship of raw material received from the mines, or lots of semi-finished products or finished products supplied to a dealer or user, a tank of petrol supplied to the petrol pump and so on. Thus, these procedures are to be applied to accept or

reject the lot supplied. Hence, these procedures are also known as '**Lot-by-Lot Acceptance Sampling Plans'**. In the following lessons (Chapters 4 and 5), various types of such plans, their design, operation and properties are discussed along with suitable examples.

14.2. The Acceptance Sampling Problem

Any product has cross various stages to reach to the customer, namely:

(1) Designing,
(2) Selection of Raw material,
(3) Production (include both semi-finished and finished products),
(4) Marketing and
(5) Transportation.

'Inspection' is a process which is to be applied in all the above mentioned stages of a product to ensure the quality of the product supplied. Acceptance Sampling is concerned with inspection and decision making regarding products, to accept or reject the same. This is the most important aspect of Statistical Quality Control known as 'Testing the quality of produced units'. This is to be done usually by the inspection department, involving qualified inspectors to determine the quality characteristics of the product, whether these quality characteristics are as per the quality standards or Not? In olden days (that is in 1930's or 1940's) these procedures are applied to only incoming products and hence are known as '**Incoming or Receiving Inspection' which is to be applied on incoming products / Raw material**.

In recent years, it has become a compulsory activity of supplier to improve their process performance which is popularly known as '**Statistical Process Control'.** Thus '**Acceptance Sampling'** is considered as a primary quality assurance tool. For example, consider a manufacturing company receives a shipment of products/raw material from a supplier. This product / raw material is frequently used in the company's manufacturing process. A sample of products / raw material is selected randomly from the supplied lot and some quality characteristics are measured from the sampled products and based on the information given by the sample, we accept or reject the lot under consideration. That is, based on the information provided by this sample, decision is taken about the lot, whether to accept the lot or reject the lot. This decision process sometimes is known as '**Lot Sentencing'**.

Accepted lots are put into production and rejected lots may be returned to the supplier / vendor. Sometimes rejected lots are recommended for 100% inspection and all the remaining products in the lot are inspected for their quality. Sometimes, rejected lots are disposed by lowering the cost, but lowering the cost does not imply the quality of the product will be increased. Because, the quality of the product is poor, and hence, the cost is reduced. It is not **a suggestible situation to accept the poor quality product since the cost is low. One has to develop the habit of rejecting poor quality products.**

It is important to note that for producing the good product of defective product, we require same time of production same raw material and same number of machine and human hours. All these will become a waste if the product manufactured is a defective

one. **Wastage is to be minimized so that production will increase and hence, the cost of production will be reduced.** This is the basic philosophy behind all statistical tools applicable in industry.

14.3. Objectives, Assumptions, Advantages and Disadvantages of Acceptance Sampling plans

Although it is customary to consider the acceptance sampling as **'Receiving Inspection'** activity, there are other uses of sampling methods applied while the product is undergoing the production. For example, the manufacturer will sample and inspect its own product at various stages of production and lots that are accepted are sent for further processing and **lots rejected are reworked or scrapped**. The following are the objectives of Acceptance Sampling Plans:

14.3a. Objectives of Acceptance Sampling Plans:

- 1. The basic objective of Acceptance Sampling Plan is **to accept the lot or Reject the lot** and not to estimate the lot's quality. These plans are designed not to estimate the lot's quality. Thus chief application of Acceptance Sampling plans to **'Sentence the lot'** under inspection and to decide the lot is eligible for the marketing or not.
- 2. Another objective of acceptance sampling is to serve as '**Audit Tool'** to ensure the output from a process conform the quality standards / requirements or Not?
- 3. Acceptance sampling Plans are designed simply to determine the lots of products are eligible for marketing or not? But not to provide any tool which provide to improve the quality of the product.

Rejected lots are to be **Scraped / reworked after 100% inspection** of the remaining units in the lot. Generally, there are three approaches to be applied for sentencing a lot. Namely:

1. Accept the lot with no inspection.
2. Accept the based on 100% inspection – that is every item in the lot is inspected and remove all the defective items found and replace them with good items. This activity is usually known as "**rectifying Inspection**".
3. Acceptance Sampling – that is a random sample of items are selected from the lot and accept or reject the lot based in the sample units selected.

Among the above, first one is not suggestible because, accept the lot without inspection is not a proper approach. Of course this can be applied when the manufacturing process so good that defective items are almost never occurring or there is no economic justification to look for a defective item. The second approach, that is 100% inspection is suggested when the component is extremely critical and passing any defectives would result a high failure involve heavy losses in subsequent stages or when the manufacturing process is of inadequate or low quality requirements. In majority of the situations, approach three, that is Acceptance Sampling is preferred. These methods are applied in the following situations.

1. When the cost of 100% inspection is extremely high.
2. When 100% inspection is technically not feasible or would require long time, such that the production schedule is affected seriously.
3. When there are many items to be inspected 100% and inspection error rate is much high that 100% inspection might cause a higher percentage of defective items / units passed than would occur with the use of acceptance sampling procedures.
4. When the testing is based on destructive Procedures. Many life testing experiments are destructive in nature. That is, the item put for the test will fail or not usable after the test. Such tests are '**Destructive tests'**.
5. When there is a need for continuously monitoring the process is necessary even though the manufacturer's production process is satisfactory.
6. When no inspection will provide an unsatisfactory quality level and 100% inspection is costly, Acceptance Sampling provide satisfactory solution to ensure the supply of quality products.

It is important to note that, acceptance sampling has no direct effect to improve the quality of the product. But, has indirect effects to improve the production process as explained below.

1. Continuous rejection of products produced from a manufacturing process makes the manufacturer to change the present production process to a high quality production process which can produce high quality products. Continuous rejection of products may provide psychological pressure on the manufacturer to produce qualitative products.

Or

2. The customer will shift to a new high quality product by rejecting the bad quality if the manufacturers supply continuously low quality products. This is because of the fact that no customer is prepared to accept bad quality products. Since he has paid the cost, he has every right to demand for a 'good quality ' product. At the same time has right to reject the bad / low quality products.

Hence, Acceptance sampling procedures have indirect effect on the quality of the products. Thus **Acceptance Sampling is a "middle ground" between the two extreme procedures namely, '100% inspection' and 'No inspection' or '0% inspection'**. Now we proceed to discuss various Assumptions of Acceptance Sampling Plans.

14.3b. Assumptions

Acceptance Sampling plan is mainly depending on the formation of lots. Basically, it is assumed that the lots are formed from a large consignment and we assume that all lots are of equal quality and there is no specific reason to doubt any lot with regards to its quality. Further it is important to note that an inspection lot is not necessarily a shipment lot. Usually, inspection lot is formed for the purpose of inspection under the supervision of quality control engineer / inspector. Several principles should be followed in forming lots. They are:

1. **Lots should be homogeneous:** The products / units in the lot should be homogeneous in all respects. That is products must be produced from the same machine, by the same operator using same / common raw material at approximately same time. For example,

all tablets produced from the same mix in the same batch by a machine. Or All cool drink bottles filled with same batch of mixture of cool drink. Usually cool drink or any mixture is prepared in large quantity and filled in small bottles by a filling machine. These filled bottles are stored in containers which can store 24, 36, 48 bottles. Each consignment can be considered as a lot. All bottles contain same juice mix prepared in the same batch on a particular day. Thus all the lots are homogeneous with regards to the proportion of ingredients, taste, color, thickness and so on. The juice mix may differ from day to day or from mix to mix. As far a s possible, lots are to be formed such that products in the lot should be uniform / homogeneous.

2. **Larger lots are preferred over Smaller ones:** It is usually preferable to have larger lots than smaller ones, because larger lots are more economically efficient to inspect than smaller ones.

3. **Lots should be conformable to the materials –handling systems used in both vendor and consumer facilities:** In addition, the items in the lots should be packed so as to minimize shipping / transportation and handling risks. Further, it should be easy to select a sample / section of units as random sample.

14.4. Analogy with Testing of Hypotheses, Producers Risk and Consumers risk

Acceptance sampling is similar to the usual testing of hypothesis in the Statistical inference. In the usual testing of hypothesis, first we frame the Null hypothesis and alternative Hypothesis. Then, collect the data of experimental observations and calculate the test statistic using the collected sample observations / data. If the test statistic is less than or equal to the theoretical value (that is the table value) we accept the Null hypothesis framed. Otherwise, we reject the null hypothesis and accept the alternative Hypothesis framed.

In acceptance sampling also, we follow the same steps as followed in usual testing of hypothesis, explained above. Here also we frame the Null Hypothesis and Alternative Hypothesis as follows:

Null Hypothesis: H_0 : The lot under consideration is a good lot. In other words, the lot is eligible for the shipment / transportation to the market.

Alternative Hypothesis: H_1 : The lot is a bad lot. In other words, the lot is rejected or not eligible for the shipment / transportation to the market.

Le the lot is of size ‘N’ items and a sample of ‘n’ items are selected from the lot and counted for number of defective items in the sample. If number of defective items are less than or equal to some pre-determined number ‘C’, we accept the null hypothesis and allow the lot for

transportation to the market. Otherwise, we reject the lot from transporting to the market. That is we accept the alternative hypothesis.

Thus, Acceptance sampling procedure is similar to the usual testing of hypothesis procedure learnt, in the Statistical Inference. Since it is nothing but the testing of hypothesis, here also two types of errors occur can occur, namely, Type – I and Type – II errors.
The probability of committing Type – I error is usually referred '**The Producer's Risk'** and the probability of committing Type – II error is referred "**Consumer's Risk'** in Quality control, which are defined as follows:

Producer's Risk: Pr[Rejecting the good lot] = Pr[Rejecting H_0 | H_0 is true] = α and
Consumer's Risk : Pr[Accepting a bad lot for market] = Pr[Accepting H_0 | H_0 is false] = β.

We know that α is called as size of the test and (1 – β) is called as the power of the test. Tests are developed by using Neymann Pearson Lemma, such that for a given α , the power of the test (1 – β) is maximum. Such tests are called '**Most powerful Tests'**. Here also, we are developing Acceptance sampling procedure, such that its power is maximum, for a given size α. That is Acceptance sampling procedures are developed such that consumer's risk is minimum, for a given Producer's Risk α. Thus acceptance sampling procedures are nothing but most powerful tests, which makes the consumer's risk as minimum as possible. Since consumer is the prime person in business, his risk is to be minimized. But producer will have a risk of α%. That is usually fixed at 5% (0.05) or 1% (0.01) level.

14.5. Fundamental concepts, various types of acceptance Sampling Plans.

Now, we proceed to discuss about some fundamental concepts and related terms, through which one can measure the performance of Acceptance Sampling plans. They are :

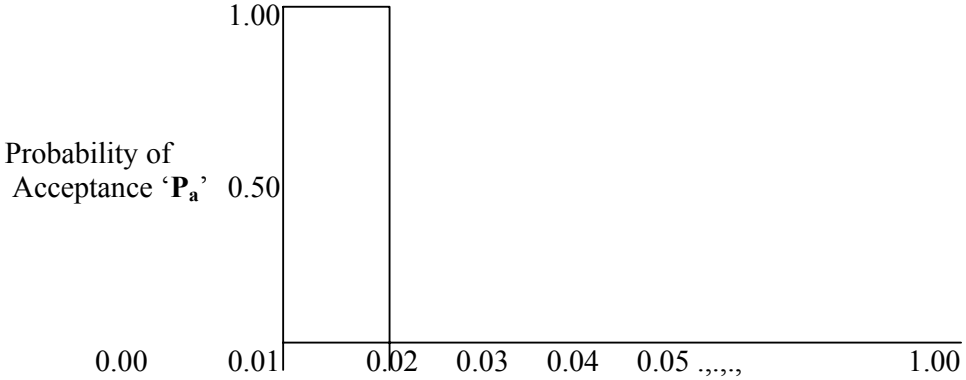
1. Operating Characteristic (OC) Curve,
2. Average Sample Number (ASN) Curve,
3. Acceptable Quality Level (AQL),
4. Lot Tolerance percent Defective (LTPD),
5. Rectifying Inspection,
6. Average outgoing Quality (AOQ) and Average Outgoing Quality Level (AOQL) and
7. Average Total Inspection (ATI) .

These concepts are to be discussed in detail before going to discuss various types of Acceptance sampling plans.

1. **Operating Characteristic (OC) Curve**: An important measure of the performance of an acceptance sampling plan is through its Operating characteristic Curve. This curve plots the probability of accepting the lot denoted by '$\mathbf{P_a}$' versus the lot fraction defective denoted by '**p**'. Thus, OC curve displays the discriminatory power of the Acceptance Sampling plan. That is, it shows the probability that a lot submitted with certain fraction defective 'p' either accepted or rejected. An ideal OC curve runs horizontally at a probability of acceptance $\mathbf{P_a = 1.00}$ until the level of the lot quality that is considered "bad" is reached, at which point the curve drops vertically to a probability of acceptance $\mathbf{P_a = 0.00,}$ and then run horizontally

again for all fraction defectives greater than the undesired level**. Thus an ideal OC curve looks like a Z – shaped curve as shown below.**

Fig. (14.5.1): Graph of an ideal OC curve.



Unfortunately, the ideal OC curve given above in fig,(14.5.1) can almost never be obtained in practice. However, we select such acceptance sampling plans, whose OC curve is closer to the Z – shape, as shown above.

Further there are two types of OC curves. Namely Type – A and type – B OC curves. The type – A OC curve is used to calculate probabilities of acceptance for an isolated lot of an infinite size. To calculate the probability of acceptance, P_a we use Hyper-geometric distribution probability mass function in the construction of Type – A OC curve. Whereas to construct the type – B OC curve, we assume that the samples came from a large lot or the sample is drawn from a stream of lots selected at random from a process. To calculate probability of acceptance P_a , we use Binomial distribution probability mass function in Type – B OC curve. It is important to note that type – A OC curve will always lie below the type – B OC curve. The type – A OC curve is for an infinite lot, however, mathematically identical with type – B OC curve. In general, type – B OC curve is used as approximation to type – A OC curve when lots are large in size, that is more than 10 times larger than the sample size. Usually it is understood that in coming lectures, OC curve means it is the type – B OC curve. The probability of acceptance for a lot calculated for the type – B OC curve will always be higher than they would have been calculated if the type – A OC curve had been used instead. However, the difference is considerably large or significant when the lot size is small when compared to the sample size. Finally, it is important to note that OC curves are purely theoretical concepts and indicate the proportion of lots that would be accepted in a series of lots from a randomly operating process or in a series of identical lots.

2. Average Sample Number (ASN) Curve:

The average sample number (ASN) curve in acceptance Sampling plans play a vital role when the sample size selected from the lot 'n' is a variable. Many quality control engineers are interested to know the ASN curve to determine number of sampling units to be selected from a

lot. Further details of ASN curves are discussed at appropriate places in future lessons. This concept is used in Sampling plans like, Double, Multiple and Sequential Sampling plans. In single Sampling plan the sample size is a constant and is determined based on the OC curve selected. But, ASN curve will play an important role when the sample size changes from lot to lot in making decisions.

3. Acceptable Quality Level (AQL):

A consumer often establishes a sampling plan for continuing supply of products, or raw-material or components with reference to a quality level known as 'Acceptable Quality Level' (AQL). The supplier has to supply products at the AQL of the consumer and AQL represents the poorest level of quality for the vendor's process that the consumer would consider to be acceptable as a process average. It is important to note that AQL is a property of the vendor's manufacturing process and it is not the property of the Sampling Plan. The consumer will often design the sampling procedure so that the OC curve gives a high probability of acceptance at the AQL. Further, it is important to note that, the AQL is not usually intended to be a specification on the product, nor is it a target value for the supplier's production process.

It is simply considered as the **Standard value**, against which the lot's quality is judged.

4. Lot Tolerance percent Defective (LTPD):

Usually, the consumer will also be interested in the other end of the OC curve. That is, in the protection that is obtained for individual lots of poor quality. In such situations, the consumer may establish a '**Lot Tolerance Percent Defective (LTPD)**'. The LTPD represents the poorest level of lot's quality, that the consumer is willing to accept the individual lot. That is he is prepared to adjust in accepting the lot. This represents the maximum number of percent defectives in the lot that the consumer can tolerate and accept the lot as good quality. Further, this LTPD is also not the characteristic of the Acceptance sampling Plan but, usually, arrived through an agreement between the Producer and the consumer. Thus LTPD represents the poorest level of lot's quality that the consumer is prepared to adjust / tolerate. This is because of the fact that, no supplier can supply 100% defect free units. Because of random causes of variation, however careful the supplier is, 0.27% defective items are inevitable. Even though the process is under control, using the Normal Probability Law, it is a fact that 0.27% items will be defective. Still we consider that the process is under control and allow the process to continue production. Thus any supplier cannot supply all 100% defective free products. To make this possible, if at all any defective items reaches to the consumer with manufacture defects, the items are replaced with good ones, at free of cost. Or some supplier will give guarantee period to the products that '**Any manufacture defective items are reached, to the customer, will be replaced with a defect free or good unit at free of cost and transportation within a specific period of time**'. LTPD is also sometimes called as "**Rejectable Quality Level (RQL)**" or "**Limiting Quality Level (LQL)**".

5. Rectifying Inspection:

In acceptance Sampling procedures, it is an important concept to discuss what happens? When a lot is rejected? Usually rejected lots require a corrective action after 100% inspection of remaining units in the lot. Thus requires extra testing cost and time to check all the units in the rejected lots. After 100% inspection and discovering all the defective units in the lot, replace the defective items with good ones and send the lot for transportation to the market.

Thus the rejected lots will become 100% good lots and hence, will become better quality lots. Only thing is that the producer has to spend extra testing cost and require additional inspectors to inspect all the remaining units in the rejected lots. Thus some delay might happen to inspect all units of rejected lots. Such inspection of replacing defective units with good ones or defect free units is known as '**Rectifying Inspection'**. Thus rectifying inspection will improve lot's quality to a better level. This makes the fraction defective 'p' in the rectified lot will become Zero. Defective units found can be reworked if possible, or scrapped. Thus on the average, the quality level of the lots will improve with rectifying inspection. Rectifying inspection programs can be used either at receiving inspection, in-process inspection of semi-finished products or at final inspection of finished goods. The objective of in-plant usage of Rectifying inspection is to give assurance regarding average quality of material used in the next stage of production or manufacturing operation of products. Thus Rectifying inspection can be used at every stage of the production process.

6. Average outgoing Quality (AOQ) and Average Outgoing Quality Level (AOQL):

Average Outgoing Quality (AOQ) is widely used when we adopt the rectifying inspection in acceptance sampling plans. The AOQ is the quality of the lot that would be obtained over a long sequence of lots from a manufacturing process with fraction defective 'p'. It is simple to develop the formula for AOQ. For example, consider lots of size 'N' and a sample of 'n' items are drawn randomly from N items and counted for number of defective items in the selected sample. Then we have:

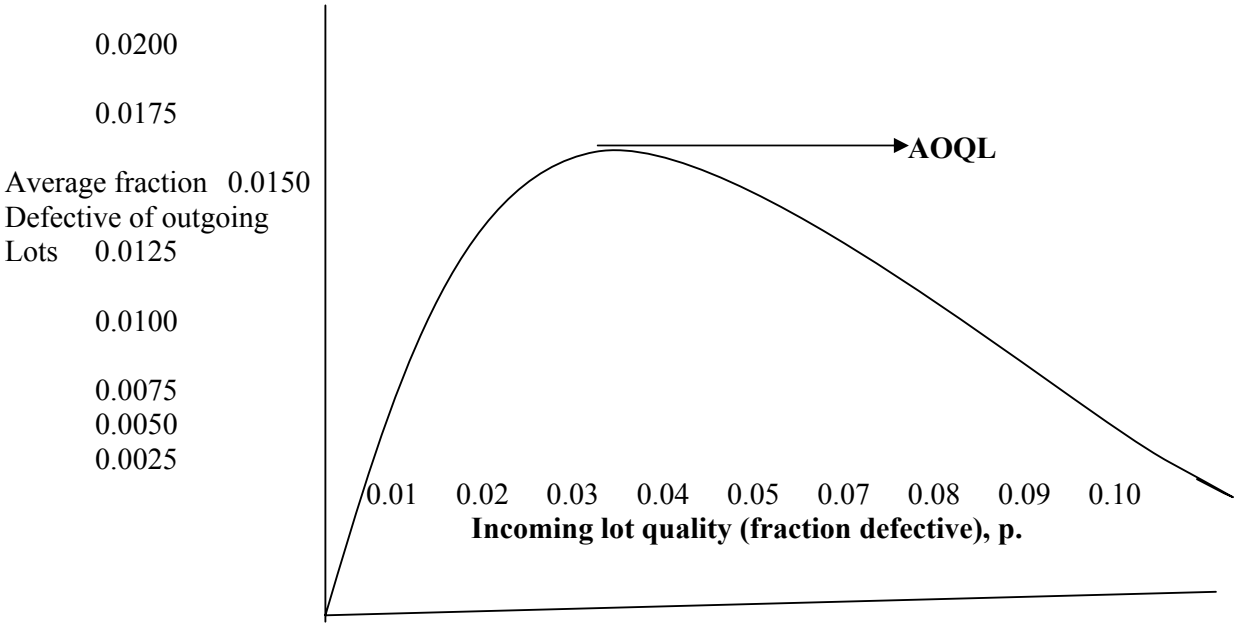
1. Selected 'n' items in the sample , after inspection contains no defective items, because all the defective items discovered in the sample are replaced with good ones.
2. If the lot is rejected, all (N – n) items contain no defective items. This is because, rejected lots are inspected 1005 and units found defective are replaced with good ones.
3. If the lot is accepted, number of defective items in the lot un-sampled are p(N – n), because, the process fraction defectives is 'p'.

Thus expected number of defective items in the outgoing lots are $\mathbf{P_a p(N - n),}$ where P_a represent the probability of accepting the lot under Acceptance Sampling Plan. Thus Average Outgoing quality (AOQ) through an average fraction defective as follows:

$$\mathbf{AOQ = [P_a p(N - n)] / N} \qquad (14.5.1)$$

The AOQ will vary as the fraction defective of the incoming lots varies. The curve that plots AOQ against incoming lots quality is called AOQ Curve. If we draw the AOQ curve, it reaches to a maximum point and falls as shown in the following figure (14.5.2). That maximum point that AOQ curve reaches is known as Average outgoing Quality Limit (AOQL), which gives the upper limit of lot's quality in a long run that a supplier can supply products.

Fig. (14.5.2): Example of an AOQ Curve and AOQL.



7. Average Total Inspection

Another important measure relative to rectifying inspection is the total number of units inspected under a given acceptance Sampling plan, when the rectifying inspection is used.

If no defective items are there in the lots, then no lot will be rejected and the amount of inspection per lot is the sample size 'n' itself. If the supplied lots contain all 100% defective items, then entire lot of N items are to be inspected, because 1005 inspection is applied. If the lot's quality is in between $0 < p < 1$, the average amount of inspection per lot vary between 'n' and 'N'. If the lot is of quality p and the probability of accepting the lot is P_a . Then, the average total inspection (ATI) per lot will be:

$$\mathbf{ATI = n + (1 - P_a)(N - n).} \qquad (14.5.2)$$

ATI play an important role when the testing procedure is destructive one. Further it helps to estimate the inspection time and number of inspectors required in the inspection department.

14.5a. Types of Acceptance Sampling Plans

There are many ways of classifying the Acceptance Sampling Plans. One major classification is (1) Acceptance sampling Plans for Attributes and (2) Acceptance sampling plans for Variables.

Attribute Sampling plans are further classified into (i) Single sampling plans, (ii) Double sampling plans (iii) multiple sampling plans and (iv) Sequential Sampling plans.

Variable sampling plans are further divided into (i) plans that control the lot or process fraction defective, (ii) Plans that control lot or process parameter (Usually the Mean), (iii) Continuous Sampling plans (iv) Chain Sampling plans and (v) Skit Lot Sampling plans.

Another type of classification of Acceptance sampling plans is: (1) Civil Sampling plans and (2) Military Standard (MIL STD) plans.

There is third type of classification of Acceptance sampling Plans, namely (1) AOQL based plans and (2) LTPD based plans.

Details of these plans are discussed in the forth coming lessons.

14.6. Summary

In this lesson, we have discussed the need, assumptions and applications of acceptance Sampling Plans. The analogy between acceptance sampling plans and usual testing of hypothesis is discussed and the important concepts of (1) Producer's Risk and (2) Consumers Risk are defined. Some fundamental concepts like OC, ASN, Curves, AOQ, AOQL, LTPD and ATI are discussed. An important concept of 'Rectifying Inspection' is introduced and discussed related topics. Finally, various types of Acceptance Sampling plans along with their classifications are discussed.

14.7. Self Assessment Questions

1. Explain the need of Acceptance Sampling Plans with suitable Examples.
2. Explain Producer's and consumer's risk with suitable examples.
3. Explain the analogy between Testing of Hypothesis and Acceptance Sampling Plans.
4. Explain various precautions to be taken in the formation of lots.
5. Explain the importance of OC curve and explain an Ideal OC Curve.
6. Distinguish between type – A and type – B OC curves.
7. Explain the terms (1) AQL (2) LTPD (3) AOQ and AOQL (4) Rectifying Inspection.
8. Explain the importance of ASN and AIT curves in Acceptance Sampling plans.
9. Explain the role of inspection in controlling the quality.
10. Explain various classifications of Acceptance Sampling Plans.

14.8. Further Readings

1. Montgomery, D.C., "Introduction to Statistical Quality Control", John Wiley (Asia) 2001.

2. Biswas.S., "Statistical Quality Control-Sampling Inspection and Reliability", New Central Book Agency Pvt. Ltd. 2003.

3.Wordsworth "Modern Methods for Quality Improvement", John Wiley and sons 2005.

4.Acheson J. Duncan "Quality control and Industrial Statistics",D.B. Taraporevala Sons & co. 1970

UNIT – IV
Lesson – 15

Attribute Sampling Plans

15.0. Objective

After going through this lesson you will be in position to understand:

- The meaning and need for Attribute Sampling Plans, their applications.
- Single sampling plan for attributes.
- Derivation of OC and ASN functions for Single sampling Plan.
- Designing a Single sampling Plan with a specified OC Curve.
- Double Sampling Plans, Comparison with single sampling plan.
- Derivation of OC and ASN functions.

Structure

15.1 Introduction

Here, we consider the situation where the quality characteristic of the product undergoing inspection is an attribute. That is, the product is dichotomized as 'good' or 'bad'; 'defective' or 'non-defective'; 'conformance to Standards' or 'Non-conformance' and so on, based on one or more quality characteristics, which are attributes. Such inspection procedures are popularly known as "**go – no – go"** basis test procedures to be applied in the inspection process. That is in go – no – go test procedures, we allow the product to 'go' in it is good and 'no' if the product is defective or bad or non-conforms to standards. Hence such test procedures are known as **"go – no – go"** inspection procedures. In such Procedures, we allow the stream of products to pass

through a gate designed as per requirements. For example, diameter of outer piston ring, or diameter of the bullet and so on, a gate of specified dimension is fixed and allow the products to pass through that gate. If the products are not up to the standards, the product cannot pass through the gate and hence can be removed from the stream as a defective product. Such test procedures are called "go – no – go" inspection procedures. When such inspection procedures are in force, one can apply attribute sampling plans.

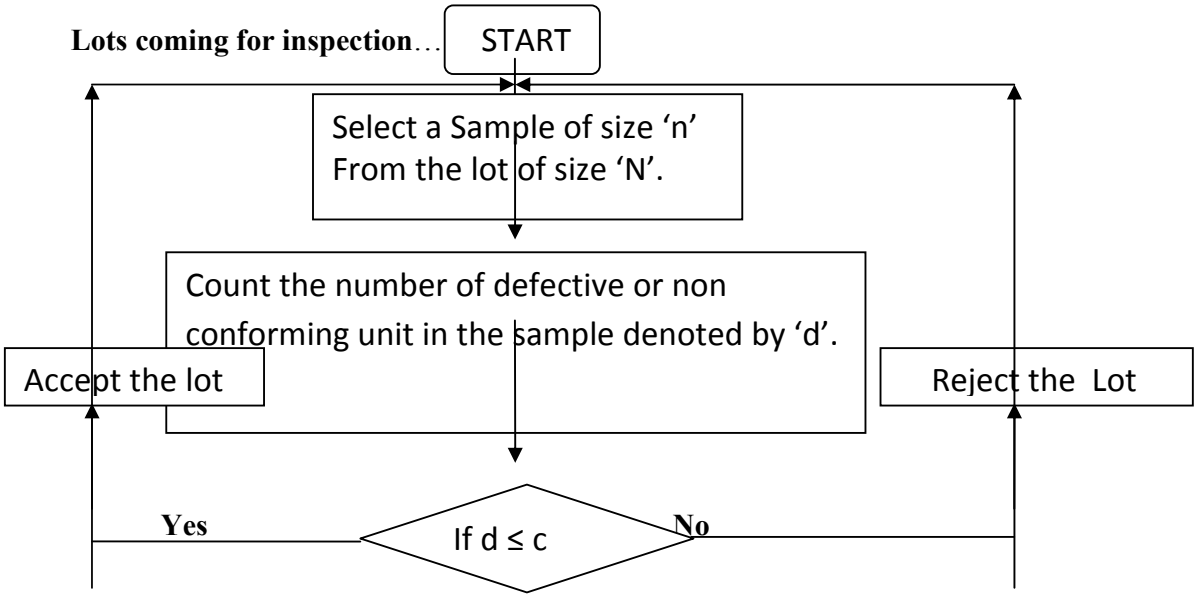
15.2. Definition of Single Sampling Plan

Let the lot of size 'N' is submitted for inspection. A single sampling plan is defined by the sample size 'n' and the acceptance number 'c'. The procedure of Single Sampling Plan is explained as follows:

Step – 1: Select a random sample of 'n' items from the submitted lot of size 'N'.
Step – 2: Count the number of non-conforming or defective units in the selected sample and denote this by a random variable 'd'.
Step - 3: If number of defective units detected in the sample, d is less than or equal to the Acceptance number c ('d ≤ c '), we accept the lot for transportation to the market or allow the lot to the next stage of production.
Step – 4: If number of defective units detected, 'd' is greater than the acceptance number 'c' (d > c) we reject the lot from sending to the market of to the next stage of production.

Above explained procedure is schematically represented as follows in fig. (15.2.1).

Fig. (15.2.1): Schematic Representation of Single Sampling Plan.



Here, the sample size **'n'** and the acceptance number **'c'** are called the parameters of the Single Sampling plan.

Example (15.2.1): Let N= 500 , n =50 and c = 3. Then select a random sample of size 50 from the given lot and count number defective items in the sample of size 50. This is denoted by 'd'. If d = 0,1,2 or 3. Then replace these defective items found with good items and accept the lot of 500 units for marketing or pass to the next stage of production. If number of defective items detected is greater than d, then reject the lot and recommend for 100% inspection. After the 100% inspection, that is inspecting all the remaining 450 units, replace all the defective units with good ones and send the lot to the market or to the next stage of production in the process.

This procedure explained in the example (15.2.1) is called **'Rectifying Inspection Scheme'.**

15.3. Derivation of OC and ASN functions of Single Sampling plans.

It is already mentioned that the Operating Characteristic (OC) curve play an important role to determine the performance of the Sampling plan under consideration. Further, to draw the OC Curve, consider the lot's quality 'p' on X-axis and the probability of accepting the Lot 'P_a' on Y – axis and draw the curve for various points of the pair of points (p.P_a). In order to calculate the probabilities, P_a , we proceed as follows:

Let the size of the lot is denoted by 'N', submitted for inspection. Further we assume that the 'N' is large (theoretically infinite). Then, the number of defectives 'd' in a random sample of size 'n' items, follow Binomial Distribution with parameters 'n' and 'p', where, p is the fraction defective of items in the lot. Thus, theoretically we can apply Binomial distribution to calculate the probabilities acceptance of the lot. This is because of the fact that the procedure is similar to draw lots of size 'N' items at random from a theoretically infinite process and then to draw random sample of 'n' items from these lots. Thus, sampling from the lot in this manner is equivalent of sampling from the process directly. Then, the probability of detecting exactly 'd' defective items in the sample of 'n' units is given by:

$$P[\text{ d defectives}] = f(d) = \binom{n}{d} p^d (1-p)^{n-d}, \quad d = 1,2,.,.,n. \qquad (15.3.1)$$

$$= [\, n! / d!\,(n-d)!\,]\, p^d\, (1-p)^{n-d}, \quad d = 1,2,.,.,n. \qquad (15.3.2)$$

Above equation is the probability mass function of the Binomial Distribution.
Thus the probability of accepting the Lot =

$$\mathbf{P_a = P[\, d \le c\,] = \sum_{d=0}^{c} \binom{n}{d} p^d (1-p)^{n-d}.} \qquad (15.3.3)$$

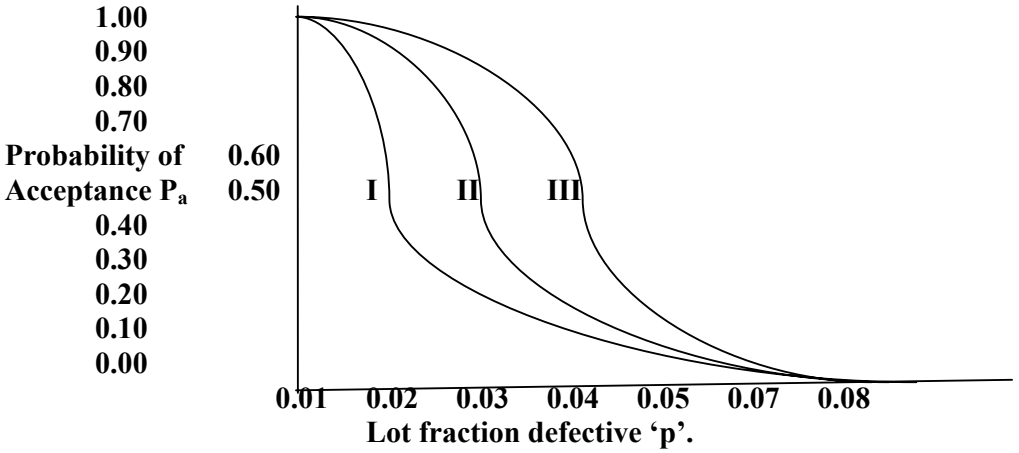
Example (15.3.1): For example consider N = 500, n = 50, c = 3 and let p 0.01. then the probability of acceptance P_a is calculated as follows:

$$P_a = P[\, d \le 3\,] = \sum_{d=0}^{3} \binom{50}{d} (0.01)^d (0.99)^{50-d}.$$

$=[50! / 0!50!] (0.01)^0(0.99)^{50} + [50! / 1!49!] (0.01)^1(0.99)^{49} + [50! / 2!48!] (0.01)^2(0.99)^{48}$
$+[50! / 3!47!] (0.01)^3(0.99)^{47}$
$= 0.60501+50(0.01)(0.6112)+1225(0.00010)(0.61729)+19600(0.000001)(0.62353)$
$=0.60501+0.30560+0.07562+0.01222 =$ **0.99845**.

On similar lines, we have to calculate 'p_a' values for each value of 'p' and then plot each pair (p,P_a) to get the OC curve. Thus an OC curve looks as follows:

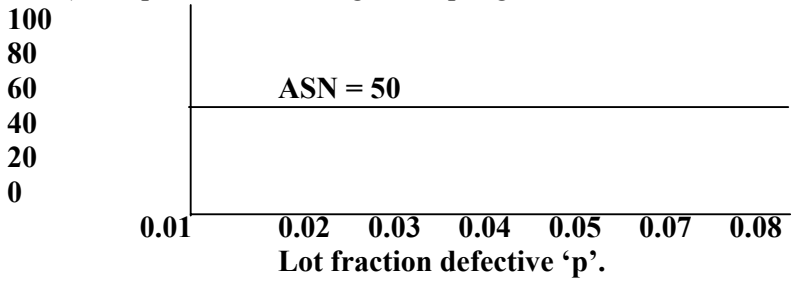
Fig. (15.3.1) OC curves for Sampling plans I,II and III.



In the above graph since, Sampling plan I is closer to Z shape, we prefer Sampling plan I to other two plans II and III.

15.3a. ASN Curve: Average sample number in the single sampling Plan is fixed, at 'n' we get ASN curve as straight line at n. In single sampling plan, calculation of ASN is simple, because sample size 'n' is determined by the plan itself. Hence the curve will be straight line as shown in the following figure.

Fig. (15.3a.1): Graph of ASN in Single Sampling Plan.



15.4. Designing a Single Sampling Plan with a specified OC curve.

In general, it is desirable to determine the sampling plan satisfying the given shape of OC curve. That is we have to determine the parameters of the Single Sampling Plan, namely the sample size 'n' and the acceptance number 'c', whose OC curve is designed / pre-determined. To do this, it is required that the OC curve passes through two designated points.

It is important to note that one point is not enough to fully specify the sampling plan; however, two points are sufficient. In general, it does not matter, which two points are specified. Suppose that we wish to construct a sampling plan such that the probability of acceptance is
$1 - \alpha$, for lots with fraction defective p_1 , and the probability of acceptance is β for lots with faction defective p_2 . Assuming Binomial distribution, for type – B OC curve, we can determine the values of 'n' and 'c' using the following two equations. Namely:

$$\mathbf{1 - \alpha} = \sum_{d=0}^{c} \binom{n}{d} p1^{d} (1 - p1)^{n-d}. \qquad (15.4.1)$$

$$\mathbf{And\ \beta} = \sum_{d=0}^{c} \binom{n}{d} p2^{d} (1 - p2)^{n-d}. \qquad (15.4.2)$$

Using the above equations (15.4.1) and (15.4.2) we can form two simultaneous equations involving two constants 'n' and 'c'. Solving these two simultaneous equations, we can determine the values of the parameters 'n' and 'c' for the Single Sampling Plan. Thus a single sampling plan can be designed, for a specified / given OC curve.

To solve above equations, many simple methods are available. For example, we can use '**binomial nomograph'**, representing binomial probabilities and probability of c ,or fewer occurrences in 'n' trials (p), we can determine the values of the constants 'n' and 'c'. In addition to the above graphical procedure, we can also use tabular procedure, explained by Duncan in the year 1986, for determining sampling plans for a specified OC curve.

Even though, any two on the OC curve can be used, to determine the parameters of the acceptance sampling plans, it is convenient to use AQL and LTPD points for this purpose. Thus consider p_1 = AQL and p_2 = LTPD, the parameters of the sampling plans can be determined. This is more appropriate because, we are taking into account the two important risks points, namely Producer's Risk point and Consumers Risk point, to determine the parameters of the sampling plan. That is, in the above equations (15.4.1) and (15.4.2), α represents **Producer's Risk** and β represents the **Consumer's Risk**.

Thus one can design a sampling plan, by choosing the parameters of the plan, namely 'n' and 'c' for a single sampling plan, such that one can obtain the OC curve, which c an pass through the desired points. This helps the consumer to put psychological pressure on the manufacturer, to supply the products according to the standards specified.

15.5. Double Sampling Plan and comparison with single sampling plan.

The idea of single sampling plan for attributes can be extended in many ways. One of such extensions is '**Double Sampling Plan'**; the procedure of sentencing the lot based on a single sample may be extended to two samples. That is a double Sampling Plan is defined as a procedure, under certain circumstances stated, a second sample is required before the is sentenced. A double Sampling plan is specified by the five parameters, namely:

n_1 = Size of the first Sample.
n_2= Size of the Second Sample.

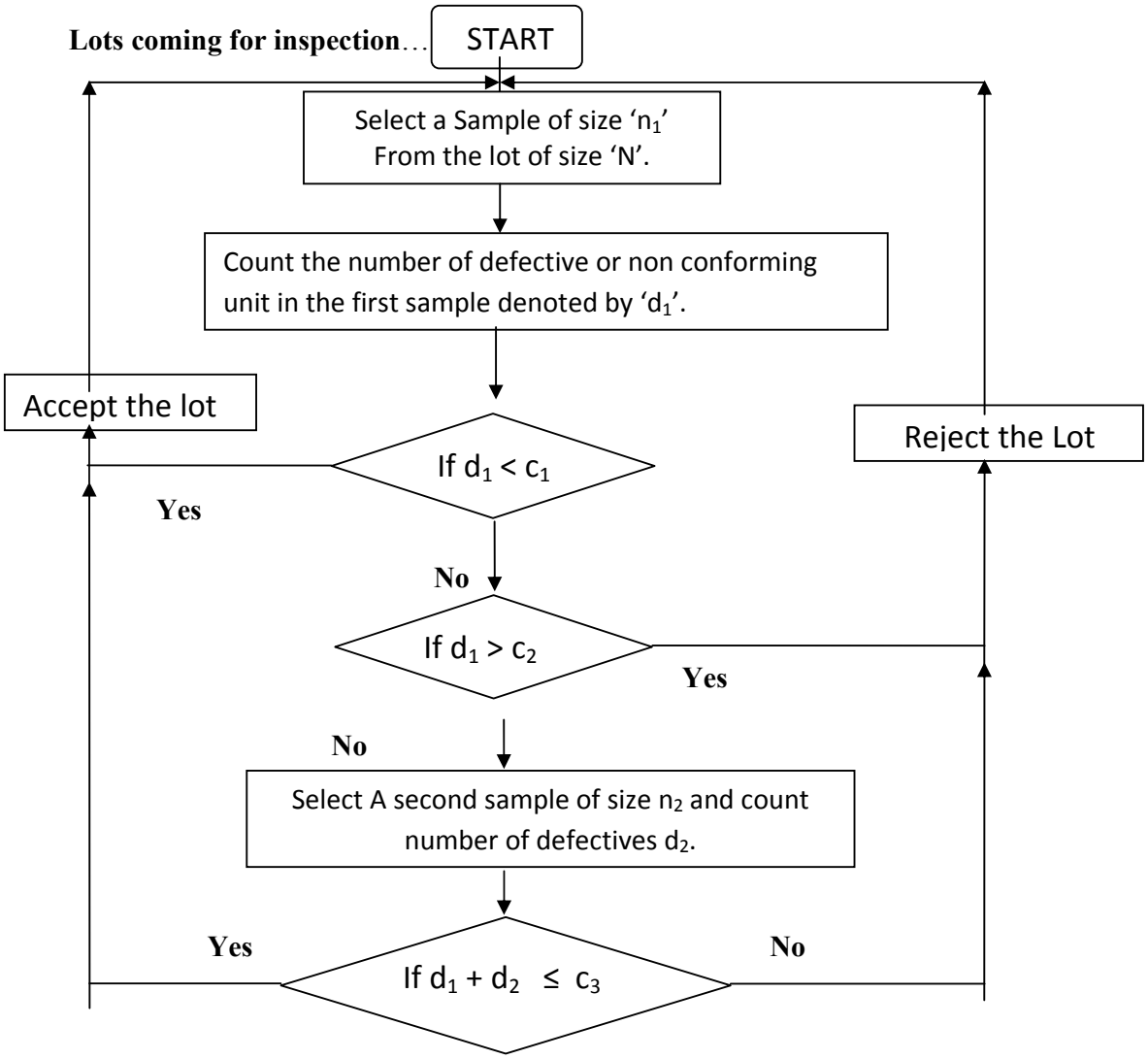
c_1 = Acceptance number of the first sample,
c_2 = Acceptance number of the second sample and
c_3 =. Acceptance number for double sample.

Double Sampling procedure is explained as follows:

Select first a Random sample of size n_1 from the lot of size N and search for number of defective items in the first sample selected, denote this detected items by d_1 . If $\mathbf{d_1 < c_1}$, We accept the lot for marketing. If $\mathbf{d_1 > c_2}$, we reject the Lot and sent for 100% inspection. If $\mathbf{c_1 < d_1 < c_2}$, we select another sample of size n and search for number of defective items denoted by d_2 . If $\mathbf{d_1 + d_2 \leq c_3}$, we accept the lot. If $\mathbf{d_1 + d_2 > c_3}$, we reject the lot. In practice, usually c_3 is considered as c_2 itself.

The basic advantage of Double sampling Plan over the single sampling plan is that it may reduce the total number units to be inspected. Further, it is giving a second chance to the lot, before rejecting straight away using a single sample. The, double sampling procedure explained above represented schematically as follows:

Fig. (15.5.1): Schematic Representation of Double Sampling Plan.



Example (15.5.1): Let N = 600, n_1 =30 n_2 = 60 c_1 = 2 c_2 = 4 c_3 = 4, then the procedure of double sampling Plan is explained as follows:

Select, first a random sample of 30 items from the lot of 600 items and count for number of defectives in the first sample denoted by d_1 . If d_1 is less than c_1 =2 , accept the lot. That is, if the first sample contain, 0 or 1 defective items, accept the lot.

If the first sample contain 3 defective items, instead of simply rejecting the lot (as in the single sample case), we take another sample (second time) of size 60 from the remaining (N – n) that is (600 – 30) = 570 units and count number of defective items in the second sample denoted by d_2 . If total number of defective items in the first and second sample put together, that is d1 + d_2 is less than or equal to c_3 = 4, we accept the lot. Otherwise, that is, if $d_1 + d_2$ is greater than 4, we reject the lot. Thus total sample size will become 30 items, if, number of defectives items are 0 or 1. If, number of defectives is equal to 3, then, we take another sample of 60 items, that is, total sample became 90. Thus sometimes decision is taken, on a sample of 30 and sometimes decision is taken based on 90 items.

15.5a: Advantages and disadvantages of Double sampling Plans.

The following are advantages and disadvantages of Double Sampling Plans, when compared with single Sampling Plans. The principle advantage of Double sampling plan over Single sampling plan is that, it may **reduce the total number of inspected units / items**. Suppose that the first sample taken under double sampling is smaller than the sample that would be required using the single sampling plan, that offers the consumer the same protection. In all cases, where the decision is taken based on the first sample only, reduces the number of items inspected and hence, **reduces the inspection cost** when compared to the single sampling plan. It is also possible to reject the lot without completing the second sample of size n_2 units. As and when number of defective units reaches the number c_3, we can stop sampling and **reject the lot** and recommend for 100% inspection. This technique of reducing the second sample size is called '**Curtailment on the Second sample'**. Thus, use of Double sampling often result in reducing the total number of inspection units and hence reduces the **inspection cost and time**. The double sampling plan has another advantage over single sampling plan that, it provides a second chance to the lot before rejection. Thus double sampling plan has the psychological advantage of providing a second chance to the lot to prove its worth. This may have some satisfaction to the supplier or the manufacturer because decision is taken not based on single sample but it is based on the second sample also.

Double sampling plans have the following disadvantages. They are:

(1) Usually double sample will result larger number of units to be inspected, when compared to single sampling plan. Thus increases the inspection cost and time in some situations, to obtain same protection provided by single sampling plan.

(2) Double sampling plan, is not economical, requires more inspection time and also costly which increases the cost.

(3) It should be used very carefully; otherwise there is a chance of increasing inspection time and cost. Unless otherwise, we use Curtailment on the second sample, we cannot reduce the inspection units to be inspected.
(4) Double sampling plans, sometimes create administrative problems in storing and handling the units selected by the first sample. These units have to wait until decision is taken, which is taken, after the selection of the second sample. Units selected in the first sample, has to wait until final decision is taken.
(5) Calculation of OC and ASN and ATI are complicated, when compared to single sampling plans.

Now we proceed to derive OC and ASN functions / curves for the Double Sampling Plan schemes.

15.6. Derivation of OC and ASN functions and design of Double Sampling Plan.

As we have derived OC and ASN functions for single sampling plans earlier, on similar lines, now we proceed to derive the same for double sampling plans. The method of derivation will be slightly complicated than single sampling plan, because while calculating the probabilities, we have to consider situations under both samples. Here also, the performance of double sampling plan is summarized by means of it Operating Characteristic (OC) curve.

Now we proceed to derive the type – B OC curve for double sampling plan as follows:

Let P_a represent the probability of accepting the lot. This can occur in two ways, namely, the lot can be accepted based on the first sample, this can occur when $d_1 < c_1$, and this probability is denoted by P^I_a . Similarly, the lot can be accepted based on the second sample and this probability is denoted by P^{II}_a . Then, we have: $\mathbf{P_a = P^I_a + P^{II}_a}$. Let 'p' represent the lot faction defective. Then, $P^I_a = \sum_{d1=0}^{c1} \binom{n}{d1} p^{d1}(1-p)^{n-d1}$ (15.6.1)

To obtain the probability of acceptance of the lot on the second sample, we must list all possible number of ways the second sample can be obtained. The second sample is drawn, only when $\mathbf{c_1 < d_1 \leq c_2}$. Then calculate the probability of $\mathbf{d_1 + d_2 < c_3}$, and then calculate P^{II}_a , using binomial probability mass function . Adding P^I_a and P^{II}_a , we obtain the value of P_a . Plot (p, P_a) values on the graph to obtain the OC curve for the given Double sampling plan. Thus it can be observed that the calculation of OC curve, for double sampling plan involve more calculations, when compared to the single sampling plan.

The above explained procedure is explained with the following example:
Example (15.6.1): Calculate the probability of accepting the lot p_a for the double sampling plan with the parameters $n_1 = 50$, $n_2 = 100$, $c_1 = 1$, $c_2 = 3$ and $c_3 = 3$. Consider p =m 0.05.
Solution: We know that: $\mathbf{P_a = P^I_a + P^{II}_a}$,
Where, $\mathbf{P_a}$ = Probability of accepting the lot based on both the samples.
$\mathbf{P^I_a}$ = Probability of accepting the lot based on the first sample.
$\mathbf{P^{II}_a}$ = Probability of accepting the lot based on the second sample.

Using Binomial probability mass function, we can calculate the probabilities as follows:

$\mathbf{P^I_a} = \sum_{d1=0}^{1} \binom{50}{d1}(0.05)^{d1}(0.95)^{50-d1}$ = (0.076940) + 50(0.05)(0.08099)

= 0.076940 + 0.20248 = 0.27942.

Similarly, $\mathbf{P^{II}_a}$ is calculated as follows:

The need for second sample is when $1 < d_1 \leq 3$. Thus we have the following two cases to accept the lot based on defects in the first and second samples.

Case – I: Let $d_1 = 2$ and $d_2 = 0$ or 1, so that $d_1 + d_2 \leq 3$ and $n_2 = 100$.

$P[d_1 = 2, d_2 \leq 1] = P(d = 2) \times P(d_2 \leq 1)$

$= [50! / 2!48!] (0.05)^2 (0.95)^{48} \times [\sum_{d2=0}^{1} \binom{100}{d2}(0.05)^{d2}(0.95)^{100-d2}]$

$= [1225 (0.0025) (0.08526)] + [0.00592 + (100)(0.05)(0.00623)]$

$= (0.26111)(0.03115)$

$= 0.00813.$

Case – II: Let $d_1 = 3$ and $d_2 = 0$ such that $d + d \leq 3$. Then,

$P[d_1 = 3 \text{ and } d_2 = 0] = P(d_1 = 3) \times P(d_2 = 0)$

$= [50! / 3!47!] (0.05)^3(0.95)^{47} \times [100! / 0! 100!] (0.05)^0(0.95)^{100}$

$= 19600(0.00013)(0.08974) \times (0.00592) = (0.22866)(0.00592) = 0.00135.$

Hence $P^{II}_a = P(\text{Case} - 1) + P(\text{Case} - II) = 0.00813 + 0.00135 = 0.00948.$

Therefore $P_a = P^I_a + P^{II}_a$ = 0.27942 + 0.00948 = 0.2889.

To draw the OC curve, calculate other points on the OC curve, on similar lines explained in the above example (15.6.1). In the example (15.6.1) we have calculate one pair of point (p, p_a), that if $p = 0.05$, $p_a = 0.2889$. like this, we have to calculate various pairs of points, to draw the OC curve for double sampling plan.

Now we proceed to explain the Average Sample Number (ASN) Curve for double sampling plan, as follows:

We have discussed in single sampling plan, the ASN curve is a straight line because the sample size n is a constant. But, in double sampling plans sample size is not a constant. Sometimes, it is n_1 and sometimes it is $n_1 + n_2$, based on the value of d_1. That is, the probability of drawing a second sample varies with the fraction defective 'p' in the incoming lot.

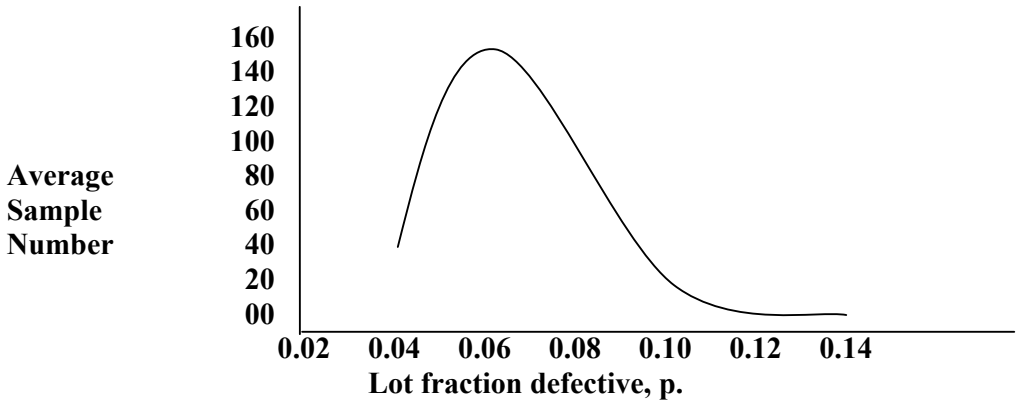
Further, with complete inspection of the second sample, the average sample size in double sampling plan is equal to the size of the first sample times the probability the there will only one sample plus the size of the combined sample times the probability that a second sample will be necessary. Therefore, a general formula for the average sample number (ASN) in double sampling, if we assume curtail sampling is not used, we have:

$$\mathbf{ASN = n_1 p_1 + (n_1 + n_2)(1 - p_1) = n_1 + n_2 (1 - p_1)} \qquad (15.6.2)$$

Where, p_1 is the probability of making a lot disposing decision on the first sample.
That is a lot is disposed only when it is either 'accepted' or 'rejected'. Thus we have:
P(lot disposed on the first sample) = p_1 = P(lot is accepted on the first sample) +
P (lot is rejected on the first sample) = P($d_1 < c_1$) + P ($d_1 > c_2$) (15.6.3)

If we plot ASN values along with lot fraction defective values p, we obtain the Average sample number (ASN) curve as given below:

Fig. (15.6.2): Average sample number curve for double sampling plan.



15.6 a. Designing a double sampling plan.

As we have discussed the method of designing single sampling plan, now, we proceed to discuss the design of double sampling plan for a specified OC curve. It is already discussed that in designing the plan, we have to decide the parameters of the plan for a given values of p_1 , p_2 , α and β, just as done in designing single sampling plan. Taking two points on the OC curve, we have determined the parameters 'n' and 'c' of single sampling plan. But designing double sampling plan, we have to determine five parameters namely n_1 , n_2 , c_1 , c_2, and c_3 . Thus, designing the double sampling plan is a more complicated procedure. First of all, it is not enough to simply specify p_1 , p_2 , α and β to get a unique double sampling plan. This means that we must draw upon other considerations in making final decision about the plan.

The popular procedure adopted is to assign fixed relationships between n_1 and n_2, that lead to good plans. If we set n_2 as an integral multiple of n_1, it is possible to design double sampling plans. The probabilities involved in double sampling plan, can be well approximated by the Poisson distribution. This means that, with a fixed relationship between n_1 and n_2, the probabilities of acceptance for a given set of c_1 and c_2 values will be simply a function of $\mathbf{pn_1}$, where, p is the incoming lot fraction defective. Hence, plans for which the p_2, and p_1 , bear a constant ratio can be made to have an identical OC curves (within the approximation given by the Poisson distribution) by simply varying the values of n_1. To see this, suppose for a given plan, that p_1 is the probability of acceptance for lots of p_2 quality. Then, if the p_1 and p_2 are both multiplied by a factor 'f', their ratio is being maintained at 'r'.

That is if we divide n_1 by 'f', the values of p_1 and p_2 as well as the OC curve will not change, because the quantity **pn_1's** are held constant. Thus we can determine the values of the parameters of the double sampling plan, namely n_1 , n_2 , c_1 , c_2, and c_3. The above procedure will lead to an extensive table of sampling plans. However, in general, all plans will be equally good. To simplify the procedure, we can eliminate plans that seem to be intuitively bad. For example, if n_2 is taken equal to $2n_1$, then it would seem to be intuitively good to have, c_2 at least 3 times c_1 . If c_2 , were less than 3 times c_1 , it would mean that, we would accept a lot on the first sample of n_1 items with a higher percentage of defective items than we would accept on the combined sample of $3n_1$ items. Similarly, we consider c_3 is equivalent to c_2 in many practical problems.

15.7. Summary

In this lesson, we have discussed two important Acceptance Sampling procedures, namely single sampling plan and double sampling plan. We have discussed the parameters of the plans, their construction and derivation of OC and ASN curves. We have discussed various advantages and disadvantages of double sampling plan and compared this plan with singles sampling plan. Further, we have discussed that double sampling plan is the extension of the idea of single sampling plan. In single sampling plan we take decision about the lot based on the single sample where as in double sampling plan, the decision is taken based on two samples. This can provide a psychological advantage of giving a second chance to the lots, before sentencing the them. Further, we have discussed the method of designing Single and double sampling plans for a specified OC curve.

15.8. Self Assessment Questions

1. Define a single sampling plan.
2. Explain the method of deriving OC curve for a single sampling plan.
3. Explain the ASN curve for a single sampling plan.
4. Give schematic representation of single sampling plan.
5. Explain the method of designing a single sampling plan for a specified OC curve.
6. Define a double sampling plan.
7. Give the schematic representation of a double sampling plan.
8. Explain the method of construction of OC and ASN curves for a double sampling plan.
9. Discuss various advantages and disadvantages of double sampling plan over a single sampling plan.
10. Calculate P_a for a double sampling plan whose parameters are given as follows:

    (a) $n_1 = 50, n_2 = 100, c_1 = 2$, $c_2 = 4$ $c_3 = 4$ and $p = 0.04$
    (b) $n_1 = 30, n_2 = 60, c_1 = 1$, $c_2 = 3$ $c_3 = 4$ and $p = 0.05$.
    (c) $n_1 = 50, n_2 = 100, c_1 = 1$, $c_2 = 3$ $c_3 = 3$ and $p = 0.05$.
    (d) $n_1 = 30, n_2 = 60, c_1 = 1$, $c_2 = 3$ $c_3 = 4$ and $p = 0.01$.

11. Explain the method of designing a double sampling plan for a specified OC curve.

12. Critically compare ASN curves of single and double sampling plans.

15.9. Further Readings

1. Montgomery, D.C., "Introduction to Statistical Quality Control", John Wiley (Asia) 2001.

2. Biswas.S., "Statistical Quality Control-Sampling Inspection and Reliability", New Central Book Agency Pvt. Ltd. 2003.

3.Wordsworth "Modern Methods for Quality Improvement", John Wiley and sons 2005.

4.Bertrand L. Hansen "Quality control – Theory and Applications" Prentice-Hall of India-2005.

5.Acheson J. Duncan "Quality control and Industrial Statistics" ,D.B. Taraporevala Sons & co. 1970

UNIT – IV
Lesson – 16

Lot – by - Lot Acceptance Sampling Plans

16.0. Objective

After going through this lesson you will be in position to understand:

- The meaning and need for lot-by-lot Acceptance Sampling Plans.
- Advantages and disadvantages of Lot-by-lot acceptance sampling plans.
- Single sampling plans for attributes and its OC, AOQ and AOQL.
- Double, multiple sampling plans and their OC, ASN, ATI curves.
- Item –by-item Sequential Sampling Plans for attributes and their properties.

Structure

16.1. Introduction

Lot-by-Lot Acceptance Sampling plans are particularly used in situations, where, a multiple number of units are presented for inspection in static form at essentially one geographic location and where the most logical method of inspection is with attributes type. The inspection procedure is **go-no-go** measuring type, that is if the product is 'good' , we allow the product to go to the next stage of production otherwise, we stop the product. In other words, if the product is good, we allow the product to 'go' if it is bad, we say, no, to the product to go. Such testing procedures are popularly, known as go-no-go test procedures, through appropriate **'gates'**, where, product is required weight, or diameter or length or size, we allow to **go** through the gate otherwise we stop the product **not to go**, through the gate.

16.2.Lot-by-lot acceptance Sampling Plans and their assumptions

Lot-by-lot Acceptance procedures applied when the products in the lot are inspected based on 'go-no-go' type inspection procedures. This can be done through various gates, where the product is judged as 'good' or 'bad' based on measuring and testing the quality characteristics of the products under the test. In recent days, such types of sampling plans are attracting more and more users particularly, in business, industry and dairy. This is explained with an example, related in dairy as follows:

Let there are 'K' suppliers supplying 'milk' in cans to the milk collection centers, which in turn will be supplied to the dairy. Since, the milk is collected from different suppliers and there is more chance that quality of the milk may vary widely from supplier to supplier. Hence, maintaining the quality of the collected milk at a uniform level, is the prime concern of the dairy. To do this, when a supplier supply the cans of milk, the quality of that lot or can of milk, a sample of milk selected and is measured by placing a **'lactometer'** and measured the quality. If the measurement of the lactometer satisfies the quality then the can of milk is determined as 'good' and is accepted. Otherwise, it is determined 'bad can of milk' and hence is rejected.

To conduct test procedures, through a test procedure, or gate are called '**go-no-go'** test procedures and are applied easily by non-technical person. Thus, Lot-by-lot acceptance sampling plans are popularly used, because of it efficiency, easy to understand and simple to understand. A vendor without technical knowledge can also apply these procedures easily. Hence, these plans are popularly used in those fields like, Business, marketing and industry where, test procedures are **non-destructive in nature**.

Usually, when lot-by-lot acceptance sampling plans are applied, there will be an agreement between the supplier and the receiver on the following:

(i) Lots accepted by the sampling plan will be accepted as it is. If any defective units are found in the sampling, defective units identified are to be replaced with good ones by the supplier.

(ii) Rejected units are returned back to the supplier, at free of cost or cost bared by the supplier, for rectification or rework and

(iii)The resubmitted lots will be re-inspected only for those defects causing original rejection or any other defects which might have been caused by the rectification process.

There are variations to these agreements in some inspection plans. Some receivers can use the option of screening the rejected lots and charging these costs from the supplier if screening involves additional testing cost. This option is exercised most often, when the material is urgently needed by the user. Sometimes, supplier will be give an offer to reduce the cost on rejected lots instead of sending the rejected lot back to the supplier for re-inspection and rework or rectification. It is important to note that reducing the cost of the product will not increasing the quality of the lot, but preparing the user to adjust with the low quality product and customer will have some psychological satisfaction that the cost of the product is reduced. This makes the user to adjust with the low quality product. In fact, it is important to cultivate the habit of '**rejecting the bad lot'** rather than accepting the same because cost is reduced. This will make the supplier to supply good products only and hence the quality of the will be increased. At

least the supplier will improve the production and makes the supplier to supply good quality products in future.

Assumptions of Lot-by-Lot Acceptance Sampling Plans: Lot-by-Lot sampling plans can be used to give assurance regarding the mean or standard deviation of the quality of the product or process or material. Here, we use Type-B OC curve, to determine the performance of Lot-by-Lot acceptance Sampling Plans. That is, in these sampling plans, we assume that the process is operating in-control and random causes alone are acting in the production process and lots can be viewed as random samples from the process. This implies that, the samples taken from the lots under a given sampling plan can be viewed mathematically as if they came directly from the production process. Hence, we can use Type – B oc curve, because the assumptions of Type – B OC curve are satisfied. That is, we assume that the lot sizes N are finite. Further, in these sampling plans, we also assume that the items in the lot are normally distributed, with respect to quality characteristic under consideration. The assumption of normality for finite size lots is valid because, these lots are assumed as samples with finite size from the process production which is usually of large (countable infinity) which follows Normal distribution. In this connection we recall the result that Binomial distribution converges to Poisson distribution which in turn converges to Normal Distribution under Stated assumptions. That is Normal Distribution is viewed as limiting case of Binomial Distribution.

Lot-by-Lot acceptance sampling plans are also considered for (i) Attributes and (ii) variables. In both the cases, we can use Single, or Double sampling plans, introduced in the previous lesson. Now, we proceed to discuss the construction of Lot-by-Lot Attribute Single sampling Plans.

16.3. Construction of Lot-by-Lot Attribute Acceptance Single Sampling Plans.

In this plan, products in the Lot are dichotomized as defective or non-defective depending on the attribute quality characteristic like having the desired smoothness or diameter of the ball point pen or desired breaking strength or smoothness of the chalk-pieces while writing on the black board or producing desirable sound produced when a cracker is fired and so on. Here, we usually apply Binomial Distribution and we know that, Poisson distribution can be considered as limiting case of Binomial distribution when, the probability of a defective unit $p \leq 0.10$ and the mean $np < 5$. Under these assumptions, we use Poisson Distribution Probabilities in the construction of Lot-by-Lot Attribute single sampling plan. Now we proceed the step-by-step procedure, for deriving the OC curve for a single Sampling Plan for Variable data.

Step – 1: First Set-up the following table with the headings and P_a column mentioned below:

n	np	p	P_a	pP_a
			0.98	
			0.95	
			0.70	
			0.50	
			0.20	
			0.05	
			0.02	

Where: n = Sample size, np= number of defectives,
p= fraction defective, P_a = Probability of acceptance and
pP_a = Average Outgoing Quality = AOQ.

Step – 2: Search Table F, in Appendix under the given acceptance number 'c' until the desired P_a (or the closest value to the desired P_a) is located. If the exact value is not found, the value in the column P_a is to be changed to the new value which correspond the selected value.
Step – 3: Place the np value associated with the selected P_a in the np column.
Step – 4: Divide the np value with n yields the value of p, i.e. np / n = p.
Step – 5: The pare (p , P_a) will form a point with coordinated p and P_a , on the OC curve.
Step – 6: using these values of p and P_a we can determine the average outgoing Quality for the Sampling plan under consideration.

Example (16.3.1): Determine the OC curve, AOQ and AOQL for a single sampling plan, for n = 300 and c = 5.
Solution: Step – 1: Calculate the other values in the following table.
Table (16.3.1): Starting table for the determination of OC curve for single sample plan.

n	np	p	P_a	AOQ =pP_a
300			0.98	
300			0.95	
300			0.70	
300			0.50	
300			0.20	
300			0.05	
300			0.02	

Step – 2: Fr n = 300 and c = 5 from the table F given in the appendix discloses a P_a value, thus 0.983 is closest value found from the table closer to 0.98. Similarly, np value associated the value of P_a = 0.983 is 2.0. Therefore, p = np / n = 2.0 / 300 = 0.00667 and AOQ = pP_a = 0.0067(0.983) = 0.0655. On similar lines, fill the other values in the table as follows:

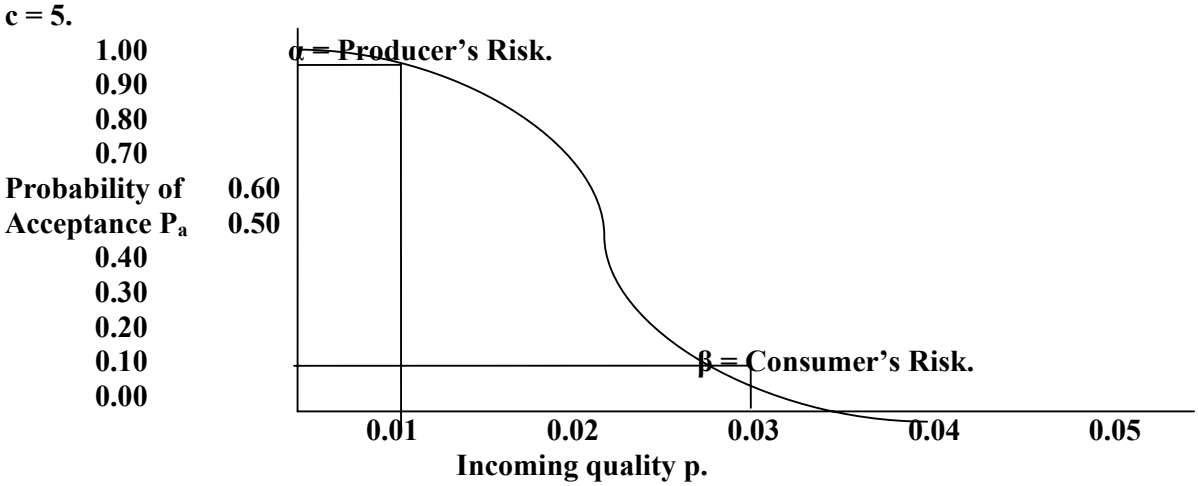
Table (16.3.2): Calculations of OC and ASN curves for single sample plan.

n	np	p=np/n	P_a	AOQ =pP_a
300	2.0	0.00667	0.983	0.00655
300	2.6	0.00867	0.951	0.00824
300	4.4	0.01467	0.720	**0.01056=AOQL**
300	5.6	0.01867	0.512	0.00956
300	7.8	0.02600	0.210	0.00546
300	10.5	0.03500	0.050	0.00175
300	12.0	0.04000	0.020	0.00800

In the above table, entries second and third columns namely np , P_a are to be noted from the table F given in the appendix and calculation of other columns are explained in the headings of the respective columns.

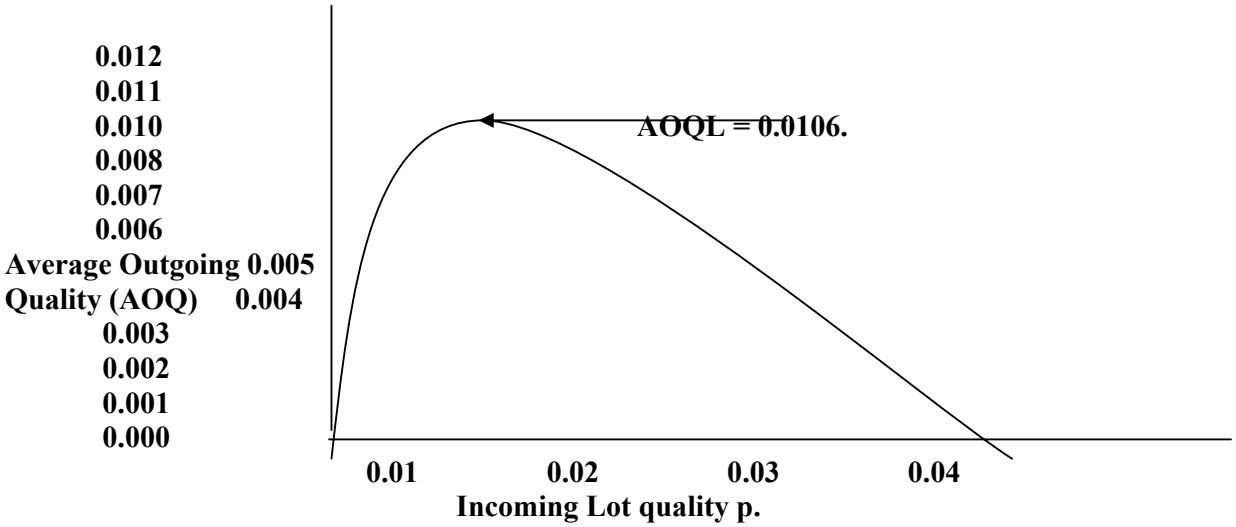
To draw OC curve, consider the pair (p ,P_a) and for AOQ curve consider the Pair (p, pP_a), we obtain the following curves.

Fig. (16.3.1): Operating Characteristic Curve for a Single Sampling Plan with n = 300 and c = 5.



In the graph, α = Producer's Risk = the risk of rejection which the producer takes with the application of the plan. Usually, set at P_a = 0.95 when α = 0.05. Similarly, β = Consumer's Risk = the risk of acceptance which the consumer takes with the application of the plan. The quantity, β is also known as Lot Tolerance Faction Defective (LTFD), which gives the maximum tolerance limit that a consumer can adjust with the quality of the lot. Now we draw the AOQ Curve as follows by considering the Pair (p, pP_a).

Fig. (16.3.2): Graph of average Outgoing Quality and Average Outgoing Quality Limit for a Single Sampling Plan.



On similar lines of single sampling plans for controlling the process mean or process variability using ranges. Further, we can extend the idea of single sampling plans to double, multiple, or sequential sampling plans which are discussed in the forth coming sections.

16.4. Double and Multiple Lot-by-Lot Attribute Acceptance Sampling Plans.

Double Sampling Plans: The idea of single sampling plan for attributes can be extended in many ways. One of such extensions is '**Double Sampling Plan'**; the procedure of sentencing the lot based on a single sample may be extended to two samples. That is a double Sampling Plan is defined as a procedure, under certain circumstances stated, a second sample is required before the is sentenced. A double Sampling plan is specified by the five parameters, namely:

n_1 = Size of the first Sample.
n_2= Size of the Second Sample.
c_1 = Acceptance number of the first sample,
c_2 = Acceptance number of the second sample and
c_3 =. Acceptance number for double sample.

Double Sampling procedure is explained as follows:

Select first a Random sample of size n_1 from the lot of size N and search for number of defective items in the first sample selected, denote this detected items by d_1 . If $\mathbf{d_1 < c_1}$, We accept the lot for marketing. If $\mathbf{d_1 > c_2}$, we reject the Lot and sent for 100% inspection. If $\mathbf{c_1 < d_1 < c_2}$, we select another sample of size n and search for number of defective items denoted by d_2 . If $\mathbf{d_1 + d_2 \leq c_3}$, we accept the lot. If $\mathbf{d_1 + d_2 > c_3}$, we reject the lot. Usually c_3 is considered as c_2 itself, in many practical situations.

Multiple Sampling Plans: A multiple sampling plan is an extension of double sampling, in which decisions are taken based on more than two samples. That is to sentence a lot we take

more than two samples. For example, consider a multiple sampling plan, with N=5 stages as follows:

Stage Number i .	Sample Number n_i.	Cumulative Sample SizeΣn_i	Acceptance Number A_i	Rejection Number R_i.
1	$20=n_1$	20	$0 = A_1$	$3 = R_1$
2	$20=n_2$	40	$1= A_2$	$4= R_2$
3	$20=n_3$	60	$3= A_3$	$5= R_3$
4	$20=n_4$	80	$5= A_4$	$7= R_4$
5	$20=n_5$	100	$8= A_5$	$9= R_5$

Multiple Sampling plan is operated as follows:

First take a sample of size 20 from the lot and count for number of defectives d_1. If $d_1 = 0$, accept the lot. If $d_1 > 3$ Reject the lot. If $0 < d_1 \leq 3$, take a second sample of size $20 = n_2$ and count number of defectives d_2 . If $d_1 + d_2$ is $\leq R_1 = 1$, accept the lot. If $d + d > R_2 = 4$, reject the lot. If $R_1 < d_1 + d_2 \leq R_1$, take the third sample of size $n_3 = 20$ and count number of defective items d_3. On similar lines, we have to continue the procedure until the N = 5 Stages in the example.

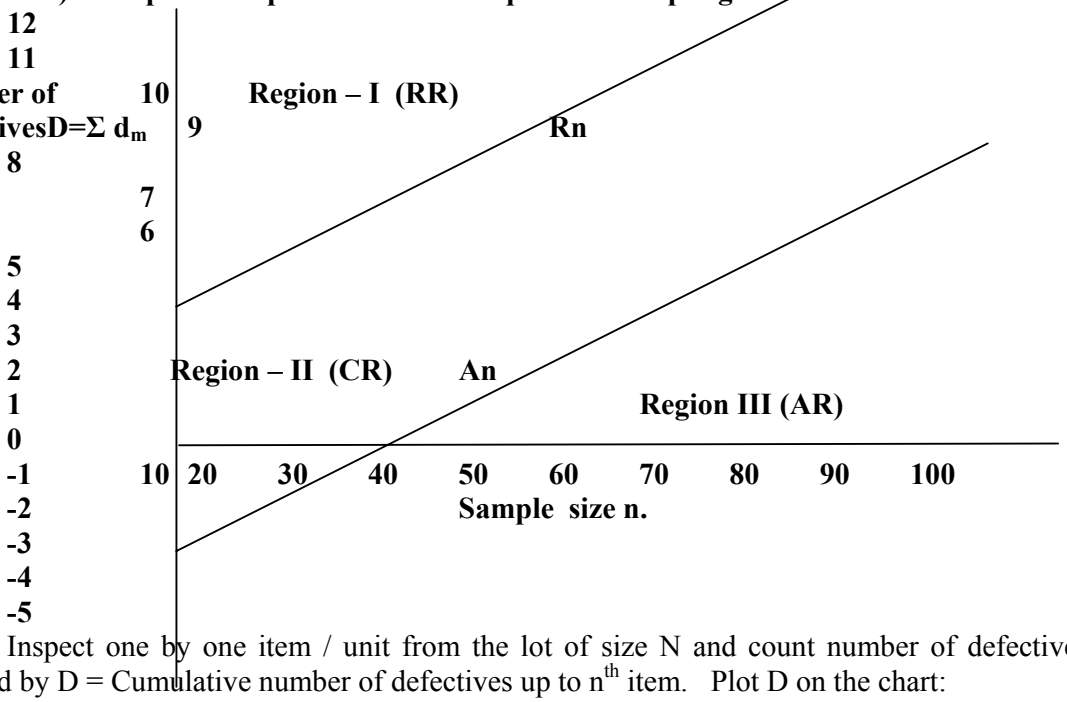
The construction of OC and ASN curve for multiple sampling plan is a straight forward by simply extension of the approach used in double sampling explained in the previous lesson (15.6). Basic advantage of multiple sampling plans is that, the samples required at each are usually smaller than those in single or double sampling plans. Hence, they are economical and requires less inspection cost and time. Thus they are used popularly in those situations where, destructive test procedures are used in the inspection process.

16.5. Item-by-item Sequential Sampling Plans for attributes.

The concept of double sampling plan and multiple sampling plan is extended to '**Sequential Sampling plan'** which is also known popularly as **'item-by-item Sequential Sampling plan'. This procedure is based on the Sequential Probability Ratio Test (SPRT) procedure. There is an important result connected SPRT is that: An SPRT will terminate with Probability One.** In this procedure, we take a sequence of item-by-item sample is drawn from the lot and allow the sample number 'n' to be determined entirely by the results of the present sample. That is, in this procedure, the sample number 'n' is not fixed but a random variable. Theoretically, sequential sampling procedure is to be continued indefinitely, until the lot is inspected 100%. In other words, Sequential will be terminated when n = N the lot size. In this procedure, the sample space is divided into three regions, namely:

(i)Rejection Region (RR)
(ii)Continuation Region (CR) and
(iii)Acceptance Region (AR), as shown in the following figure.

Fig. (16.5.1): Graphical Representation of Sequential Sampling Plan.



Inspect one by one item / unit from the lot of size N and count number of defectives denoted by D = Cumulative number of defectives up to n^{th} item. Plot D on the chart:

Situation – 1: If d lies in the Region – I, that is in the acceptance Region (AR), Stop sampling and accept the lot for the next stage.

Situation – 2: If, D lies in the continuation region – II, take another item from the lot and inspect.

Situation – 3: If D lies in the Region – III, that is in the Rejection Region (RR), Stop sampling and reject the lot.

In the above chart, An is called the Acceptance Line (AL) and Rn is called the Rejection Line (RL), which are calculated for specified values of $\mathbf{p_1}$ **,** $\mathbf{(1-\alpha)}$**,** $\mathbf{p_2}$ **and** $\boldsymbol{\beta}$ values and are calculated as follows:

Acceptance Line = AL = An = $\mathbf{-h_1 + sn}$, n = 1,2,3.,.,.N (16.5.1)

And Rejection Line = RL = Rn = $\mathbf{h_2 + sn}$, n = 1,2,3,.,.,.N. (16.5.2)

Where: $\mathbf{h_1 = [\log(\frac{1-\alpha}{\beta})]K}$ (16.5.3)

$\mathbf{H_2 = [\log(\frac{1-\beta}{\alpha})]K}$ (16.5.4)

$\mathbf{K = \log(\frac{p2(1-p1)}{p1(1-p2)})}$ (16.5.5)

And $\mathbf{s = \log[(1-p_1)/(1-p_2)]/K}$ (16.5.6)

Operating Characteristic (OC) Curve and Average Sample Number (ASN) Curve Average Outgoing Quality (AOQ) and Average Total Inspected (ATI) for Sequential Sampling Plan can be calculated as follows:

OC Curve: The OC curve for the sequential sampling plan can be easily obtained by determining two points at $(p_1 , 1- \alpha)$ and (p_2 , β) on the curve. A third point near the middle of the curve is p = s and $P_a = h_2 / (h_1 + h_2)$. Joining these three pairs of points, we obtain the OC curve for the given values of $\mathbf{p_1 , (1 – \alpha), p_2}$ **and** $\boldsymbol{\beta}$**.**

ASN Curve: The ASN curve for the Sequential sampling plan is calculated as follows:

$$\mathbf{ASN = P_a (A/C) + (1 – P_a)(B/C)} \quad (16.5.7)$$

Where, $\mathbf{A = \log [\beta / (1 – \alpha)]}$ (16.5.8)

$$\mathbf{B = \log [(1 – \beta)/\alpha]} \quad (16.5.9)$$

And $$\mathbf{C = p \log (p_2 /p_1) + (1 – p) \log [(1 – p_2) / (1 – p_1)]} \quad (16.5.10)$$

AOQ Curve: The Average Outgoing Quality Curve for Sequential Sampling Plan is given approximately given by: $\mathbf{AOQ \cong pP_a .}$ (16.5.11)

ATI Curve: In order to obtain the average total inspection (ATI) can be obtained for the sequential sampling plan, it is important to note that the amount of sampling is A/C when a lot is accepted and N, when the lot is rejected. Hence we have:

$$\mathbf{ATI = P_a(A / C) + (1 – P_a) N.} \quad (16.5.12)$$

In the above expressions P_a = the probability of accepting the Lot.

16.6. Summary

In this lesson, we have considered Lot-by-Lot sequential plans based on Attributes along with their Construction and Properties. First we have considered the construction of Single Sampling Plan and discussed its OC, AOQ and AOQL. Next, we have discussed the double, multiple and item-by-item Sequential plans. Their construction, OC, ASN, AOQ and ATI functions, their derivations are discussed.

16.7. Self Assessment Questions

1. Explain Lot-by-lot acceptance Sampling Plans along with their assumptions.
2. Explain the method of construction of OC curve for Lot-by-lot Attribute Single sampling plan.
3. Explain the method of construction of AOQ and AOQL in Lot-by-Lot Attribute Single Sampling plan.
4. Explain Lot-by-Lot Double and Multiple Sampling plans.
5. What are Sequential Sampling Plans? Explain the Decision Procedure in these plans.
6. What are item-by-item Sequential Sampling Plans?
7. Determine OC and ASN curves for Item-by-item Sequential Sampling Plans.
8. Determine the ATI and AOQ of Item-by-item sequential sampling plan.
9. Explain the method of construction of Acceptance and rejection lines of Sequential sampling plans for a given $p_1 , (1 – \alpha), p_2$ and β values.

10. Explain AR, CR and RR of Sequential Sampling plans.

16.8. Further Readings

1.Bertrand L. Hansen “Quality control – Theory and Applications” Prentice-Hall of India-2005.
2.Acheson J. Duncan “Quality control and Industrial Statistics” ,D.B. Taraporevala Sons & co. -1970

UNIT – IV
Lesson - 17

Military Standard – 105 Plans

17.0. Objective

After going through this lesson you will be in position to understand:

- The meaning and need for AQL based Military Standard Plans.
- Applications, specifications and procedure of MIL-STD105 plan.
- Various inspection Schemes and their shifting rules .
- Types of MIL-STD 105 and its versions and extensions.

Structure

17.1. Introduction to Military Standard Plans

It is already mentioned in the beginning lessons that the origin of Statistical Quality control techniques is the Second World War applied by United States (US) Government, in supplying the war material, food products and medicine supplied to the military personal fighting in the war field. After winning the war, by successfully applying these Statistical tools and quality control techniques, people got belief on these newly introduced tools, spread like **wild fire** to other countries in the rest of the world, from United States. The procedures applied by the government to maintain Standards of military supplied material are called **"Military Standard (MIL-STD)"** plans and are applied recently, in the inspection of all Industry.

17.2. MIL-STD 105 acceptance Sampling Plans and their assumptions

Military Standard (MIL-STD) Plans assure more quality levels than plans discussed earlier lessons known as **"Civil Plans"**. There are many MIL-STD plans available among them

MIL-STD 105 is the most popularly used plan as an inspection plan applied in almost all industry, as Lot-by-Lot Acceptance Sampling Plan. MIL-STD Plans are two types. Namely: **(1) MIL-STD plans for Attributes** and (2) **MIL-STD plans for Variables**. Since, these plans are basically Lot-by-Lot Acceptance Sampling Plans, the assumptions made for Lot-by-Lot Acceptance Sampling Plans are applied here also. That is, Lots are of Size N are formed from a continuous production of Infinite Size and there is no specific reason to doubt any Lot. Lots are **formed uniformly, and all lots are of Uniform Quality level 'p'.**

MIL-STD 105 plan is a Lot-by-Lot acceptance Sampling Plan by attributes applicable for parts, components, Subassembly units, assemblies, or end products/ items under consideration. Thus these plans are used in almost all stages of production of items, that is starting from the selection of Raw material, selecting the designs for the product, intermediary production stages, before transportation of production to the market, and After receiving the Lots by the receiver. Hence, MIL-STD 105 plans are most popularly used Lot-by-Lot Variable Acceptance Sampling Plans.

The **Acceptable Quality Level (AQL)** is the expression for standard quality in MIL-STD 105 plans. Here, we define a **'nominal value p'** which can be expressed in terms of percent defective or number of defects per hundred units, which is comfortable to apply for the Lot of products. The quantity 'p' is also referred as the maximum percent defective or defects per hundred units, which may be tolerated as a quality average by the customer / receiver.
This definition of 'p' as '**an average'**, sometimes, may lead to situations to accept the lots with lower quality, than the desired standards. A more accurate definition would be that it is the quality level where the producer takes a comparatively smaller risk of rejection of the product of quality which is equal to or better than AQL. Hence MIL-STD plans are usually referred as '**AQL based Plans'**. The AQL is generally specified in the contract or by the authority responsible for sampling. Different AQLs may be designed for different types of defects. For example, we can differentiate defects as **(i) Critical defects, (ii) Major Defects and (iii) Minor defects.** It is usual practice that we choose an AQL of 0% for Critical defects, an AQL of 1% for Major defects and an AQL of 2.5% for minor defects. Now we proceed to discuss various versions of MIL-STD Plans in the following Section.

17.3. Various Versions of MIL-STD 105 Inspection Schemes based on AQL

Standard military sampling procedures for inspection by attributes were developed in the World War II. Army Ordinance tables and procedures were developed in 1942, and these later (with some minor modifications) became **'Army Service Forces Tables'**. Navy tables were issued in 1945 and were adopted by Indian Navy in 1949 as a joint Army-navy Standard 'JANA-STD' latter on became MIL-STD plans. The original version of the MIL-STD attribute sampling plan introduced in 1950, is known as MIL-STD 105A. Since, then there are four revisions with minor modifications in 105A standards issued as, MIL-STD 105B, 105C. MIL-STD 105D is the outcome of a study by an American-British-Canadian Working group that sought common standards for the three countries. MIL-STD 105Dwas issued by the US government in the year 1963 and the latest revision released in the year 1989 is known as MIL-STD 105E. MIL-STD 105E is a collection of various sampling schemes; and hence, referred as **"Acceptance**

Sampling System". ANSI/ASQC Z1.4 is a Civilian Plan similar to MIL-STD 105E. The standard was also adopted by the International Organization for Standardization ISO 2859.

Standards are provided for three types of sampling: (1) Single Sampling (ii) Double Sampling and (iii) Multiple Sampling.

The primary focal point of MIL-STD 105E is the Acceptable Quality Level (AQL) determined by the user / receiver. Usually, these plans are indexed with respect to a series of AQLs ranges from 0.10% to 10% for fraction defective and ranges from 10 to 1000 defects per 100 units for number of defects per unit. It is important to note that, for the smaller AQL levels, the same sampling plan can be used to control either fraction defective or a number of defects per unit. Usually, AQLs are arranged in an increased order in the range given above and AQL is approximately, 1.585 times the preceding one. The sample size used in MIL-STD 105E is determined by the lot size and by the choice of inspection level. Now we proceed to discuss various inspection levels, rules required to shift between different levels of inspection are discussed in the following section.

17.4. Various types of inspection Levels and their shifting rules

Usually, there are three Inspection Levels denoted by Level – 1, Level –II and Level – III. In general, Level – II is referred as Normal Inspection. The level-I refers the reduced inspection, usually require one-half of the amount of inspection when compared to level – II, which is to be used when less discrimination is needed. The level – III is is the tightened inspection, require about twice as much inspection amount as required when inspection Level – II is used and this is to be used, when more discrimination is needed. In some special plans we use four inspection levels denoted by S-1,S-2,S-3 and S-4. These special Inspection schemes we use very small samples and should be used when small sample sizes are necessary and when large sampling risks can or must be tolerated. In MIL-STD 105E plans, we generally use three levels of inspection known as:

(i) **Tightened Inspection, (ii) Reduced Inspection and (iii) Normal Inspection**.

For each type of Sampling Plan, rules are framed when to shift from Normal to Tightened or Normal to Reduced, or Tightened to Normal or reduced to Normal and so on. The rules of shifting different levels of inspection are explained as follows:

Rule – 1: Normal to Tightened or Shifting from Level – II to Level – III: When normal inspection

(Level – II) is in effect, tightened inspection (Level – III) is instituted, when two out of five consecutive lots or batches have been rejected on original submission.

Rule – 2: Tightened to Normal or Shifting from level-III to Level - II: When tightened inspection (Level – III) is in effect, normal inspection (level – II) is instituted, when five consecutive lots or batches are accepted on original inspection.

Rule – 3: Normal to Reduced or Shifting from Level –II to Level –I: When Normal inspection (Level – II) is in effect, reduced inspection (Level – III) is instituted provided all four of the following conditions are satisfied.

(a) When the preceding 10 lots have been on Normal inspection and none of the lots have been rejected on original inspection.
(b) When the total number of defectives in the samples from the preceding 10 lots is less than or equal to the applicable limit number specified in the standard.
(c) When production is at a steady rate; that is, no difficulty such as machine beak-downs, material shortages, or other problems have recently occurred.
(d) When, reduced inspection is considered desirable by the authority responsible for sampling.

Rule – 4: Reduced to Normal or Shifting from Level – III to Level – II: When reduced inspection (Level – III) is in effect, Normal Inspection (Level – II) is instituted provided any of the following conditions are satisfied.

(a) When, a Lot or Batch is rejected.
(b) When the sampling procedure terminates with neither acceptance nor rejection criteria have been met, the lot or batch is accepted, but normal inspection is reinstituted starting with the next lot.
(c) When the Production is irregular or delayed.
(d) Other conditions warrant that normal inspection be instituted.

Rule – 5: Discontinuance of Inspection: In the event that 10 consecutive lots remain on tightened inspection, inspection under the provision of MIL-STD 105E should be terminated, and action should be taken at the supplier's level / Production Level to improve the quality of submitted lots.

17.5. Procedure of MIL-STD 105E plan and its OC Curves

Now, we proceed to explain the Step-by-step procedure for using MIL-STD 105E Plans.
Step – 1: Choose the AQL.
Step – 2: Choose the Inspection Level.
Step – 3: Determine the lot size.
Step – 4: Find the appropriate sample size code letter from table (A – 4) given in the appendix.
Step – 5: Determine the appropriate type of sampling plan to use (Single or Double or Multiple).
Step – 6: Enter the appropriate table to find the type of plan to be used.
Step – 7: Determine the corresponding normal, reduced inspection plans to be used when required.

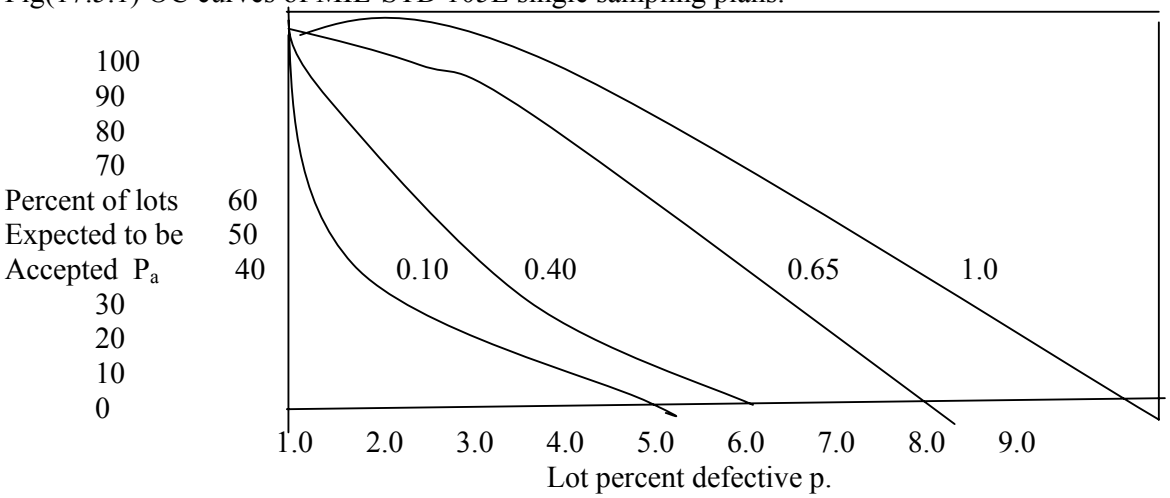
Above explained procedure is explained with an example as follows:

Example (17.5.1): Suppose that the product is submitted in lots of size N = 2000 and the acceptable Quality Level (AQL) is 0.65%. We will use the standard to generate normal, tightened and reduced single sampling plans for this situation.

It is observed from the table A – 4 in Appendix, for lots of size n = 2000, under general inspection level, indicate that the appropriate sample size code is 'K'. Therefore, from table A-5, for single sampling plan we have n = 125 and c = 1. Note that in switching from normal to tightened inspection, the sample remains the same but the acceptance number is reduced by one. This general strategy is used in MIL-STD 105E plans is Tightened Inspection (A-6) If the normal inspection acceptance number is 1, 2, 3, the acceptance number for the corresponding tightened inspection plan is reduced by one. If the normal inspection acceptance number is 5,7,10 or 14, the reduction in the acceptance number for tightened inspection is two (A-7). If the Acceptance number is 21 the reduction is three.

The OC Curves for MIL-STD 105E for single sampling plans. These OC Curves are all type – B OC Curves. The OC curves, for matching double and multiple sampling plans are roughly comparable with OC curves of single sampling plans. Following figure (17.5.1) gives OC Curves for MIL-STD 105E plans. It is important to note that in MIL-STD 105E we use 2,3,5,8,13,20,32,50,125,500,800,1250 and 2000. Thus not all sample sizes are possible. Further in MIL-STD 105E sample sizes are related to Lot sizes.

Fig(17.5.1) OC curves of MIL-STD 105E single sampling plans.



Further, it is important to note that the civil plans ANSI/ASQC Z1.4 or ISO 2859, are the counterparts of MIL-STD 105E plans and MIL-STD 105E plans are AQL based plans adopted by military Personnel.

17.6. Summary

In this lesson, we have introduction to military standard plans and discussed various military standard plans along with their assumptions. Further, MIL-STD 105 sampling plans are discussed and introduced various versions of MIL-STD 105 plans. MIL-STD 105E is the latest version introduced in the year 1989. WE have discussed various steps involved in

applying MIL-STD 105E plans and discussed that MIL-STD 105E are Acceptable Quality Level (AQL) based plans. Operating characteristic (OC) curves for MIL-STD 105E plans are drawn for various AQL levels.

17.7. Self Assessment Questions

1. What are Military Standard plans? Explain their origin.
2. Explain Various Versions of MIL-STD 105 plans.
3. Explain various steps involved in the procedure of MIL-STD 105E plans.
4. Explain various levels of inspection schemes.
5. Explain different rules in shifting between different levels of inspection.

17.8. Further Readings

1. Montgomery, D.C., " Introduction to Statistical Quality Control", John Wiley (Asia) 2001.
2. Biswas.S., "Statistical Quality Control-Sampling Inspection and Reliability", New Central Book Agency Pvt. Ltd. 2003.
3.Wordsworth "Modern Methods for Quality Improvement", John Wiley and sons 2005.
4.Acheson Jj. Duncan "Quality control and Industrial Statistics" ,D.B. Taraporevala Sons & co. 1970

UNIT – IV
Lesson – 18

Dodge and Romig Acceptance Sampling Plans

18.0. Objective

After going through this lesson you will be in position to understand:

- The meaning and need for LTPD based Military Standard Plans.
- Distinction between AQL based and LTPD based plans.
- Other Lot-by-Lot acceptance sampling plans for attributes.
- Split-Risk plans for attributes.

Structure

18.1. Introduction to Dodge and Romig Plans.

In the previous lesson 17, we have discussed various military standard 105 plans, their versions and applications. These plans are Acceptable Quality Level (AQL) based plans and have universal acceptance. These plans are satisfactorily applied by military and army people jointly and are meeting the requirements of almost all any situation, where sampling inspection by attributes is required by these people. There are many other plans also, which may fit a particular situation better than other plans. One such a plan is Dodge and Romig Acceptance Sampling Plans. These Procedures were designed for the use within the Bell Telephone Systems and were developed by two Bell Telephone Engineers, namely H.F. Dodge and H.G. Romig. Later on, these plans were published for general applications. These plans consists of Single and Double Sampling Plans based on two criteria, namely: (1) Lot Tolerance Percent Defective (LTPD) and (2) Average Outgoing Quality Limit (AOQL). Thus we have four types of Dodge Romig Acceptance Sampling Plans. For each of these approaches, to develop

sampling plan designs, separate tables are available for Single and Double Sampling plans. These plans along other plans are discussed in this lesson.

18.2. Distinction between AQL based and LTPD based Plans.

Sampling plans that emphasize LTPD protection, proposed by H.F. Dodge and H.G. Romig, are often preferable to AQL oriented sampling plans, like, in MIL-STD 105E, particularly, when we deal with critical components or systems involving many small sub-parts or sub-systems. For example, consider An Airplane, or Computer or A T.V. set, or an Auto-mobile or Telephone Exchange, which consists of many sub-parts or systems. In such situations, in the past people use to apply AQL based plans. Now a days, the trend has been changed and are emphasizing on parts per million (ppm). This leads to a new approach popularly known as "Six Sigma Concept" which will be discussed in detail in the last lesson. In this direction, other sampling plans, like, LTPD based or AOQL based plans are more applicable for complicated systems involving many small sub-parts. This can be explained with the following example: Comparison of different AQL levels and the corresponding defective parts per million are explained as follows:

Acceptable Quality Level	**Defective parts per million (ppm).**
10%	**100,000**
1%	**10,000**
0.1%	**1,000**
0.01%	**100**
0.001%	**10**
0.0001%	**1**

Thus, even a small AQLs imply large number of defective parts per million (ppm). Thus, in complex systems, if we apply AQL plans is a disadvantage because many parts are defective, but the system is accepted as good one. For example, AQL at 0.01% means 100 defective parts are defects, still the system is a good one. This, in fact is not a desirable situation. Hence, there is a need to ensure Zero defects per million.

Further, consider a printed Mother Boards circuits consisting of 1000 circuits each manufactured by a process operating at 0.001%defectives. If we use AQL plans, we have to accept the mother board even if 10 circuits are failed, but all the circuits on the Mother board must operate for the computer card function properly. Hence, we must concentrate on 0% defective items plans but not the AQL plans. Thus there is an obvious need to develop sampling plans that emphasize LTPD protection, even when, the process average fallout is low. Thus, there is a need to use, Dodge and Romig LTPD or AOQL Acceptance sampling are used in such situations. Further, Dodge-Romig AOQL plans are designed so that the average total inspection for a given AOQL and for a specified process average 'p' will be minimized. Similarly, LTPD plans are designed, so that the Average Total Inspection (ATI), is minimum.

This makes the Dodge-Romig Inspection plans very popular, particularly used for in-plant inspection of semi-finished products within the production process. Now, we proceed to explain these plans in detail in the following section.

18.3. Procedures of Dodge and Romig plans.

First we explain Dodge-Romig AOQL based plans introduced in the year 1959, and then introduce, LTPD based plans.

18.3a: AOQL Plans: Dodge-Romig AOQL plans are designed so that the average Total Inspection for a given AOQL is minimized. Further, these plans apply only to those programs, that submit rejected lots to 100% inspection. Unless rectifying inspection is used, the AOQL concept is meaningless. Further, to use these plans, we must know the process average, that is, the average fraction nonconforming of the incoming products. When the supplier / the Vendor is relatively new, then, we usually do not know the its process average. In such situations, we estimate the same, from a preliminary sample or from the data provided by the supplier / Vendor. Until such estimate is available, we use the largest possible process average available in the table. Dodge-Romig AOQL plans are available, for values of AOQL = 0.1%, 0.25%, 0.50%, 0.75%, 1%, 1.5%, 2%, 2.5%, 3%, 4%, 5%, 7% and 10%. For each of these AOQL values, Six classes of values for the process average are specified. Table values are provided for Single and Double sampling plans and are given in table A – 8 in the appendix. These plans have been designed such that, the average total inspection at the given AOQL and process average is as minimum as possible.

Example (18.3a.1): Suppose, we wish to inspect the computer mother boards of personal computers. The boards are shipped in lots of size N = 5000. The vendor's process average is 1% nonconforming. Determine a single sampling plan for the AOQL = 3%.

Solution: Here, N = 5000, Incoming process quality p = 0.01 and AOQL = 0.03. From the tables we can determine that, for a single sampling plan, we have: n = 65 and c = 3.

18.3b: LTPD Plans: On similar lines of AOQL plans, Dodge-Romig tables are provided for various LTPD values of 0.5%, 1%, 2%, 3%, 4%, 5%, 7%, and 10%. These plans are designed such that the probability of lot acceptance at the given LTPD is 0.1. Dodge-Romig LTPD plans are designed for Single and Double sampling plans and are given in the table A- 9, in Appendix.

Example (18.3b.1): Consider the problem in the example (18.3a.1) of Computer Mother Boards in lots of size N = 5000. Let the supplier / Vendor process average is 0.25% of nonconforming. Determine the single Sampling Plan with LTPD = 1%.
Solution: Here, N = 5000, Incoming process quality p = 0.25% and LTPD = 0.01. From the tables we can determine that, for a single sampling plan, we have: n = 770 and c = 4.

18.4. Additional Lot-by-Lot Acceptance sampling plans.

There are other Lot-by-Lot Acceptance sampling plans available for attributes. Some of such plans are (1) Unit Sequential Sampling Plans for Inspection by Attributes, (2) Multiple-sampling plans for inspection by attributes (3) item-by-item Sequential Probability Ratio

Sampling Plans, (4) Split Risk Plans lot-by-Lot acceptance sampling by attributes and (5) Continuous Acceptance Sampling Plans by attributes. All these plans discussed here are attribute sampling plans. We can also discuss such plans for variable characteristics of the products. These variable sampling plans are discussed in the next unit.

NOTE: These Plans listed in 18.4 are not there in the syllabus prescribed and hence are not discussed here in this lesson. Basic objective of mentioning of these plans here is to indicate there are other Lot-by-Lot acceptance sampling plans available in literature. Those readers who are interested on these plans can refer books mentioned in the Additional References.

18.5. Disadvantages in Lot-by-Lot acceptance sampling plans.

In order to apply Lot-by-Lot acceptance sampling plans, formation of items in the form of lots is a must because our decisions are mainly regarding to accept or reject the Lot. Thus formation of Lots will become a problem, when the production is continuous in nature. In many production processes, the formation of Lot or collection of units in the form of lots / banks will create many production problems. This creates, serious problems and interferes the smooth flow of the products through the production line to completion. Further, it also requires in-process inventory and hence adds substantially to the producer's investment and requires space and security problems for the inventory control.

Attempts have been made, notably by US government, to apply lot-by-lot inspection to continuous productions. But, most of these attempts have not resulted with satisfactory results. Thus, MIL-STD 105 plans cannot be applied to continuous production. This is because of the fact that determination and formation of lots in continuous production is not advisable. Thus there is a need to develop suitable plans applicable, when the production is continuous in nature. Such plans are popularly known as "Continuous Sampling Plans" discussed in the next Unit, that is Unit – 5.

18.6. Summary

In this lesson Dodge-Romig sampling plans are discussed, which are based on Average Out going Quality Limit (AOQL) and Lot Tolerance Percent Defective (LTPD). Critical comparison is made between AQL plans and LTPD plans. Procedures of AOQL and LTPD plans are discussed. Procedures of Single sampling plans and double sampling plans are discussed and finally the unit –IV is concluded with the discussion and problems in the formation of lots, which is a must in lot-by-lot acceptance sampling plans.

18.7. Self Assessment Questions

1. What are Dodge-Romig Sampling plans? How they are distinct from MIL-STD 105 plans?
2. Distinguish between AQL and LTPD based plans with suitable examples.
3. Explain Dodge-Romig AOQL based plans for attributes with an example.
4. Explain Dodge-Romig LTPD based plans for attributes, with an example.
5. Explain disadvantages of Lot-by-Lot acceptance sampling plans.

18.8. Further Readings

1. Montgomery, D.C., " Introduction to Statistical Quality Control", John Wiley (Asia) 2001.
2. Biswas.S., "Statistical Quality Control-Sampling Inspection and Reliability", New Centgral Book Agency Pvt. Ltd. 2003.
3.Wardsworth "Modren Methods for Quality Improvement", John wiley and sons 2005.
4.Acheson j. Duncan "Quality control and Industrial Statistics" ,D.B. Taraporevala Sons & co. 1970

UNIT – V
Lesson – 19

ACCEPTANCE SAMPLING PLANS BY VARIABLES

19.0. Objective

After going through this lesson you will be in position to understand:

- The meaning and need for Acceptance Sampling Plans by variables, their assumptions, advantages and disadvantages.
- Applications of Lot-by-Lot acceptance sampling by variables.
- Comparison between Variable and Attribute sampling plans.
- Derivation of a variable sampling plan for a specified OC Curve.
- Other variable Sampling Plans available in literature.

Structure

19.1. Introduction

In the previous Unit – IV, we have discussed Acceptance Sampling Plans when the quality of the product is determined based on an attribute or classify the product as good or bad, or when we count number of defects per unit. When the quality characteristic is a 'Variable', like, inner or outer diameter of a piston ring, weight or length, lifetime of the unit or bursting strength of soft drink bottles and so on, we use "**Acceptance Sampling by Variables**". These charts may be used as substitute for Attribute Sampling Plans. Basic objectives of these charts are to reduce (1) the Average Total Inspection (2) the Inspection time and (3) the inspection cost.

19.2. Objectives, Assumptions, Advantages and Disadvantages of Acceptance sampling plans by variables

It may be possible to use as a substitute for an attributes sampling plan, a sampling plan based on sample measurements such as the sample mean and standard deviation of the sample. Such plans are known as **"Variable sampling Plans"**.

Objectives: Basic objective of Variable sampling plans is that to reduce the inspection cost and inspection time. Further, to propose more accurate methods than approximate methods and to reduce the sample size for inspection.

Assumptions: The quality characteristic of the units is measurable on a continuous scale and its distribution and its form is known. Usually we assume that the distribution is Normal with mean μ and variance σ^2. If mean and variances are not known, these can be estimated from the sample measurements. Variable Sampling plans can be applied under the following conditions:

1. When the inspection characteristic under consideration must be a variable.
2. When attribute inspection is excessively costly, with respect to inspection time destructive nature of inspection procedures and so on.
3. When attributes inspection will not give enough information, that is, the extent of variation from the desired value is desired.
4. When the variable under consideration should be approximately normal in its distribution.

Under above circumstances, one can use Acceptance sampling plans for Variable quality characteristics.

Advantages: These variable sampling plans have primary advantage over attribute sampling plans that same Operating Characteristic (OC) Curve can be obtained with a smaller sample in the former ones than required in the latter ones. The precise measurements required by a variable sampling plan will probably cost more than the simple classification items as 'good' or 'bad'; 'defective' or 'non-defective'; 'conformance' or 'non-conformance' units required by attribute sampling plans. But the reduction of sample size may significantly reduce this additional cost and hence, over all inspection cost will be reduced. For example, if require inspecting 50 units in attribute sampling plans, may reduce to 30 units required in variable sampling plans. Thus we require 1.65 times less sample size in Variable sampling plans. Further, in variable sampling plans, since the sample size is reduced enormously, we require less inspection time to dispose the lot. Thus one can reduce the inspection cost and inspection time. When inspection process involves destructive test procedures, like in the measurement of breaking strength of a brick, glass shell in an electric bulb, or firing cracker to measure the sound produced, or Stencil strength and so on, reduction of sample size reduces the units to be inspected and hence, reduces the wastage of units used in the process of inspection. Thus, Variables sampling will effectively results in saving of the cost, time and reduces the wastage.

This is particularly significant, when the units are costly like T.V. Color tubes or Computer Mother Boards or VDUs and so on, and test procedures are destructive in nature. Another advantage of variable sampling plans is that measurement data usually provide more information about the manufacturing process or about the lot, than the attribute data. Generally, numerical measurement of characteristics, are more useful than simple classification of item as defective or non-defective.

Disadvantages: The Principal disadvantage of Variables sampling plans is that a separate sampling plan is to be employed for each quality characteristic. For example, we have to

develop one plan for inner diameter and another plan for outer diameter of the piston rings in Automobile spare parts. But we can have one attribute plan for both the characteristics. The product can be classified as defective, with respect to inner diameter or outer diameter or both. This will become cumbersome when we deal more Variables say, five variables or six variables or more. We have to construct so many Variables sampling plans. Where as one attribute plan is enough for all these characteristics.

The Second disadvantage of Variable sampling plans is that, it is also theoretically possible, although not very likely, that under these plans, a lot will be rejected even though all the products in that lot are good items. A supplier unfamiliar with the nature of these plans, would undoubtedly protest strongly against these plans because his product is rejected in this manner, even though all the products in the lot of good quality. Even though this situation is very rarely occur, but one cannot say that it is impossible.

The third drawback of variable sampling plans is that, it is necessary that the distribution of quality each characteristic must be of specified form which is known. If the distribution is not known it is difficult to develop the variable sampling plan. In such situations, it is better to use attribute sampling plans rather than variable sampling plans.

Finally, for some quality characteristics, the assumption of Normal distribution may not be true and one has to identify the exact form of the distribution. Otherwise we may commit wrong decisions/ conclusions. It is remembered that the quality characteristic must be of known form to use variable sampling plans.

19.3. Lot-by-Lot Acceptance Sampling Plans for Variables.

There are three procedures available for the construction of acceptance sampling plans for Variables which are explained as follows:

Procedure – I:

Lot-by-Lot acceptance sampling plans by variables are to be applied if the quality characteristic for the inspection can be measured on a continuous scale and the acceptance on a lot-by-Lot basis is desired. For applying these sampling plans, select a random sample of size 'n', from the given lot of size 'N'. Lt x_1, x_2,.,., x_n are the observations on the quality characteristic measured from the selected sample. Then calculate the sample mean $\bar{x} = \sum_{i=1}^{n} xi/\ n$, and the sample standard deviation $\sigma = S = \sqrt{\sum_{i=1}^{n}(xi - \bar{x})^2 / n - 1}$.

The following Criteria for Variable sampling plans are applied:

For upper specification limit (U) = $\bar{x} + K\sigma \leq U$ (19.3.1)

For Lower specification limit (L) = $\bar{x} - K\sigma \geq L$ (19.3.2)

For two-sided limits, both (19.3.1) and (19.3.2) are to be applied along with $\sigma \leq MSD$.

Where: $\bar{x}$ **= sample mean or the average.**

K = Acceptance constant (normal deviate) which, in conjunction with the sample size, which reflects the risks of acceptance and rejection inherent in the sampling plan.

σ= The measure of variability, which is estimated from the sample standard deviation or estimated from the ranges = $\bar{R}$ /d_2 as done in $\bar{x}$ control charts.

MSD = maximum Standard Deviation.

Now, we proceed to explain the other procedure of constructing Variable sampling Plans.

Procedure – II:

Take a random sample of n items from the lot and compute the statistic Z_{LSL},

Where:

$$\mathbf{Z_{LSL} = (\bar{x} - LSL) / \sigma .} \qquad (19.3.3)$$

Here, $\bar{x}$ = sample mean or the average.

σ = Sample Standard deviation.

LSL = Lower Specification for the quality characteristic.

Note that, Z_{LSL} given in equation (19.3.3) simply expresses the distance between the sample mean $\bar{x}$ and Lower specification Limit (LSL) in Standard deviation **σ** units. The larger the value of Z_{LSL}, the farther the sample average $\bar{x}$ from the LSL and hence, the smaller is the lot fraction defective 'p'. If there is critical value of 'p' of interest that should not exceed with the stated probability, we translate this value of 'p' into a "Critical Distance" (say K), for Z_{LSL} . Thus if $Z_{LSL} \geq K$, we accept the lot because, the sample data imply that the lot mean is sufficiently far above the LSL which ensure that, the lot fraction of non-conforming or defective units is satisfactory. On the other hand, if $Z < K$, implies that, the sample mean $\bar{x}$ is closer to the LSL and hence Lot should be Rejected.

Similarly, we can define Z_{USL} , as follows:

$$\mathbf{Z_{USL} = (USL - \bar{x}) / \sigma .} \qquad (19.3.4)$$

Here, $\bar{x}$ = sample mean or the average.

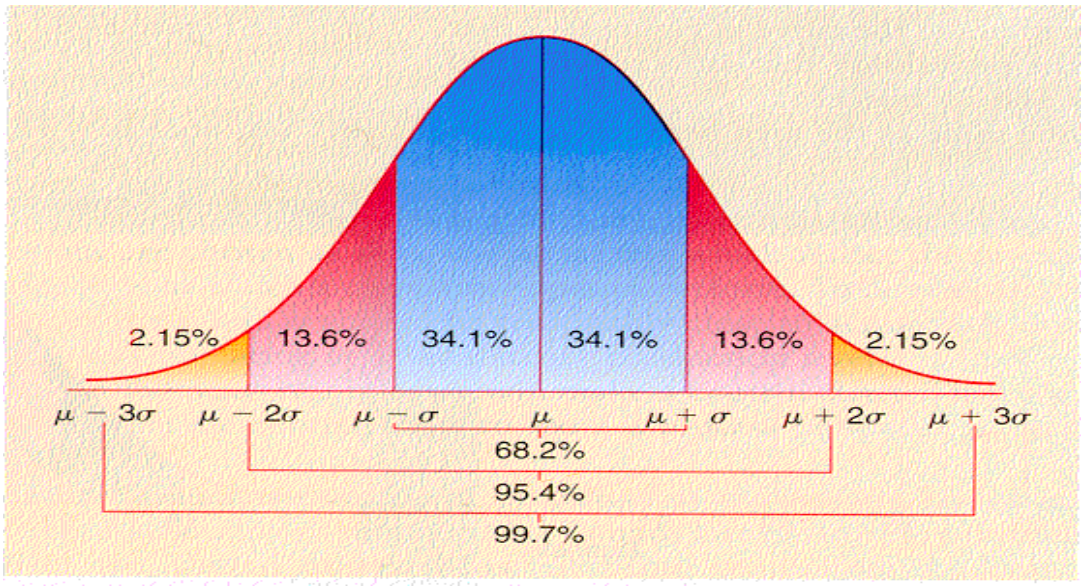
σ = Sample Standard deviation.

USL = Upper Specification for the quality characteristic.

If $Z_{USL} \leq k$, accept the Lot, otherwise, that is, if $Z_{USL} > K$, Reject the Lot. In those situations, where both LSL and USL are to be applied, we have to use both Z_{LSL} and Z_{USL}, simultaneously.

Procedure – III: Take a random sample of size 'n' items/units from the lot and calculate Z_{LSL} using equation (19.3.3). Using this Z_{LSL} we can estimate the process fraction defective of the lot or the process. This is because of the fact that, the process faction defective 'p' and the process mean and the Standard deviation are related. Their relation is explained in the following figure.

Fig. (19.3.1): Relationship of the process or lot fraction defective to the mean and S.D. in Normal Distribution.



P LSL USL

p

Where p = area under the curve representing process or Lot fraction defective.

LSL = Lower Specification Limit,

μ = Process or the lot average and

σ = Process or Lot Standard Deviation.

Use Z_{LSL} or Z_{USL} to estimate to estimate the process fraction defective of the lot or the process as the area under the standard normal curve below Z_{LSL} or above Z_{USL} are similar.

Using the Symmetric property of Standard Normal distribution we have:

$$\mathbf{Q_{LSL} = Z_{LSL}\ [\sqrt{n} / (n-1)]} \qquad (19.3.5)$$

Or $$\mathbf{Q_{USL} = Z_{USL}\ [\sqrt{n} / (n-1)]} \qquad (19.3.6)$$

Thus Q_{LSL} or Q_{USL} is a better estimate of the process or lot's fraction defective 'p'.

Let ''$\hat{p}$' is the estimated value of 'p' obtained by using the formula (19.3.5) or (19.3.6). If the estimated $\hat{p}$ exceeds a specified value '**M**', reject the lot; otherwise accept the lot. Here '**M**' is called maximum allowable fraction defective. This procedure is to be used for Lower specification limit (LSL) L or Upper Specification Limit (USL) U are specified. When both LSL and USL are specified then we have to use $\hat{P}_{LSL} + \hat{P}_{USL} \leq$ **M, the lot will be accepted. Otherwise reject the lot. Here** $\hat{p}_{LSL}$ **= process or lot fraction defective estimated using** Z_{LSL} **and** $\hat{P}_{USL}$ **= process or lot fraction estimated using** Z_{USL} **.**

19.4. Caution in the use of Variable Acceptance Sampling Plans

It is important to note that lot of care and caution is to be taken before using variable Acceptance sampling Plans. This is because of the fact that the quality characteristic under consideration must have a distribution in known form to use appropriate sampling plan.

Furthermore, Variable sampling plans assume that the parameter of the quality characteristic of interest follow normal distribution. This assumption is very important and critical, because, all variable sampling plans require that, there must be some method of converting a sample mean and Standard Deviation into a lot or process fraction defective. If the parameter of interest is not normally distributed, estimates of the fraction defective based on the sample mean and sample Standard deviation will not be same as if the parameter were normally distributed. The difference between these estimated fraction defectives may be large when we are dealing with very small fractions defective.

For example, if the mean of normal distribution fall three sigma times below a single upper specification limit, the lot will contain no more than is 0.135% defective. On the other hand, if the quality characteristic in the lot or process is very non-normal, and the mean lays three standard deviations below the specification limit, it is entirely possible that 1% or more of the items in the lot might be defective.

It is possible to use variables acceptance sampling plans, when the parameter of our interest does not follow normal distribution. But, the form of the distribution must be known or there must be a method of determining the fraction defective from the sample mean and the sample standard deviation or the other appropriate sample statistic connecting them. In such situations only it is possible to develop Variable acceptance sampling plans.

19.5. Designing a Variable Sampling Plans for a specified OC curve

It is easy to design a variable sampling plan by using any one of the three procedure explained above. To design a Variable Sampling plan, for a specified OC Curve, consider two points $(p_1, 1 - \alpha)$; (p_2, β) on the OC Curve of our interest. Let p_1 and p_2 be two levels of lot or the fraction nonconforming, which correspond to acceptable and rejectable levels of quality, respectively.

Using the nomograph given in the following figure (19.4.1), we can find the sample size 'n' and the critical value 'K' to meet the set of conditions p_1, $(1 - \alpha)$, p_2 and β; for both σ known and σ unknown cases. The nomograph contains separate scales for sample sizes for the two cases. Further we can observe that, the sample size requires a large sample size when σ is unknown than the case when the sample size when σ is known case, when the same value of K is used. In addition, for a given sampling plan, the probability of acceptance for any value of the fraction defective can be found from the nomograph. By considering all these points, one can determine the Variable Acceptance Sampling Plan, having the specified OC Curve. Refer table A-10 in appendix.

Fig. (19.4.1): Nomograph for designing Variable Acceptance Sampling Plans.

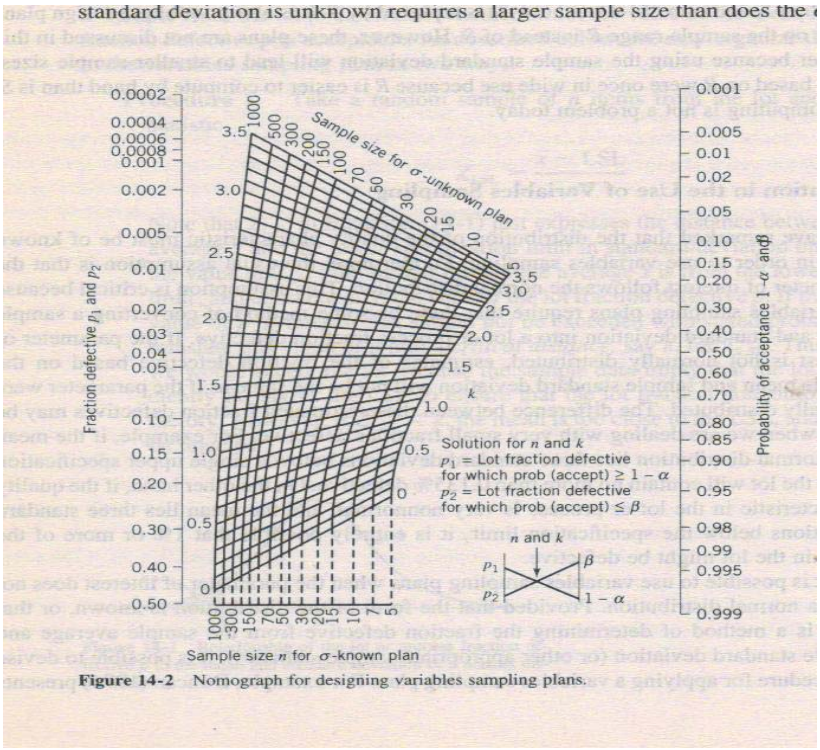


Figure 14-2 Nomograph for designing variables sampling plans.

Example (19.4.1): A soft drink manufacturing company buys non-returnable empty bottles, to fill soft drink from a supplier. The company established a lower specification limit for the bursting strength of the empty bottle at 225 psi. If 1% or less of the empty bottles burst below this limit, the company wishes to accept the lot with probability 0.95. Whereas, 6% or more of the empty bottles burst below this limit, the company would like to reject the lot with probability 0.90. Find the suitable Variable Acceptance Sampling Plan for the given problem.

Solution: In the given problem, it is given that $p_1 = 0.01$; $1 - \alpha = 0.95$; $p_2 = 0.06$ and $\beta = 0.10$. To determine the Sampling plan, draw a line connecting the point $p_1 = 0.01$ on the fraction defective scale to the point $1 - \alpha = 0.95$ on the probability of acceptance scale. Similarly draw another line joining $p_2 = 0.06$ and $P_a = \beta = 0.10$. The intersection point of these two lines will determine the sample size 'n' and 'K' values. From the above nomograph given in fir (19.4.1) we can observe that n = 40 bottles. Take a random sample of 40 empty bottles from the lot and measure the bursting strengths. From the collected data, compute sample mean $\bar{x}$ and the sample standard deviation $\sigma = S$ and as explained in section 19.3. Then calculate:
$Z_{LSL} = [\bar{x} - LSL] / S$ and accept the lot if $Z \geq K = 1.9$. Otherwise reject the lot. If σ is known, then comedown straight down word from the same intersection point towards x – axis. We can observe that the sample size n = 15.

This indicates that when σ is known, the sample size reduced from 40 to 15 approximately 2.67 times less when compared to the case when σ is unknown case. After determining the values of n and K, we can used any one of the procedure explained above.

19.6. Summary

In this lesson we have discussed the need of Variable sampling plans when the quality characteristic is a variable. Variable sampling plans have advantages over attribute sampling that sample size n is reduced in variable sampling plans when compared to attribute sampling plans. This reduces the inspection time, inspection cost and wastage of units in testing when testing is destructive in nature. Further variable sampling plans are more accurate than attribute sampling plans and give more information about the process mean and Variation. We have also discussed various procedures for the construction of variable sampling plans and to determine a sampling plan for a specified O.C. curve.

19.7. Self Assessment Questions

1. What are variable Sampling Plans? How they are distinct from Attribute Sampling plans?
2. Explain the need for Variable sampling plans along with their assumptions.
3. Discuss various advantages and disadvantages of Variable sampling plans.
4. Explain various caution to be taken before constructing a Variable Sampling plan.
5. Critically compare Variable and attribute sampling plans.
6. Explain various methods of construction of Variable sampling plans.
7. Explain the use of nomograph in the construction of variable sampling plans.
8. Explain the role of normal distribution in the construction of variable sampling plans.
9. Explain the method of estimating the fraction defective of the process using sample mean and standard deviation.
10. Explain the method of construction of a Variable sampling plan for a specified OC Curve.

19.8. Further Readings

1. Montgomery, D.C., “Introduction to Statistical Quality Control”, John Wiley (Asia) 2001.
2. Biswas.S., ”Statistical Quality Control-Sampling Inspection and Reliability”, New Central Book Agency Pvt. Ltd. 2003.
3.Wardsworth “Modern Methods for Quality Improvement”, John Wiley and sons 2005.
4.Acheson J. Duncan “Quality control and Industrial Statistics” ,D.B. Taraporevala Sons & co. 1970

UNIT –V
Lesson – 20

MIL-STD 414 AND SEQUENTIAL SAMPLING PLANS

20.0. Objective

After going through this lesson you will be in position to understand:

- The concept of Acceptance Sampling Plans by variables used in military.
- General description and applications of MIL-STD 414 plan.
- Notations of Sequential Sampling Plans for variables.
- Procedure of sequential sampling plans for variables.
- Other variable Sequential Sampling Plans available in literature.

Structure

20.1. Introduction

Earlier, we have discussed MIL-STD Acceptance Sampling plans used by army and Navy people, based on attribute inspection procedures. Thus, MIL-STD 105 series of plans are attribute based plans. The department of defense standard for sampling inspection by variables is Military Standard 414 (MIL-STD – 414), which is used as an alternative to the attribute sampling plans MIL-STD 105 series discussed in Unit – IV. In this lesson we discuss MIL-STD 414 plans along with Sequential Acceptance Sampling Plans used to monitor the process average and the process standard deviation or variation.

20.2. General Description of MIL-STD 414 Pans

Military Standard 414 is a Lot-by-Lot acceptance Sampling Plan for Variables. The Standard was introduced in the year 1957. The focal point of these standards is Acceptable Quality Level (AQL), which ranges from 0.04 to 15. MIL-STD 414 consists of **five levels** of inspection and Level – IV is designated as "Normal Level'. Inspection Level –V gives a steeper OC Curve than the inspection Level- IV. When reduced inspection costs are necessary and when greater risks can or must be tolerated, lower inspection Levels-I or II or III can be used. Like attribute standard, it uses the sample size code letters, but the same letter does not mean

same sample size in both standards. In MIL-STD 414 Standards, sample sizes are function of Lot sizes and the inspection level. Provision is made for normal, tightened and reduced inspection in MIL-STD 414 plans. In all these sampling plans in this standard assume that the quality characteristic under consideration is a normally distributed random Variable.

MIL-STD 414 provides Operating Characteristic (OC) curves for all plans, but offers Single Sampling Plans only. MIL-STD 414 is divided into **four sections** which are explained as follows:

Section – A: This section is a general description of the sampling plans. It explains various terms used in the Standard, defines various classes of defects and indicates the method of selection of the Sample from the Lot or from the process. A special part of Section – A provide for mixed attributes and variable sampling plans, when lots have been previously screened to meet specifications and in other characteristics of the units in the lot.

Section – B: This section of the plan gives variables sampling plans based on sample Standard Deviation for the case in which the process or the lot Standard Deviation is Known.

Section – C: This section gives Variables Sampling Plans based on the sample Ranges or on the average of ranges of sample subgroups of 5 items / units each, when the process or lot Standard Deviation is not known.

Section – D: This section provide variable sampling plans based on the sample mean for the case when the process or lot Standard Deviation is known.

Operating Characteristic Curves are given in section –A, only for plans of Section – B, but, the plans of Section – C and Section - D are so matched with those of section – B that their OC Curves are essentially the same. No OC Cures are given for mixed sampling plans. Variable sampling plans are given for single and double specification limits.

In the year 1980 American National Standards Institute (ANSI) and the American Society for Quality Control (ASQC) released an updated civilian version of MIL-STD 414, known as ANSI/ASQC Z1.9. The principle advantage of ANSI/ASQC Z1.9 standard is that it is possible to start the inspection by using an attribute sampling scheme from MIL-STD 105 plans or from ANSI/ASQC Z1.4, and collect sufficient information to use variables sample inspection, and then shift to variable inspection scheme, while maintaining the same AQL code letter combination. It would be possible to shift back to the attributes scheme, if the assumption of the variables scheme appeared not to be satisfied. Thus we can apply Attribute and variables sampling plans simultaneously and gained information can be best utilized for monitoring any production process statistically. As in MIL-STD 414, ANSI/ASQC Z1.9 also assumes that the quality characteristic must be a normally distributed random variable. This is an important assumption for all variables sampling plans and hence it is advisable to apply a test for normality before using these variable Acceptance Sampling Plans.

20.3. Use of MIL-STD 414

MIL-STD 414 can be used as an alternative to MIL-STD 105 plans when we are interested to monitor quality characteristics of the products are measurable. For example, consider the maximum temperature of operation for a certain device is specified as 209^{o} F. A lot of 40 units are submitted for inspection. It is already mentioned that the inspection level – IV is normal inspection. Let the Acceptable Quality Level (AQL) = 1.0% is to be used for this plan. From table A-11, in appendix, we can observe that Sample code letter is B for the lot size N = 40. From the table A-12, appendix, we can observe that the k value for the Sample code B at AQL = 1.0% is 1.53. The M value from table A-12 is 3.32. Select a random sample of 5 units and calculate the sample mean $\bar{x}$ and the sample standard deviation s. Let the collected data are as follows: 205, 201, 184, 197, and 188. Then Sum of X = $\sum X$ = 975, Sum of Squared Measurements $\sum X^2$ = 190,435. Sample Mean $\bar{x}$ = 975/5 = 195; Variance of X = 77.5 and s = 8.81. From the problem we know that the upper specification limit U = 209. Therefore, the quantity (U - $\bar{x}$) / s = (209 – 195)/ 8.81 = 1.5891. Therefore, the estimate of lot percent defective p_u = 2.19 (from the table A-10). The Acceptable Criteria [(U - $\bar{x}$) / s] must be greater than or equal to K. In the Problem 1.5891 > k = 1.53. Similarly, Acceptable criteria for P_u must be less than or equal to M, that is 2.19 < 3.32. Thus we accept the lot from which the sample is drawn. The natural question arises is that why should we have two different methods yielding the same answer? That is there are two acceptable criteria provided above.

The only logical answer is that MIL-STD 414 represents the combined effort of all departments of defense and each department has its own method. If the lot is good, it should be accepted, whatever may be the method? Thus in both the methods / criteria the lot is accepted. This was demonstrated in the above example. In MIL-STD 414, we can also use Tightened or Reduced inspection levels as the situation demands. The conditions for instituting reduced inspection are as follows:

Condition – 1: When the preceding 10 lots have been under Normal inspection and none has been rejected.

Condition – 2: When the estimated percent defective for each of these preceding lots is less than the applicable lower limit or for certain sampling plans, the estimated lot Percent defective is equal to zero for a certain specified number of consecutive lots.

Condition – 3: When the production is at a steady rate.

Similarly, Normal inspection is reinstated if any one of the following conditions occurs under reduced inspection.

Condition – 4: When a lot is rejected.

Condition – 5: When the estimated process average is greater than AQL.

Condition – 6: When the production becomes irregular or delayed.

Condition – 7: other conditions or as desired by the engineer / inspector that the normal inspection should be reinstated.

Tightened inspection is to be instituted when the estimated process average computed from the preceding 10 lots is greater than the AQL, and when more than the certain number, T, of these lots have estimates of the percent defective exceeding the AQL. Usually T may be

considered as 5 or 10 or 15 lots. Further, the estimated process average is the weighted arithmetic mean of the percent defectives of the preceding lots designated.

Similarly, Normal inspection is reinstated, if the estimated process average of lots under tightened inspection is equal to or less than the AQL.

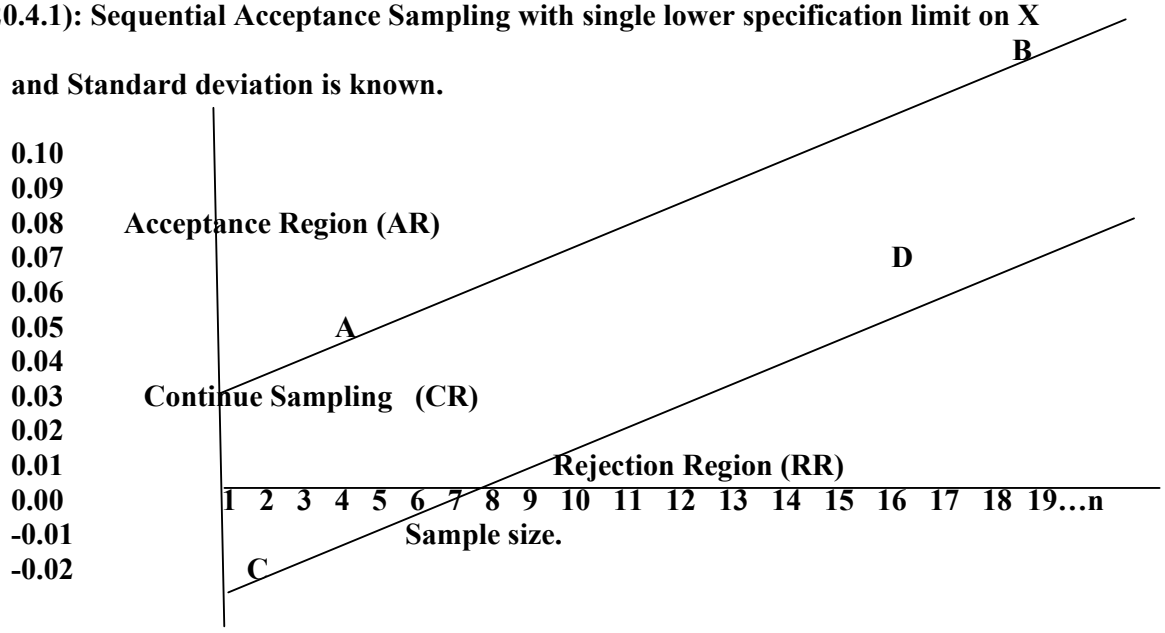
20.4. Sequential Acceptance Sampling Plans for Variables

As we have discussed the Sequential Sampling Plans for attributes, we can also consider the same plans in variables inspection. The usual assumptions are that the quality characteristics are normally distributed and the process / the lot standard deviation if known. The item-by-item Sequential Sampling Plan by variables plots Cumulative Sums of the measurements on the quality characteristics. The procedure of Sequential Acceptance Sampling plan is explained as follows:

Procedure: A unit is taken randomly from the lot and measurement X is made of the given quality characteristic. Let this be denoted by X_1 . The result X_1 is plotted on a chart similar to the fig. (20.4.1):

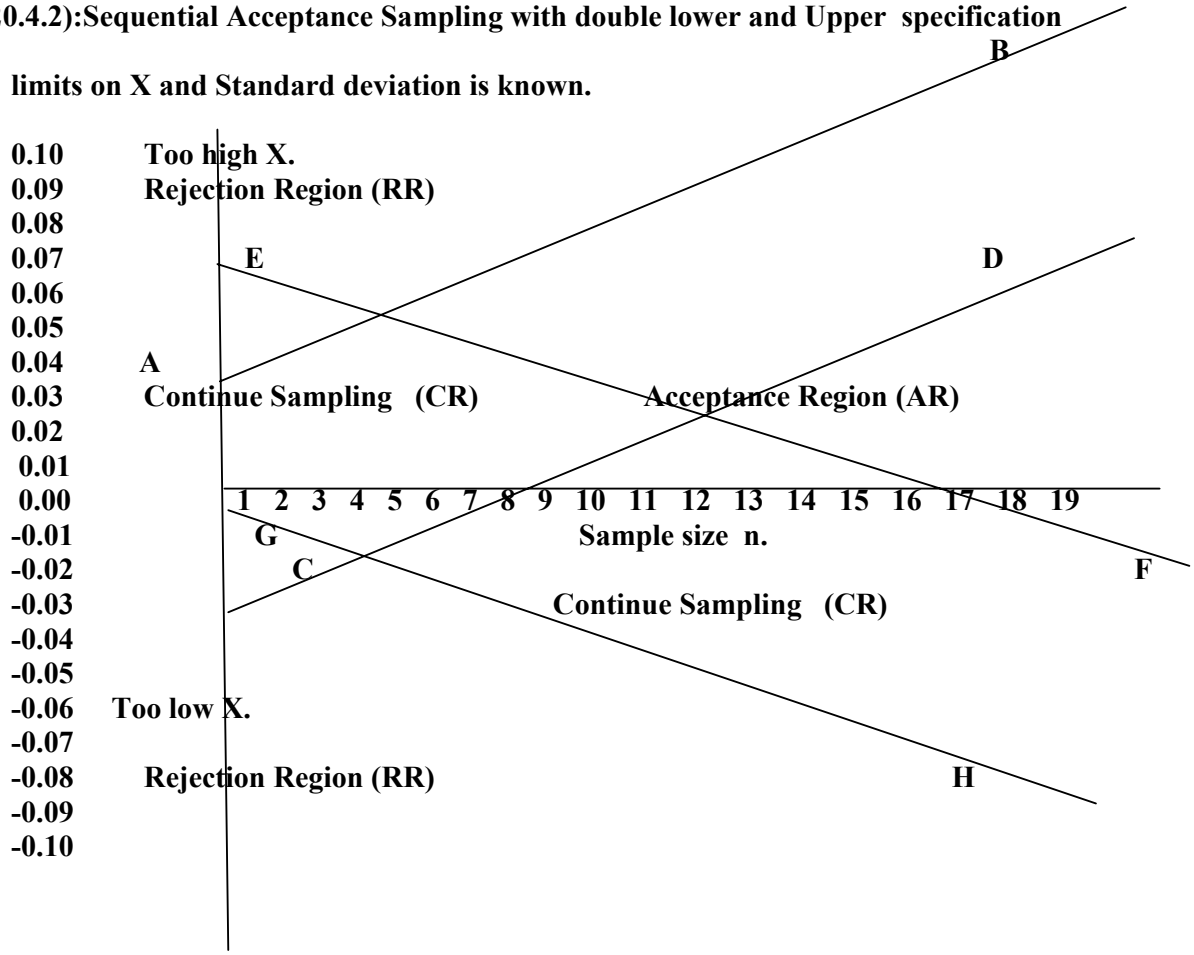
If X_1 falls on or above the line AB and high values of X are desirable, the lot is immediately accepted. If X_1 falls on or below the line CD, the lot is rejected. If X_1 falls between the lines AB and CD, we take another unit from the lot and calculate Total $T = X_1 + X_2$. Then T is plotted against n = 2. If the Sum T is plotted on or above the Line AB, Accept the lot. If the sum T lies on or below the line CD, reject the lot. If the sum T lies in between the lines AB and CD, take the third sample and measure X_3 Calculate $T = X_1 + X_2 + X_3$. Repeat this procedure Until the sum T falls in the Acceptance Region or Rejection Region, for some n. Ultimately, if T falls in the AR, accept the lot. If it falls in the RR, Reject the lot.

Fig. (20.4.1): Sequential Acceptance Sampling with single lower specification limit on X and Standard deviation is known.



In sampling plans of this kind, it is possible that the Acceptance and Rejection lines to be opposite as described in fig. (20.4.1), the lower line CD may be Acceptance Line and the upper line AB may be Rejection Line. This will be true when if low values of X are desirable. For two sided specifications, both higher as well as lower values are undesirable, we obtain the graph as follows:

Fig. (20.4.2):Sequential Acceptance Sampling with double lower and Upper specification limits on X and Standard deviation is known.



In the above graph, the lines AB and GH are Rejection lines and lines CD and EF are acceptance lines. For example diameter of a bullet should not be too low or should not be too large and it should lie in between two specification limits namely Upper specification limit (USL) and Lower Specification Limit (LSL). Similarly with the case of distance between electric plug points or plug pins. The distance should not be too large above upper limit (U) or too small, below a lower limit (L). Too high or too low quality characteristic will lead to rejection.

20.5. Other variable Sequential Sampling Plans

Sequential Sampling plans for attributes; with an unknown process / lot standard deviation follows much the same procedure as when the process / lot standard deviation is

known. Given an X the quality characteristic and a single tolerance grade $\bar{X}$, we can find Sequential Plan that will have specified values for α and β . the statistic used in such plans is

$$\mathbf{U} = [\sum(\boldsymbol{X} - \bar{\boldsymbol{X}}) / \sqrt{\sum(\boldsymbol{X} - \bar{\boldsymbol{X}})^2}] \qquad (20.5.1)$$

Unfortunately, there is no simple formula for drawing Rejection and Acceptance Lines. These formulas are beyond the scope of the syllabus and hence are not discussed here. Those who are interested can refer books given in further references given in 20.8.

Similarly Lot-by-Lot Sequential sampling plans to give assurance regarding the variability of the process or lot are also available. We can also use Sequential-Probability-Ratio plans for testing process / lot variability can be used. We can also use tightened and reduced inspection plans in Variable Sampling Plans. These plans are beyond the scope of the syllabus and hence are not discussed here. Those who are interested can refer books given in further references given in 20.8.

20.6. Summary

In this lesson, we have discussed the general description and uses of MIL-STD 414 plans for variables acceptance sampling plans. Further, discussed the concept of Sequential sampling plans applicable for variable data. Both Upper Specification and Lower Specification Limits are discussed along with double specification limits. Brief discussion is also provided other Variable Sequential Sampling Plans in the last section of this lesson.

20.7. Self Assessment Questions

1. Distinguish between MIL-STD 105 plans and MIL-STD 414 plans.
2. Explain the MIL-STD 414 plans along with its assumptions.
3. Explain the Variable Sequential Sampling Plan with Lower Specification Limit.
4. Explain the Variable Sequential Sampling Plan with double Specification Limits.
5. Draw Acceptance and Rejection lines for single and double specification limits.

20.8. Further Readings

1. Montgomery, D.C., “Introduction to Statistical Quality Control”, John Wiley (Asia) 2001.
2. Biswas.S., ”Statistical Quality Control-Sampling Inspection and Reliability”, New Central Book Agency Pvt. Ltd. 2003.
3.Wordsworth “Modern Methods for Quality Improvement”, John Wiley and sons 2005.
4.Acheson J. Duncan “Quality control and Industrial Statistics”, D.B. Taraporevala Sons & co. 1970

UNIT –V
Lesson – 21

Chain Sampling and Continuous Sampling Plans

21.0. Objective

After going through this lesson you will be in position to understand:

- The concept of Chain Sampling Plans and their applications.
- OC curves for chain Sampling plans.
- Notations of Continuous Sampling Plans and their Types.
- Procedure of Multi Level sampling plans for variables.
- Other Continuous Sampling Plans available in literature.

Structure

21.1. Introduction

In the previously discussed Sequential sampling plans, the sample size is a random variable and we have to continue the sampling until the cumulative statistic falls in the Acceptance Region or Rejection Region. But theoretically, we know an important theorem in Sequential analysis that the sequential procedure terminates with probability one for some finite value of the sample size 'n'. Some situations, in which testing is destructive and units are very costly, sampling plans with smaller sample size required. That is, the sample size 'n', should be as small as possible, because the units are very costly like, T.V. Color Tubes, Computer Mother Boards, and Cathode Ray tubes or X – ray tubes and so on. These units are costly and if testing procedure is a destructive one, one has to waste huge amount of money in testing or measuring some quality characteristics like life times or breaking strength, or bursting Strength and so on. In such situations we use "Chain Sampling Plans", which we are going to discuss in the coming sections.

21.2. General description of Chain Sampling Plans

H.F. Dodge in 1955 suggested an alternative procedure to Sequential Sampling Plans known as **"Chain Sampling Plans"**. These sampling plans are to be used when the units are costly and testing procedure is destructive in nature. In such situations, the sample size should be very, very small and the acceptance number will be zero. But, sampling plans with zero acceptance number are often undesirable, however, in such situations the OC curves are always convex throughout. This means that, the probability of lot acceptance begin to drop very rapidly as the lot faction defective becomes greater than zero. This is often unfair to the supplier / producer producing the products. Further, in situations, where rectifying inspection is used, it can require the consumer to screen large number of lots that are essentially of acceptable quality.

Chain Sampling Plans are used as substitute for ordinary single sampling plans, with zero acceptance numbers applicable in special circumstances. The chain sampling plan makes use of the cumulative results of several proceeding lots. Chain sampling plans are denoted by ChSP. The objective of ChSP is to alert the shape of the OC curve near the origin, so that it has more desirable shape. That is, it is more difficult to reject lots with very small fraction defectives. The general Procedure of Chain sampling similar with that is Single sampling plan is denoted by ChSP – I and is described as follows:

General Procedure of ChSP-I:

Step – 1: From each lot, select a sample of size 'n' and observe the number of defectives in the selected sample.

Step – 2: If the sample has Zero defectives, accept the lot.

Step – 3: If the sample has two or more defectives, reject the lot.

Step – 4: If the sample has one defective, accept the lot, provided there have been no defectives in the previous 'i' lots.

21.2a: Conditions under which Chain sampling is applied:

The proper use of **Chain Sampling Plans**, requires the following conditions, which are to be satisfied, namely:

1. The lot should be one of a series in a continuing stream of lots, from a process in which there is repetitive production under similar conditions.
2. The lots of products submitted for inspection must be in the order of their production.
3. Lots should be usually being expected to be of essentially of same quality.
4. The sampling agency should have no reason to believe that the current lot is of poor quality than those immediately proceeding.
5. There should be a good record of quality performance on the part of the supplier / vendor / producer.
6. The sampling agency must have confidence in the supplier, in that the supplier will not take advantage of its good record and occasionally send a bad lot, when such a lot would have the best chance if acceptance.

Under the above stated conditions, Chain Sampling will result good efforts to reduce the sample size, and can reduce the inspection cost and time. Since wastage in the inspection is reduced, the production cost also reduces considerably, when Chain Sampling is

implemented. Now we proceed to concentrate on the construction of OC curves for Chain Sampling Plans in the following section.

21.3. OC Curves for Chain Sampling Plans.

Construction of OC Curves with the acceptance number C = 0, have very different shape than the OC curves of sampling plans with the acceptance number C > 0. Generally, Sampling plans with C = 0 have OC curves that are convex throughout their range. As a result, of this shape, the probability of acceptance of the lot begins to drop very sharply, even for small values of the lot fraction defective. This is very hard for the vendor / producer / supplier, and in some situations, it is extremely uneconomical for the consumer. This is because of the fact that, if rejected lots are to be returned to the supplier/producer, then large number of lots are to be unnecessarily returned to supplier / vendor, perhaps this may create unnecessary delay in the production. If the consumer screens or 100% inspect all rejected lots, a large number of lots that are of acceptable quality can be found rejected and screened. In order to avoid such situations we consider the acceptance number C =0. Such Sampling Plans with Zero acceptances are known as "Chain Sampling Plans". Under above stated conditions, Chain Sampling plans with the acceptance number C = 0, works considerably better than single sampling plans, discussed earlier in the previous unit. The construction of OC curves for Chain Sampling plans is approximately similar to Single sampling plan OC curves. The Points on the OC curve of a ChSP-I plan are given by the equation:

$$P_a = P(0,n) + P(1,n)[P(0,n)]^i \qquad (21.3.1)$$

Where, P(0,n) and P(1,n) are the probabilities of obtaining zero or one defectives, respectively, out of a randomly selected sample of size n from a lot. In the calculations of Probabilities we use the Binomial Probability mass function. Now, we illustrate these calculations of OC curve for Chain sampling Plan with the following Example:

Example (21.3.1) Construct Operating characteristic Curves for chain sampling plans with n = 5 and C = 0, p = 0.10 and i=1,2,3,4 and 5.

Solution: In the problem, it is given that n = 5 and C = 0, p = 0.10 and i=1,2,3,4 and 5. That is in chain sampling plan – I (ChSP-I), to construct the O.C. curve, first we have to calculate the probabilities P(0,n) and P(1,n), as follows:

$P(0,n) = \frac{n!}{d!\,(n-d)!}\; p^d (1-p)^{n-d} = \frac{5!}{0!\,(5)!}\; (0.10)^0 (1-0.10)^5 = (0.90)^5 = 0.59049$

Similarly, $P(1,n) = \frac{n!}{d!\,(n-d)!}\; p^d (1-p)^{n-d} = \frac{5!}{1!\,(4)!}\; (0.10)^1 (1-0.10)^4 = 5(0.10)(0.90)^4$

$= 5\,(0.10)(0.65610) = 0.32805.$

Using equation (21.3.1), we calculate Probability for acceptance of the lot P_a as follows:

If i=1, we have $P_a = P(0,n) + P(1,n)\,[P(0,n)]^1 = 0.59049 + 0.32805\,(0.59049)^1 = 0.78420.$

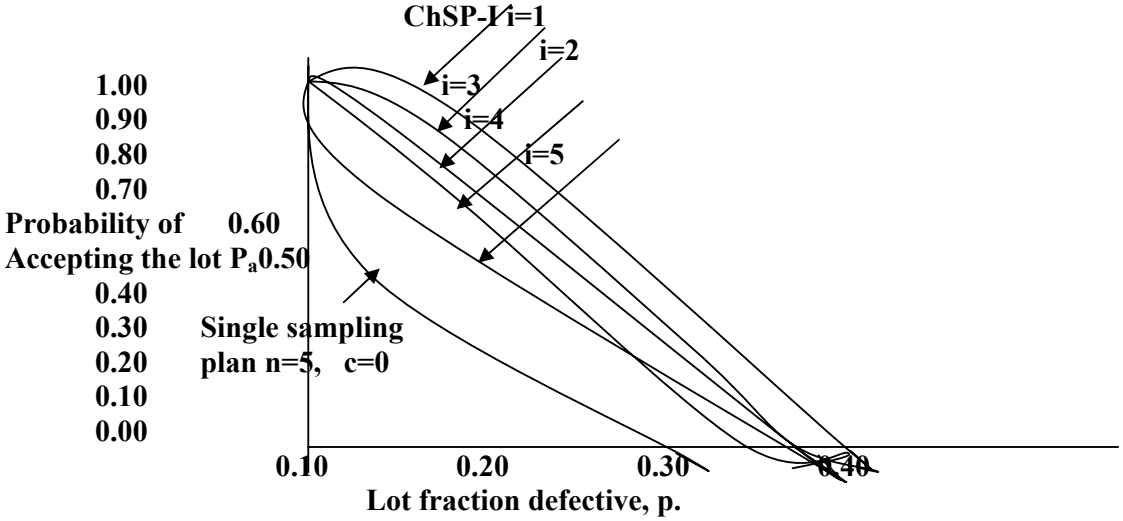
Similarly, i=2,3,5 are calculated as follows:

If i=2, we have $P_a = P(0,n) + P(1,n)\,[P(0,n)]^2 = 0.59049 + 0.32805\,(0.59049)^2 = 0.704874.$

If i=3, we have $P_a = P(0,n) + P(1,n)\,[P(0,n)]^3 = 0.59049 + 0.32805\,(0.59049)^3 = 0.658033.$

If i=4, we have $P_a = P(0,n) + P(1,n)\,[P(0,n)]^4 = 0.59049 + 0.32805\,(0.59049)^4 = 0.630377$.
If i=5, we have $P_a = P(0,n) + P(1,n)\,[P(0,n)]^5 = 0.59049 + 0.32805\,(0.59049)^5 = 0.614041$.
To draw O.C. Curves for ChSP- I for various values of 'i', consider lot fraction defective 'p' on x-axis and Probability of lot acceptance p_a on y – axis, we the following figure (21.3.1).

Fig. (21.3.1) Operating Characteristics for Chain sampling Plans with i=1,2,3,4and 5.



Thus construction of OC Curves for ChSP-I is similar to Single sampling plan and decision should be carefully taken, when the acceptance number C = 0. Now, we proceed to discuss other sampling plans, which are used when formation of lots are difficult.

21.4. Need for Continuous Sampling Plans.

Previously discussed Acceptance Sampling Plans are useful when the products submitted for inspection are in the form of Lots. Hence, they are known as Lo-by-Lot Acceptance Sampling Plans. In all these plans, there is an explicit assumption that, the product must be formed into lots and the purpose of the sampling plan to sentence the individual lots. However, in many manufacturing process, particularly, when the production a product involve a complex assembly process, like, computers, Auto mobiles, Aero planes, or ships, do not result in a natural formation of lots. Production of such products is to be performed through an assembly line and hence is not possible to form lots. In such situations, where production is

Continuous, the following two approaches may be used to form lots. Namely:

Procedure – 1: In this procedure, we allow the accumulation of products produced at a given point in the assembly process. This procedure has the disadvantage of creating an in-process inventory at various points of assembly and requires additional space and requires additional security measures. This is very inefficient approach to manage at different assembly lines involved in the production process of the product.

Procedure – II: In this procedure, we arbitrarily mark off a given segment of production as a lot. The disadvantage of this procedure is that, if a lot is ultimately rejected, and 100% inspection of the lot is subsequently required, it may be necessary to recall products from production manufacturing operations, which are in the further downstream. Thus, the disassembly of semi-finished products and their destruction, may be partial, is unavoidable.

Because of above explained problem, there is a need to develop some special sampling plans which can avoid grouping the produced items into lots. Such special sampling plans suitable for continuous production are known as **"Continuous Sampling Plans" proposed by H.F. Dodge in the year 1943.** The procedure of continuous sampling plans is explained as follows:

Procedure: The procedure of Continuous sampling plan consists of alternating sequence of sampling inspection and 100% inspection or screening. The plan usually begins with 100% inspection and when a stated number of units are found to be free from defects, sampling inspection is inspected. Such a predetermined number is usually known as "**Clearance Number"** and is denoted by the letter **'i'**. Sampling inspection is continued, until a specified number of defective units are found, at which time again 100% inspection is resumed. Continuous sapling plans are rectifying inspection sampling plans and the quality of the products is improved by the partial screening of semi-finished products/finished products. That is, whenever a defective is found, is to be replaced with a good unit at any stage of production process.

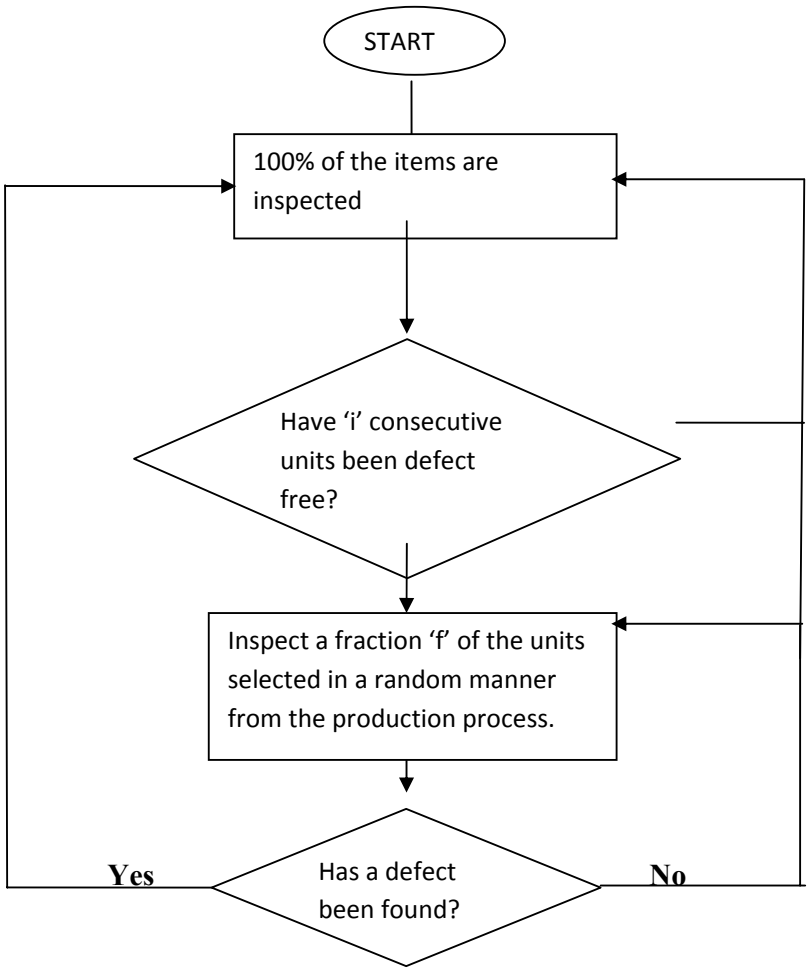
The procedure of Continuous sampling plan – 1, denoted as CSP-1 and is explained as follows:

21.4a. Continuous Sampling Plan – 1 (CSP-1): The continuous sampling plan procedure proposed by H.F. Dodge in the year 1943 is known as CSP – 1. At the start of the plan, all units are inspected 100% and as soon as **'i'** consecutive units are free from defects, 100% inspection is discontinued and only a fraction **'f'** of the units are inspected. These sample units are selected one at a time at random from the flow of production. If any sample unit is found defective, immediately 100% inspection is resumed. The number 'i' is called the clearance number and 'f' is called as the sampling fraction. All defective units found are either reworked or replaced with good ones.

A CSP-1 plan has an overall AQL and the value of AOQL depends on the values of the Clearance number 'i' and the sampling fraction 'f'. The same AOQL can be obtained by different combinations of the clearance number 'i' and the sampling fraction 'f'. The choice of 'i' and 'f' are usually based on practical considerations in the manufacturing process. Further, i and f may be influenced by the workload of the inspector and the operator in the system. It is usually common in practice that, the quality inspector will do the sampling and place burden of 100% inspection on the persons involved in the manufacturing process. As a thumb rule, it is not a good idea to choose the value of f smaller than 1 /200 because the protections against bad quality in a continuous run of production then become very poor.

The procedure of CSP-1, is schematically represented in the following figure (21.4a.1).

Fig. (21.4a.1): Schematic Representation of CSP-1:



21.4b. Properties of Continuous Sampling Plan – 1 (of CSP – 1):

The average number of units inspected in a 100% screening sequence following the occurrence of a defective is denoted by 'u' and is given by:

$$\mathbf{u = (1 - q^i) / pq^i} \quad . \qquad (21.4b.1)$$

Where, q = 1 – p, and p is the fraction defective produced when the process is operating in control. The average number of units passed under the sampling inspection procedure is denoted by 'v' and is given by:

$$\mathbf{v = 1 / fp} \quad . \qquad (21.4b.2)$$

The average fraction of total manufactured units inspected in the long run denoted by AFI and is given by:

$$\mathbf{AFI = (u + fv) / u + v} \quad . \qquad (21.4b.3)$$

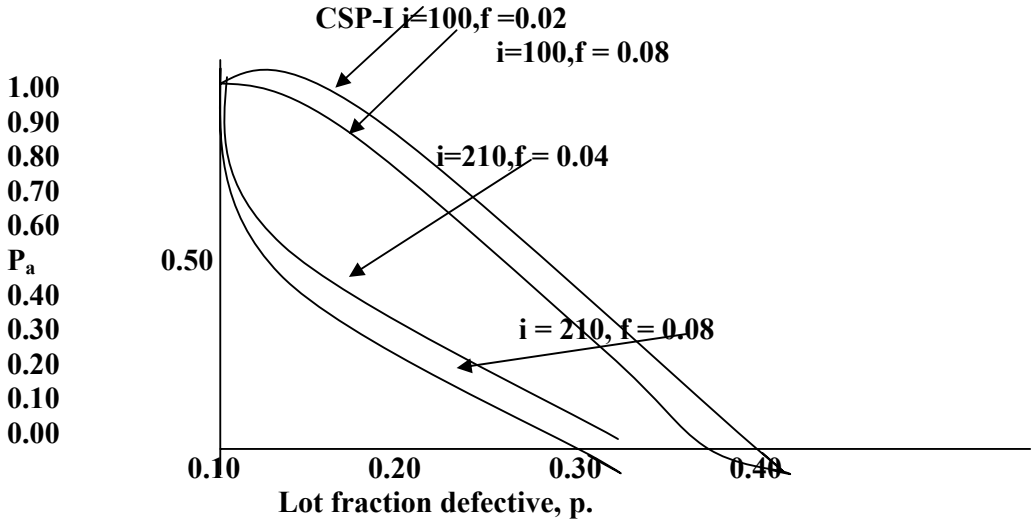
The average fraction of manufactured units passed under the sampling procedure is denoted by P_a and is given by:

$$\mathbf{P_a = v / (u + v)} \quad . \qquad (21.4b.4)$$

When 'P_a ' is plotted as a function of 'p' we obtain the Operating Characteristic (OC) curve for a continuous sampling plan – 1 (CSP – 1).

It is important to note that the OC curve for a lot-by-lot acceptance sampling plan gives the percentage of lots that would be passed under sampling inspection where as OC curves for continuous sampling plan gives the percentage of units passed under sampling inspection. Operating Characteristic curves varies as the values of 'I' and 'f' are changing and based on the shape of the OC curves, we have to select the parameters of the Continuous sampling plans namely: 'i' and 'p'. OC Curves for various values of 'i' and 'p' are given in the following figure (21.4b.1). One can observe from the graphs of various OC curves, in fig. (21.4b.1), It is important to note that, for moderate to small values of 'f ','i' has much more effect on the shape of the curve than does 'f'.

Fig. (21.4b.1) Operating Characteristic Curves for Continuous sampling Plans (CSP – 1).



Now we proceed to discuss various types of continuous sampling plans in the following section.

21.5. Other Continuous Sampling Plans

There are various versions for the original version of Dodge CSP-1. One version was designed to meet the objection that the occurrence of one single isolated defective unit sometimes does not warrant return to 100% inspection. This is particularly true when we are dealing with small or minor defects. To meet this objection, Dodge and Torrey in 1951 proposed CSP-2 and CSP – 3, which are explained as follows:.

CSP – 2: Under CSP-2, 100% will not be reinstated when the production is under sampling inspection until, two defective sample units have been found within the space of 'K' sample units of each other. It is common practice to choose K equal to the clearance number 'i'. CSP – 2 plans are indexed by specific AOQLs that might be obtained by different combinations of the clearance number 'i' and the sampling fraction 'f'.

CSP – 3: This procedure is similar to the procedure of CSP – 2, but is designed to give additional protection against spotty production. It requires that, after a defective unit has been found, in sampling inspection, the immediately following four units should be inspected. If any of these four units is defective, 100% inspection is immediately reinstituted. If no defectives are found the sampling plan under CSP – 2 may be continued.

Thus shifting rules from sampling inspection to 100% inspection will change from CSP - 1,2 and 3. This gives some protection against random and spotty production of defective units from a production process. It is important to note that however careful the workers are, however precise the instruments are, however quality the raw material is, we cannot produce two identical items in this world. Some variation or the other will be present between the products. All out effort is to control / eliminate '**Assignable Causes of Variation'** but not the '**Random Causes of Variation'**.

Basic criticism on all continuous sampling plans is that, the abrupt transition between sampling inspection and 100% inspection is done in all these plans. To avoid this criticism, Liberman and Soloman in 1955 designed another type of sampling plans known as "**Multilevel Continuous sampling plans**" whose procedure is explained as follows:

21.5a. Multilevel Continuous Sampling Plans: Multilevel Continuous Sampling Plans also begins with 100% inspection as it is done in CSP – 1. Switch to sampling inspection of a fraction 'f' of the production, as soon as the clearance number 'i' has been reached. However, when under sampling inspection at the rate of 'f', a run of 'i' consecutive sample units are found free of defects, the sampling continue at the rate of 'f^2 ' . If further a run of 'i' consecutive sample units under sampling inspection at the rate of 'f^2' are defect free then shift the sampling inspection rate to 'f^3' and so on. This procedure may be continued as far as the sampling agency wishes. If at any stage, a defective unit is found, sampling inspection rate may be reversed to next lower level from the present level. That is if we are under the sampling inspection at the rate of 'f^5', if a defective unit is found, the reverse the sampling inspection rate to 'f^4' and so on.

This type of Multilevel Continuous sampling plans are very popularly used in the standard manufacturing processes produces good quality products. This procedure also reduces the total units inspected to a great extent when compared to other Acceptance Sampling Plans if the production process is of good standards and maintain good quality methods in the production. During poor periods of production, in these procedures, naturally increases the total inspection.

21.6. Summary

In this lesson, we have discussed Chain Sampling Plans applicable for lot-by-Lot acceptance sampling plans, their need, procedure of the plans and construction of OC Curves are discussed. Further, the need of continuous Sampling Plans, proposed by H.F. Dodge is discussed and the problems in the formation of Lots in Lot-by-Lot acceptance sampling plans are discussed. The procedure of ChSP – 1 is explained along with the construction of OC Curves, and other characteristics of continuous Sampling plans are discussed. Various versions of Continuous Sampling Plans like CSP – 2 and CSP – 3 , along with their inter relationships are discussed. Finally, the concepts of Multilevel Continuous Sampling Plan, their need, the procedure, and their applications are discussed.

21.7. Self Assessment Questions

1. Explain the need of Chain Sampling plan, with an example.
2. Explain the procedure of ChSP – 1. How they are distinct from Sequential Sampling Plans.
3. Explain the method of construction of OC Curves for Chain Sampling Plans.
4. Explain various conditions under which chain Sampling is Applied.
5. Explain the need for Continuous Sampling Plans.
6. Explain the procedure of CSP-1.
7. Explain various versions of Continuous Sampling Plans and compare them critically.
8. Draw the schematic representation of CSP -1.
9. What are multi-level plans and discuss their applications.
10. Explain the procedure of Multilevel Continuous Sampling Plans.

21.8. Further Readings

1. Montgomery, D.C., "Introduction to Statistical Quality Control", John Wiley (Asia) 2001.
2. Biswas.S., "Statistical Quality Control-Sampling Inspection and Reliability", New Central Book Agency Pvt. Ltd. 2003.

3.Wordsworth "Modern Methods for Quality Improvement", John Wiley and sons 2005.

4.Acheson J. Duncan "Quality control and Industrial Statistics" , D.B. Taraporevala Sons & co. 1970

UNIT –V
Lesson – 22

MIL-STD 1235 and Skip Lot Sampling Plans

22.0. Objective

After going through this lesson you will be in position to understand:

- The concept MIL-STD1235 Sampling Plans and their applications.
- Other Continuous Sampling plans.
- Notations of Skip-lot Sampling Plans and their Types.
- O.C Curves for Skip-lot sampling plans.

Structure

22.1. Introduction

The concept of Continuous acceptance sampling introduced in the previous lesson, has been applied by the department of Defense in their sampling plans. This Military Sampling Plan using the concept of continuous Sampling plan is known as MIL-STD1235 plans. The standards of MIL-STD – 1235 provides for five different types of Continuous Sampling Plans. These Five Continuous Sampling Plans are CSP – 1, CSP-F, CSP – 2, CSP – V, and CSP –T Plans which are different parts of MIL-STD 1235C Version. In the Syllabus, we have MIL-STD 1235b plans and hence this version is explained in the following section.

22.2. General Description of MIL-STD 1235B plans

The Standard established in MIL-STD 1235B is Continuous sampling Plans and procedures for inspection by attributes. When this standard is referenced, in a contact, Specification, inspection Standard or Similar document, the provisions of MIL-STD 1235B Standard is applicable for all attribute type continuous Sampling Plans and Procedures. This

MIL-STD-1235B is applicable to all inspection procedures which are non-destructive in nature and the production of the product is continuous in nature and the produced product is moving on a conveyor belt. Further, there exists ample space, equipment and manpower at or near the site of inspection to permit rapid 100% inspection when required. These MIL-STD-1235B plans designed for various entities like, end items, components, raw materials, data or records, and any other entities satisfying above said conditions.

MIL-STD- 1235B consists of five types of Continuous Sampling Plans, namely: CSP-1, CSP-F, CSP-2, CSP-T and CSP –V. The features of these five plans are explained as follows:

a) CSP-1: CSP-1 is a single level Continuous Sampling Procedure which provide for alternating sequence of 100% inspection and Sampling inspection with no limit as to the number of such sequences. CSP – 1 requires a return to 100% inspection whenever a non-conforming unit is discovered during the sampling inspection. One has to choose the suitable from tables the Clearance number 'I' and the sampling fraction 'f'.

b) CSP – F: This is a variation of the CSP – 1 plan in that CSP-F plans are applied to a relatively short run of product, thereby permitting smaller clearance numbers to be used. CSP-F is a single level continuous sampling procedure, which provides for alternating sequence of 100% inspection and sampling inspection. CSP-F is equivalent to the application of CSP-1 plan to a smaller clearance number 'i' to be used. This plan is used in situations involving short production runs, or it may be applied to more production intervals at a time, in situations involving time consuming inspection operations or the unit is costly like T.V. color Tube or X – ray equipment, where a larger clearance number could cause a production bottle-neck. AOQ and AOQL for CSP-F relate to the long run average and the limit respectively over many periods of application of the plan, which in fact are the same as the expected values, respectively, for a single application of the plan.

c) CSP-2: This plan is a modification to the CSP-1 in that 100% inspection resumes only after a prescribed number of defect-free units any two defective sample units. Instead of immediately shifting from sampling to 100% inspection with a single defect sample unit, we wait until another defect with the interval of 'K' units. If any defective unit is found within 'i' sampled units, then shifting to 100% inspection is made. This gives additional protection against spotty or sudden production of defective units. That is CSP-2 requires a return to 100% inspection when ever two defective units are found separated by fewer than 'i' consecutive sampled units, but does not require return to 100% inspection if 'i' or more consecutive defect-free sample units separate two defective units.

d) **CSP-T:** This plan is a Multilevel Continuous Sampling Procedure which provides for reducing the sampling frequency 'f' upon demonstration of superior product quality. We can increase the fraction 'f' to 'f^2' or 'f^3' when the production is continuously good and steady. When there is a hint for decreasing the quality, we can reduce the sampling fraction to the next lower level which increases the sampling frequency. CSP-T requires a return to 100% inspection when ever, a non-conforming unit discovered during

sampling inspection, but provides for a reduced sampling frequency upon demonstration of superior product quality. CSP-T shall not be used for inspection for "**Critical Defects**". Classification of defects or defectives; are discussed in detail in the next section.

e) **CSP-**V: This plan is also a single level Continuous Sampling Procedure, which is an alternative to CSP-T. In these plans, a provision is there to reduce the clearance number, when the production is of good quality and meets the standards prescribed by the plan. CSP-V is a single level Continuous sampling procedure, which provide for alternating sequences of 100% inspection and Sampling inspection. CSP-V require a return to 100% inspection whenever a non-conforming unit is discovered during sampling inspection, but, provide a procedure for reducing the clearance number upon demonstration of good or superior product quality. It can be beneficially applied in those situations when there is no advantage to reduce sampling frequencies, for example, when the inspector has more ideal time, if the sampling frequency is reduced.

There are other versions of MIL-STD – 1235 plans namely, MIL-STD -1235A1, MIL-STD-1235 C. Since, the syllabus consists of only MIL-STD -1235B plan, explanation is provided on for the same. Those who are interested on other plans can refer further readings provided at the end of this lesson in Section 22.8.

22.3. Classification of Defects and Defectives

So far, we have considered classifying the units into defective or non-defective; good or bad; conforming or non-conforming units. But, there is a need to classify the units as defective or not according to their seriousness of the defects. Some defects may be minor and some may be serious in nature. There is a need to classify the units based on the seriousness or severity. Defects will normally be grouped into one or more of the following classes; however, defects may be grouped into other classes or into sub-classes within these classes. The Classification of defects and defectives are as follows:

22.3a. Method of classifying defects: Usually, defects are classified into three categories, namely:

(1) Critical Defects: A critical defect a defect that judgment and experience indicate is likely to results in hazardous or unsafe conditions for individuals using or depending or maintaining the unit / the product. In other words, Critical Defect is a defect that judgment and experience indicate is likely to prevent the performance of the intended job or tactical function of a major end item / unit/ system like, a ship, aircraft, tank, missile, or space craft or a computer.

(2) Major Defects: A major defect is defect other than Critical, that is likely to result in failure or materially reduce the usability of the unit / product for its intended purpose.

(3) Minor Defects: A minor defect is a defect, that is not likely to reduce materially the usability of the unit or product for its intended purpose. Thus a minor defect is a departure from established standards having little bearing on the effective use of operation of the unit.

Now, we proceed to classify the defectives as follows:

22.3b:Method of classifying defectives: A defective is a unit/product/system which consists of one or more defects. These defects may be critical or major or minor. Depending on the nature of the defects, the product can also be classified into three categories of defects, as follows:

(1) Critical Defective: A unit / product is said to be critical defective if it consists of one or more critical defects and may also contain major and / or minor defects.
(2) Major Defective: A unit / product is said to be a major defective if it contains one or more major defects and may also contain minor defects, but contains no Critical defects.
(3) Minor Defective: A unit/ product is said to be minor defective, if it contains one or more minor defects but should not contain any critical or major defects.

Classification of defects / defectives plays an important role in determining the suitable sampling plan in various Acceptance Sampling Plans. Now we proceed to concentrate on other type of sampling plans known as 'Skip-Lot Sampling Plans' in the following section.

22.4. Skip-Lot Sampling Plans

In this section, we explain the development and evaluation of Lot-by-Lot inspection plans in which provision is made for inspecting some fraction of the submitted lots. The idea introduced in continuous sampling plans can also be applied to sequence of lots instead of units from a continuous production. Such plans are known as "**Skip-Lot Sampling plans**" introduced by H.F. Dodge in the year 1956. These sampling plans should be used only when the quality of the submitted product is good ad demonstrated by Vendor's quality history. These plans are initially proposed as an extension of Continuous Sampling Plans (CSP) type applied to individual products.

In other words, a **Skip-Lot Sampling Plan, denoted by SkSP,** which is the application of continuous sampling Plan to lots rather than to individual units of production on an assembly line. The first version of Skip-Lot Sampling Plan proposed by Dodge, required a single determination or analysis to ascertain the lot's acceptability or un-acceptability. Such plans are denoted by SkSP – 1. The next version SkSP – 2 is the extension of SkSP – 1, which is the next logical step applied in SkSP -2. That is each lot to be sentenced is sampled according to a particular attribute lot inspection plan known as "**Reference Sampling Plan**". The procedure of SkSP – 2 is explained as follows:

22.4a. Procedure of SkSP – 2 Plan: A Skip-Lot Sampling Plan of the type SkSP – 2 uses a specified lot inspection Plan, known as " Reference Sampling Plan" together with the following rules:

Rule – I: Begin with normal inspection, using the reference plan. At this stage of operation, every lot is inspected.
Rule – II: When 'i' consecutive lots are accepted on normal inspection, switch to skipping inspection by skipping some fraction of lots. In skipping inspection a fraction 'f' of

the lots are inspected.

Rule – III: When a lot is rejected, on skipping inspection, return to normal inspection.

The quantities "i" and "f" are called the parameters of the Skip-Lot sampling Plan SkSP - 2. In general, the clearance number 'i' is a positive integer and the sampling fraction 'f' lies in the interval (0,1). That is $0 < f < 1$; when the sampling fraction $f = 1$, the skip-lot sampling plan reduces to the original reference sampling plan. Thus skip-lot sampling plans are an effective acceptance sampling procedure, and may be useful as a procedure through which one can reduce the inspection, when the quality of the submitted lots are of good quality.

22.5. OC and ASN functions of Skip-Lot Sampling Plans

The parameters of the skip-lot sampling plan are the clearance number 'i' and the sampling fraction 'f'. Let P denote the probability of acceptance of a lot from the original reference sampling plan. Then P_a (f,i) is the probability of acceptance of the lot under skip-lot sampling plan SkSP – 2, and is given by:

$$\mathbf{P_a\,(f,i) = [fP + (1 - f)\,P^i\,] / [f + (1 - f)\,P^i\,]}\ . \qquad (22.5.1)$$

It can be easily verified and can be shown that if $f_2 < f_1$, and for a given reference number 'i' and a specified reference sampling plan, $\mathbf{P_a\,(f_1,i) \le P_a\,(f_2,i)}$. Furthermore, for an integer, Clearance number $i < j$, a fixed value of f and a given reference sampling plan, we can prove that $\mathbf{P_a\,(f,j) \le P_a\,(f,i)}$.

These are the special properties of Skip-lot sampling plans, with reference plans with n =20 and c=1 are shown in the following figures (22.5.1) and (22.5.2).

Fig. (22.5.1) Operating Characteristics curves of Skip-lot Sampling Plans (SkSP – 2).

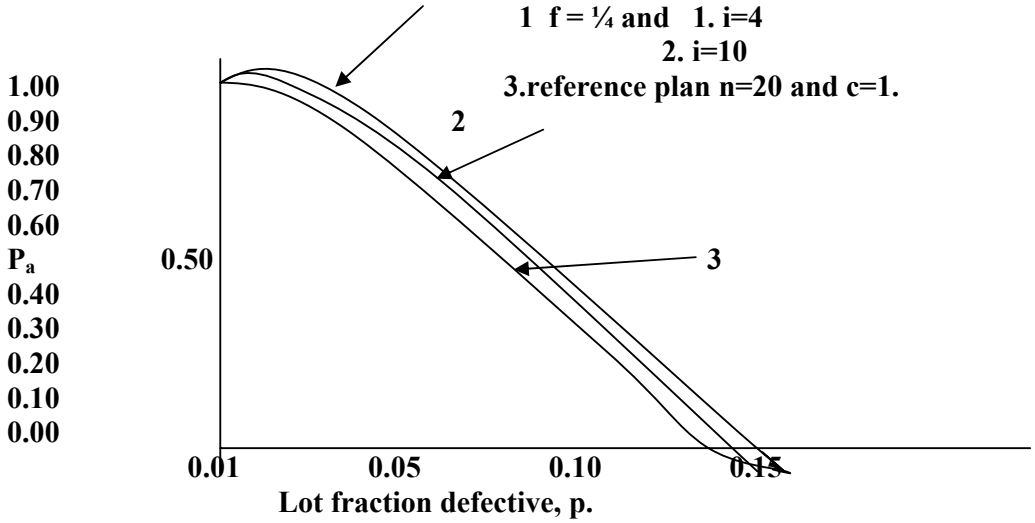
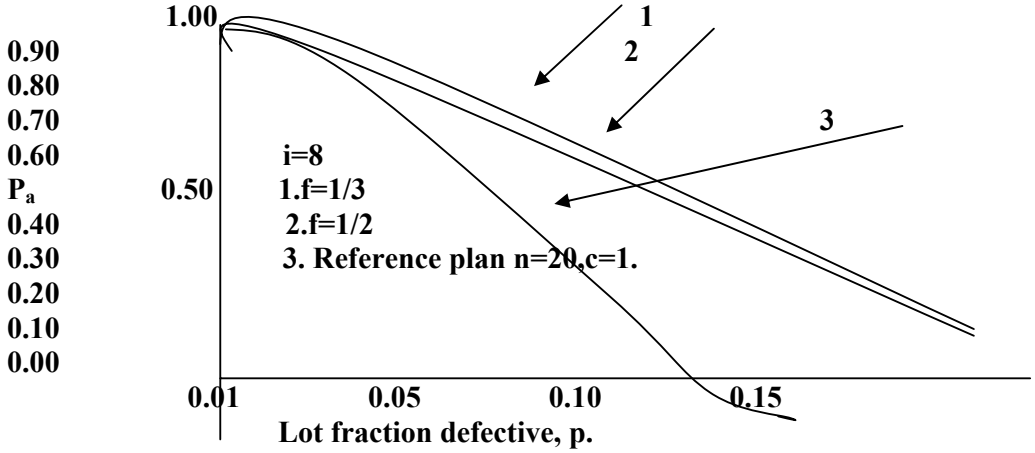


Fig. (22.5.2) Operating Characteristic Curves for Skip-lot sampling Plans (SkSP – 2).



A very important property of skip-lot sampling plan is that the average amount of inspection required. In general, skip-lot sampling plans are basically used to reduce the average amount of inspection required. The Average Sample Number (ASN) of a skip-lot sampling plan is given by:

$$\mathbf{ASN\ (SkSP) = ASN(R)F} \qquad (22.5.2)$$

Where F is the average fraction of submitted lots that are sampled and ASN(R) is the Average Sample Number of the Reference Sampling Plan. It can be shown that

$$\mathbf{F = f / [(1 - f)P^i + f]} \qquad (22.5.3)$$

Since F is in the interval (0,1), that is $0 < F < 1$, it follows that:

$$\mathbf{ASN\ (SkSP) < ASN(R)} \qquad (22.5.4)$$

Hence, Skip-lot sampling plans always yields a reduction in the average sample number required. For those situations, in which incoming lots of good quality from a standard production, the reduction in the ASN is very significant in Skip-lot Sampling Plans. Finally it is very important to note that skip-lot sampling plans should be carefully used only for those situations in which there is sufficient history of vendor / producer can supply high quality products and can supply the lots with very good quality. Furthermore, if the vendor / supplier is supplying in a highly erratic manner and there is great deal of variability from Lot – to- Lot, Skip-lot sampling plan should not be used for such erratic quality lots. If we know that the production process is statistically in control and the process capability is adequate to ensure virtually, defect free production, then only Skip-lot sampling are advisable. Otherwise, this may reduce the inspection but quality of the products is not maintained at the desired level. To produce defect free production or Zero defect production, there is a need to introduce some new concepts known as **"Total Quality Management" (TQM)** and "**Six-Sigma**" Concepts which are introduced in the last lesson – 23.

22.6. Summary

In this lesson, we have introduced the concepts of Military Standard Plans used by defense people for Continuous Sampling Plans. Such plans are known as MIL-STD 1235 plans and discussed the procedure of MIL-STD 1235B plans. Further, discussed various classifications of defects and defectives and finally introduced Skip-Lot Acceptance Sampling plans. The procedure, construction of OC and ASN functions for SkSP-2 sampling plans are discussed along with their applications. The concept of defect free production or zero defectives is introduced and to achieve this we required two more new concepts known as **"Total Quality Management"** and **"Six-Sigma"** concepts which are introduced in the last lesson – 23.

22.7. Self Assessment Questions

1. What are MIL-STD 1235 plans? Discuss their applications.
2. Explain various categories of MIL-STD 1235 sampling plans.
3. Explain various parts of MIL-STD 1235B Plans.
4. Distinguish between a defective and a defect with suitable examples.
5. Explain various categories of defects and defectives with suitable examples.
6.Explain the Concept of Skip-Lot sampling plan with an example.
7. Distinguish between Continuous Sampling Plans and Skip-Lot sampling plans.
8. Explain the procedure of SkSP – 2 sampling plan.
9. Explain the OC and ASN functions of SkSP sampling plans.
10. What is meant by Reference Sampling Plan? Explain its role in SkSP plans.

22.8. Further Readings

1. Montgomery, D.C., "Introduction to Statistical Quality Control", John Wiley (Asia) 2001.
2. Biswas.S., "Statistical Quality Control-Sampling Inspection and Reliability", New Central Book Agency Pvt. Ltd. 2003.
3. Wordsworth "Modern Methods for Quality Improvement", John Wiley and sons 2005.
4. Acheson J. Duncan "Quality control and Industrial Statistics" , D.B. Taraporevala Sons & co. 1970.

UNIT –V
Lesson – 23

Total Quality Management and Six Sigma Concepts

23.0. Objective

After going through this lesson you will be in position to understand:

- The meaning and need for Total Quality Management.
- Dimensions of Quality ad its philosophies.
- Management Strategies.
- Concepts and tools of Total Quality management.
- The concept of Six Sigma and its uses.

Structure

23.1. Introduction

Basic objective of all the techniques learnt in all the previous lessons is to control the quality of the industrial output. Further, we also know that, however careful we are, however precise the instruments are, and however quality the raw material is, one cannot produce two identical items in this world. Some variation or the other which is physically observable or measurable or not, which are inherently present in the repetitive processes. If the variation is purely due to random causes, which is uncontrollable and hence, we ignore such variations and consider all the products are of same quality. If the variations are significant, one can identify the cause for such variation and perhaps one can remove such causes of variation known as assignable causes of variation from the production process, if such causes are detected and identified. We must understand the concept of Quality in depth and understand the philosophies of quality to introduce the concepts of Total Quality Management (TQM) and Six-sigma. Now

we proceed to discuss various dimensions of quality and its related philosophies in the following section.

23.2. Dimensions of Quality

The quality of a product can be evaluated in several ways. It is often very important to differentiate various dimensions of quality. Garvin in 1987, provides a excellent discussion on eight dimensions or components of the quality of a product, which are explained as follows:

1. **Performance: (Will the product do the intended job?)**

Usually any customer will evaluate the product to determine, whether the product will perform its intended job properly or not before purchasing the same. That is he wants to evaluate the product to determine whether it will perform certain specific functions and to determine who well it perform them. For example, to purchase a cell phone, we see how the piece identifies the networks, the incoming signals, clarity in sound and voice, outgoing signals, and its operations and so on. We can compare different cell phones with respect to the performance of above mentioned jobs, the execution speed and perfectness in the execution of instructions given by the user.

2. **Reliability (How often does the product fail?):**

Any customer, before purchasing the product will naturally interested to know how often, the product will fail? It is important to note that no product will work for ever, that is infinitely many years. Any product however quality it is, has to fail on one day or the other. The complex products such as many appliances, automobiles, or airplanes, or computers usually require some repair over their service life of the product. For example, if we purchase a Personal Computer (PC), we are interested to know or to estimate the expected time that the PC require occasional repair. If any product requires frequent repairs, we say that the product is un-reliable. That is the reliability of the product is another important dimension of the quality of the product.

3. **Durability (How long the product last?):**

This measures the effective service life of the product the customer, going to purchase. Customer obviously wants the product which he is going to purchase, perform satisfactorily over a long period of time. It is important to note that no product in this world can serve infinitely many years, but fails in a certain finite time. If this life time is reasonably long, for example, if we purchase an auto-mobile, roughly it should serve the customer for 10 or 15 years. Based on this period only, Life time taxes are paid usually. Thus durability is considered as another important dimension of the quality of the product.

4. **Serviceability (How easy to repair the product?):**

Many customers think, how quickly and economically a repair if it occurs or routine maintenance can be done to the product, they are going to purchase. For example, a prospective customer of a bank will concentrate on how long it takes, to deposit or withdraw the amount or how long it takes to get a credit card or a loan amount, or how long it takes to make corrections for errors occurred in your current month bill and so on. Similarly, if we purchase a

car, how long it takes to regular monthly maintenance? Is the important concept to concentrate on? Thus serviceability is another dimension of the quality of the product.

5. **Aesthetics (What does the product look like?):**

This is the visual appeal of the product, often a customer's take into account factors such as style, color, shape, packing alternatives, and other sensory features smoothness and so on. For example, to purchase a mobile, we see the color, smoothness, size, weight attractive look and the cost are some of the characteristics a customer will look for. Hence, how the product look for is another important characteristic / dimension of the quality of the product we are going to purchase.

6. **Features (What does the product do?):**

Usually, any customer will be interested to purchase, high quality product that have added features. For example, in a mobile, what are the additional features of the product? Can we send SMS? Or connect to internet and get net information? Can the product include the MP3 player and Audio player? Can we add memory cards of different sizes; can we connect to our Personal Computer? And so on. Thus additional features of the product, that is apart from the regular job, what jobs additionally the product can do? Is another dimension of the quality. Features of the product are makes the product more popular one and a customer prefer such additional feature products to purchase. Thus features of the product are another dimension of the quality of the product.

7. **Perceived Quality (What is the reputation of the company or its product?):**

In many cases, customer will also concentrate on the good will of the company, from which the product is produced. The reputation of the company directly influenced, by the failures of the product, which are highly visible to the public. For example, if book a luggage, the transport agency, should not damage the product and deliver. It should be properly delivered intact to the customer without any damage done in the transportation. For this proper packing its position while transporting are to be properly done. Thus, any customer will prefer to go for such transportation agency which can give assurance for the safe delivery of the product to the customer. Thus, goodwill of the company or the perceived Quality of service rendered by the organization is another dimension of the quality of the product from which it is produced or from which the service is rendered.

8. **Conformance to Standards (Is the product made exactly as the designer intended?)**

Generally any customer think of high quality product as one that exactly meet the requirements placed on it. For example, we take a medicine with a prescribed quantity of the medicine in mg., will the tablet contains exactly the same mg. or not? If it is high mg. the medicine may give reaction or if it is of low mg. may not give expected effect in the patient. Thus the drug manufacturing company should concentrate on producing medicine should concentrate on the power of the chemical or medicine in the produced product. Thus any customer is interested to know the manufactured products produced from a process meet the designer requirements or standards prescribed? Auto mobile piston rings should not be slightly, larger or slightly small in diameter as prescribed, will not suit to our vehicle. If it is of required dimension, then only it suits and fit properly to our automobile. Otherwise, the piston ring is of bad quality. Thus Conformance to standards is another important aspect of quality of the

product under study. Thus manufactured products, which do not exactly meet the designer's requirements, can cause serious problems in the functioning of the unit.

In order to consider all the dimensions of quality, we require some recent tools and techniques popularly known as "**Total Quality Management**" whose meaning, related tools and philosophies are discussed in coming sections of this lesson.

23.3. Meaning and tools of Total Quality Management (TQM)

Thus, to control the quality of a product, we should concentrate on all the dimensions explained above. To achieve this all the persons involved in various stages of production that is from designing stage to the ultimate use by the customer should work in a coordinated manner. Maintaining the quality is not only the responsibility of a single person or the management or the worker working on the machine to produce the product. This is team work and every person should work together for achieving this common 'goal'. Even though Statistical techniques are very important and critical tools available for quality control and improvement, which are to be effectively used most effectively and must be implemented by all the people involved at every stage of production process. The management system must direct the implement the quality improvement philosophy and ensure its implementation in all aspects of the production process. This concept, of implementation of the concept of Quality and improvement, by the management in its framework, is quite recent one and is known as "**Total Quality Management (TQM)",** or "**Company-wide Quality Control (CWQC**)" or "**Total Quality Assurance (TQA**)".

Now, we proceed to discuss some key elements and tools of TQM, some important quality philosophies, the link between quality and productivity, the notation of Six-Sigma and legal implications of quality, in this lesson.

23.3a. Quality Philosophies and Management Strategies:

Basically there are three philosophies exists for quality improvement.

They are:
(i) Dr. W. Edwards Deming's Philosophy,
(ii) Dr. Joseph M. Juran's Philosophy and
(iii) Dr. Armand V. Feigenbaum's Philosophy, which are briefly explained as follows.

(i) Dr. W. Edwards Deming's Philosophy:

During the World War – II, Dr. Deming worked for the war department and the census Bureau. Later on, he became a consultant to Japanese industries and convinced their top management the power of Statistical tools and importance of quality as a competitive weapon to become number one in the world market. The philosophy of Dr. Deming consists of the following 14 points, which are briefly explained as follows:

(1). Create Constancy of purpose of improving product design , performance and services.
(2). Adopt a new philosophy of rejecting poor workmanship.
(3). Do not rely on mass inspection to control quality.

(4). Do not award business to any supplier on the basis of the price alone, but also consider the quality of the product and minimize total cost by working with a single supplier.
(5). Focus on Continuous improvement constantly forever every process for planning production and services.
(6). Practice modern training methods and invest in giving training all employees on the job.
(7). Practice modern supervision methods and supervision should not be passive but should be active, adopt and institute leadership.
(8). Drive out fear in employees to suggest for improvement to the management.
(9). Team work among different organizational units is essential for effective quality improvement and productivity improvement.
(10). Eliminate targets, slogans and numerical goals for the workforce or the management. Concentrate not on the quantity but on the quality of production.
(11). Eliminate numerical quotas and work standards and eliminate the annual rating or merit system.
(12). Remove barriers that discourage employees from doing their jobs.
(13). Institute an ongoing program of training, self improvement and education for all employees.
(14). Create a structure in top management that will vigorously advocate and implement the above 13 points. Put everybody in the company to work accomplishing the transformation.

In the above 14 points, we can observe that there should be strong emphasis on the change and the important role played by the management in guiding this change in the process of production.

(ii) Dr. Joseph M Juran's Philosophy:

Dr. Juran is one of the founding father of Statistical Quality Control and is less focused on Statistical methods, than Dr. Deming. The Juran philosophy is based on organization for change and the implementation of improvement through **"Managerial Breakthrough**". Dr. Juran is a firm believer that management action is required to improve the Quality. He believes that 80% of quality improvement can only be dealt by the management and only 20% can be dealt with by the employees or workers. Dr. Juran was invited to speak to Japanese industry leaders, as they began their industrial transformation in the early 1950's.

Dr. Juran worked with Dr. Walter A. Shewhart at AT&T Bell Telephone Laboratories and played a key role in introducing 'Quality Improvement" moment.

(iii) Dr. Armand V. Feigenbaum's Philosophy:

Dr. Feigenbaum is the first person to introduce the concept of "**Company- wide Quality Control**". Many Japanese companies used the name "**Total Quality Control**", to describe their efforts. This philosophy later leads to the new concept known as "**Total Quality Management**". Dr. Feigenbaum philosophy is more concerned with organizational structure and a system approach to improve quality than he is with the "**Statistical Methods'**. He is initially suggested that much of the technical capability can be concentrated by a" Specialized Department". This is in contrast to the more modern view that knowledge and use of statistical tools need to be wide spread.

23.3b.Meaning and Tools of Total Quality Management

The **Total Quality management (TQM) is a** management approach to long-term success through customer satisfaction**. TQM** means, a Strategy for implementing and managing quality improvement activities on an organization-wide or Company-wide basis. The TQM began in the early 1980's with the three philosophies explained above in 23.3a. The TQM evolved into a wider spectrum of concepts and ideas; involving participative organizations and work culture, customer focus, supplier quality improvement, integration of the quality system with business goals and many other activities to focus all elements and employees of an organization around the basic goal of "**Quality Improvement**". TQM approach to quality improvement has "**Quality Councils"** or "**Quality Circles**". A High-level teams that concentrates with strategic quality initiatives, where as Work-force level teams which focus on '**routine production' or 'business activities'**. The cross sectional teams will address specific quality improvement issues.

In a TQM effort, all members of an organization participate in improving Process, Products, Services and the work culture in which they work. TQM is an integrative philosophy of management for continuously improving the quality of products, process and the services. This concept is used around the world in almost all fields.

The TQM programs have the following tools, namely:

(1) Six – Sigma approach.
(2) Just-in-time (JIT) approach.
(3) Agile or Lean manufacturing approach
(4) Poka – Yoke,
(5) Operate a pull type
(6) Plan-Do-Check-Act (PDCA) cycle
(7) House of quality
(8) 5 S principles
(9) Quality Function Development (QFD)
(10). Pareto chart and analysis
(11 Scatter diagrams
(12) Failure Mode and Effects Analysis (FEMA)
(13) Cause and Effect or Fish-bone or Ishikawa diagrams
(14) Cost Benefit Analysis
(15) Histograms
(16) Root Cause Analysis
(17) Suppliers-Inputs-Process-Outputs-Customers (SIPOC) Analysis
(18) Taguchi methods
(19) Taguchi Loss Function
(20) Housekeeping and so on.

23.4. The concept of Six-sigma and its uses

The concept of Six-Sigma is very recent one, which is a tool of Total Quality Management (TQM). High –technology products with many complex components typically

have many opportunities for failure or defect to occur. In the year i980, Motorola Company developed the Six-Sigma concept as one of the tools of TQM for the development of such complex products. The focus of six-sigma is to reduce the variability in the key product quality characteristic to the level at which failures or defects are extremely unlikely. That is very, very less chance of defects of failures the products.

In Shewhart control charts, we basically use 3σ control limits, which means that if the quality characteristic is Normally distributed, the probability of producing a product defect free is 0.9973, which means that 2700 **parts per million (ppm)** are defective items. This is the concept of "**Three Sigma (3σ) Quality Performance**" and it actually sounds very good . However, suppose we have complex system with 100 sub-components with 3σ quality performance, then the probability of producing a defect free sub-parts / units is:
0.9973 x 0.9973x0.9973x.,.,., 0.9973(100 times) = $(0.9973)^{100}$ = **0.7631**. That is about 23.7% of the products produced under 3σ quality performance are defectives. If we take complex Systems consisting of one million sub-parts, then 66,810 parts are defects under 3σ performance, which is not at all a desirable situation.

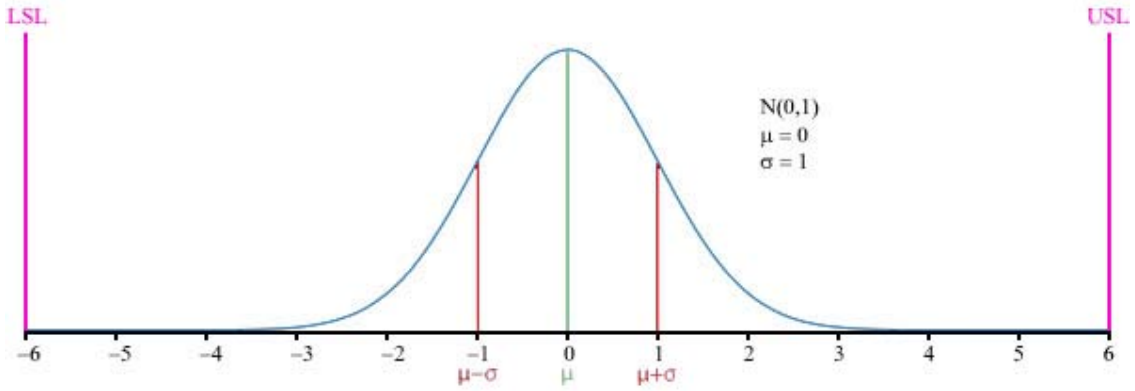
The Motorola Six-sigma concept, is to reduce the variability in the process, so that the specification limits are 6σ or six times the standard deviation from the mean on either direction. That is under 6σ (Six-sigma0 performance, the probability of producing a non-defective will be 0.9999998. or the probability of producing a defective is 1 – 0.9999998 = 0.0000002, that is 0.2 defective parts per million (ppm) which is much better performance than the 3σ performance or Shewhart control limits performance.

When 6σ concept we initially developed, an assumption was made that when the process reached the six-sigma quality level, the process mean was still subject to disturbance that could cause it to shift by as much as 1.5 standard deviations off the target. A comparison between various sigma limits are given below along with ppm, which explains the concept of six-sigma clearly.

Specification limits.	Percentage of area inside the limits.	ppm defectives.
± 1sigma	68.27	3,17,300
± 2sigma	95.45	45,500
± 3sigma	99.73	2,700
± 4sigma	99.9937	63
± 5sigma	99.999943	0.57
± 6sigma	99.9999998	0.002

Above explained concepts are represented graphically as follows in figure (23.4.1).

Fig.(23.4.1): Six-Sigma Concept in Normal Distribution.



That is under six-sigma performance, if we produce 1000 million parts, there is a possibility of 2 defective units or the probability if getting a defective unit is 0.0000002.

The concept of Six-sigma came from statistics known as "Process Capability Studies" and the goal of six-sigma is to improve all process to that level of quality or better. Six-sigma also aims of reducing total cost and improving quality. In recent years, some practitioners have combined Six-sigma idea with "**Lean Manufacturing**". The lean manufacturing process concentrates the process flow, wastage and business and operational excellence.

23.5. Quality Costs and legal aspects of Quality

It is important to note that reducing the waste is one important aspect of reducing the cost of production. Further, we require same raw material, machine time, technology, manpower and inspection time, to produce a good item as well as a defective or non-conforming item / unit. Thus "**Do it right for the time itself"** is most opt and it reduces raw material, time and money. If the produced product is a defective one, entire effort will become waste. Hence, for reducing the wastage, one has to do perfectly, precisely, with all care and excellence. This makes the cost of producing will become as least as possible.

When come to the **quality costs**, means the money incurred on inspection, wastage, if the testing is destructive one and expenditure on appointment of inspectors or supervisors their salaries and insurances and so on. In some organizations, quality costs are 4% or 5% of sales of their product where as in some organizations, it can be as high as 35% or 40% sale of their products. However, quality costs are to be reduced to a maximum extent. This can be done with "**Pareto Analysis**", which consists of identifying and analyzing quality costs by category wise or by product wise or by type of defect wise or nonconformity wise and reduce each type to its lowest.

Coming to **legal aspects of quality**, consumerism and product liability are two important reasons for the recent and important trends involved in business strategy. The rise of consumerism is an important and crucial part of quality, because a large number of failures in the field of consumer products may occur and makes the user unhappy. Si9nce he has paid the cost

of the product, it is the manufacturing process or management responsibility to satisfy the customer. In recent business trends most important and crucial role is played by the consumer. Ultimately, Consumer is the '**God**' for any business and industry. Thus manufacturers are always vitally concerned about 'field failures' because , heavy external failure costs and the related threat to their competitive position in the market. Hence, most producers or manufacturers have to make product improvements and its quality which results reduction of field failures. For example, automobile tires now have over 10 times the life of many of their early predecessors. Similarly, Integrated - circuits or computer Mother-boards, the technology has been improved greatly and reduced the failure of electronic equipment or Personal Computers (PCs) considerably. Basically**, every product of today is superior to that of yesterday**.

The second aspect of the legal problem is that, consumer can tolerate for minor defects and aesthetic problems but consider very seriously the product working conditions and the product performance. Product liability is a major social, market and economic force and the legal aspects or obligations of manufacturers or sellers to compensate heavily for the injuries or damages caused by the defective product. The direction of the law has always been that manufacturers or sellers are likely to incur a liability, when they have been Unreasonably careless or negligent in what they have designed or produced or manufactured.

In recent years, the courts have placed a more stringent rules in effect which are known as "**Strict Liability**" . The **first responsibility,** lies on the manufacturer or producer or seller of the product, requiring immediate responsiveness to unsatisfactory quality through product service, repair or replacement of the defective product. By producing a product both manufacturer and seller must accept the responsibility for the ultimate use of that [product – not only for its performance, but also for its environmental effects on the society, the safety aspects of its use, the related health problems arise because of environment pollution and other aspects.
Note: Total Quality Management and Six-sigma concepts are Separate topics to be discussed and one can write separate book on these tools and techniques. The basic objective of this lesson is to give introduction to TQM and to list tools used. Hence we are not discussing the details of these tools. Those who are interested can refer further references given at the end of this lesson 23.8.

23.6. Summary

In this lesson, we have concentrated two important aspects namely (1) Total Quality Management and (2) Six-sigma concept. Both these concepts are very recent and evolutionary and crucial concepts involves the Social element, legal aspects and the reduction of various costs involved in the production of the product. The meaning of TQM is discussed along with an exhaustive list of tools of TQM. These concepts are very recent and are used by all industry throughout the world, in these days. These concepts are introduced in an elementary basis and are to be discussed in depth. There are many things to learn and discuss, but because of limitations of the syllabus, we have not discussed these concepts in depth.

23.7. Self Assessment Questions

1. Explain the need of Quality Improvement with suitable examples.
2. What is meant by TQM? Explain briefly.

3. Explain the Deming's philosophy of quality.
4. Explain briefly various philosophies related to the quality.
5. List out various tools of TQM.
6. What is meant by Six-sigma concept.
7. Distinguish between three-sigma and six-sigma concepts with suitable examples.
8. Discuss various quality costs involved in the production of a product.
9. Discuss various legal aspects of quality.
10. Explain TQM and Six-sigma concepts briefly.

23.8. Further Readings

1. Montgomery, D.C., "Introduction to Statistical Quality Control", John Wiley (Asia) 2001.
2.Deming,W.Edwards "Out of the Crisis" 1986.
3.Ishikawa , Kaoru "What is Total Quality Control? The Japanese way"- 1985.
4.Jain K.C. and Chitale, A.K. " Quality Assurance and TQM" Khanna Publisher, 1998.
5. Sharma S.C. "Inspection, Quality Control and Reliability" Khanna Publisher, 1998.

APPENDIX

Table A-1: Constants for $\bar{X}$ and R- Charts.

Appendix VI Factors for Constructing Variables Control Charts

Observations in Sample, n	Chart for Averages: Factors for Control Limits A	A_2	A_3	Chart for Standard Deviations: Factors for Center Line c_4	$1/c_4$	Factors for Control Limits B_3	B_4	B_5	B_6	Chart for Ranges: Factors for Center Line d_2	$1/d_2$	d_3	Factors for Control Limits D_1	D_2	D_3	D_4
2	2.121	1.880	2.659	0.7979	1.2533	0	3.267	0	2.606	1.128	0.8865	0.853	0	3.686	0	3.267
3	1.732	1.023	1.954	0.8862	1.1284	0	2.568	0	2.276	1.693	0.5907	0.888	0	4.358	0	2.575
4	1.500	0.729	1.628	0.9213	1.0854	0	2.266	0	2.088	2.059	0.4857	0.880	0	4.698	0	2.282
5	1.342	0.577	1.427	0.9400	1.0638	0	2.089	0	1.964	2.326	0.4299	0.864	0	4.918	0	2.115
6	1.225	0.483	1.287	0.9515	1.0510	0.030	1.970	0.029	1.874	2.534	0.3946	0.848	0	5.078	0	2.004
7	1.134	0.419	1.182	0.9594	1.0423	0.118	1.882	0.113	1.806	2.704	0.3698	0.833	0.204	5.204	0.076	1.924
8	1.061	0.373	1.099	0.9650	1.0363	0.185	1.815	0.179	1.751	2.847	0.3512	0.820	0.388	5.306	0.136	1.864
9	1.000	0.337	1.032	0.9693	1.0317	0.239	1.761	0.232	1.707	2.970	0.3367	0.808	0.547	5.393	0.184	1.816
10	0.949	0.308	0.975	0.9727	1.0281	0.284	1.716	0.276	1.669	3.078	0.3249	0.797	0.687	5.469	0.223	1.777
11	0.905	0.285	0.927	0.9754	1.0252	0.321	1.679	0.313	1.637	3.173	0.3152	0.787	0.811	5.535	0.256	1.744
12	0.866	0.266	0.886	0.9776	1.0229	0.354	1.646	0.346	1.610	3.258	0.3069	0.778	0.922	5.594	0.283	1.717
13	0.832	0.249	0.850	0.9794	1.0210	0.382	1.618	0.374	1.585	3.336	0.2998	0.770	1.025	5.647	0.307	1.693
14	0.802	0.235	0.817	0.9810	1.0194	0.406	1.594	0.399	1.563	3.407	0.2935	0.763	1.118	5.696	0.328	1.672
15	0.775	0.223	0.789	0.9823	1.0180	0.428	1.572	0.421	1.544	3.472	0.2880	0.756	1.203	5.741	0.347	1.653
16	0.750	0.212	0.763	0.9835	1.0168	0.448	1.552	0.440	1.526	3.532	0.2831	0.750	1.282	5.782	0.363	1.637
17	0.728	0.203	0.739	0.9845	1.0157	0.466	1.534	0.458	1.511	3.588	0.2787	0.744	1.356	5.820	0.378	1.622
18	0.707	0.194	0.718	0.9854	1.0148	0.482	1.518	0.475	1.496	3.640	0.2747	0.739	1.424	5.856	0.391	1.608
19	0.688	0.187	0.698	0.9862	1.0140	0.497	1.503	0.490	1.483	3.689	0.2711	0.734	1.487	5.891	0.403	1.597
20	0.671	0.180	0.680	0.9869	1.0133	0.510	1.490	0.504	1.470	3.735	0.2677	0.729	1.549	5.921	0.415	1.585
21	0.655	0.173	0.663	0.9876	1.0126	0.523	1.477	0.516	1.459	3.778	0.2647	0.724	1.605	5.951	0.425	1.575
22	0.640	0.167	0.647	0.9882	1.0119	0.534	1.466	0.528	1.448	3.819	0.2618	0.720	1.659	5.979	0.434	1.566
23	0.626	0.162	0.633	0.9887	1.0114	0.545	1.455	0.539	1.438	3.858	0.2592	0.716	1.710	6.006	0.443	1.557
24	0.612	0.157	0.619	0.9892	1.0109	0.555	1.445	0.549	1.429	3.895	0.2567	0.712	1.759	6.031	0.451	1.548
25	0.600	0.153	0.606	0.9896	1.0105	0.565	1.435	0.559	1.420	3.931	0.2544	0.708	1.806	6.056	0.459	1.541

For $n > 25$

$$A = \frac{3}{\sqrt{n}}, \quad A_3 = \frac{3}{c_4\sqrt{n}}, \quad c_4 \approx \frac{4(n-1)}{4n-3},$$

$$B_3 = 1 - \frac{3}{c_4\sqrt{2(n-1)}}, \quad B_4 = 1 + \frac{3}{c_4\sqrt{2(n-1)}},$$

$$B_5 = c_4 - \frac{3}{\sqrt{2(n-1)}}, \quad B_6 = c_4 + \frac{3}{\sqrt{2(n-1)}}.$$

Table A-2: Constants for Median and Mid Range Charts.

TABLE I–B. Factors for Median and Mid-range Charts.

$\tilde{X}'$ = median of parent universe

$\tilde{X}$ = median of sub-group

$\bar{\tilde{X}}$ = mid-range of sub-group

$\tilde{X}_i$ = median of all individuals in sample

$\overset{\approx}{X}$ = median of sub-group medians

$\overset{\approx}{\bar{X}}$ = median of sub-group mid-ranges

$\tilde{R}$ = median of sub-group ranges

Standards Given:

$$\tilde{d}_2\sigma' = \tilde{R}'$$

$$\tilde{A}_2\tilde{R}' = 3\sigma_{\tilde{x}}, \qquad \bar{\tilde{A}}_2\tilde{R}' = 3\sigma_{\bar{\tilde{x}}}, \qquad \tilde{A} = \frac{A}{\sqrt{E_{\tilde{x}}}}$$

$$\tilde{D}_3\tilde{R} = D_3R, \qquad \tilde{D}_4\tilde{R} = D_4R, \qquad \bar{\tilde{A}} = \frac{A}{\sqrt{E_{\bar{x}}}}$$

Estimates:

$$\tilde{X}' = \overset{\approx}{X} \text{ or } \tilde{X}_i \text{ or } \bar{\tilde{X}}$$

$$\sigma = \frac{\tilde{R}}{\tilde{d}_2}$$

3σ Limits for

$$\tilde{X} \quad \overset{\approx}{X} \pm \tilde{A}_2\tilde{R} \text{ or } \tilde{X}_i \pm \tilde{A}_2\tilde{R}$$

$$\bar{\tilde{X}} \quad \overset{\approx}{\bar{X}} \pm \bar{\tilde{A}}_2\tilde{R}$$

$$R \quad \tilde{D}_3\tilde{R} \text{ and } \tilde{D}_4\tilde{R}$$

Note; $E_{\tilde{x}} = \sigma_x^2/\sigma_{\tilde{x}}^2$.

Factors for Median and Mid-range Charts Using Median Range Normal Distribution Theory

n	$\tilde{A}$	$\bar{\tilde{A}}$	$\tilde{A}_2$	$\bar{\tilde{A}}_2$	$\tilde{d}_2$	$\tilde{D}_3$	$\tilde{D}_4$
2	2.121	2.121	2.224	2.224	0.954	0	3.865
3	2.014	1.806	1.265	1.137	1.588	0	2.745
4	1.637	1.637	0.829	0.829	1.978	0	2.375
5	1.615	1.532	0.712	0.679	2.257	0	2.179
6	1.387	1.458	0.562	0.590	2.472	0	2.055
7	1.385	1.402	0.520	0.530	2.645	0.078	1.967
8	1.233	1.358	0.441	0.486	2.791	0.139	1.901
9	1.240	1.322	0.419	0.453	2.916	0.187	1.850
10	1.216	1.293	0.369	0.427	3.024	0.227	1.809

* From Paul C. Clifford, "Control Charts Without Calculations," *Industrial Quality Control*, May 1959, p. 44. Reprinted with permission.

Table A-3: Critical Values for ANOM Charts(5% and 1%).

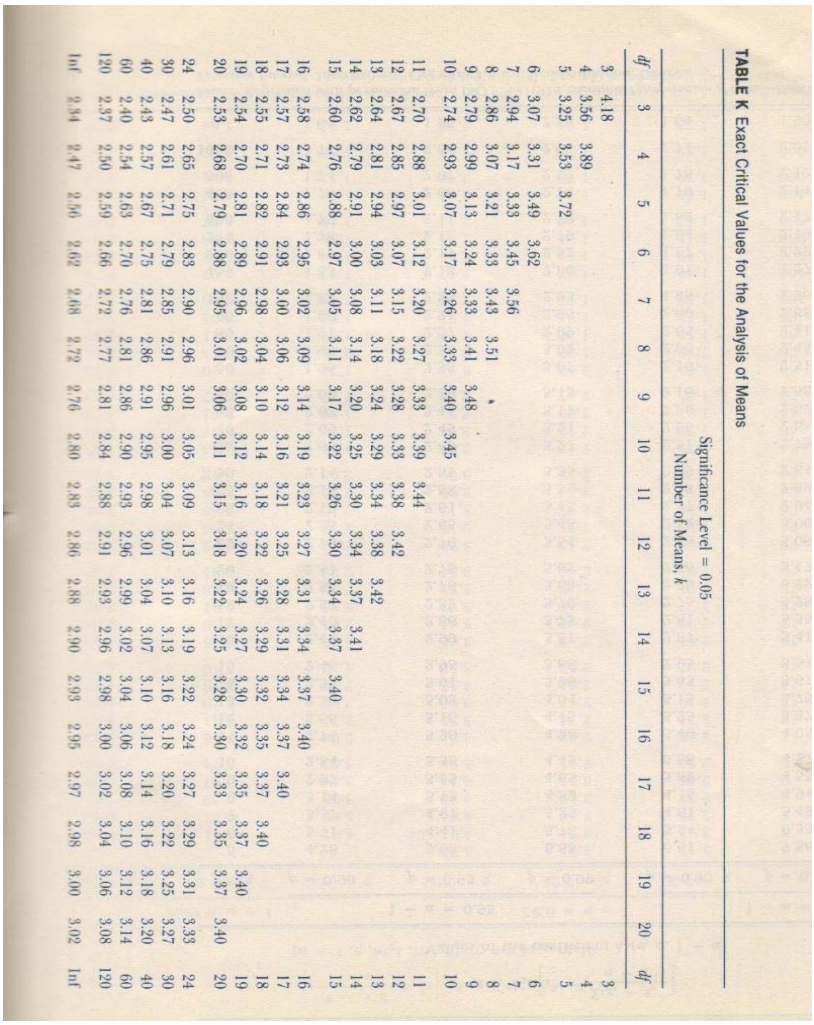


TABLE K Exact Critical Values for the Analysis of Means

Significance Level = 0.05

df	Number of Means, k																			
	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	df
3	4.18																		3
4	3.56	3.89																	4
5	3.25	3.53	3.72																5
6	3.07	3.31	3.49	3.62															6
7	2.94	3.17	3.33	3.45	3.56														7
8	2.86	3.07	3.21	3.33	3.43	3.51													8
9	2.79	2.99	3.13	3.24	3.33	3.41	3.48												9
10	2.74	2.93	3.07	3.17	3.26	3.33	3.40	3.45											10
11	2.70	2.88	3.01	3.12	3.20	3.27	3.33	3.39	3.44										11
12	2.67	2.85	2.97	3.07	3.15	3.22	3.28	3.33	3.38	3.42									12
13	2.64	2.81	2.94	3.03	3.11	3.18	3.24	3.29	3.34	3.38	3.42								13
14	2.62	2.79	2.91	3.00	3.08	3.14	3.20	3.25	3.30	3.34	3.37	3.41							14
15	2.60	2.76	2.88	2.97	3.05	3.11	3.17	3.22	3.26	3.30	3.34	3.37	3.40						15
16	2.58	2.74	2.86	2.95	3.02	3.09	3.14	3.19	3.23	3.27	3.31	3.34	3.37	3.40					16
17	2.57	2.73	2.84	2.93	3.00	3.06	3.12	3.16	3.21	3.25	3.28	3.31	3.34	3.37	3.40				17
18	2.55	2.71	2.82	2.91	2.98	3.04	3.10	3.14	3.18	3.22	3.26	3.29	3.32	3.35	3.37	3.40			18
19	2.54	2.70	2.81	2.89	2.96	3.02	3.08	3.12	3.16	3.20	3.24	3.27	3.30	3.32	3.35	3.37	3.40		19
20	2.53	2.68	2.79	2.88	2.95	3.01	3.06	3.11	3.15	3.18	3.22	3.25	3.28	3.30	3.33	3.35	3.37	3.40	20
24	2.50	2.65	2.75	2.83	2.90	2.96	3.01	3.05	3.09	3.13	3.16	3.19	3.22	3.24	3.27	3.29	3.31	3.33	24
30	2.47	2.61	2.71	2.79	2.85	2.91	2.96	3.00	3.04	3.07	3.10	3.13	3.16	3.18	3.20	3.22	3.25	3.27	30
40	2.43	2.57	2.67	2.75	2.81	2.86	2.91	2.95	2.98	3.01	3.04	3.07	3.10	3.12	3.14	3.16	3.18	3.20	40
60	2.40	2.54	2.63	2.70	2.76	2.81	2.86	2.90	2.93	2.96	2.99	3.02	3.04	3.06	3.08	3.10	3.12	3.14	60
120	2.37	2.50	2.59	2.66	2.72	2.77	2.81	2.84	2.88	2.91	2.93	2.96	2.98	3.00	3.02	3.04	3.06	3.08	120
Inf	2.34	2.47	2.56	2.62	2.68	2.72	2.76	2.80	2.83	2.86	2.88	2.90	2.93	2.95	2.97	2.98	3.00	3.02	Inf

Significance Level = 0.01

df	Number of Means, k																	df
	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
3	7.51																		3
4	5.74	6.21																	4
5	4.93	5.29	5.55																5
6	4.48	4.77	4.98	5.16															6
7	4.18	4.44	4.63	4.78	4.90														7
8	3.98	4.21	4.38	4.52	4.63	4.72													8
9	3.84	4.05	4.20	4.33	4.43	4.51	4.59												9
10	3.73	3.92	4.07	4.18	4.28	4.36	4.43	4.49											10
11	3.64	3.82	3.96	4.07	4.16	4.23	4.30	4.36	4.41										11
12	3.57	3.74	3.87	3.98	4.06	4.13	4.20	4.25	4.31	4.35									12
13	3.51	3.68	3.80	3.90	3.98	4.05	4.11	4.17	4.22	4.26	4.30								13
14	3.46	3.63	3.74	3.84	3.92	3.98	4.04	4.09	4.14	4.18	4.22	4.26							14
15	3.42	3.58	3.69	3.79	3.86	3.92	3.98	4.03	4.08	4.12	4.16	4.19	4.22						15
16	3.38	3.54	3.65	3.74	3.81	3.87	3.93	3.98	4.02	4.06	4.10	4.14	4.17	4.20					16
17	3.35	3.50	3.61	3.70	3.77	3.83	3.89	3.93	3.98	4.02	4.05	4.09	4.12	4.14	4.17				17
18	3.33	3.47	3.58	3.66	3.73	3.79	3.85	3.89	3.94	3.97	4.01	4.04	4.07	4.10	4.12	4.15			18
19	3.30	3.45	3.55	3.63	3.70	3.76	3.81	3.86	3.90	3.94	3.97	4.00	4.03	4.06	4.08	4.11	4.13		19
20	3.28	3.42	3.53	3.61	3.67	3.73	3.78	3.83	3.87	3.90	3.94	3.97	4.00	4.02	4.05	4.07	4.09	4.12	20
24	3.21	3.35	3.45	3.52	3.58	3.64	3.69	3.73	3.77	3.80	3.83	3.86	3.89	3.91	3.94	3.96	3.98	4.00	24
30	3.15	3.28	3.37	3.44	3.50	3.55	3.59	3.63	3.67	3.70	3.73	3.76	3.78	3.81	3.83	3.85	3.87	3.89	30
40	3.09	3.21	3.29	3.36	3.42	3.46	3.50	3.54	3.58	3.60	3.63	3.66	3.68	3.70	3.72	3.74	3.76	3.78	40
60	3.03	3.14	3.22	3.29	3.34	3.38	3.42	3.46	3.49	3.51	3.54	3.56	3.59	3.61	3.63	3.64	3.66	3.68	60
120	2.97	3.07	3.15	3.21	3.26	3.30	3.34	3.37	3.40	3.42	3.45	3.47	3.49	3.51	3.53	3.55	3.56	3.58	120
Inf	2.91	3.01	3.08	3.14	3.18	3.22	3.26	3.29	3.32	3.34	3.36	3.38	3.40	3.42	3.44	3.45	3.47	3.48	Inf

Source: Reprinted by permission of Nelson, L. S. (1983), "Exact Critical Values for Use with the Analysis of Means," *Journal of Quality Technology*, Vol. 15, pp. 40–44.

Table A-4: MIL-STD 105E Sample Code Letters.

Table 13-4 Sample Size Code Letters (MIL STD 105E, Table 1)

Lot or Batch Size	Special Inspection Levels				General Inspection Levels		
	S-1	S-2	S-3	S-4	I	II	III
2 to 8	A	A	A	A	A	A	B
9 to 15	A	A	A	A	A	B	C
16 to 25	A	A	B	B	B	C	D
26 to 50	A	B	B	C	C	D	E
51 to 90	B	B	C	C	C	E	F
91 to 150	B	B	C	D	D	F	G
151 to 280	B	C	D	E	E	G	H
281 to 500	B	C	D	E	F	H	J
501 to 1200	C	C	E	F	G	J	K
1201 to 3200	C	D	E	G	H	K	L
3201 to 10000	C	D	F	G	J	L	M
100001 to 35000	C	D	F	H	K	M	N
35001 to 150000	D	E	G	J	L	N	P
500001 to 500000	D	E	G	J	M	P	Q
500001 and over	D	E	H	K	N	Q	R

Table A-5: Master Table for Normal Inspection-Single Sampling (MIL STD 105E)

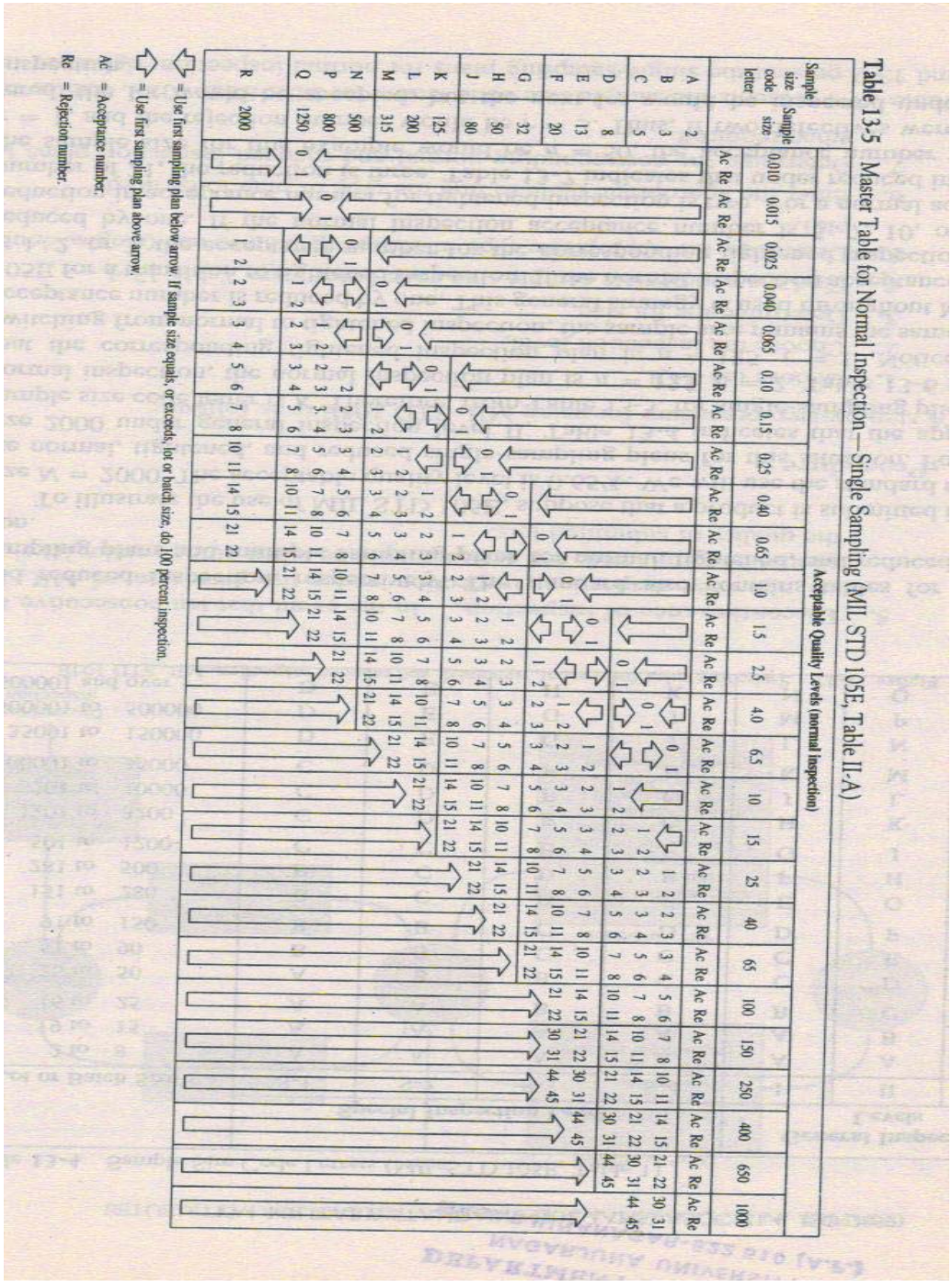


Table 13-5 Master Table for Normal Inspection—Single Sampling (MIL STD 105E, Table II-A)

Acceptable Quality Levels (normal inspection)

Sample size code letter	Sample size	0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
		Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A	2																										
B	3																										
C	5																										
D	8																										
E	13																										
F	20																										
G	32																										
H	50																										
J	80																										
K	125																										
L	200																										
M	315																										
N	500																										
P	800																										
Q	1250																										
R	2000																										

⇩ = Use first sampling plan below arrow. If sample size equals, or exceeds, lot or batch size, do 100 percent inspection.

⇧ = Use first sampling plan above arrow.

Ac = Acceptance number.

Re = Rejection number.

Table: A-6 Master Table for Tightened Inspection-Single Sampling (MIL STD 105E)

Table 13-6 Master Table for Tightened Inspection—Single Sampling (MIL STD 105E, Table II-B)

Sample size code letter	Sample size	Acceptable Quality Levels (tightened inspection)																									
		0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
		Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28
B	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42
C	5	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42	↑
D	8	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42	↑	↑
E	13	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	27 28	41 42	↑	↑	↑
F	20	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑
G	32	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑
H	50	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑
J	80	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑
K	125	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
L	200	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
M	315	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
N	500	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
P	800	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
Q	1250	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
R	2000	0 1	↑	↓	1 2	2 3	3 4	5 6	8 9	12 13	18 19	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
S	3150			1 2																							

↓ = Use first sampling plan below arrow. If sample size equals or exceeds lot or batch size, do 100 percent inspection.

↑ = Use first sampling plan above arrow.

Ac = Acceptance number.

Re = Rejection number.

Table: A-7 Master Table for Reduced Inspection-Single Sampling (MIL STD 105E)

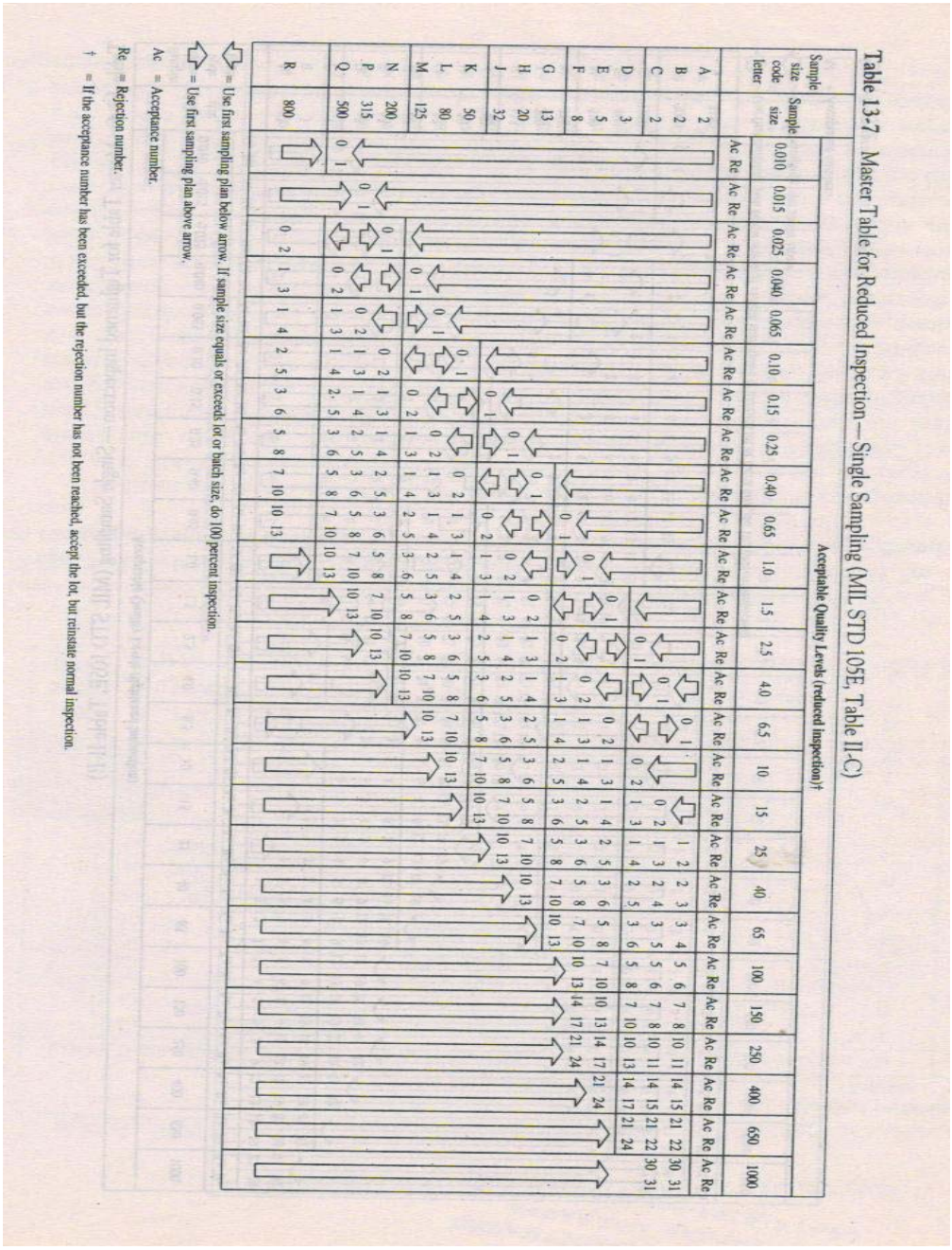


Table 13-7 Master Table for Reduced Inspection—Single Sampling (MIL STD 105E, Table II-C)

Acceptable Quality Levels (reduced inspection)†

Sample size code letter	Sample size	0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
		Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A	2																										
B	2																										
C	2																										
D	3																										
E	5																										
F	8																										
G	13																										
H	20																										
J	32																										
K	50																										
L	80																										
M	125																										
N	200																										
P	315																										
Q	500																										
R	800																										

⇩ = Use first sampling plan below arrow. If sample size equals or exceeds lot or batch size, do 100 percent inspection.

⇧ = Use first sampling plan above arrow.

Ac = Acceptance number.

Re = Rejection number.

† = If the acceptance number has been exceeded, but the rejection number has not been reached, accept the lot, but reinstate normal inspection.

Table A-8: Dodge Romig Inspection Table – Single Sampling Plans for AOQL=3%.

Table 13-8 Dodge-Romig Inspection Table—Single-Sampling Plans for AOQL = 3.0%

	Process Average																	
	0–0.06%			0.07–0.60%			0.61–1.20%			1.21–1.80%			1.81–2.40%			2.41–3.00%		
Lot Size	*n*	*c*	LTPD %	*n*	*c*	LTPD %	*n*	*c*	LTPD %	*n*	*c*	LTPD %	*n*	*c*	LTPD %	*n*	*c*	LTPD %
1–10	All	0	—	All	0	—	All	0	—	All	0	—	All	0	—	All	0	—
11–50	10	0	19.0	10	0	19.0	10	0	19.0	10	0	19.0	10	0	19.0	10	0	19.0
51–100	11	0	18.0	11	0	18.0	11	0	18.0	11	0	18.0	11	0	18.0	22	1	16.4
101–200	12	0	17.0	12	0	17.0	12	0	17.0	25	1	15.1	25	1	15.1	25	1	15.1
201–300	12	0	17.0	12	0	17.0	26	1	14.6	26	1	14.6	26	1	14.6	40	2	12.8
301–400	12	0	17.1	12	0	17.1	26	1	14.7	26	1	14.7	41	2	12.7	41	2	12.7
401–500	12	0	17.2	27	1	14.1	27	1	14.1	42	2	12.4	42	2	12.4	42	2	12.4
501–600	12	0	17.3	27	1	14.2	27	1	14.2	42	2	12.4	42	2	12.4	60	3	10.8
601–800	12	0	17.3	27	1	14.2	27	1	14.2	43	2	12.1	60	3	10.9	60	3	10.9
801–1000	12	0	17.4	27	1	14.2	44	2	11.8	44	2	11.8	60	3	11.0	80	4	9.8
1,001–2,000	12	0	17.5	28	1	13.8	45	2	11.7	65	3	10.2	80	4	9.8	100	5	9.1
2,001–3,000	12	0	17.5	28	1	13.8	45	2	11.7	65	3	10.2	100	5	9.1	140	7	8.2
3,001–4,000	12	0	17.5	28	1	13.8	65	3	10.3	85	4	9.5	125	6	8.4	165	8	7.8
4,001–5,000	28	1	13.8	28	1	13.8	65	3	10.3	85	4	9.5	125	6	8.4	210	10	7.4
5,001–7,000	28	1	13.8	45	2	11.8	65	3	10.3	105	5	8.8	145	7	8.1	235	11	7.1
7,001–10,000	28	1	13.9	46	2	11.6	65	3	10.3	105	5	8.8	170	8	7.6	280	13	6.8
10,001–20,000	28	1	13.9	46	2	11.7	85	4	9.5	125	6	8.4	215	10	7.2	380	17	6.2
20,001–50,000	28	1	13.9	65	3	10.3	105	5	8.8	170	8	7.6	310	14	6.5	560	24	5.7
50,001–100,000	28	1	13.9	65	3	10.3	125	6	8.4	215	10	7.2	385	17	6.2	690	29	5.4

Table A-9: Dodge Romig Single Sampling Plan Tables for LTPD = 1%.

inspection sampling when the process average exceeds one-half the desired LTPD.

Table 13-9 Dodge-Romig Single-Sampling Table for Lot Tolerance Percent Defective (LTPD) = 1.0%

	Process Average																	
	0			0.011%–0.010%			0.11–0.20%			0.21–0.30%			0.31–0.40%			0.41–0.50%		
			AOQL			AOQL			AOQL			AOQL			AOQL			AOQL
Lot Size	*n*	*c*	%	*n*	*c*	%	*n*	*c*	%	*n*	*c*	%	*n*	*c*	%	*n*	*c*	%
1–120	All	0	0	All	0	0	All	0	0	All	0	0	All	0	0	All	0	0
121–150	120	0	0.06	120	0	0.06	120	0	0.06	120	0	0.06	120	0	0.06	120	0	0.06
151–200	140	0	0.08	140	0	0.08	140	0	0.08	140	0	0.08	140	0	0.08	140	0	0.08
201–300	165	0	0.10	165	0	0.10	165	0	0.10	165	0	0.10	165	0	0.10	165	0	0.10
301–400	175	0	0.12	175	0	0.12	175	0	0.12	175	0	0.12	175	0	0.12	175	0	0.12
401–500	180	0	0.13	180	0	0.13	180	0	0.13	180	0	0.13	180	0	0.13	180	0	0.13
501–600	190	0	0.13	190	0	0.13	190	0	0.13	190	0	0.13	190	0	0.13	305	1	0.14
601–800	200	0	0.14	200	0	0.14	200	0	0.14	330	1	0.15	330	1	0.15	330	1	0.15
801–1000	205	0	0.14	205	0	0.14	205	0	0.14	335	1	0.17	335	1	0.17	335	1	0.17
1,001–2,000	220	0	0.15	220	0	0.15	360	1	0.19	490	2	0.21	490	2	0.21	610	3	0.22
2,001–3,000	220	0	0.15	375	1	0.20	505	2	0.23	630	3	0.24	745	4	0.26	870	5	0.26
3,001–4,000	225	0	0.15	380	1	0.20	510	2	0.23	645	3	0.25	880	5	0.28	1,000	6	0.29
4,001–5,000	225	0	0.16	380	1	0.20	520	2	0.24	770	4	0.28	895	5	0.29	1,120	7	0.31
5,001–7,000	230	0	0.16	385	1	0.21	655	3	0.27	780	4	0.29	1,020	6	0.32	1,260	8	0.34
7,001–10,000	230	0	0.16	520	2	0.25	660	3	0.28	910	5	0.32	1,150	7	0.34	1,500	10	0.37
10,001–20,000	390	1	0.21	525	2	0.26	785	4	0.31	1,040	6	0.35	1,400	9	0.39	1,980	14	0.43
20,001–50,000	390	1	0.21	530	2	0.26	920	5	0.34	1,300	8	0.39	1,890	13	0.44	2,570	19	0.48
50,001–100,000	390	1	0.21	670	3	0.29	1,040	6	0.36	1,420	9	0.41	2,120	15	0.47	3,150	23	0.50

Table A-10: Factors for Acceptance Control Charts.

TABLE L Factors for Acceptance Control Charts

	$\alpha = \beta = 0.05$			$\alpha = \beta = 0.01$			$\alpha = \beta = 0.001$		
n	$C_{0,.05}$	$C_{2,.05}$	$C_{3,.05}$	$C_{0,.01}$	$C_{2,.01}$	$C_{3,.01}$	$C_{0,.001}$	$C_{2,.001}$	$C_{3,.001}$
2	1.837	1.629	2.302	1.356	1.202	1.698	0.815	0.723	1.020
3	2.050	1.211	2.314	1.657	0.979	1.870	1.216	0.718	1.372
4	2.178	1.057	2.363	1.837	0.892	1.994	1.455	0.706	1.579
5	2.264	0.974	2.409	1.960	0.843	2.085	1.618	0.696	1.721
6	2.328	0.919	2.447	2.050	0.810	2.155	1.738	0.686	1.827
7	2.378	0.879	2.479	2.121	0.784	2.211	1.832	0.677	1.909
8	2.418	0.849	2.506	2.177	0.765	2.257	1.907	0.670	1.977
9	2.452	0.825	2.529	2.225	0.749	2.295	1.970	0.663	2.032
10	2.480	0.806	2.549	2.264	0.736	2.328	2.023	0.659	2.079
11	2.504	0.790	2.568	2.298	0.725	2.357	2.068	0.652	2.121
12	2.525	0.775	2.583	2.329	0.715	2.382	2.108	0.647	2.156
13	2.544	0.762	2.597	2.355	0.706	2.404	2.143	0.642	2.187
14	2.560	0.752	2.610	2.378	0.699	2.425	2.174	0.639	2.216
15	2.575	0.742	2.621	2.399	0.691	2.442	2.202	0.634	2.241
16	2.589		2.632	2.419		2.458	2.227		2.264
17	2.601		2.642	2.436		2.474	2.250		2.286
18	2.612		2.650	2.452		2.487	2.272		2.304
19	2.623		2.659	2.467		2.501	2.291		2.323
20	2.632		2.667	2.480		2.513	2.309		2.339
21	2.641		2.674	2.492		2.524	2.325		2.355
22	2.649		2.681	2.504		2.534	2.341		2.369
23	2.657		2.687	2.515		2.543	2.355		2.382
24	2.664		2.694	2.526		2.553	2.369		2.395
25	2.671		2.700	2.535		2.562	2.385		2.408

Table A-11: Sample Size Code Letters for MIL STD 414.

Table 14-1 (Table A-2. MIL STD 414) Sample Size Code Letters

	Inspection Levels				
Lot Size	1	II	III	IV	V
3 to 8	B	B	B	B	C
9 to 15	B	B	B	B	D
16 to 25	B	B	B	C	E
26 to 40	B	B	B	D	F
41 to 65	B	B	C	E	G
66 to 110	B	B	D	F	H
111 to 180	B	C	E	G	I
181 to 300	B	D	F	H	J
301 to 500	C	E	G	I	K
501 to 800	D	F	H	J	L
801 to 1,300	E	G	I	K	L
1,301 to 3,200	F	H	J	L	M
3,201 to 8,000	G	I	L	M	N
8,001 to 22,000	H	J	M	N	O
22,001 to 110,000	I	K	N	O	P
110,001 to 550,000	I	K	O	P	Q
550,001 and over	I	K	P	Q	Q

A-12: Master Table for Normal and Tightened Inspection based on AQL (MIL STD 414).

Table 14-2 Master Table for Normal and Tightened Inspection for Plans Based on Variability Unknown (Standard Deviation Method) (Single-specification Limit—Form 1)(Table B-1, MIL STD 414)

Sample Size Code Letter	Sample Size	Acceptable Quality Levels (normal inspection)													
		.04	.065	.10	.15	.25	.40	.65	1.00	1.50	2.50	4.00	6.50	10.00	15.00
		k	k	k	k	k	k	k	k	k	k	k	k	k	k
B	3	↓	↓	↓	↓	↓	↓	↓	▼	▼	1.12	.958	.765	.566	.341
C	4	↓	↓	↓	↓	↓	↓	↓	1.45	1.34	1.17	1.01	.814	.617	.393
D	5	↓	↓	↓	↓	↓	↓	1.65	1.53	1.40	1.24	1.07	.874	.675	.455
E	7	↓	↓	↓	↓	2.00	1.88	1.75	1.62	1.50	1.33	1.15	.955	.755	.536
F	10	↓	↓	↓	2.24	2.11	1.98	1.84	1.72	1.58	1.41	1.23	1.03	.828	.611
G	15	2.64	2.53	2.42	2.32	2.20	2.06	1.91	1.79	1.65	1.47	1.30	1.09	.886	.664
H	20	2.69	2.58	2.47	2.36	2.24	2.11	1.96	1.82	1.69	1.51	1.33	1.12	.917	.695
I	25	2.72	2.61	2.50	2.40	2.26	2.14	1.98	1.85	1.72	1.53	1.35	1.14	.936	.712
J	30	2.73	2.61	2.51	2.41	2.28	2.15	2.00	1.86	1.73	1.55	1.36	1.15	.946	.723
K	35	2.77	2.65	2.54	2.45	2.31	2.18	2.03	1.89	1.76	1.57	1.39	1.18	.969	.745
L	40	2.77	2.66	2.55	2.44	2.31	2.18	2.03	1.89	1.76	1.58	1.39	1.18	.971	.746
M	50	2.83	2.71	2.60	2.50	2.35	2.22	2.08	1.93	1.80	1.61	1.42	1.21	1.00	.774
N	75	2.90	2.77	2.66	2.55	2.41	2.27	2.12	1.98	1.84	1.65	1.46	1.24	1.03	.804
O	100	2.92	2.80	2.69	2.58	2.43	2.29	2.14	2.00	1.86	1.67	1.48	1.26	1.05	.819
P	150	2.96	2.84	2.73	2.61	2.47	2.33	2.18	2.03	1.89	1.70	1.51	1.29	1.07	.841
Q	200	2.97	2.85	2.73	2.62	2.47	2.33	2.18	2.04	1.89	1.70	1.51	1.29	1.07	.845
		.065	.10	.15	.25	.40	.65	1.00	1.50	2.50	4.00	6.50	10.00	15.00	
		Acceptable Quality Levels (tightened inspection)													

All AQL values are in percent defective.

↓ Use first sampling plan below arrow, that is, both sample size as well as k value. When sample size equals or exceeds lot size, every item in the lot must be inspected.