LINEAR COMPONENTS AND CIRCUIT ANALYSIS

(DSEL11)

(BSC ELECTRONICS-I)

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UNIT-I **LESSON-1**

AC FUNDAMENTALS

OBJECTIVES

This lesson explains you the concepts of

- Units and definitions
- Alternating current and voltage, j operator
- AC Generator
- Phasor notation and Phasor algebra
- Forms of representation of AC voltage

Structure of the Lesson

1.a) Units and definitions

INTRODUCTION

In order to understand the circuit analysis, one should know the units (SI), definitions, Alternating current and voltage, AC voltage generation, derivation of AC voltage expression, etc. The AC is to be specified by its RMS or average or peak value. This lesson explains phasor, phasor algebra, phasor notation of AC quantities, j operator etc. It also explains the other forms of representation of AC quantities like Polar form, Rectangular form because these representations help to understand the phasor algebra very easily.

1.1 Units

The International System of Units (SI) is the widely used system. As far as the electric circuit theory is concerned, the commonly used units are listed in table 1.1

Table 1.1

Quantity	Unit	Symbol
Electrical charge	Coulomb	C
Electric Potential	Volt	V
Resistance	Ohm	Ω
Conductance	Siemens	S
Inductance	Henry	H
Capacitance	Farad	F
Frequency	Hertz	Hz
Force	Newton	N
Energy, Work	Joule	J
Power	Watt	W
Magnetic flux	Weber	Wb

Decimal multiples and submultiples of S.I. Units are given below in table 1.2. The concepts of electrical charge, current, electric field, were developed before the invention of electron. In those days, no body has any idea about the nature of electrical charge. So the parameters were defined as described in those days.

Table 1.2

Factor	Prefix	Symbol
10^9	Giga	G
10^6	Mega	M
10^3	Kilo	K
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	P

Electrical charge (q)

It is considered as the property associated with matter due to which it produces and experiences electric and magnetic effects. The fundamental unit of charge is Coulomb (C). It is that charge which when placed at a distance of 1 meter from an equal charge in vacuum repels it with a force of 9×10^9 Newton. Now we know that Coulomb of charge is produced by 10^{19} electrons or an electron possesses a charge of -1.602×10^{-19}C. We have two types of charges. Positive and Negative.

Electric current

Transfer of charge or charge in motion constitute current. It has both a numerical value and direction.

The rate of flow of charge is called electric Current. It's unit is Ampere (A).

1 Ampere (A) = $\frac{1\,\text{Coulomb}}{1\,\text{second}}$, i.e., $A = \frac{C}{S}$

Electric filed (E)

The region surrounding the charge in which its electrical effects are perceptible is called the Electric filed. The electric filed is represented by electric flux lines, which are drawn to indicate the strength of the electric filed at the any point around the charged body. Electric filed at a point is specified by a vector called Electric intensity.

Thus if a test charge q_0 at a point in electric filed experiences a force $\overline{F}$, then

$$\vec{E} = \frac{\vec{F}}{q_o}$$

The unit of electric filed intensity $\overline{E}$ is

$$\frac{Newton}{Coulomb} = \frac{Kg \times meter - \sec^{-2}}{ampere \times \sec} = Kg \times meter \times \sec^{-3} \times ampere^{-1}$$

Electric potential

The potential at any point in an electric filed is numerically equal to the work done in bringing a positive charge of 1Coulomb from infinity to that point against the electric field. It is expressed in volt.

$$1\,\text{Volt} = \frac{1\,\text{Joule}}{1\,\text{Coulomb}}$$

Potential Difference

The work done in moving a unit charge from point A to point B in an electric filed produced by a charge +q is called the ***Potential Difference*** between these points. It is expressed in volts.

A potential difference of 1 volt exists between two points if one Joule of work is done in shifting a charge of one Coulomb from one point to the other.

Voltage is represented by V.

Electro Motive Force (EMF)

It is the work required to lift unit charge from lower potential to higher potential inside the cell. It is also measured in volts.

A voltage can exist between two electrical terminals whether a current is flowing or not.

1.b) Alternating current and voltage

1.2 AC voltage generation

Alternating Voltage: AC voltage can be defined as one that continuously varies in magnitude and periodically reverses in polarity.

AC voltage can be generated either by rotating a coil within a stationary magnetic filed or by rotating a magnet within a stationary coil as shown in Fig.1.1 (a) or Fig.1.1 (b). AC generator is used to produce AC voltage. It is a device to convert mechanical energy into electrical energy. It is based on the principle of electro magnetic induction.

The magnet, which is stationary, is known as **Stator**. A copper wire would on an iron frame (armature coil) which rotates in the filed is known as **Rotor**. The ends of the coil are connected to two flat brass rings known as Slip rings. The brushes are connected to a load in the external circuit,. When the rotor rotates, the magnetic flux linked with it changes. Voltage is induced in the coil and current flows through the load.

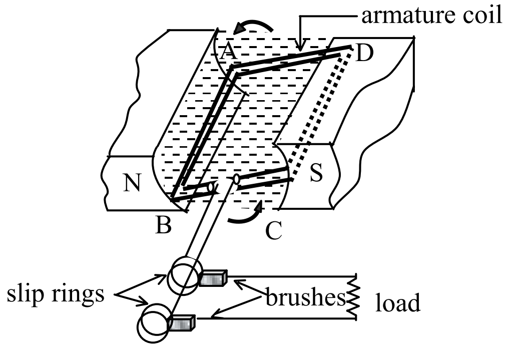

Fig 1.1(a)

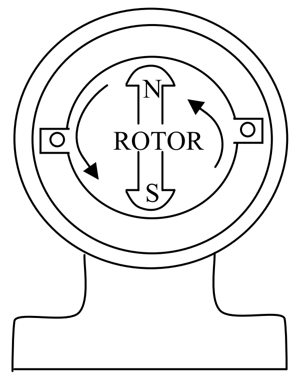

Fig 1.1(b)

Consider the coil; at the beginning, is in vertical position. Then the emf induced is zero. When the coil starts rotating in the anticlockwise direction, the side AB moves downward and CD moves upward. According to Fleming's Right Hand rule, the induced current flows from A to B and from C to D. Hence, during the first half rotation of the coil, current flows in the coil in the direction CDAB. During the second half rotation, the current flows in the direction BADC. Moreover, the magnitude of induced e.m.f. and current also change. In this way, alternating current is produced by the generator.

1.3 AC Voltage expression derivation

Consider a rectangular coil of area A containing N turns rotating with angular velocity 'ω' rad/sec in a uniform magnetic field as shown in Fig.1.2.

Let the time be measured from the instant of coincidence of the plane of the coil with Y-axis. Let the coil assume the position after moving in counterclockwise direction for 't' seconds through an angle θ.

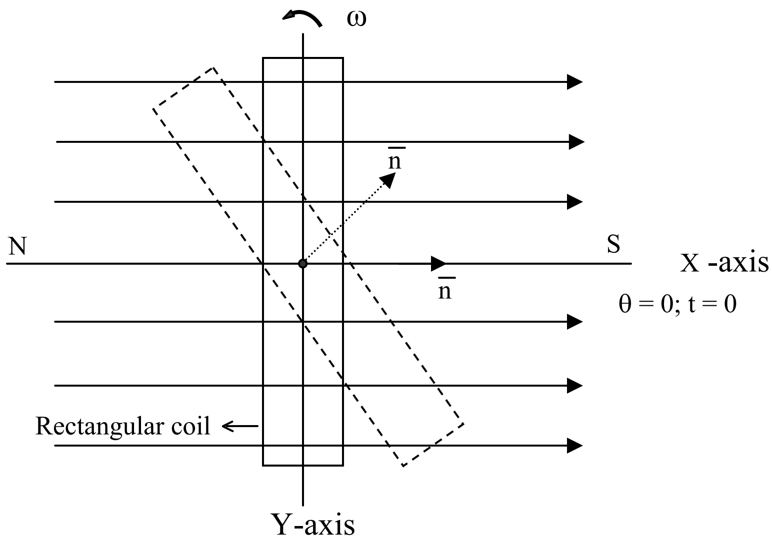

Fig 1.2

$\theta = \omega t$

Magnetic flux linked with one turn of the coil=BA cos θ

Hence, magnetic flux linked with N turns of the coil = BAN cos θ

or ϕ = BAN cos ωt

According to Lenz's law, induced e.m.f. due to rotation is

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}[BAN \cos \omega t]$$

$$e = BAN\omega \sin \omega t \qquad \text{-----------} \quad (1.1)$$

When θ = 90^0, sin θ =1;

Hence e = BANω, is maximum. So we call it as E_{max}.

Now equation (1.1) be comes

$$e = E_{max} \sin \omega t$$

$$\text{or } e = E_{max} \sin \theta \qquad \text{-----------} \quad (1.2)$$

Here E_{max} is the maximum value of induced voltage; 'e' is the instantaneous value for any angle 'θ' .Dividing eq.(1.2) on both sides by R, the resistance of the coil, the produced current is given by

$$i = I_{max} \sin \theta \qquad \text{-----------} \quad (1.3)$$

When the coil is vertical, θ = 0, e = 0

When the coil is horizontal, θ = 90^0, e = E_{max}

When the coil is again vertical, θ = 180^0, e = 0

When the coil is horizontal, θ = 270^0, e = - E_{max}

When the coil returns to its original position, θ = 360^0, e = 0

Thus the e.m.f induced in the coil of the generator varies sinusoidally, i.e., just like a ***sine*** curve. Further, the induced emf increases from '0' to positive maximum and decreases from maximum to '0'. This is called ***Positive Half Cycle.*** Again the emf increases from '0' to negative maximum and then decreases from negative maximum to zero. This is called ***Negative Half Cycle.*** Hence we can conclude that

i) In one full cycle, the magnitude of e.m.f changes like a sine wave.

ii) For every half cycle, the direction of e.m.f. is reversed. Hence this voltage is called ***Alternating Voltage*** and ***Alternating e.m.f***. The current so produced also reverses its direction for every half cycle. Thus it is called ***Alternating Current.***

If we plot a graph between e.m.f. induced and time, a curve of sine wave shape is obtained as shown in Fig.1.3. Such an e.m.f is called ***Sinusoidal emf*** or ***Sinusoidal Voltage.***

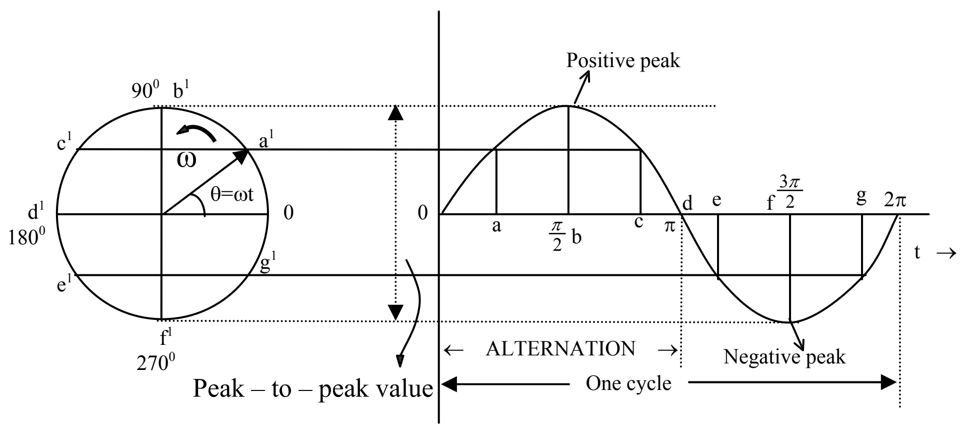

Fig 1.3

Amplitude: The maximum positive or negative value, which an alternating quantity attains during one cycle, is called the Amplitude or peak value or crest value or maximum value.

Time period: The time taken by an alternating quantity to complete one cycle is known as *Time Period* (T).

Frequency The number of cycles completed per second by an alternating quantity in called the *Frequency* **'*f*'**. Its unit is Hertz (Hz)**.**

1.4 Average value and its derivation

The average value of an AC is given by that steady current which transfers across any circuit the ***same charge*** as is transferred by that AC during the same time.

The average value of a sine wave over a complete cycle is zero. Hence, the half cycle is used for the estimation of average value.

Derivation of $I_{ave} = \dfrac{I_{\max}}{\Pi/2}$

We know that instantaneous value of AC current $i = I_{\max} \sin\theta$

Average value of AC current = $I_{ave} = \int_0^{\Pi} \dfrac{i d\theta}{\Pi - 0} = \dfrac{1}{\Pi} \int_0^{\Pi} I_{\max} \sin \theta d\theta$

$$= \frac{I_{\max}}{\Pi} \int_0^{\Pi} \sin\theta d\theta$$

$$= \frac{I_{\max}}{\Pi} \left[-\cos\theta\right]_0^{\Pi} = \frac{I_{\max}}{\Pi}.2$$

$$I_{ave} = \frac{I_{\max}}{\Pi/2}$$

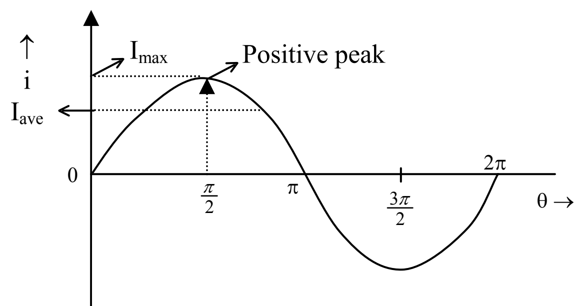

Fig 1.4

(or) $I_{ave} = 0.637 I_{\max}$ ------------ (1.4)

Similarly $E_{ave} = 0.637 E_{\max}$

1.5 RMS value and its derivation

The RMS Value or effective value or virtual value of an AC is given by that ***steady current (DC)*** which when flowing through a given circuit for a given time produces the ***same amount of heat*** as produced by the AC when flowing through the same circuit for the same time.

The effective value of an AC current is equal to the square root of mean of the squares of instantaneous currents. Hence it is known as Root Mean Square value [RMS].

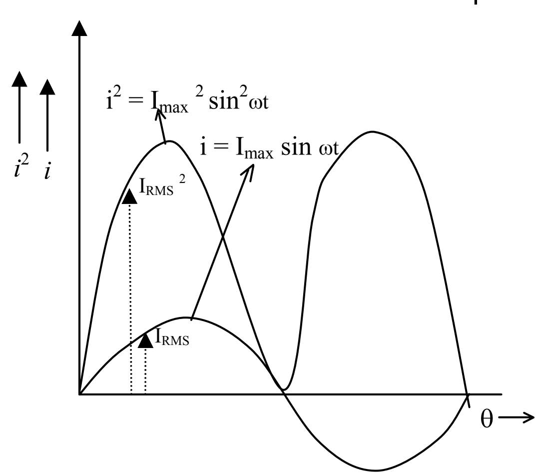

Fig 1.5

$$\left.\begin{array}{l}\text{The mean of the squares of the instantaneous}\\ \text{values of current over one complete cycle}\end{array}\right\} = \int_0^{2\Pi} \frac{i^2 d\theta}{(2\Pi - 0)}$$

$$I_{RMS}^2 = \int_0^{2\Pi} \frac{I_{\max}^2}{2\Pi} \sin^2 \theta d\theta$$

$$= \frac{I_{\max}^2}{4\Pi} \int_0^{2\Pi} [1 - \cos 2\theta] d\theta$$

$$= \frac{I_{\max}^2}{4\Pi} [\theta - \frac{1}{2}(\sin 2\theta)]_0^{2\Pi}$$

$$I_{RMS}^2 = \frac{I_{\max}^2}{4\Pi} .2\Pi = \frac{I_{\max}^2}{2}$$

$$I_{RMS} = \frac{I_{\max}}{\sqrt{2}} = 0.707 I_{\max}$$

$$I_{RMS} = O.707 I_{\max} \qquad \text{---------------} \quad (1.5)$$

Note: The most common method of specifying the amount of a sine wave of voltage or current is by stating its value at 45^0, which is 70.7% of the peak. This is its root mean square value, abbreviated as RMS. Unless indicated otherwise, all sine wave AC measurements are in RMS value. For example the AC supply voltage, for domestic purposes, is specified by its RMS value i.e., 230V.

Ex: Calculate the peak voltage of mains supply.

$$V_{peak} = \frac{230}{0.707} = 1.414 \times 230 = 325.22 \text{ V.}$$

Form Factor

The ratio of RMS value to the average value is the ***Form Factor.***

Form Factor $K_f = \dfrac{0.707 I_{\max}}{0.637 I_{\max}} = 1.11$

1.6 j operator

The vector operator 'j' may be defined as an operator, which rotates any complex number (vector) $\overline{A}$ by 90^0 in counterclockwise direction. If $\overline{A}$ is a pure real number, then the rotation sends $\overline{A}$ into $j\overline{A}$ on the positive imaginary axis. Similarly j^2 advances $\overline{A}$ by 180^0; j^3

advances $\overline{A}$ by 270^0 and j^4 advances $\overline{A}$ by 360^0. Representation of *'j' operator* is shown in Fig1.6

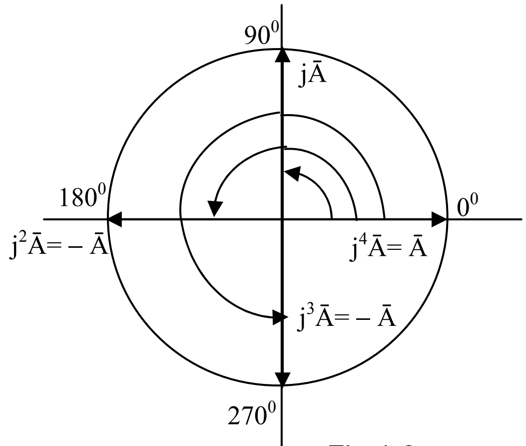

Fig 1.6

Hence application of the operator 'j' to the vector $\overline{A}$ produces 90^0 steps of rotation of the vector in counterclockwise direction, without affecting its magnitude. In mathematics, $i = \sqrt{-1},$ but in electronics $j = \sqrt{-1}$ because 'i' is reserved for current.

1.7 Phasor Notation

Phasor

It is defined as a directed line segment rotating in counterclockwise direction at a constant angular velocity ω rad./sec. A Phasor is a quantity that has magnitude and direction. The length of the arrow indicates the magnitude of the alternating voltage, in RMS, peak, or any AC voltage as long as the same measure is used for all the phasors. The angle of the arrow with respect to the horizontal axis indicates the phase angle.

The terms phasor and vector are used for a quantity that has direction, requiring an angle to specify the value completely. However, a vector quantity has direction in space, while a phasor quantity varies in time.

A phasor corresponds to the entire cycle of voltage without extra details of a whole cycle, phasor represent the alternating voltage or current in a compact form that is easier for comparing phase angles. A phasor is denoted in print by italic bold-faced **I.** Following figures Fig 1.7(a) and 1.7(b) show the waveform diagram and phasor diagram.

For phasor arrows, the angles shown represent differences in time. One sinusoid is chosen as the reference. Then the timing of the variations in another sinusoid can be compared with the reference by means of the angle between the phasor arrows.

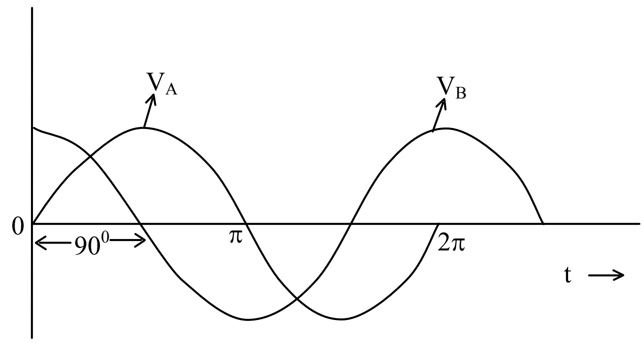

Fig 1.7(a) Waveform diagram

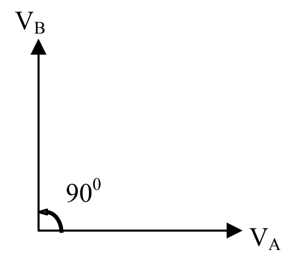

Fig 1.7(b) Phasor diagram

1.8 Polar Form and Rectangular Form

In Polar form of representation, a vector $\overline{A}$ can be represented as $\overline{A} = A\angle\theta$.
This represents a vector of numerical value 'A' having a phase angle of 'θ' with the reference axis.

In Rectangular form of representation, also known Cartesian form of representation, a vector is expressed algebraically in terms of its rectangular components. Symbolically, the vector A may be represented as

$$A = a_1 + jb_1$$

where a_1 is its horizontal component and b_1 is its vertical component, shown in Fig 1.8.

Numerical value of A is $\sqrt{a_1^2 + b_1^2}$. Angle with X-axis $\theta = \tan^{-1}\left(\frac{b_1}{a_1}\right)$.

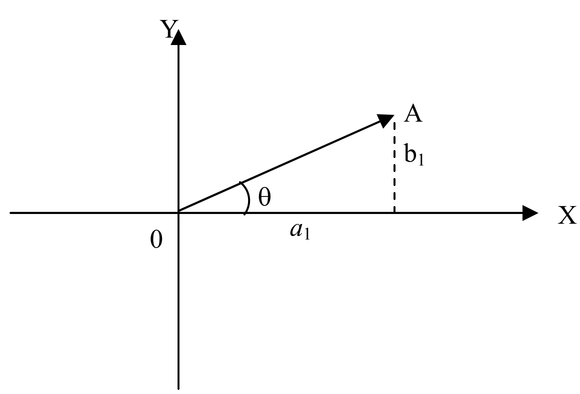

Fig 1.8

Note: Rectangular form is useful in adding or subtracting phasors, while polar form is useful to express the product or quotient of two phasors.

1.9 Phasor Algebra

If Zs are any phasor quantities, r_1 and r_2 are their magnitudes

then $\bar{Z}_1 = r_1 \angle \theta_1 = r_1 e^{j\theta_1}$

$\bar{Z}_2 = r_2 \angle \theta_2 = r_2 e^{j\theta_2}$

Hence $\overline{Z_1} \cdot \overline{Z_2} = r_1 e^{j\theta_1} \cdot r_2 e^{j\theta_2}$

$= r_1 \cdot r_2 \cdot e^{j(\theta_1 + \theta_2)}$

$= r_1 r_2 \angle(\theta_1 + \theta_2)$

$$\frac{\overline{Z_1}}{\overline{Z_2}} = \frac{r_1 e^{j\theta_1}}{r_2 e^{je_2}} = \frac{r_1}{r_2} e^{j\angle(\theta_1 - \theta_2)} = \frac{r_1}{r_2} \angle(\theta_1 - \theta_2)$$

Addition and subtraction of phasors can be performed in phasor notation conveniently using rectangular form.

$\bar{Z}_1 = x_1 + jy_1; \ \bar{Z}_2 = x_2 + jy_2$

then $\bar{Z}_1 + \bar{Z}_2 = x_1 + jy_1 + x_2 + jy_2 \quad = (x_1 + x_2) + j(y_1 + y_2)$

then $\bar{Z}_1 - \bar{Z}_2 = x_1 + jy_1 - (x_2 + jy_2) \quad = (x_1 - x_2) + j(y_1 - y_2)$

1.10 Power Factor

Power is the rate of doing work. Power defines the rate at which electrical energy is consumed in an electrical circuit. It is equal to the product of voltage and current. In an AC circuit both the applied e.m.f and the current vary continuously with time and they are usually out of phase.

Suppose that the current and voltage in some portion of a circuit are given by

$i = I_{\max} \sin \omega t$ ------------- (1.6)

$v = V_{\max} \sin(\omega t + \phi)$ ------------- (1.7)

where ϕ is the phase angle. The instantaneous power is given by

$p = vi = V_{\max} I_{\max} \sin \omega t \sin(\omega t + \phi)$ ----------- (1.8)

The average power is given by

$$p = \frac{1}{T}\int_0^T vi\,dt$$

$$= \frac{I_{max}V_{max}}{T}\int_0^T \sin\omega t \cdot \sin(\omega t + \phi)\,\mathrm{dt}$$

$$= \frac{V_{max}I_{max}}{T}\left[\cos\phi\int_o^T \sin^2 \omega t\,dt + \sin\phi\int_0^T \cos\omega t \cdot \sin\omega t\,dt\right]$$

$$= \frac{V_{max}I_{max}}{2}\cos\phi = \left(\frac{V_{max}}{\sqrt{2}}\right)\left(\frac{I_{max}}{\sqrt{2}}\right)\cos\phi$$

(or) $P = V \cdot I \cdot \cos\phi$ ------------- (1.9)

where V and I are the RMS values.

$$power\ \ factor\ = \frac{Averagepower}{Apparentpower}$$

$$\cos\phi = \frac{P}{VI}$$ ------------- (1.10)

In AC circuits, the useful power depends not only upon the current and voltage in the circuit but also upon the phase difference between them, according to the relation given in equation (1.9). The terms '$\cos\phi$' is called the ***Power Factor*** of the circuit. In a pure resistor $\phi = 0$; $\cos\phi$=1; in a pure inductor $\phi = 90$; $\cos\phi = 0$ and in a pure capacitor $\phi = 90^0; \cos\phi = 0$.

1.11 Summary

The alternating current, which is sinusoidal in nature, can be generated by an AC generator or dynamo. It can be generated either by rotating a coil in a stationary magnetic filed or by rotating a magnet in a stationary coil. It is based on the principle of electromagnetic induction. The ac voltage is generally represented by the equation e = E_{max} sinωt, where ω is the angular frequency of the coil. The terms associated with alternating current are peak value, average value, RMS value, peak-to-peakvalue, form factor, power factor. Phasors are used to represent ac quantities in a compact form. Phasor is like a vector. It is a directed line segment. It rotates in counterclockwise direction with constant angular velocity 'ω' rad/sec. Phasors can be added, subtracted, multiplied and divided. To represent a phasor quantity, rectangular form and polar form of representation can also be used in addition to trigonometric form and exponential form. To represent more than one phasor on phasor diagram, each and every phasor should possess the same frequency.

1.12 Key Terminology

Alternating current – Alternating voltage – RMS Value – Average value – Maximum Value – Phasor – Rectangular notation – Complex number.

Numerical Problems:

1. An alternating voltage has the equation υ=1.414 sin 377t. What are the values of

a) Peak value **b)** RMS Value **c)** Average value

d) Frequency **e)** Instantaneous voltage when t =3 ms.

Solution: We know that ac voltage is given by

$$v = V_{Max} \ \sin \omega t$$
$$v = 14.14 \ \sin 377t$$

On comparing the above equations, we get

a) Peak value V_{Max} **=**14.14 V

b) RMS value $V_{Rms} = \dfrac{V_{Max}}{\sqrt{2}} = \dfrac{14.14}{1.414} = 10V$

c) Average value $V_{Ave} = \dfrac{V_{Max}}{\Pi/2}$

$= 0.637 \times V_{Max} = 0.637 \times 14.14V = 9.00V$

d) Frequency $\omega = 377$

$$2\Pi f = 377 \quad ; \quad f = \frac{377}{2\Pi} = 60Hz$$

e) Instantaneous voltage

When t = 3 ms

$$v = 14.14 \times \sin\left(377 \times 3 \times 10^{-3}\right)$$
$$= 14.14 \times \sin 1.131$$
$$= 14.14 \times 0.904$$
$$= 127.8V$$

2. Represent the following equations in the from of phasor

i) $v = 150\cos(500t + 45^0)$ ii) $i = 3\times10^{-3}\cos(2000t - 60^0)$

Solution: (i) **(ii)**

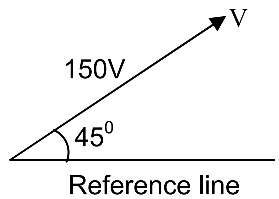

Fig 1.9(a)

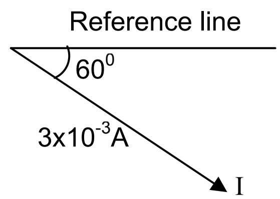

Fig 1.9(b)

3. A sinusoidal voltage of peak value 240 V gives rise to a current of maximum value 16 A. This current is also sinusoidal and lags behind the Voltage by 45^0. Represent the current and voltage waveforms and draw the phasor diagram.

Solution:

Taking the voltage as reference,

$$v = V_m \sin \varpi t = 240 \sin \varpi t.$$

Similarly $i = I_m \sin(\omega t + \phi)$

$$= 16\sin(\omega t - 45^0)$$

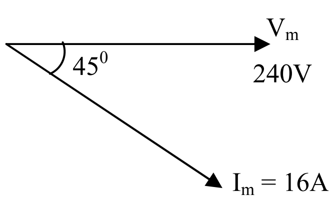

Fig 1.10(a) Phasor diagram

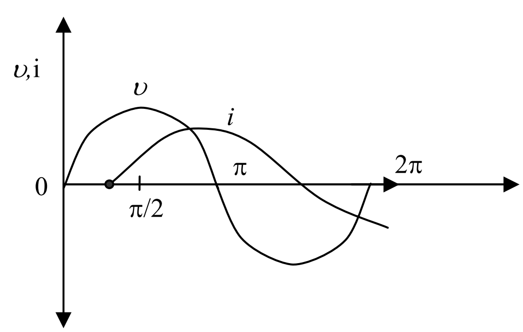

Fig 1.10(b) Waveform diagram

4. The graph in the following figure shows the variation of voltage with time. Calculate

(i) RMS value of this voltage

(ii) Frequency of this voltage

(iii) Calculate the RMS current which would flow through a 5Ω resistor, if this voltage were connected across it.

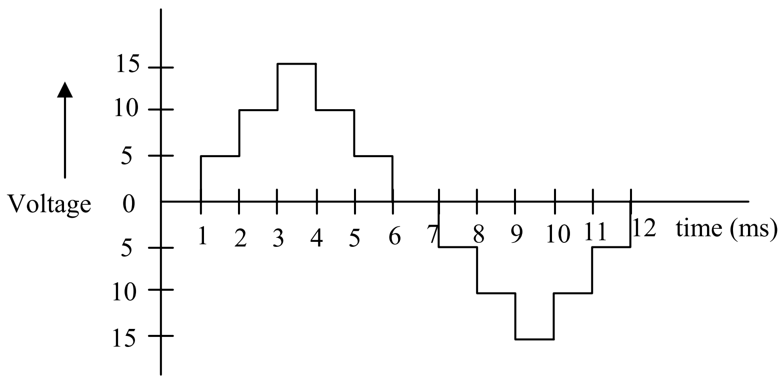

Fig 1.11

Solution:

The graph is symmetrical about the time axis, we can consider the positive half cycle only.

Mean value of v^2 is

$$= \frac{0^2 + 5^2 + 10^2 + 15^2 + 10^2 + 5^2}{6} = \frac{475}{6}$$

$$V_{RMS} = \sqrt{\frac{475}{6}} = 8.89\,V$$

Time period $= T = 12m\sec = 12 \times 10^{-3} \sec$

Frequency $= f = \frac{1}{T} = \frac{1}{12 \times 10^{-3}} = \frac{1000}{12} = 83 \cdot 3Hz$

$$I_{RMS} = \frac{V_{RMS}}{R} = 1.779A$$

1.13. Self Assessment Questions

A. Long Answer Type Questions

1) What is an AC generator? How can you generate AC voltage using this generator? Derive an expression for alternating voltage.
2) Define RMS value, Average value and derive relations for them.
3) What is phasor and phasor representation?

B. Short Answer Type Questions

1) State the RMS value and obtain an expression for it.

2) State the Average value and obtain an expression for it.

3) What is ' j' operator and explain?

4) Obtain an expression for AC voltage.

5) What is a phasor? Explain the phasor notation.

6) State and explain power factor and form factor.

7) Derive an expression for power factor of an AC waveform.

C. Numerical problems

1) An AC e.m.f. is given by

$$e=220 \sin(157t+\pi/2)$$

Calculate: (i) Frequency (ii) Time period (iii) Peak value (iv) Average value (v) RMS value.

[Ans: (i) 25Hz (ii) 0.04sec (iii) 220V (iv) 140.13V (v) 155.6V.]

2) An e.m.f. of 100 $\cos(\omega t + \pi/3)$ is applied across a pure resistance of 10Ω. Calculate the maximum instantaneous power and average power.

[Ans: 1000W; 500W].

1.14 References

1. Electrical Technology (Vol. I)
----- B.L.Theraja, A.K.Theraja
2. Circuit analysis
----- Umesh Sinha

UNIT – I LESSON – 2

RESISTORS

Objectives

This lesson explains you the concept of

(i) Resistance, Laws of Resistance

(ii) Types of Resistors

(iii) Uses, Combinations

Structure of the Lesson

2.1 Concept of Resistance (Introduction)

The opposition to the flow of current is called ***Resistance***. The number of free electrons and ions in a material contribute to current flow. As metals have good number of free electrons, they offer very little resistance to the flow of electric current through them. This means that they are good conductors of electricity. Germanium, Silicon etc have less number of free electrons. Resistance means characteristic value of a circuit. The term resistance describes a part or component of electrical equipment, which is designed to perform a certain function.

2.2 Laws of Resistance

The resistance offered by a conductor depends:

i) directly on its length (ℓ)

ii) inversely on its cross-section area (A)

iii) on the nature of the conductor material

i.e $R \alpha \frac{l}{A}$ or $R = \frac{\rho l}{A}$ or $\rho = \frac{RA}{l}$ ---------- (2.1)

where 'ρ' is a constant depends on the nature of conductor material, known as ***Resistivity***. Its unit is ohm-meter. It is defined as the resistance between the opposite faces of a unit cube of that material. Conductance (G) is the reciprocal of resistance. It measures the ease with which it allows the current to flow through it.

$$G = \frac{1}{R} = \frac{1}{\rho}\frac{A}{l} = \sigma \frac{A}{l} \qquad \text{---------- (2.2)}$$

The unit of conductance is Siemens or mho. In equation (2.2), 'σ' is the reciprocal of resistivity, known as the Conductivity.

$$\sigma = \frac{1}{\rho} = \frac{Gl}{A} \qquad \text{---------- (2.3)}$$

The unit of conductivity is Siemens/meter.

2.3 Ohm's law and its limitation

Ohm's law is used when the resistance (R) is the unknown quantity and current (I) and voltage (V) are known. The ohm's law states that "current flowing through a conductor is directly proportional to the potential difference applied across its ends".

$$I \,\alpha\, V \quad \text{or} \quad I = \frac{V}{R} \qquad \text{----------- (2.4)}$$

Limitation

Ohm's law cannot be applied to circuits consisting of electronic valves or transistors because these elements are not bilateral. It can't be applied to circuits consisting of non-linear elements such as powdered carbon, electric arc, thyrite etc.

2.4 Voltage – Current characteristics

Ohm's law states that voltage and current in a conductor of resistance 'R' are directly proportional to each other. It can be understood with the help of the following circuit shown in Fig. 2.1.

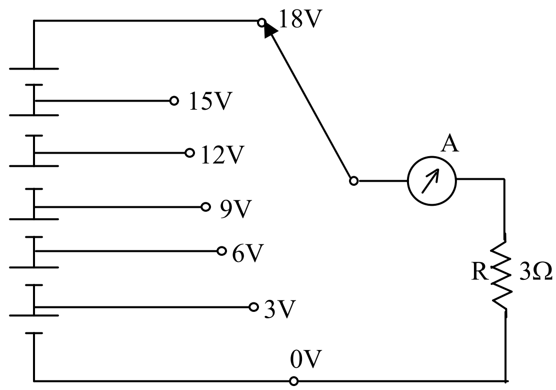

Fig. 2.1

Table 2.1

Volts	Ohms	Amperes
0 V	3	0 A
3 V	3	1 A
6 V	3	2 A
9 V	3	3 A
12 V	3	4 A
15 V	3	5 A
18 V	3	6 A

When voltage V is changed from 0 to 18V, the current meter 'A' shows current values directly proportional to voltage as shown in table 2.1. This relationship can be shown graphically by taking voltage values along X-axis and current values along Y-axis. This graph shown in Fig.2.2 is known as Volt-Ampere (V – I) characteristic of a resistor. This characteristic curve shows how much current the resistor allows at different voltages.

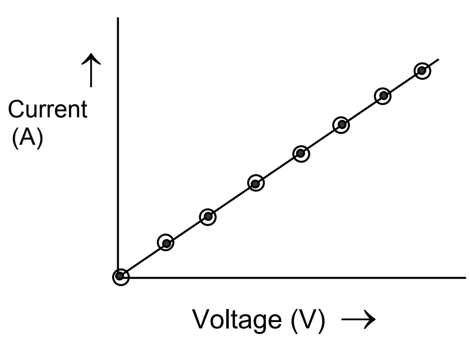

Fig. 2.2 V–I Characteristic of a simple resistor

2.5 Types of Resistors

There are two types of resistors used in electronic circuits. They are (i) Fixed and (ii) Variable.

2.5.1 Fixed Resistors

The resistors, whose resistance value is fixed, are known as Fixed Resistors. Fixed resistors may be

a) Wire-wound resistors
b) Carbon – composition resistors
c) Metal – film resistors

2.5.1(a) Wire-wound resistors

These are constructed from a long wire, which is usually made of nickel chromium, wound on a ceramic core. The entire assembly is coated with a ceramic material. Such resistors are available in power ratings from 5 Watts to several hundred Watts and resistance values from 1Ω to 100KΩ. These can be either fixed or variable type. They are used where large power dissipation is necessary. It is shown in Fig.2.3. Wire wound resistors have inductance because of wire windings, have poor performance at radio frequencies. Resistance and wattage values are stamped on the body of the resistors.

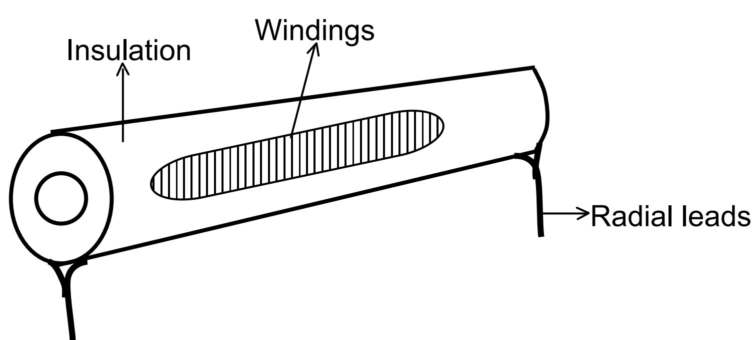

Fig. 2.3 Wire wound resistor

2.5.1(b) Carbon – composition resistors

These are made of finely – divided carbon, mixed with a powdered insulating material in suitable proportion. The resistance element is a simple rod of pressed carbon granules, which is enclosed in a plastic case for insulation and mechanical strength. The two ends of carbon resistance element are joined to metal caps with leads of tinned wire for soldering. Such resistors are available in power ratings of $\frac{1}{10}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2$ Watt. By changing composition, these can produce a wide range of resistance values from 1Ω to 20MΩ. These resistances are noisy. Some manufacturers stamp the value and wattage on the body of resistors. Some manufacturers use the colour code. In manufacturing of composition resistors, exact resistance value cannot be reproduced. i.e. a resistor stamped or coded value of 1000 ohms may have any value between 900Ω and 1100Ω. These variations are specified in terms of a colour band called Tolerance band. A 10% tolerance band indicate that the resistance value may be anywhere between 900Ω to1100Ω.

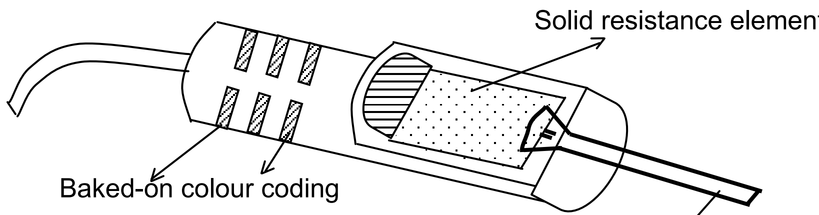

Fig. 2.4

2.5.1(c) Metal - film resistors

These are also referred to thin-film resistors. They consist of a thin metal coating deposited on an insulating support. The high resistance values are due to thinness of the film. These are free from inductance effects. These are preferred in high frequency circuits.

2.5.2 Variable Resistors

A resistor may change its value due to ageing. For proper performance of the circuit, its value has to be restored. The resistors whose resistance value is not fixed but varying are called ***Variable Resistors***. They can be wire wound or carbon type. Variable resistors may be in the form of

a) Potentiometer
b) Rheostat
c) Decade Resistance Box (DRB)

2.5.2(a) Potentiometer

This consists of a circular carbon composition resistance element, a sliding arm and a shaft. As the shaft rotates, the point of contact of the sliding arm on the circular carbon element changes. Hence the resistance between arm terminal B and terminals A , C of the stationary resistance changes as shown in Fig.2.5. Resistance values ranges from 20Ω to 22MΩ.

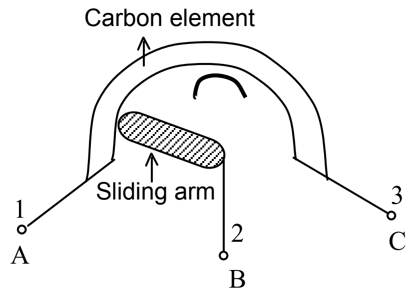

Fig. 2.5(a)

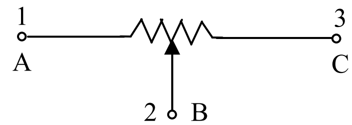

Fig. 2.5(b) Symbol

2.5.2(b) Rheostat

It is a potentiometer with different architecture. This consists of a long fine resistance wire wound on an insulating hollow cylindrical core, terminated at two ends. Above this assembly, a movable jockey is arranged.

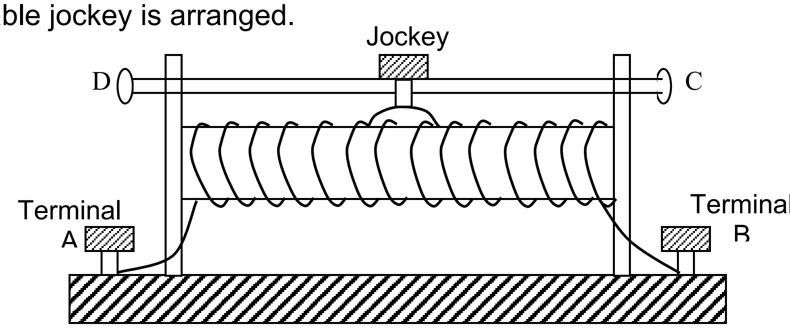

Fig. 2.6

The resistance between the jockey contact and one of the two terminals A and B can be changed. These are used to control high currents. The rheostat is shown in Fig. 2.6.

2.5.2(c) Decade Resistance Box (DRB)

This is also a variable resistor, but its resistance can be fixed at any integer value of resistance starting from 1Ω. This box consists of carbon resistors, along with step switches for x1, x10, x100, x1000 etc. ranges. This may be called as DRB, which is shown in Fig 2.7.

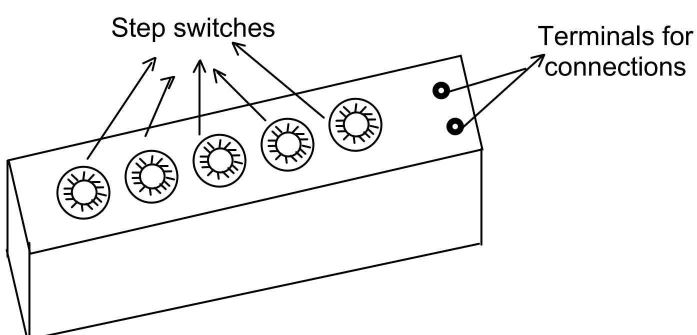

Fig. 2.7

2.6 Colour Code

Manufacturer uses colour code to mark the resistance value. The system is based on the use of colours (printed on the body of the resistor) to represent numerical values. The various colours and the numbers assigned to them are shown in table 2.2.

Table 2.2

Colour	Value
Black	0
Brown	1
Red	2
Orange	3
Yellow	4
Green	5
Blue	6
Violet	7
Grey	8
White	9

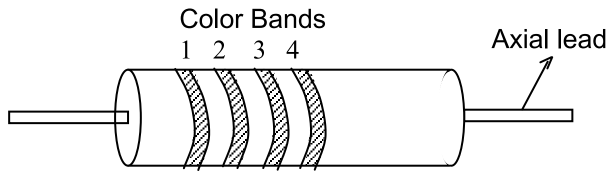

Fig. 2.8

Generally colour bands are printed around the body of the resistor near one end of it. Each colour stands for a digit. Often there are four bands. The first three bands give the resistance value and the fourth band gives the tolerance value.

Starting from left to right, the first band close to the edge indicates the first digit in numerical value; the second band gives the second digit; the third band is the decimal multiplier i.e., it gives the number of zeros after the two digits.

Tolerance: The percentage variation of resistance between the marked value and the actual value of a resistor is the tolerance of a resistor. If the fourth band on the body of a resistor is gold, then its tolerance value is 5% of its colour coded value. On the other hand, if the fourth band is silver, then the tolerance value is 10% of its colour coded value. If the fourth band is absent, it indicates a tolerance value of 20%.

Example:

The colours of a resistor from one edge are: Red, Black, Orange, Gold. Then its resistance value is

Red, Black, Orange = $20 \times 10^3 = 20{,}000\Omega = 20K\Omega$

Tolerance value (gold band) = 5% of $20{,}000\Omega$

$$= \frac{5}{100} \times 20{,}000 = 1000\Omega = 1K\Omega$$

The resistor may have any value between $19K\Omega$ and $21K\Omega$.

2.7 Combination of Resistors

The electronic components (resistors) can be connected in any circuit in two ways: (i) Series (ii) Parallel

2.7 (a) Resistors in Series

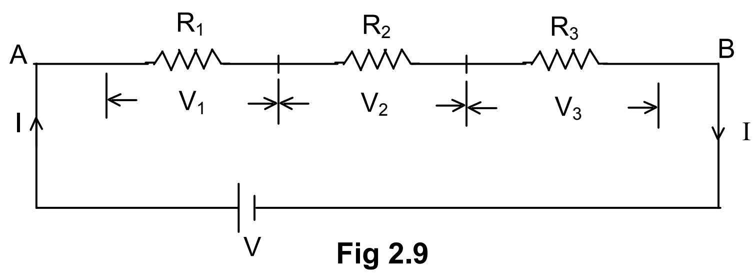

Fig 2.9

In series combination of resistors, resistors are connected end to end one after the other, as shown in Fig.2.9.

Let V be voltage source connected to the series combination of resistors R_1, R_2 and R_3. Let I be the current flowing in the combination. It is same in all resistors. But voltage drop across each resistor (i.e. V_1, V_2, V_3) are different. Let R_{eq} be the equivalent resistance between the terminals A and B.

From Fig 2.9, we can write

$$V = V_1 + V_2 + V_3$$

$$\text{or} \quad IR_{eq} = IR_1 + IR_2 + IR_3$$

$$\text{or} \quad IR_{eq} = I[R_1 + R_2 + R_3]$$

$$\text{or} \quad R_{eq} = (R_1 + R_2 + R_3) \qquad \text{-----------} \quad (2.5)$$

Any number of resistors can be connected in series, then $R_{eq} = R_1 + R_2 + R_3 + \ldots + R_n$

2.7 (b) Resistors in Parallel

In parallel combination of resistors, resistors are connected side by side across a voltage source as shown in Fig. 2.10

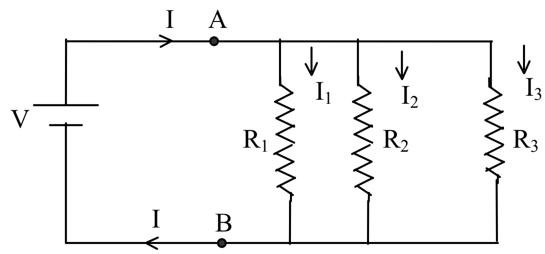

Fig 2.10

Let V be voltage source connected across the parallel combination. In this case, voltage developed across each resistor is the same but the current through each resistor is different. Let R_{eq} be the equivalent resistance between the terminals A and B.

From Fig. 2.10, we can write

$$I = I_1 + I_2 + I_3$$

or $$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

or $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad \text{----------} \quad (2.6)$$

Any number of resistors can be connected in parallel, then

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots\ldots\ldots + \frac{1}{R_n}$$

2.8 Effect of Temperature

The resistance of the conductors increases with increase in temperature (Metals, alloys). The temperature-resistance graph is a straight line. Metals have a positive temperature coefficient of resistance. The resistance of some materials like semiconductors decreases with increase in temperature. At high temperatures, some of these may even become good conductors. These materials are said to possess a negative temperature coefficient of resistance. Similarly, insulating materials also possess negative temperature coefficient of resistance.

Temperature Coefficient of Resistance

It is the ratio of increase in resistance per degree rise of temperature to the original resistance and denoted by 'α'.

$$\alpha = \left(\frac{R_t - R_0}{R_0 t} \right) /^0 C$$

or $$R_t = R_0 [1 + \alpha t]$$

$R_0 =$ Resistance of metallic conductor at 0^0C

$R_t =$ Resistance of metallic conductor at t^0C

2.9 Linear and Non-linear resistors

The resistor, in which voltage and current are directly proportional to each other, is called ***Linear Resistor.*** These resistors obey Ohm's law. The V-I characteristic is a straight line (Ex: Metals) as shown in Fig. 2.11(a)

The resistor, in which voltage and current are not directly proportional to each other, is called ***Non-linear Resistor.*** These resistors does not obey Ohm's law. The V-I characteristic is not a straight line (Ex: diodes) as shown in Fig. 2.11(b).

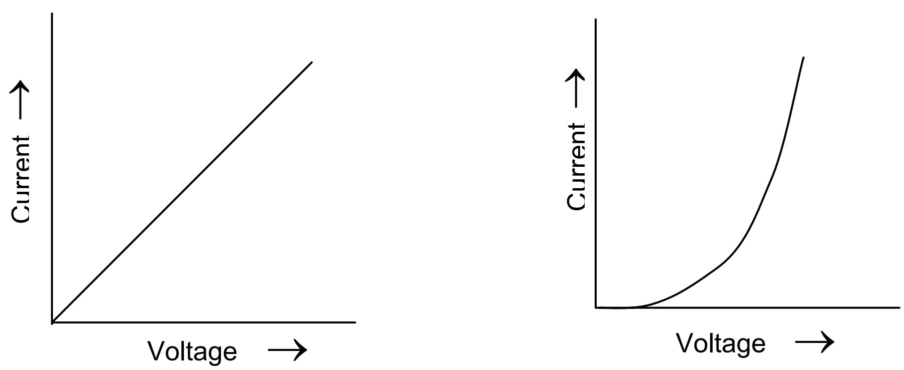

Fig. 2.11(a) Linear Resistor V-I curve   Fig. 2.11(b) Non-Linear Resistor V-I curve

2.10 Static and Dynamic Resistance

The Static resistance is defined as the ratio of voltage to current at a specified point (operating point) on V-I characteristic curve. It is denoted by R, and is given by R=V / I.

The dynamic or incremental resistance 'r' is defined as the reciprocal of the slope of the V-I characteristic shown in Fig. 2.12 and is given by $r = \frac{\Delta V}{\Delta I}$

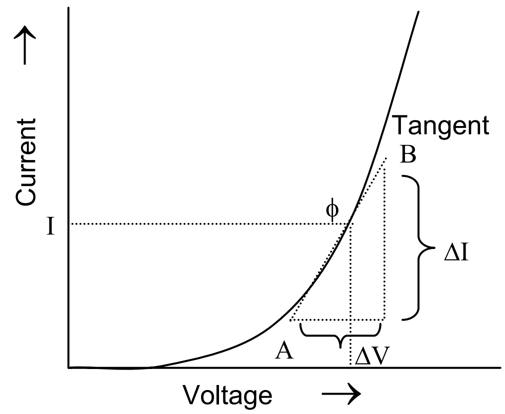

Fig 2.12.

2.11 Summary

Resistance is the opposition to the flow of electric current. Resistor is the basic electronic component, which offers resistance. Resistance is measured in ohms. The resistance of any material depends on its length, cross-section area and temperature. The usefulness of a resistor is determined by its electrical rating. Resistors may be either fixed type like wire wound, metal film, carbon composition or variable type like potentiometer, rheostat, DRB, etc. Carbon resistors are generally colour coded to indicate their resistance value in ohms. Resistors can be combined either in series or in parallel in a circuit. As per the behaviour, the resistors may be either linear or non-linear.

1.12 Key Terminology

Resistance – Resistors – Ohm's Law – V-I characteristic.

Solved Numerical Problems

Example 1

Two meter resistance wire of cross section area 0.5 mm^2 has resistance of 2.2 ohms. Calculate the resistivity of the material of the wire.

Solution: We know that

Resistivity $\rho = \frac{RA}{l}$; Given $R = 2.2\Omega$

$$\mathrm{A} = 0.5mm^2 = 0.5\times10^{-6}m^2$$

l = 2 meters

$$\therefore \rho = \frac{2.2\times0.5\times10^{-6}}{2} = 5.5\times10^{-5}\text{ ohm-meter.}$$

Example 2

The resistance of a conductor at 20^0C is 3.15 ohms and at 100^0C is 3.75 ohms. Calculate the temperature coefficient of resistance of the conductor and its resistance at 0^0C.

Solution: Let R_1, R_2 be the resistances of conductor at t_1^0C and t_2^0C, then we can write

$$R_1 = R_0(1+\alpha t_1) \quad \text{-----------} \quad (1)$$

$$R_2 = R_0(1+\alpha t_2) \quad \text{-----------} \quad (2)$$

Hence $\dfrac{R_1}{R_2} = \dfrac{1+\alpha t_1}{1+\alpha t_2}$

So $\alpha = \dfrac{R_2 - R_1}{R_1 t_2 - R_2 t_1}$

Given $R_1 = 3.15\Omega$; $t_1 = 20^0C$; $R_2 = 3.75\Omega$; $t_2 = 100^0C$

$$\alpha = \frac{3.75-3.15}{(3.15\times100)-(3.75\times20)} = \frac{0.6}{240} = 0.0025/^0C$$

Substituting the value of 'α' in eq.(1) we get

$$R_0 = \frac{R_1}{1+\alpha t_1} = \frac{3.15}{1+0.0025\times20} = 3\Omega$$

Example 3

A resistor has colour band sequence of red, green, yellow and gold. Find the value of resistance and percentage tolerance.

Solution:

Colour	Code
Red	2
Green	5
Yellow	10^4
Gold	$\pm 5\%$

Value of resistance $= 25\times10^4 \pm 5\%$ of $25\text{x}10^4$

$$= 250K\Omega \pm 5\times\frac{250000}{100}$$

=250KΩ ± 12500 = 250KΩ ± 12.5 KΩ

=262.5KΩ to 237.5 KΩ.

Example 4

Find the equivalent resistance of the network of Fig.2.13 between terminals A and B.

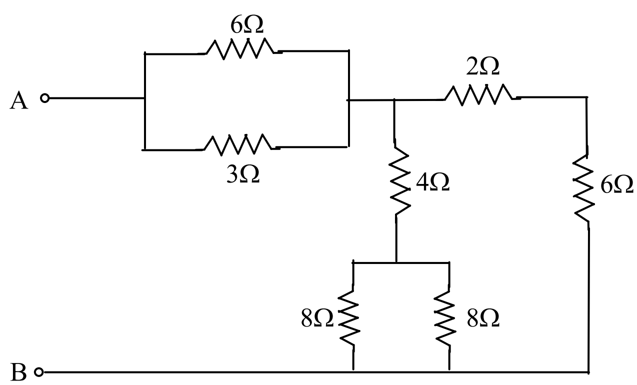

Fig 2.13

Solution

Using series combination and parallel combination rules of resistors, the above circuit can be reduced to

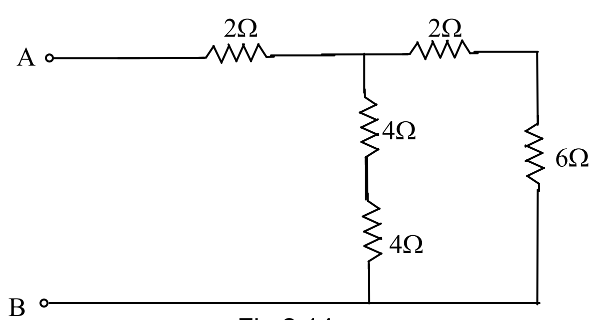

Fig 2.14

R_{AB} = [6 || 3] + [4+ (8 || 8)] || [2+6]

$$= \frac{6\times3}{6+3} + \left[4+\frac{8\times8}{8+8}\right] \Big\| [8]$$

= 2 + 8||8

$$= 2 + \frac{8\times8}{8+8} = 2+4 = 6\Omega$$

Example 5:

Find the total current supplied by the source of the network shown in Fig. 2.15

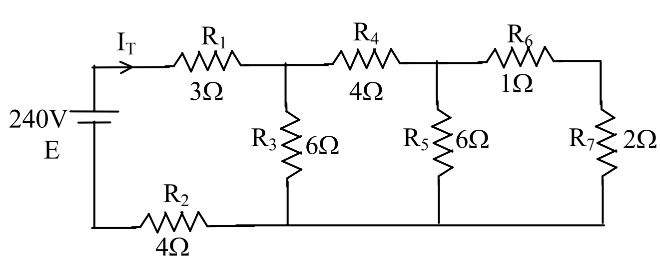

Fig 2.15

Solution:

$$I_T = \frac{E}{Total\ resistance\ as\ viewed\ from\ the\ battery\ terminals}$$

Total resistance $R_T = R_1 + \left[R_3 \| \left(R_4 + R_5 \| \left(R_6 + R_7\right)\right)\right] + R_2$

$$= 3 + \left[6 \| \left(4 + 6 \| 3\right)\right] + 4 \ = 3 + \left[6 \| \left(4 + 2\right)\right] + 4 = 3 + \left[6 \| 6\right] + 4 = 3 + 3 + 4 = 10\Omega$$

$$I_T = \frac{240V}{10\Omega} = 24A$$

Self-Assessment Questions

(A) Long Answer Questions

1) What are different types of resistors used? Briefly explain the preparation, properties and uses of carbon, metal film and wire wound resistors.
2) Compare the main characteristics of carbon composition and wire wound resistors. Explain the effect of temperature on the resistance of a conductor.
3) What is a colour coding system? How a resistance can be specified by using it? Explain with examples.

(B) Short Answer Questions

1) State Ohm's law and give its limitations.
2) Explain the colour code of resistors.
3) Explain the effect of temperature on resistance of conductors.
4) Discuss briefly the types of resistors.

(C) Numerical Problems

1) The dimensions of a carbon back are 1.0cm $\times$ 1.0cm $\times$ 50cm. Find the resistance between (i) two square ends (ii) two opposite rectangular faces. The resistivity of carbon is 3.5×10^{-5} ohm-meter. **Ans: (i) 0.175Ω; (ii) 7×10^{-5} Ω.**
2) A carbon resistor has coloured strips in sequence given by yellow, violet, brown, gold. What is its resistance? **Ans: 470Ω ± 5%**

2.14 References

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UNIT-I LESSON-3

INDUCTORS

Objectives

This lesson explains you the concepts of

- Inductance, Inductive Reactance
- V-I relationship
- Types, Energy stored
- Mutual Inductance
- Transformer, Types and applications

Structure of the Lesson

3.1 Concept of Inductance

3.2 Mutual Inductance

3.3 Coefficient of coupling

3.4 Phase relationship & V-I relationship.

3.5 Inductive Reactance and Susceptance

3.6 Energy stored in an Inductor

3.7 Types of Inductors

3.8 Combination of Inductors

3.9 Transformer Principle and working

3.10 Types of Transformers

3.11 Applications

3.12 Summary

3.13 Key Terminology

3.14 Self-Assessment questions

3.15 References

Introduction

Inductor is one of the basic electronic components commonly used in electronic circuits. It is a coil of wire wound on a core of some suitable material. Inductance is that property of a circuit or component, which opposes a change in current. Because of this property, it is also called a '***choke***'. Inductor stores energy in the form of electromagnetic field. These are used mainly as choke, in tuning circuits and in transformers.

3.1 Concept of Inductance

The property of a coil due to which it opposes any change of current through it is called Inductance [L]. Its unit is Henry. The phenomenon of the production of an induced e.m.f in a circuit itself due to change of current through it is called ***Self-Induction***. The e.m.f induced is called the ***Back e.m.f***. The inductor opposes any change in current passing through it and thus exhibits the property of electrical inertia. This is just like a mass, opposes any change in motion because of the property of inertia.

Self-inductance 'L' of a coil is given by

$$L = \frac{\mu_0 \mu_r A N^2}{l}$$

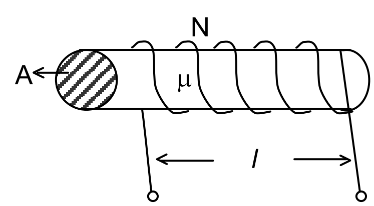

Fig 3.1

where

$\mu_0 \rightarrow$ Permeability of free space

$\mu_r \rightarrow$ Relative permeability of the core material

$A \rightarrow$ Cross sectional area

$N \rightarrow$ No. of turns

$l \rightarrow$ Core length.

Henry: A coil has an inductance of one Henry if an e.m.f of one volt is induced in it, when current through it changes at the rate of 1A/s.

3.2 Mutual Inductance (M)

When two coils are placed so close to each other that the expanding and collapsing magnetic flux of one coil links with the other, an induced e.m.f is produced in the other coil. These two coil are said to have Mutual Inductance (M). Its unit is also Henry (H).

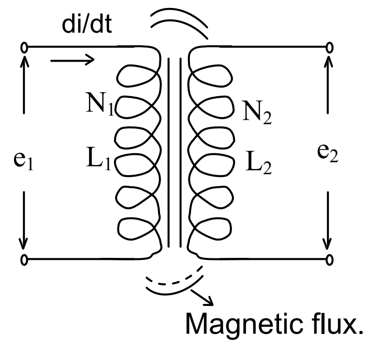

Fig 3.2

As shown in Fig.3.2, the rate of current change through the first coil is di/dt. This will produce a changing magnetic flux through it, which will link partly or fully with the second coil. Hence an induced e.m.f e_2 (called Mutually Induced EMF) will be produced in the second coil. It is given by

$$e_2 = M(di/dt)$$

If di/dt = 1A/s; e_2 = 1V then M = 1H.

Henry: Two coils have a mutual inductance of one Henry, if a current change of one Ampere/sec. in one coil induces one volt in the other.

3.3 Coefficient of Coupling

Two coils are said to be magnetically coupled if full or part of the flux produced by one coil links with the other, then coupling between the two coils is zero. The coupling effect is measured in terms of coefficient of coupling, given by the formula

$$k = M\sqrt{L_1 L_2}$$

where

$L_1, L_2 \rightarrow$ Inductances of first and second coil

$M \rightarrow$ Mutual inductance between the two coils.

When magnetic flux produced by one coil does not link with the other coil, then K = 0. If all flux produced by one coil links with the other, K = 1.

Derivation for coefficient of coupling

Referring to Fig.3.2, we can write

$$L_1 = \frac{\mu_0 \mu_r N_1^2 A}{l} \quad \& \quad L_2 = \frac{\mu_0 \mu_r N_2^2 A}{l}$$

when I_1 current flows in first coil, flux produced per turn is $\phi = \frac{L_1 I_1}{N_1}$; If a fraction k_1 of ϕ_1 i.e. $k_1\phi_1$ is linked with second coil, then

$$M = \frac{k_1 \phi_1 N_2}{I_1} = \frac{k_1 N_2}{I_1} \cdot \frac{L_1 I_1}{N_1} = \frac{k_1 L_1 N_2}{N_1} \qquad \text{----------} \quad (3.1)$$

Similarly, when a current I_2 flows in second coil, the flux produced per turn is

$$\phi_2 = \frac{L_2 I_2}{N_2}$$

If a fraction k_2 of ϕ_2 i.e. $k_2\ \phi_2$ is linked with first coil, then

$$M = \frac{k_2 \phi_2 N_1}{I_2} = \frac{k_2 N_1}{I_2} \cdot \frac{L_2 I_2}{N_2} = \frac{k_2 L_2 N_1}{N_2} \qquad \text{----------} \quad (3.2)$$

Multiplying equations (3.1) and (3.2), we get

$$M^2 = k_1 k_2 L_1 L_2$$

$$M^2 = k^2 L_1 L_2 \qquad \text{where } k^2 = k_1 k_2$$

or $$k = \frac{M}{\sqrt{L_2 L_2}} \qquad \text{----------} \quad (3.3)$$

If k = 1, then it is called ***Critical Coupling***

If k > 1, then it is called ***Over Coupling***

If k < 1, then it is called ***Under Coupling***

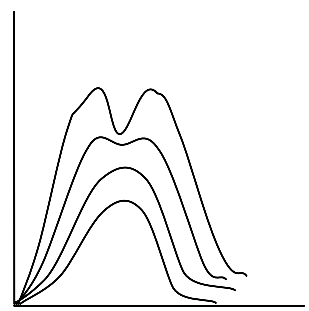

Fig 3.3

(Iron core transformers are not useful for high frequency circuits. For high frequency applications, iron core is replaced with ferrite core. For still higher frequencies, nonmagnetic materials are used as formers. To achieve this, tuned transformers are used. When tuned transformer is used, both primary and secondary are tuned to the same frequency.**)**

3.4 (a) Phase relationship between voltage and current in an Inductor

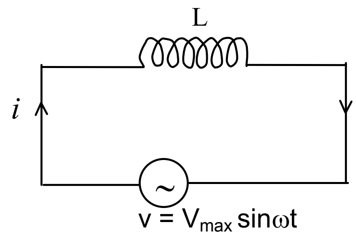

Fig 3.4(a)

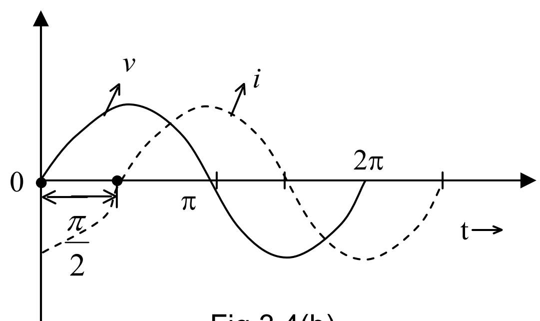

Fig 3.4(b)

Fig 3.4(c)

Let v = V_{max} sinωt be the applied voltage

i be the resultant current

But $v = L\frac{di}{dt}$

or $V_{\max} \sin \omega t = L\frac{di}{dt}$

$$\Rightarrow di = \frac{V_{\max}}{L} \sin \omega t \; dt$$

On integrating both sides we get,

$$\int di = \frac{V_{\max}}{L} \int \sin \omega t \; dt$$

$$i = \frac{V_{\max}}{\omega L} \left[-\cos \omega t \right]$$

or $i = \frac{V_{\max}}{\omega L} \cos \omega t$

$$= \frac{V_{max}}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

or $$i = I_{max} \sin\left(\omega t - \frac{\pi}{2}\right) \qquad \text{where} \quad I_{max} = \frac{V_{max}}{\omega L} \qquad \text{----------} \quad (3.4)$$

So, we that if applied voltage is represented by $v = V_{max}\sin\omega t$ then the current flowing through the inductor is $i = I_{max} \sin\left(\omega t - \frac{\pi}{2}\right)$. Thus the ***current lags behind*** the ***applied voltage by 90^0*** or the phase difference between the voltage and current is 90^0.

3.4.(b) V-I relationship

The induce e.m.f is given by

$$v = L\frac{di}{dt} \qquad \text{or} \qquad di = \frac{1}{L} v dt$$

$$\text{or} \quad \int di = \frac{1}{L}\int v dt \qquad \Rightarrow \qquad i = \frac{1}{L}\int_0^t v dt$$

The V-I relationship is linear. Hence an inductor is a linear device.

3.5 Inductive Reactance and Susceptance

From equation (3.4),

$$I_{max} = \frac{V_{max}}{\omega L} = \frac{V_{max}}{X_L} \qquad \text{----------} \quad (3.5)$$

Here ‘ωL’ plays the role of ‘resistance’. It is called ***Inductive Reactance*** (X_L) of the coil and is expressed in ohms if ‘L’ is in Henry and ‘ω’ is in red/sec.

$$X_L = \omega L = 2\pi f L \text{ ohms} \qquad \text{----------} \quad (3.6)$$

Hence X_L depends directly on frequency of the applied voltage. The reciprocal of inductive reactance is called ***Inductive Susceptance***, given by

$$B_L = \frac{1}{\omega L} \qquad \text{----------} \quad (3.7)$$

3.6 Energy stored in an Inductor

When current through an inductor is gradually changed from zero to maximum value I, then every change of it is opposed by the self-induced e.m.f produced due to this change. Energy is needed to overcome this opposition. This energy is stored in the magnetic field of the coil and is recovered when that field collapses. The value of this stored energy may be found as follows.

Let i be the instantaneous value of current and 'e' be the induced emf at that instant. We know that

$$e = L\frac{di}{dt} \qquad \text{-----------} \quad (3.8)$$

Work done in time dt in overcoming this opposition is

$$dw = eidt$$

$$dw = L\frac{di}{dt} \cdot i \cdot dt$$

$$dw = Li\,di$$

Total work done in establishing the maximum steady current of I is

$$W = \int_0^W dw = \int_0^I Li\,di = \frac{1}{2}LI^2$$

or $$W = \frac{1}{2}LI^2 \ \ Joules \qquad \text{----------} \quad (3.9)$$

This work is stored as energy of the magnetic field.

3.7 Types of Inductors

There are two types of inductors used in electronic circuits: (i) Fixed (ii) Variable.

3.7.1 Fixed Inductors

The inductors, whose inductance value is fixed, are known as Fixed Inductors.

Ex: Air-Core inductors, Iron core inductors, Ferrite core inductors.

3.7.1(a) Air-core Inductor

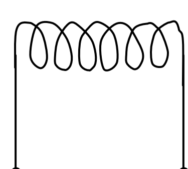

Fig 3.5

It consists of a number of turns of wire wound on a former of ordinary card board. As there is no core inside the coil, an air-core inductor has the least inductance for a given number of turns and core length.

3.7.1(b) Iron-core Inductor

In this inductor, a coil of wire is wound over a solid or laminated iron core. To avoid eddy current losses, iron core is made up of thin iron laminations pressed together but insulated from each other. These inductors are also called **Choke**, as shown in Fig 3.6(a). The core may some times surrounds the coil on its two sides as shown in Fig 3.6(b).

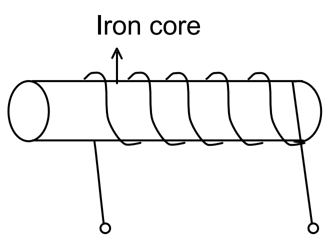

Fig 3.6(a)

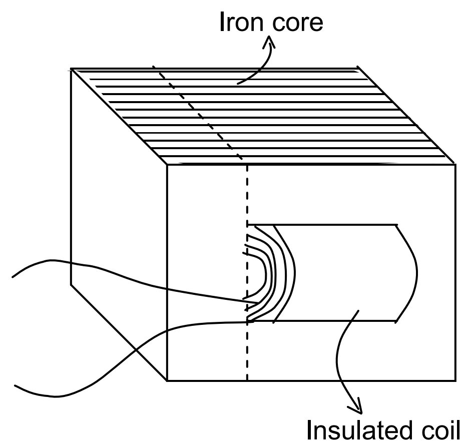

Fig 3.6(b)

3.7.1(c) Ferrite-core Inductor

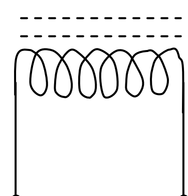

Fig 3.7

A coil of wire is wound on a solid core made of highly ferromagnetic substance called Ferrite. Ferrite is a solid material consisting of fine particles of iron powder embedded in an insulating binder. A ferrite core has minimum eddy current loss.

3.7.2 Variable Inductors

The inductance of a coil can be varied by three different methods:

(i) By using a tapped coil as shown in Fig.3.8(a). Here either more or fewer turns of the coil can be used by connection to one of the taps on the coils.

(ii) By using a slider contact to vary the number of turns used as in Fig. 3.8(b).

(iii) Fig.3.8(c) shows the symbol for a coil with a ferrite slug, which can be screwed in or out of the coil to vary its inductance.

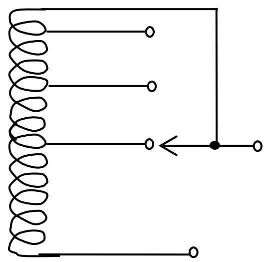
Fig 3.8(a)

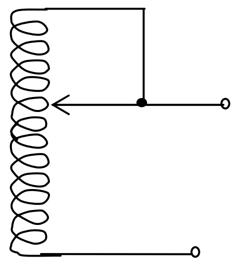
Fig 3.8(b)

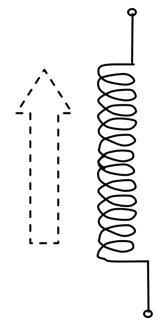
Fig 3.8(c)

3.8 Combination of Inductors

Inductors can be connected in an electronic circuit either in series or in parallel, with fluxes adding or opposing.

3.8.1 Inductors connected in Series

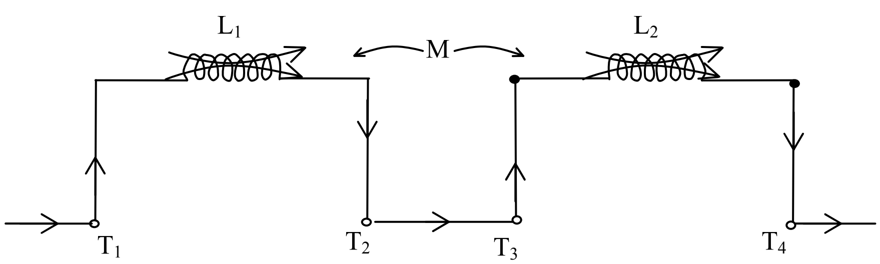

Fig 3.9(a)

Fig.3.9(a) shows a circuit of two inductors connected in series with fluxes adding. Let L_1, L_2 be the coefficient of self-inductances and let M be the coefficient of Mutual Inductance between them.

We can write for Fig.3.9(a)

$$v(t) = \frac{L_1 di}{dt} + \frac{M di}{dt} + \frac{L_2 di}{dt} + \frac{M di}{dt}$$

or $$\left(L_1 + L_2 + 2M\right)\frac{di}{dt} = v(t)$$

Here $v(t)$ is the ac voltage applied across the combination.

Hence the equivalent inductance is given by

$$L_{eq} = L_1 + L_2 + 2M \qquad \text{-----------} \quad (3.10)$$

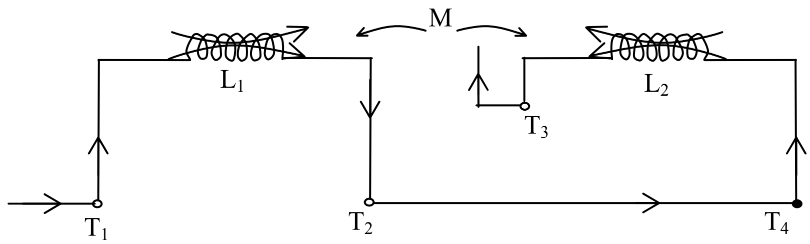

Fig 3.9(b)

The two coils in series can also be connected as shown in Fig.3.9(b) with fluxes opposing. Then for the circuit shown in Fig.3.9(b), we can write

$$\frac{L_1 di}{dt} - \frac{Mdi}{dt} + \frac{L_2 di}{dt} - \frac{Mdi}{dt} = v(t)$$

or $$\left(L_1 + L_2 - 2M\right)\frac{di}{dt} = v(t)$$

Hence the equivalent inductance is given by

$$L_{eq} = L_1 + L_2 - 2M \qquad \text{-----------} \quad (3.11)$$

3.8.2 Inductors connected in Parallel

Fig 3.10 shows two mutually coupled coils connected in parallel. Here the fluxes may be additive or subtractive.

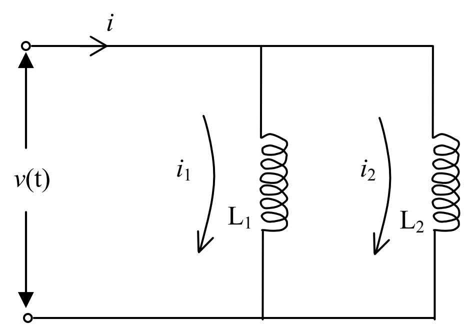

Fig 3.10

Applying KVL, we get

$$L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = v(t) \qquad \text{----------} \quad (3.12)$$

and $$L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} = v(t) \qquad \text{----------} \quad (3.13)$$

From sinusoidal time variations v,i_1 and i_2 , we can also write

$$j\omega L_1 I_1 + j\omega M I_2 = V \qquad \text{----------} \quad (3.14)$$

$$j\omega L_2 I_2 + j\omega M I_1 = V \qquad \text{---------} \quad (3.15)$$

on solving equations (3.14) and (3.15), we get

$$I_1 = \frac{j\omega(L_1 - M)V}{\omega^2(M^2 - L_1 L_2)} \quad \text{and} \quad I_2 = \frac{j\omega(L_2 - M)V}{\omega^2(M^2 - L_1 L_2)}$$

The total current is given by

$$I = I_1 + I_2 = \frac{j\omega(L_1 + L_2 - 2M)V}{\omega^2(M^2 - L_1 L_2)}$$

The input impedance is

$$Z_{in} = \frac{V}{I} = \frac{\omega^2(M^2 - L_1 L_2)}{j\omega(L_1 + L_2 - 2M)} = \frac{j\omega(L_1 L_2 - M^2)}{(L_1 + L_2 - 2M)} \qquad \text{-----------} \quad (3.16)$$

Hence, from eq.(3.16), we can conclude that the circuit shown in Fig 3.10 can be replaced by an equivalent inductance

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} \qquad \text{-----------} \quad (3.17)$$

If the fluxes are subtractive then we can write

$$L_{eq} = \frac{L_1 L_2 + M^2}{L_1 + L_2 + 2M} \qquad \text{-----------} \quad (3.18)$$

3.9 Transformer

It is a device to convert alternating current at high voltage into low voltage and vice versa. The transformer is based on the principle of electromagnetic induction.

Construction

A transformer consists of two coils, which are electrically insulated from each other and wound uniformly on the same iron core as shown in Fig.3.11. The coil to which energy is supplied is called the ***Primary coil*** (P) and that from which energy is taken is called the ***Secondary coil*** (S).

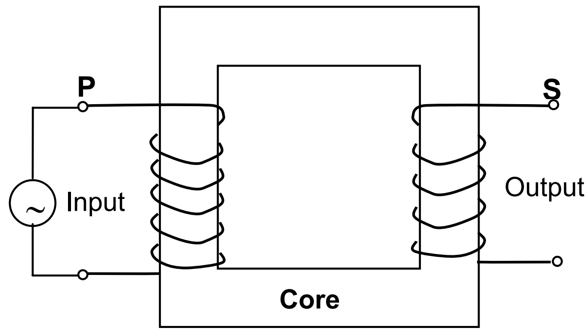

Fig 3.11

Working

An alternating current in the primary coil sets up an alternating magnetic flux in the core. This flux, linked with the secondary coil induces an alternating e.m.f in the secondary coil. In this way, power is transferred from one coil to the other via the changing magnetic flux in the core.

Efficiency (η)

It is defined as the ratio of output power to input power.

$$\eta = \frac{\text{output puwer}}{\text{input power}} \times 100$$

For an ideal transformer, output power and input powers are equal. Hence it has 100% efficiency.

Let $N_p \rightarrow$ No. of turns in primary

$N_s \rightarrow$ No. of turns in secondary

$E_s \rightarrow$ Voltage induced in secondary

$E_p \rightarrow$ Voltage applied to the primary

We can write

$$\frac{E_s}{E_p} = \frac{N_s}{N_p} = K$$

K is constant for transformer known as ***Transformation Ratio*** or ***Turns Ratio***.

Case-1

If $K>1$; $E_s>E_p$ so $N_s>N_p$.

In this case, the transformer is a ***Step-up*** transformer. A thin wire is used for secondary coil and a thick wire is used for primary coil.

Case-2

If $K<1$; $E_s<E_p$ So $N_s<N_p$.

In this case, the transformer is a ***Step-down*** transformer. A thin wire is used for primary coil and a thick wire is used for secondary coil. If losses are neglected power in primary will be equal to secondary power or $V_1I_1 = V_2I_2$.

Transformers are used not only for getting desired voltage and currents at power line frequency (50Hz), but also at high frequencies. When the transformer core selected in such that the transformer can pass audio frequencies without losses, such transformers are called ***Audio Frequency Transformers***.

3.10 Types of Transformers

The main types of Transformers are

(a) **Step-down Transformer:** It is used for conversion of high-voltage low-amperage current into low-voltage high-amperage current of the same frequency.

(b) **Step-up Transformer:** It is used for conversion of low-voltage high-amperage current into high-voltage low-amperage current of the same frequency.

(c) **Audio Transformer:** These transformers pass a range of frequencies (Audio) between two regions of a device. These are of four types:

(i) **Input Transformer:** These transformers couple or connect generator to the input the first stage of a multistage amplifier.

(ii) **Inter stage Transformer:** These transformers couple the output of one stage to the input of the other stage of a multistage amplifier. In some amplifiers, input impedance and output impedance differ very much. So for maximum transfer of signal power between stages, impedance matching transformers are used. Such transformers are called ***Inter stage transformers***.

(iii) **Output Transformer:** These transformers are used to change the impedance level of the output signal to the impedance level of the load or to provide d.c. isolation for the load.

(iv) **Driver Transformer:** This transformer is a large step-down transformer. This also couples the output of an amplifier to the input of other amplifier.

(d) **Auto Transformer:** In this transformer, there is only one winding on an open core from which separate tappings are taken. This is very widely used when a gradual change in the output voltage is required.

(e) **Constant voltage transformer:** These transformers are designed to give a constant output voltage even when the input voltage varies considerably.

(f) **Constant Current Transformer:** These transformers are designed to give a constant output current as is required for supplying energy to series street lamps.

(g) **High Current or Furnace Transformer:** These are used for welding or for furnace purposes, requires current of the order of 25,000 amperes and consists of only a few turns of thick copper wire in the primary and only one turn of a heavy copper conductor in the secondary. They operate for a short time.

3.11 Applications

1. Transformers are used in all electrical apparatus where voltage variations are required.
2. At electricity generating stations, step-up transformers are used.
3. At substations, step-down transformers are used. They are also used in night lamps.
4. In T.V., Radio sets, tape recorders, power supplies, transformers are widely used.

3.12 Summary

Inductor, the second basic electronic component, is a coil of wire wound on a core. The ability of a coil to oppose any change in current is a measure of the self-inductance 'L' of the coil. Inductors are coils of various dimensions designed to introduce specified amounts of inductance into a circuit. Ferromagnetic materials are frequently employed to increase the inductance by increasing the flux linking the coil. The ideal inductor does not dissipate the electrical energy supplied to it. It stores the energy in the form of magnetic field. Mutual inductance exists between two coils. When two coils are very close together and are tightly coupled, then the coupling coefficient K = 1: In an inductor, the applied voltage and resultant current are at a phase of 90^0. Current lags behind the e.m.f. The opposition presented by an inductance to an ac or changing current is called Inductive Reactance. It is measured in ohms.

Transformer is a device, works on the principle of electromagnetic induction. Varieties of transformers are available and are frequently used for household and industrial purposes.

3.13 Key Terminology

Self-inductance – Mutual inductance – Coefficient of coupling – Transformer.

Solved Problems

Example 1

Find the inductive reactance offered by a coil of inductance 25μH to radio frequency currents of frequencies (i)1 MHz (ii) 10 MHz.

Solution:

The inductive reactance $X_L = 2\pi f L$

(i) At f = 1 MHz = 1 $\times 10^6$ Hz; L = 25μH = 25×10^{-6}H

$X_L = 2\pi \times 10^6 \times 25 \times 10^{-6}$ =157 ohms.

(ii) At f = 10 MHz = 10 $\times 10^6$ Hz; L = 25μH = 25×10^{-6}H

$X_L = 2\pi \times 10^7 \times 25 \times 10^{-6}$ =1570 ohms.

Example 2

A coil with an inductance of 2H and a resistance of 10 ohm is connected to a battery of e.m.f 100V. What is the equilibrium current? How much energy is stored up in the magnetic field?

Solution:

$$I_{\max} = \frac{V}{I} = \frac{100}{10} = 10A$$

Energy stored = ½ L I_{Max}^2

Given L = 2H

Energy stored $= \frac{1}{\cancel{2}} \times \cancel{2} \times 10 \times 10 = 100\ Joules$.

Example 3

Two magnetically coupled coils have a coefficient of coupling 0.5. When they are connected in series, the total inductance is 100mH. When one coil is reversed, the total inductance becomes 60mH. Calculate their self and mutual inductances.

Solution:

If currents are in the same direction,

$$L_{eq} = L_1 + L_2 + 2M = 100\text{mH} \quad \text{-----------} \quad (1)$$

If currents are in the opposite direction,

$$L_{eq} = L_1 + L_2 - 2M = 60\text{mH} \quad \text{-----------} \quad (2)$$

Adding equations (1) & (2), we get

$$L_1 + L_2 = 80\text{mH} \quad \text{-----------} \quad (3)$$

Substituting this value in eq (1), we get

80mH + 2M = 100mH

or 2M = 20mH

M = 10mH ---------- (4)

Coefficient of coupling $K = \dfrac{M}{\sqrt{L_1 L_2}}$

$$0.5 = K = \frac{10 \times 10^{-3}}{\sqrt{L_1 L_2}}$$

or $\sqrt{L_1 L_2} = 20 \times 10^{-3}$

$$L_1 L_2 = 400 \times 10^{-6} \quad \text{----------} \quad (5)$$

$$(L_1 - L_2)^2 = (L_1 + L_2)^2 - 4L_1 L_2$$

$$= (80 \times 10^{-3})^2 - 4 \times 400 \times 10^{-6}$$

$$= 6400 \times 10^{-6} - 1600 \times 10^{-6} = 4800 \times 10^{-6}$$

$$\Rightarrow \quad (L_1 - L_2)^2 = 40\sqrt{3} mH \quad \text{----------} \quad (6)$$

Adding eq (3) and eq (6), we get.

$$2L_1 = 80 + 40\sqrt{3}\ mH$$

or $L_1 = \left(40 + 20\sqrt{3}\right)\ mH$

$$L_1 = 20\left(2 + \sqrt{3}\right)\ mH \quad \text{----------} \quad (7)$$

From eq (6),

$$40 + 20\sqrt{3} - L_2 = 40\sqrt{3} mH$$

$$L_2 = 40 - 20\sqrt{3}mH$$

$$L_1 = 20\left(2 - \sqrt{3}\right)\, mH \qquad \text{------------ (8)}$$

Example 4

A step-down transformer operates on a 2000V supply line and supplies a load current of 60A. The ratio of primary to secondary windings is 20:1. Assuming 100% efficiency, calculate the secondary voltage, primary current and power output.

Solution

Let I_s, I_p are secondary and primary currents; E_s, E_p are secondary and primary voltages then

$$\frac{I_p}{I_s} = \frac{E_s}{E_p} = \frac{N_s}{N_p}; \qquad \text{Given } \frac{N_s}{N_p} = \frac{1}{20}$$

E_p = 2000V ; I_s = 60A

Hence $\dfrac{E_s}{2000} = \dfrac{1}{20} \quad or \quad E_s = 100V$

$$\frac{I_P}{60} = \frac{1}{20} \quad or \quad I_p = \frac{60}{20} = 3A.$$

Power output = $I_s E_s$ = 60 × 100 = 6000 Watts.

3.14 Self-Assessment questions

A. Long Answer Questions

1. What is inductive reactance? Explain the V-I relationship in an inductor. Show the phase relationship diagrammatically.
2. Define mutual inductance, coefficient of coupling and inductive reactance. Derive an expression for the energy stored in an inductor. What are different types of inductors used in electronic circuits.
3. Describe the principle and working of a transformer.

B. Short Answer Questions

1. Derive an expression for the energy stored in an inductor.
2. Derive an expression for the coefficient of coupling for two magnetically coupled coils.
3. Obtain an expression for equivalent inductance. When two inductances are connected in series.

4. Obtain an expression for equivalent inductance when two inductances are connected in parallel.

C. Numerical Problems

1. The current in a coil changes from 20A to 12A in 0.1 sec. If the e.m.f. produced is 100V, find the inductance of the coil. **[Ans: 1.25H]**
2. Find the energy stored by the inductor in the circuit of Fig.3.12, when the current through it has reached its final value? **[Ans: 27×10^{-3}J]**

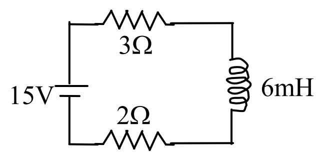

Fig . 3.12

3. Calculate the inductive reactance of coil with 50μH at a frequency of 4MHz.

[Ans: 1256 ohms]

3.15 References

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UNIT-I LESSON - 4

CAPACITORS

Objectives

This lesson explains you the concept of

- Capacitance, Capacitive Reactance
- V - I Relationship
- Types, Energy Stored
- Combination of Capacitors
- Color Coding

Structure of the Lesson

Introduction

Capacitor is the third passive device. It is one of the basic electronic components used in any electronic circuit. In ideal condition, the capacitors do not dissipate energy like the resistor but store it in electrostatic energy form that can be returned to the circuit whenever required. Capacitance is a property of an electric circuit that tends to oppose a change in voltage. In a capacitive circuit, current leads the voltage by 90^0. There are varieties of capacitors with different values and ratings.

4.1 Capacitor

It is one of the basic electronic components that have the following properties:

1. It can store electric charge
2. It opposes change of voltage in a circuit
3. It blocks d.c.
4. It allows a.c.
5. It stores energy in the form of electrostatic filed

Construction

A capacitor consists of two conducting plates separated by an insulating medium called DIELECTRIC as shown in Fig. 4.1(a)

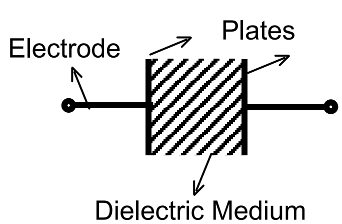


Fig 4.1(a)

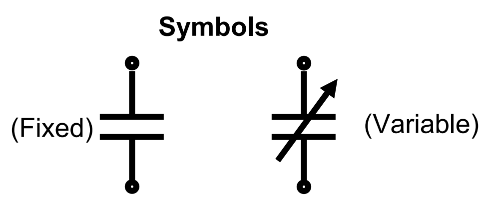


Fig 4.1(b)

4.2 Capacitance

It is the property of a capacitor to store electric charge. It may be defined as the amount of charge required to create a unit potential difference between its plates.

Explanation

Suppose we give Q Coulombs of charge to one of the plates of a capacitor and if a p.d. of V volts is established, then the capacitance is

$$C = \frac{Q}{V}$$

If Q = 1 Coulomb, V = 1 volt then C = 1 Farad.

Farad: It is the capacitance of a capacitor, which requires a charge of one Coulomb to establish a potential difference of one volt between its plates.

$1\mu F$ = 1 Micro Farad = 1×10^{-6}F

1nF = 1 Nano Farad = 1×10^{-9}F

1pF = 1 Pico Farad = 1×10^{-12}F

4.3. Capacity of a Parallel Plate Capacitor

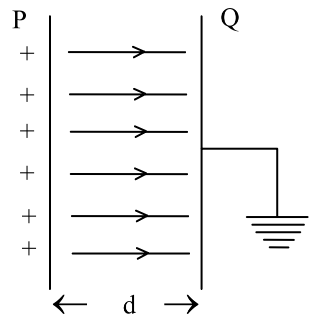


Fig 4.2

Fig.4.2 is a parallel plate capacitor. It consists of two parallel plates of area A, each (P & Q) separated by a distance 'd'. Let the plate P be positively charged and Q be earthed. Let E be the uniform electric filed strength between the plates and 'V' be the p.d. between them. Using Gauss theorem in electrostatics, we can write.

$$E = \frac{q}{\epsilon_0 A} \qquad \begin{array}{l} \epsilon_0 \rightarrow Permitivity\ of\ vacuum\ or\ air \\ q \rightarrow Charge \end{array}$$

but $E = \frac{V}{d}$

Hence $\frac{V}{d} = \frac{q}{\epsilon_0 A}$ (or) $\frac{q}{V} = \frac{\epsilon_0 A}{d}$

(or) $C = \frac{q}{V} = \frac{\epsilon_0 A}{d}$ --------- (4.1)

From Eq.(4.1), it is clear that the capacitance of a capacitor depends on:

Area of the plates

Distance between the plates

Permittivity of medium between the plates

Note: If a dielectric medium of dielectric constant K is introduced between the plates, the capacity of a capacitor increases. New capacitance

$$C_1 = KC = K \in_0 A/d$$

(or) $$K = \frac{C_1}{C} = \frac{\text{Capacitance of a capacitor with the dielectric}}{\text{Capacitance of a capacitor with air}}$$

Uses

1. To store base amounts of charges at low potentials.
2. For tuning of wireless sets.
3. To filter d.c. current and allow only a.c.
4. To create a very strong electric filed.
5. In TV and Radio sets.

4.4 Series & Parallel combinations

4.4(a) Series combination

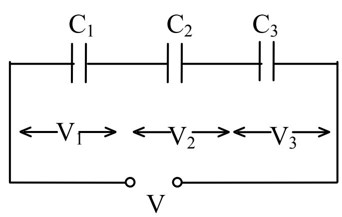


Fig 4.3

In series combination, the charge on all capacitors is the same, but p.d. across each is different.

$$V=V_1+V_2+V_3$$

But $C_{eq} = \frac{Q}{V} \Rightarrow V = \frac{Q}{C_{eq}}$

i.e., $\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$ Hence $V_1 = \frac{Q}{C_1}$; $V_2 = \frac{Q}{C_2}$; $V_3 = \frac{Q}{C_3}$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad \text{---------- (4.2)}$$

4.4(b) Parallel combination

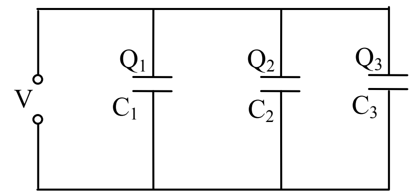


Fig 4.4

In parallel combination of capacitors, shown in Fig.4.4, p.d. across each capacitor is same, but the charge on each is different.

i.e., $Q = Q_1 + Q_2 + Q_3$ ----------- (4.3)

But $Q = C_{eq}V$

Hence $Q1 = C_1 V$; $Q_2 = C_2 V$ and $Q_3 = C_3 V$.

Equation (4.3) becomes $C_{eq}V = C_1 V + C_2 V + C_3 V$

$\Rightarrow$ $C_{eq} = C_1 + C_2 + C_3$ ------------ (4.4)

4.5 Phase difference between voltage across and current through a capacitor

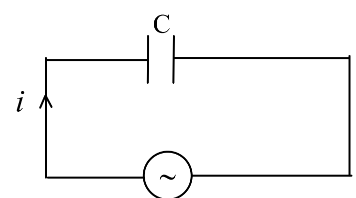


Fig 4.5

Consider the circuit shown in Fig. 4.5. Let $v = V_{\max} \sin \omega t$ be the alternating voltage, with $\omega = 2\pi f$. At any instant, electric charge on capacitor is $Q = CV$

or $$\frac{dQ}{dt} = C\left(\frac{dv}{dt}\right)$$

$$v = V_{\max} \sin \omega t \qquad \text{---------- (4.5)}$$

or $$i = C\frac{d}{dt}\left(V_{\max} \sin \omega t\right)$$

$$= C \cdot V_{\max} \cdot \omega \cdot \cos \omega t$$

$$= I_{\max} \cos \omega t \qquad \text{--------- (4.6)}$$

$$I_{\max} = CV_{\max} \cdot \omega$$

$$I_m = \frac{V_{\max}}{1/\omega C} = \frac{V_{\max}}{X_c}$$

Here 'i' is the instantaneous current.

Equation (4.6) becomes

$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

Hence, in capacitor, the current 'i' leads the applied voltage 'v' by 90^0 i.e., the phase difference between the voltage and current in a capacitor in 90^0 or $\frac{\pi^c}{2}$. The term X_c is like resistance in dc circuits, given by $X_C = \frac{1}{2\pi f C} ohms$. X_c varies inversely with f and also inversely with 'C'.

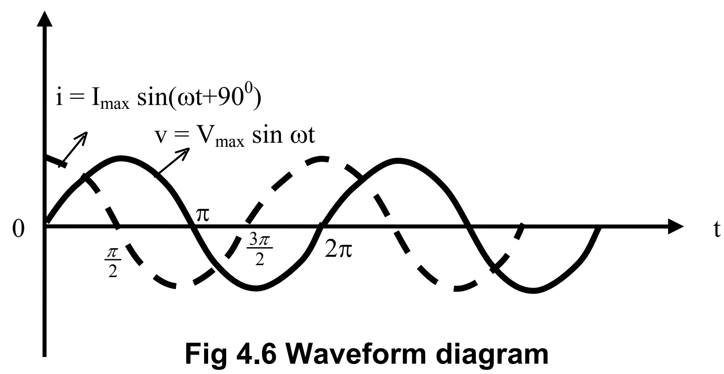


Fig 4.6 Waveform diagram

4.6 Capacitive Reactance and Susceptance

The opposition to the flow of A.C. through a capacitor is called ***Capacitive Reactance***. Its unit is ohm. It is given by $X_C = \frac{1}{\omega C}$. The reciprocal of capacitive reactance is ***Capacitive Susceptance***, given by B=ωC. These two quantities vary with frequency.

4.7 V-I relationship

The current is given by

$$i = \frac{dQ}{dt}$$

but $Q = Cv$

So, the expression for current, becomes

$$i = \frac{d}{dt}(Cv) = C\frac{dv}{dt}$$

or $$dv = \frac{1}{C} idt$$

or $$\int dv = \frac{1}{C}\int_0^t idt$$

or $$v = \frac{1}{C}\int_0^t idt$$

The I- V relationship is linear, hence a ***capacitor*** is a ***linear device***.

4.8 Energy stored in a capacitor

While charging a capacitor, p.d. across the capacitor will be developed. By definition, it is equal to the work done in carrying one coulomb of charge from plate to another.

If dq is the charge transferred, the amount of work done is:

$$dw = v\,dq$$

But $q = Cv \Rightarrow dq = Cdv.$

$\therefore dw = C.v.dv$

Total work done in giving a p.d of 'V' is

$$W = \int dw = C\int v \cdot dv$$

or $$W = C\int_0^V v \cdot dv = \frac{CV^2}{2}$$

$$\therefore W = \frac{1}{2}CV^2 Joules.$$

or $$C = \frac{Q}{V} \quad \therefore \mathrm{W} = \frac{1}{2}\frac{Q}{V}\cdot V^2 = \frac{QV}{2}$$

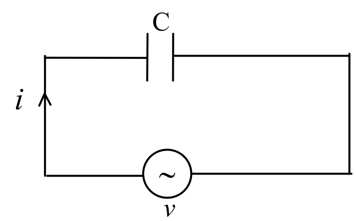


Fig 4.7

or, $$E = W = \frac{1}{2}C.\frac{Q^2}{C^2} = \frac{Q^2}{2C}\left(\because V = \frac{Q}{C}\right)$$

Energy stored in a capacitor $W = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$.

Q of a capacitor $$Q = \frac{\left(\frac{1}{\omega C}\right)I^2}{RI^2} = \frac{1}{\omega CR}$$

4.9 Types of capacitors

The capacitors may be divided into two general classes: (a) Fixed capacitors (b) Variable capacitors. Fixed capacitors may be further divided into: (a) Electrolytic capacitors (b) Non-Electrolytic capacitors.

(a) Fixed capacitors

The non-electrolytic type capacitors may be paper, mica, ceramic and polyester.

(i) Paper capacitors

This consists two tin foil sheets, which are separated by thin tissue paper. The sandwich of foil and paper is then rolled into a cylindrical shape and enclosed in a paper tube or enclosed in a plastic capsule. The lead at each end of the capacitor is internally attached to the metal foil. These capacitors have a range of 0.001μF to 2.0μF and working voltage range as high as 2000V.

(ii) Mica capacitors

This capacitor is a sandwich of several thin metal plates separated by thin sheets of mica. Alternate plates are connected together and leads attached for outside connections. The total assembly is encased in a plastic capsule or bakelite case. These capacitances have small values from 50 to 500 pF and working voltage range as high as 500V. These capacitors are used in radio circuits.

(iii) Ceramic capacitors

These capacitors have disc or hollow tubular-shaped dielectric made of ceramic material such as titanium dioxide and barium titanate. Thin coating of silver compound is deposited on both sides of the dielectric disc, which act as capacitor plates. Leads are attached to each end of the disc and the whole unit is encapsulated in a moisture proof coating. These capacitors have very large capacitances up to 0.01μF

Electrolytic capacitors

These capacitors are called Electrolytic because they use an electrolyte (borax or carbon salt) as negative plate. This capacitor consists of

(i) A positive plate of aluminium

(ii) An extremely thin insulating film of aluminium oxide (Al_2O_3) as dielectric medium

(iii) An electrolyte of borax

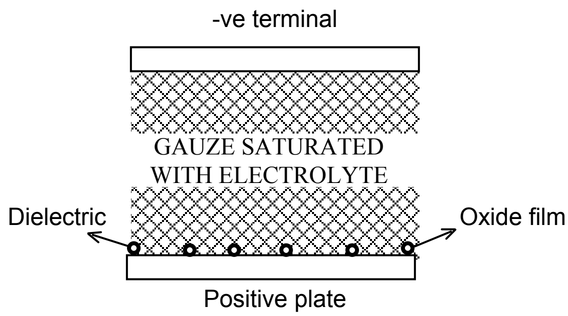


Fig. 4.8

As shown in Fig.4.8, an absorbent gauze saturated with the electrolyte is kept in contact with the dielectric. The second aluminium plate serves as a contact to the electrolyte. It forms the -ve terminal. A thin strip of aluminium is coated with a molecular thin film of Al_2O_3. It is covered with a layer of gauze soaked in an electrolytic of borax. The entire sandwich is rolled up into a compact cylinder and placed inside a metal cylinder. This enclosing cylinder contact the outside metal foil of the capacitor and serves as the -ve terminal. The outside metal cylinder is usually enclosed in a paper tube in order to insulate it from components. These capacitors give capacitance values but have large leakage currents also and have low quality factor.

Range: 1μF ---- 10,000μF

These are used in filter circuits.

Tantalum capacitors

This consists of two electrodes placed in conducting liquid. When conduction takes place, one of the electrodes acts as a cathode. The electrodes are made of tantalum while the conducting liquid is a solution of sodium phosphate or ammonium borate. A thin insulating film is made to form on the positive electrode. Acid electrolytic acts as negative electrode.

Properties

(i) They have longer life than other electrolytic condensers
(ii) These are highly reliable for a good rated value
(iii) They have low D.C. leakage value

Uses

1. These are used in circuit where high precision value is required
2. These are used in TV, Radio, Telephone and electronic equipment
3. Range of C = 100nF→ 10μF

4.9 Variable capacitors

A variable capacitor is one whose capacitance can be varied by rotating a shaft. This capacitor consists of two sets of metal plates ganged together separated from each other by air. One set of plates is stationary and is called the ***Stator***. It is insulated from the frame of the capacitor upon which it is mounted. The other set of plates, connected to the shaft and can be rotated, is called ***Rotor***. By rotating the rotor with the help of a suitable knob, rotor plates can be made to move in or out of the stator plates. Capacitance is maximum when rotor plates are fully '**in**' and minimum when '**out**'.

If 'n' is the total number of plates and 'd' is the separation between any two adjacent plates, then capacitance for air dielectric is

$$C = \frac{(n-1)\in_0 A}{d}$$

When two or more such capacitors are operated by a single shaft, it is known as a "***Dual Gang Capacitor***".

Use: Radio receivers have a variable capacitor with two or three gangs.

Trimmers

A small variable capacitor, which is often used in parallel with the main variable capacitor, is known as ***Trimmer*** or ***Padder***. It is primarily used for making fine adjustments of the total capacitance of the device.

Construction

A trimmer consists of two small flexible metal plates separated by air or mica or ceramic slab as the dielectric. The spacing between the plates can be changed by means of screw arrangement. As the screw is turned in ward, plates are compressed and its capacitance is increased.

Range: 5pF --- 30pF Padders: 10pF --- 500pF

Use: As tuning capacitors in radio receivers.

4.10 Colour coding of capacitors

As in the case of resistors, capacitors are also frequently colour coded to indicate various capacitor characteristics. The points usually covered by colour coding of capacitors are: capacitance, capacitance tolerance and temperature co-efficient. Some capacitors use colour coding to indicate the dc working voltage.

4.10(a) EIA colour code of Molded Mica capacitors: (Pico Farads)

[formerly RETMA 6-dot colour code]

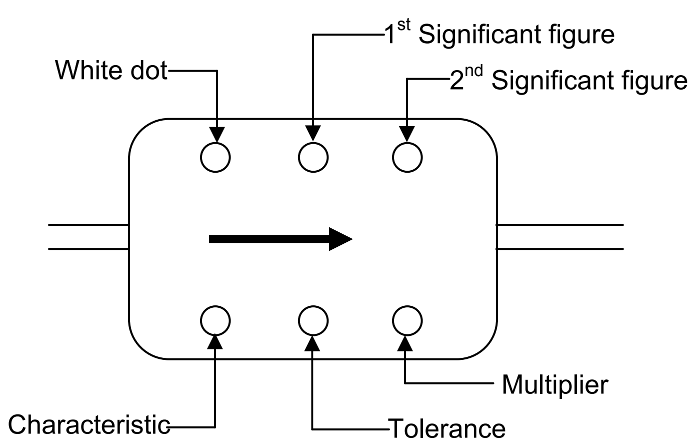


Fig 4.9

If first dot is not white, and then the system is known as RMA system. Here Dot1 gives the first significant number, Dot2 gives the second, Dot2 gives the third, Dot3 gives the rated dc voltage, Dot5 gives the tolerance, and Dot6 gives the power of decimal multiplier.

Illustration

Dots 1 to 6 are of colours yellow, violet, black, green silver and orange. Then the capacitance value in pF is 470 x 10^3 pF, 500V and 10% tolerance.

Table 4.1

Colour	Significant Figure	Decimal Multiplier	Tolerance ± %	Classifi-cation	Temp Coefft. PPM /^{0}C Not more than
Black	0	1	20	A	± 1000
Brown	1	10	-	B	± 500
Red	2	100	2	C	± 200
Orange	3	1000	3	D	± 100
Yellow	4	10000	-	E	+ 100 – 20
Green	5	-	5	-	-
Blue	6	-	-	-	-
Violet	7	-	-	-	-
Gray	8	-	-	I	+ 150 – 50
White	9	-	-	J	+ 100 – 50
Gold	-	0.1	-	-	-
Silver	-	0.01	10	-	-

4.10(b) Colour coding of Molded Tabular Capacitors: (Pico Farads)

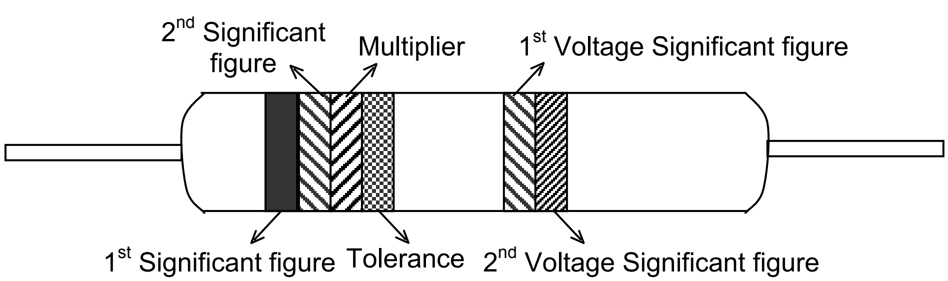


Fig 4.10

Table 4.2

Colour	**Significant Figure**	**Decimal Multiplier**	**Tolerance $\pm$ %**
Black	0	1	20
Brown	1	10	-
Red	2	100	-
Orange	3	1000	30
Yellow	4	10000	40
Green	5	10^5	5
Blue	6	10^6	-
Violet	7	-	-
Gray	8	-	-
White	9	-	10

4.11 Summary

A capacitor is an assembly of any two conductors separated by an insulating material (dielectric). Capacitance is the property of a capacitor, whereby energy may be stored in the form of an electric field between two conductors separated by a dielectric. Capacitance in a circuit opposes any change in voltage. The action of storing electricity in a capacitor is called ***Charging***. In a capacitive circuit, current leads the applied voltage by 90^0. The capacitance of a capacitor varies directly with the plate surface area and inversely with the distance between the plate surfaces. Fixed capacitors are distinguished, according to the dielectric material used, as paper, mica, ceramic, electrolytic. Electrolytic capacitors show polarity and are used principally in high-power, low-frequency filter circuits up to 600V. Capacitors connected in parallel add like resistances in series; Capacitors connected in series divide according to the parallel-resistance formula. The opposition of a capacitor presents to A.C. is called Capacitive Reactance (X_c); The formula is $X_C = \dfrac{1}{2\pi fC}$.

4.12 Key Terminology

Capacitance – Capacitor – Capacitive reactance – Phase relation.

Solved Problems

Example 1

What is the capacitance of a capacitor if a charging current of 10mA flows when the applied voltage changes 40V at a frequency of 50Hz?

Solution:

$$i = C\frac{dV}{dt}$$

Given : $i = 10mA$

$dV = 40V$

$dt = \frac{1}{50} = 0.02\sec$

Hence C = $\frac{i}{dV/dt}$

$= \frac{10\times10^{-3}}{40/0.02}$

$= \frac{10\times10^{-3}\times0.02}{40}$

$= 5\times10^{-6}F = 5\mu F.$

Example 2

A $50\mu F$ capacitor is connected to a 230V, 50Hz A.C. supply. Calculate the (i) Reactance offered by the capacitor (ii) The maximum current drawn by the capacitor.

Solution:

(i) Reactance $(X_C) = \frac{1}{2\pi fC} = \frac{7}{2\times22\times50\times50\times10^{-6}}$

$X_c = 63.69\Omega$

(ii) Maximum Current $I = \frac{V}{X_C} = \frac{230}{63.69} = 3.61A$

Example 3

The capacitance of a parallel plate capacitor is 400pF and its plates are separated by 2mm of air. What will be the energy when it is charged to 150V?

Solution:

Energy $= \frac{1}{2} CV^2$

Given $C = 400 \times 10^{-12} F;$

Voltage = 150V

Distance d = 2mm = 0.002 m

Energy Stored $= \frac{1}{2} \times 400 \times 10^{-12} \times (150)^2$

$= 200 \times 10^{-12} \times 225 \times 10^2$

$= 450 \times 10^{-8} = 4.5 \times 10^{-6} \; Joules$

Example 4

For a capacitor, with two parallel plates of area 0.1 m^2, separation 3 mm. Calculate the capacitance, electric field strength, charge on each plate if 500V is applied across the plates.

Solution:

Capacitance C = $\frac{\epsilon_0 A}{d}$; Given A = 0.1 m^2, d = 3 mm = 3 x 10^{-3} m

ϵ_0 = 8.85 x 10^{-12} F/m (Permittivity of vacuum)

Hence C $= \frac{8.85 \times 10^{-12} \times 0.1}{3 \times 10^{-3}}$ = 295 x 10^{-12} = 295 pF

Electric field Strength E = $\frac{V}{d}$ = $\frac{500}{3 \times 10^{-3}} \cong 167 \times 10^3 \; V/m$

Change on each plateQ = CV

= (295 x 10^{-12})(500)

= 0.1475 x 10^{-6} C

Q = 0.1475 μC

4.13 Self-Assessment questions

A. Long Answer Questions

1) Derive the V – I relationship in an ideal capacitor. Explain the phase difference between voltage across and current through an ideal capacitor.
2) Explain different types of capacitors
3) Describe in detail the construction of electrolytic, tantalum and ganged capacitors. Mention their applications in electronic circuits.

B. Short Answer Questions

1. What is capacitive reactance? Explain how it varies with frequency?
2. Discuss the construction and properties of electrolytic capacitors.
3. Explain capacitor colour code.
4. Describe an expression for the energy stored in a capacitor.
5. Obtain an expression for the capacitance of a parallel plate capacitor
6. Obtain expressions for equivalent capacitances when capacitors are connected in series and in parallel.

C. Numerical problems

1. The capacity of a parallel plate capacitor is 400ρF and its plates are separated by 1 mm of air. What will be the energy when charged to 300 V.
   [Ans: 18 x 10^{-6} Joules]
2. Find the total capacitance of circuit, shown in Fig.4.11. Determine the charge on each plate. Find voltage across each capacitor.

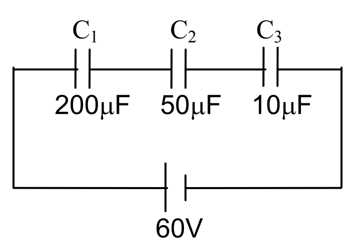


Fig 4.11

***[Ans**: C_{eq} = 8μF; $Q_{eq} = Q_1 = Q_2 = Q_3$ = 480μC; V_1 = 2.4 V, V_2 = 9.6 V, V_3 = 48 V**]**

4.14 Reference books

1) Introductory Circuit Analysis

----- Boylestad

2) Electrical Technology (Vol I)

----- B.L. Theraja & A.K.Theraja

3) Basic Radio (Vol 2)

----- Marvin Tepper

UNIT II LESSON 5

SIMPLE CIRCUITS

Objectives of the lesson

This lesson explains you the concepts of Network definitions, ideal voltage source, ideal current source, nodes and meshes in a network, division of voltage across various circuit elements, and division of current in various branches.

Structure of the lesson

In Unit I, we learnt about the concepts of voltage, current, resistance, inductance, capacitance, and reactance offered by the inductance and capacitance as a function of signal frequency. Phase shift occurs in current when it passes through an inductor or capacitor. The voltage developed across the components also suffers phase shift. With this background, we learn what a circuit or network is?, and various concepts related to the networks.

5.1 Circuit, Source and Load

A circuit is closed path. When we connect a bulb to a battery, current flows from the battery to the bulb and returns back to the battery. This is an example for closed path or circuit. Part of the electrical energy of the battery is given to the bulb and bulb converts it to light energy. We may call the circuitous path taken by the current as a ***simple circuit***. The battery is called a ***source*** and the bulb is called a ***load***. When the bulb is of lesser voltage rating, to match the requirements of the bulb, we may connect a resistor in series with the bulb. This will drop the excess voltage and allow the required amount of current through the bulb at the required voltage. We may say that we matched the load and source by introducing a resistor. The bulb, the resistor, the battery are called ***circuit elements***. Infact condensers, inductors, transformers and any other electronic or electrical devices connected in the circuit are called circuit elements. In real life, a source may be any electrical signal source and a load may be any device that can be equivalently represented by an inductance, resistance, capacitance or a combination of these. The matching load may have to be a combination of circuit components connected in a complex way. This arrangement is also called a circuit in general, even though it may contain several circuitous current paths. These various closed paths may not be in a row or a column but the circuit drawn may look like a set of grids in a network or a mesh. We adopt the word network to represent the interconnection of the various circuit components.

5.2 Voltage drop across components

When a current I passes through a resistor, voltage developed across it is given by V=RI. Likewise, the voltage developed across a capacitor is given by jX_CI and voltage developed across an inductor is given by jX_LI. The symbol j indicates a complex number and is a symbol to indicate that a phase shift is associated. From the expressions of inductive and capacitive reactances, we observe that their values are frequency dependant. An interconnection of these components will modify the frequency, amplitude and phase characteristics of the output signal. The modified signal is given to the load. One more point to be noted is that usually the signal source is a transducer, which converts a physical parameter like sound energy or heat energy into electrical energy.

Transducer:

A transducer is a device that converts energy of one form into another. Examples are microphone, loudspeaker, Platinum resistance thermometer, photocell etc. The load can also be a transducer which accepts the signal in electrical form and converts the electrical energy into other form of energy like sound, light, mechanical, magnetic etc. In a class laboratory, a function generator which allows us to vary input voltage, input frequency, shape of the signal is used for a systematic study of the given circuit.

5.3 Branch, lumped and distributed networks

Branch: The components in a circuit are joined with thin copper wires of suitable length. Two or more circuit components may be connected in series. It is usually called a ***branch*** in network terminology. Some times a branch may have single component also.

Junction: If more than two branches meet at a point, such a point is called a ***junction*** or ***node***. In addition to this junction, there will be another junction where the other ends of the various branches meet. It is called a ***ground node*** or ***reference node*** or ***datum node.*** A parallel branch network will have one ground node and in addition may have any number of junctions.

Branches connect two nodes. Of course, one or more branches may be connected between any two nodes. The current flowing through a branch is called ***branch current*** and potential difference across a branch is called ***branch voltage***. In ordinary circuits with lumped circuit elements, the various branches can be easily identified. Such networks are examples of branch networks.

Lumped networks: Any network in which the various circuit elements can be readily identified is called a ***discrete component network*** or ***lumped network***. The effect of the various components on the circuit performance can be studied easily by changing their values by substitution method.

Distributed networks: At high frequencies, twisted wires offer considerable inductive reactance. Potential difference between wires and electrodes give rise to capacitive reactance. A wire wound resistor of given value at high frequencies shows in addition to resistance, both capacitive and inductive reactances. These limit the network performance. A transmission line behaves like a resonant circuit at high frequencies. So, networks where the resistance, inductance and capacitance do not appear physically but distributed over the length of the line are called ***Distributed Networks***. Waveguides, resonance cavities, antennas are some other examples of distributed networks.

5.4 Active and passive networks

Usually batteries, d.c. sources, ac sources, signal generators etc. are called ***energy sources***. Circuit consisting one or more energy sources is called an ***active network***. Circuit that do not contain any energy source is called a ***passive network***. However, when energy source is connected to it becomes an active network. With this background, we see the features of interconnection between various circuit components i.e. of some simple circuits.

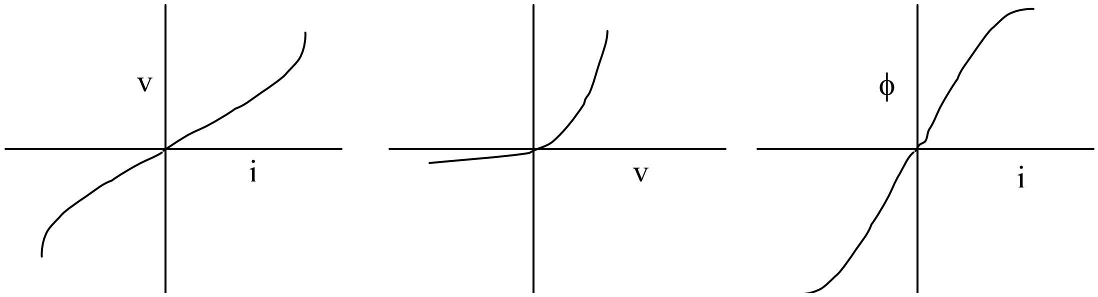
5.5 Linear and Non-linear networks

If a linear mathematical relationship exists between excitation (input) x_i given to a network and the response (output) x_o measured across it, we say that linear relationship exists between excitation and response.

$$x_o = K.x_i \quad \text{where K is a constant}$$

However, some networks modify the characteristics of the response such that there will be no linear relationship between excitation and response. Such networks, which do not modify the relationship between the excitation and response, are called ***linear networks***.

Even if the points of excitation and response are interchanged and if the same linear relation ship exits between them, such networks are called ***linear bilateral networks***. In practical networks, non-linearity exists to some extent. In some cases, non-linearity can be disregarded; linear approximation yields results that predict the behaviour of the real devices within acceptable limits. In other cases, non-linearity is annoying and special steps must be taken to avoid or eliminate its effect. Some times non-linearity is desirable or even essential; the distortion that is annoying in an amplifier is necessary in the harmonic generator for obtaining output signals that are multiples of the input signals. A dissipative element for which voltage is not proportional to current is a non-linear resistor. An ordinary incandescent lamp has a characteristic similar to that in Fig.5.5a. A semiconductor diode has a characteristic as shown in Fig.5.5b. Its non-linear characteristic is useful in discriminating positive and negative voltages. In an iron core inductor, the magnetic flux is not proportional to current (see Fig.5.5c). At large values of current, a given increment of current produces only a small increment of flux. This non-linearity is troublesome in a power transformer, but it may be useful in a control system.



a) Incandescent lamp b) Semiconductor diode c) Iron core inductor

Fig.5.5

5.6 Concept of Impedance and Admittance

A pure resistor has no ability to store energy. Its value is fixed and there is no change in resistance with increment or decrement in supplied energy. On the other hand, a capacitor and an inductor also have an ability to resist the flow of current and to store energy. The main difference between resistance of a resistor and resistance offered by capacitor and inductor is that, their resistive property changes with applied signal frequency.

Impedance: The combination of capacitive/inductive resistance (called reactance) with the resistance of resistor is called ***Impedance***. It is denoted by 'Z'. It is represented as **Z = R+jX** where the real part R is called ***Resistance*** and the imaginary part X is called the ***Reactance***. It may be defined as the ratio of voltage to current in an a.c. circuit. The unit for impedance is Ohm. There are different ways to calculate the impedance of the network. The impedance depends on type of elements, such as resistor, capacitor and inductor, and type of connection i.e. series or parallel.

Admittance: The reciprocal of Impedance is called ***Admittance***. It is represented by 'Y'. It may be defined as the ratio of current to voltage. The admittance is analogues to conduction. Unit for admittance is mho or Siemens. Mathematically

$$\mathrm{Y} = \frac{1}{\mathrm{Z}}$$

$$Y = \frac{1}{(R + jX)} = \frac{(R - jX)}{(R^2 + X^2)} = \frac{R}{(R^2 + X^2)} - j\frac{X}{(R^2 + X^2)}$$

Like, impedance, admittance is a complex quantity. It is represented as **Y = G+jB**, where the real part G is called ***Conductance*** and imaginary part B is called ***Susceptance***.
Here

$$G = \frac{R}{\left(R^2 + X^2\right)}$$

$$B = \frac{X}{\left(R^2 + X^2\right)}$$

Different cases are discussed here.

Case I: Consider the circuit given in Fig.5.6a, in which a resistor and a capacitor are connected in series, then the impedance of the circuit is

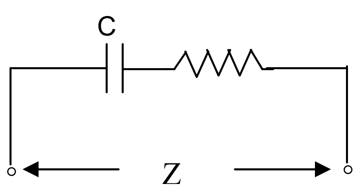


Fig.5.6a

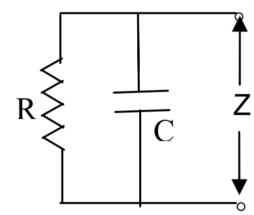


Fig.5.6b

$$Z = R + \frac{1}{j\omega C}$$

$$Y = \frac{1}{R} + \frac{1}{X_C} = \frac{1}{R} + j\omega C$$

***Case* II**: Consider the circuit given in Fig.5.6.b, in which a resistor and a capacitor are connected in parallel, then the impedance of the circuit is

$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{X_C} = \frac{1}{R} + \frac{1}{1/j\omega C} \qquad \text{----------} \quad (5.1)$$

Case III: Consider the circuit given in Fig.5.6c, in which a resistor and an inductor are connected in series, then the impedance of the circuit is

$$Z = R + X_L = (R + J\omega L)\Omega$$

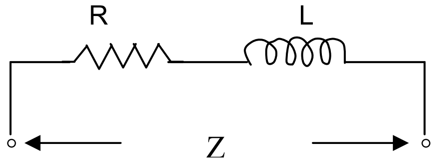


Fig.5.6c

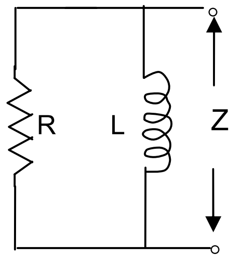


Fig.5.6d

***Case* IV**: Consider the circuit given in Fig.5.6d, in which an inductor and a resistor are connected in parallel, then the impedance of the circuit is

$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{X_L} = \frac{1}{R} + \frac{1}{j\omega L}$$

$$\therefore Z = \frac{Rj\omega L}{R + j\omega L}\Omega$$

***Case* V**: Consider the circuit given in Fig.5.6e, in which an inductor and a capacitor are connected in series, then the impedance of the circuit is

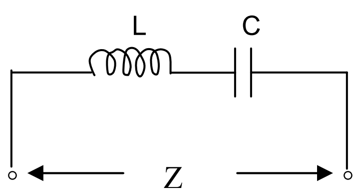


Fig.5.6e

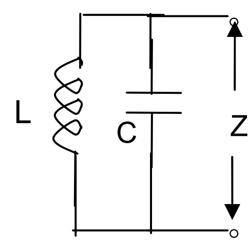


Fig.5.6f

$$Z = X_L + X_C$$

$$Z = j\omega L + \frac{1}{j\omega C}$$

***Case* VI**: Consider the circuit given in Fig.5.6f, in which an inductor and a capacitor are connected in parallel, then the impedance of the circuit is

$$\frac{1}{Z} = \frac{1}{X_L} + \frac{1}{X_C}$$

$$\frac{1}{Z} = \frac{1}{j\omega\ L} + j\omega\ C$$

***Case* VII**: Consider the circuit given in Fig.5.6g, in which a resistor, an inductor and a capacitor are connected in series, then the impedance of the circuit is

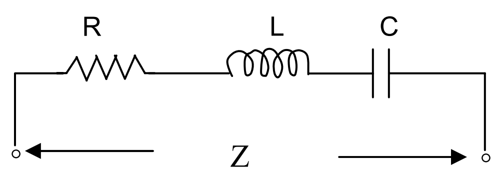


Fig.5.6g

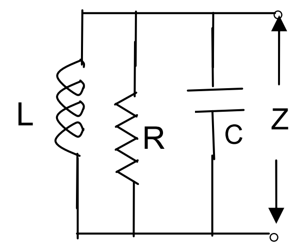


Fig.5.6h

$$Z = R + X_L + X_C = R + j\omega L + \frac{1}{j\omega C}$$

From vector analysis, the magnitude of the above relation will be

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

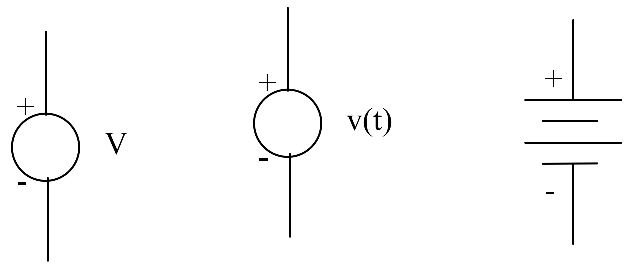
***Case* VIII**: Consider the circuit given in Fig.5.6h, in which a resistor, an inductor and a capacitor are connected in parallel, then the impedance of the circuit is

$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{X_L} + \frac{1}{X_C} = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

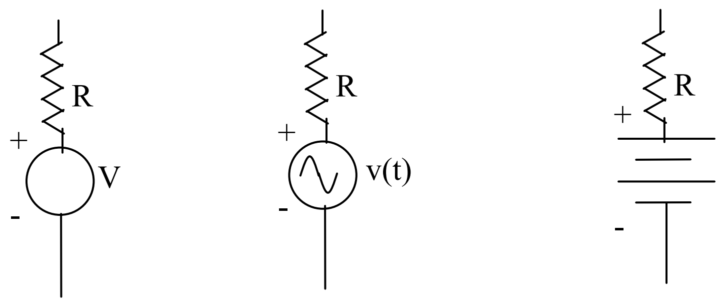
5.7 Concept of Ideal voltage and current sources

Ideal voltage source

An ideal voltage source generates voltage of a given time variation but neither the magnitude nor the time variation of the generated voltage changes with the magnitude of the current drawn from it. Thus the terminal voltage of this source remains constant for all values of output current from zero current condition to blunt short circuit. Evidently, such a performance can be achieved by an energy source only when it has zero internal resistance, inductance and



a. Ideal d.c. Voltage source   b. Ideal a.c. Voltage source   c. Ideal battery.



d.c. voltage source   a.c. voltage source   Battery.

Fig 5.7 Voltage sources

and capacitance. The symbol for ideal voltage source is generally a circle with polarity marks + and - denoting the positive and negative terminals of the source as shown in Fig. 5.7a and 5.7b. The lower case letter *v* indicates a time varying source while the upper case letter *V* indicates a time invariant source. An approximate time variation of voltage is some times sketched within the circle. Thus the ~ symbol having placed within the circle indicates a source of sinusoidal voltage. However, a battery is generally symbolized as in Fig.5.7c.

Almost all practical voltage sources fall short of the ideal and their terminal voltage falls with increase in the output current. However, most of these practical voltage sources may be approximated as an ideal voltage source with a series resistance in case of dc sources or with a series impedance in the case of AC sources.

This series resistor then accounts for the fall in terminal voltage with increase of output current. Symbol for a practical voltage source is accordingly the same as for an ideal voltage source with a series resistor R (or Impedance Z).

Ideal Current Source

An ideal current source generates current of a given time variation but neither the magnitude nor the time variation of the generated current changes with the load. Thus the output current of this source remains constant for all values of load ranging from zero resistance to infinite resistance. If this ideal current source gives zero output current, the source reduces to just an open circuit. The requirement of constant current in an ideal current source can be satisfied provided it has zero internal resistance, inductance and capacitance. The symbol for ideal current source is generally a circle associated with an arrow indicating the positive current flow as shown in Fig.5.7d and 5.7e. In this case also, the lower case latter i or i(t) indicates the time invariant source . Again approximate time variation of current may be sketched within the circle. Some times a rectangle instated of a circle is used to symbolize current source, in order to clearly differentiate it from voltage source.

Again all practical current sources fall short of the ideal and their output current falls with the increase of load resistance (or impedance). Accordingly, a practical current generator may be approximated as an ideal current generator with a shunt resistor in case of d.c. Current source or shunt impedance in case of a.c. current source. This shunt resistor then accounts for the fall of output load current with increase of load resistance. Symbol for a practical current source is the same as that for the corresponding ideal current source with a shunt resistance (impedance) as shown in Fig.5.7e.

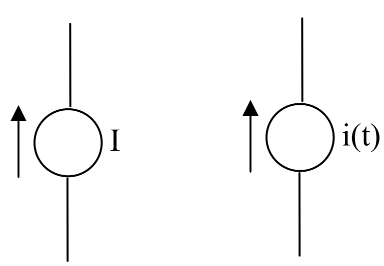


Fig.5.7d

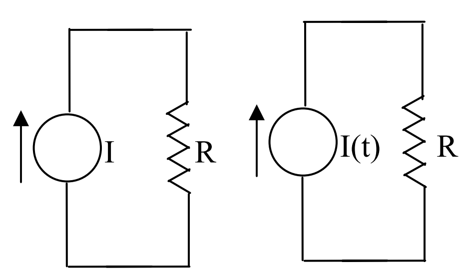


Fig.5.7e

Photo-electric cell and pentode vacuum tube amplifiers form practical current generators of common occurrence in electronic circuits.

Although ideal current and voltage sources form two altogether different types of sources, practical sources cannot be so distinctly classified. In fact, any source with suitable associated regulator circuit may be made to behave as constant voltage generator or constant current generator, closely approaching the ideal condition in either of the cases. The distinction made here is primarily for facility in network analysis and any particular energy source is treated either as a voltage source or a current source depending upon which type facilitates analysis of the network under consideration. Thus a practical voltage source may be transformed into its equivalent practical current source or *vice versa.*

Source conversion: A given voltage source with a series resistance can be converted into or replaced by an equivalent current source with a parallel resistance and vice versa.

a. Conversion of voltage source into current source

Fig.5.7f shows a constant voltage source having voltage V and internal resistance Ri. In order to convert it to constant current source; we have to follow the following procedure.

Remove the load R_L across the points A and B. Measure the voltage V across them and short-circuit them with a current meter (ideal current meters have zero internal resistance). Let the current be I. The definite value of I indicates that the voltage source has internal resistance and the short-circuit current I is evidently equal to V/R_i. This is considered as the current supplied by the equivalent current source with internal resistance R_i in parallel with it. The circuit with equivalent current source is shown in Fig 5.7g.

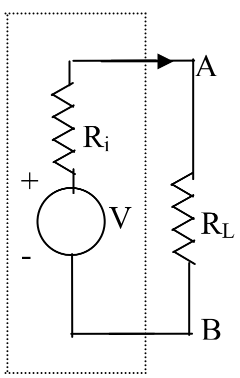


Fig 5.7f: Circuit with voltage source

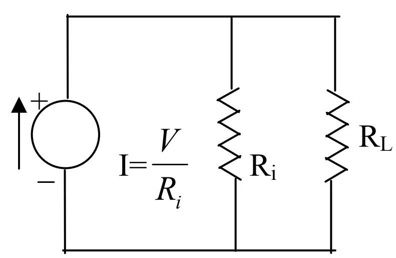


Fig 5.7g: Circuit with equivalent current source

b. **Conversion of current source into voltage source**: Fig 5.7h shows a current source whose internal resistance is Ri. To convert it into equivalent voltage source multiply the current value of the current source with its internal resistance value. It is equal to the equivalent voltage of the voltage source. Place a resistance of value Ri in series with this voltage source as shown in Fig 5.7i.. Positive terminal of the voltage source is indicated with a + sign in the direction of current flow.

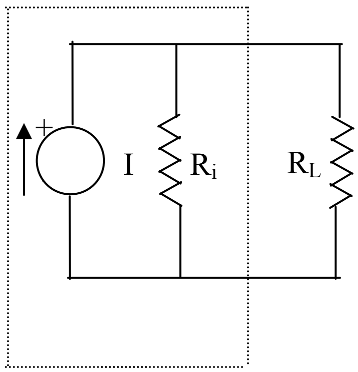


Fig 5.7h: Circuit with voltage source

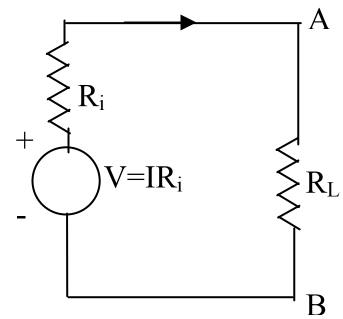


Fig 5.7i: Circuit with equivalent current source

5.8 Division of voltage across circuit components

[Voltage Divider Rule]

A voltage divider circuit is a series network, which is used to feed other networks with a number of different voltages all derived from a single input voltage source. Current flowing in a series circuit like the one shown in Fig.5.8 is obtained by dividing the source voltage with resistances (impedances) R_1 and R_2 in the circuit.

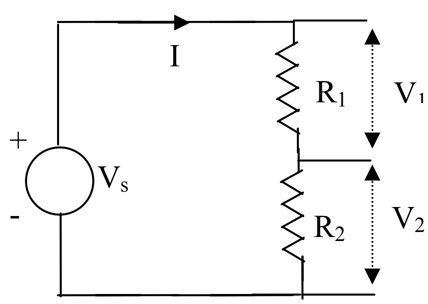


Fig 5.8

$$I = \frac{V_s}{(R_1 + R_2)}$$

The voltage drop across R_1 is

$$V_1 = R_1 \times I = R_1\left(\frac{V_s}{R_1 + R_2}\right)$$

Likewise voltage drop across R_2 is

$$V_2 = R_2 \times I = R_2\left(\frac{V_s}{R_1 + R_2}\right)$$

It can be seen that $V_s = V_1 + V_2$. We see that the voltage drops across various resistors is proportional to their resistances and also sum of individual voltage drops is equal to the source voltage.

5.9 Division of current in various branches

If two resistors are connected in parallel and if they are connected across a voltage source, same voltage drop occurs across them. However, current flowing through individual resistors differ.

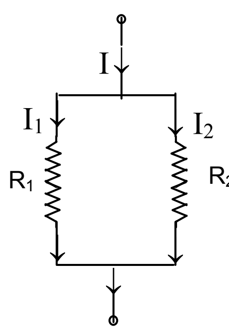


Fig 5.9a

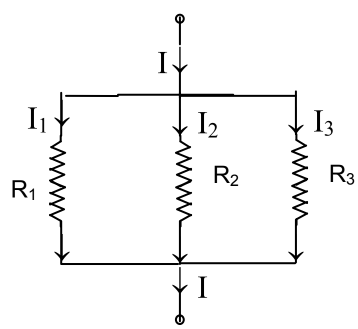


Fig 5.9b

Current flowing through a resistor in a parallelly connected resistor network as shown in Fig.5.9 is equal to the total current multiplied by the ratio of sum of resistors in the other branches to the sum of resistances in all branches. In a two-branch network shown in Fig.5.9a, current I_1 through resistor R_1 is

$$I_1 = I \times \left(\frac{R_2}{R_1 + R_2}\right)$$

and current I_2 through R_2 is

$$I_2 = I \times \left(\frac{R_1}{R_1 + R_2} \right)$$

and note that $I = I_1 + I_2$

In a three-branch network

$$I_1 = I \times \left(\frac{R_2 + R_3}{R_1 + R_2 + R_3} \right)$$

$$I_2 = I \times \left(\frac{R_1 + R_3}{R_1 + R_2 + R_3} \right)$$

$$I_3 = I \times \left(\frac{R_{11} + R_2}{R_1 + R_2 + R_3} \right)$$

We can extend this to a branch network having ‘n’ branches, where n is an integer. Remember that if there are two are more components in series in a branch, same current flows through them.

5.10 Meshes and Nodes

As mentioned in earlier sections, a network may possess various circuitous paths for current. For the convenience of analysis, each circuitous path is called a **mesh**. Current passing in a mesh is called **mesh current**. Certain components may be common to two meshes. In that case, the current passing through that current is considered as the algebraic sum of the two mesh currents. If a mesh doesn’t contain a voltage or current source, the algebraic sum of the voltage drops around the mesh will be zero. To compute the sign of voltage drop in a component, a convention is adopted for the direction of current flowing in a mesh. Some people may prefer anticlockwise direction for all the currents. Some may prefer clockwise direction. Some may prefer to have anticlockwise direction for some and clockwise direction for the other. Whatever convention is adopted, it must be followed throughout the treatment of the problem. It is good practice to assume positive currents leave the generator’s positive terminal and enters through negative terminal. Whenever a component is shared by two or more meshes, the individual mesh currents flow through that common component and the resultant current is the algebraic sum of individual mesh currents.

5.11 Summary

A circuit is closed path. Condensers, inductors, transformers and any other electronic or electrical devices connected in the circuit are called circuit elements. A source may be any electrical signal source and a load may be any device that can be equivalently represented by a combination of inductance, resistance, capacitance or a combination of all of these. In a circuit, usually a combination of circuit components connected in a complex way. This arrangement is also called a circuit or network, in general. These various closed paths may not be in a row or a column but the circuit drawn may look like a set of grids in a network or a mesh. However, each closed path in a network is termed as a mesh. When a current I, passes through a resistor, voltage developed across it is given by V = RI. Likewise, the voltage developed across a capacitance is given by $jX_C I$ and voltage developed across an inductance is given by $jX_L I$. Usually the signal source is a transducer, which converts a physical parameter like sound energy or heat energy into electrical energy.

Branch: Two or more circuit components may be connected in series or parallel. It is usually called a branch in network terminology. Some times, a branch may have single component also.

Node: If more than two branches meet at a point, such a point is called a junction or node.

Lumped network: A circuit formed with discrete components like condensers, resistors, inductors etc. is called a discrete component circuit or lumped network.

Distributed network: At sufficiently high frequencies, you cannot physically visualize some capacitances and inductances but they are very much present, distributed over the network. Such networks are called distributed networks.

Impedance: The combination of capacitive/ inductive resistance (called reactance) with the resistance of resistor is called impedance. It is denoted by Z. It may be defined as the ratio of voltage to current in an ac circuit. The unit for impedance is Ohm.

Admittance: The reciprocal of Inductance is called Admittance. It is represented by ‘Y’. It may be defined as the ratio of current to voltage.

Voltage source: practical voltage sources may be approximated as an ideal voltage source with a series resistance in case of dc sources or with a series impedance in the case of AC sources. Ideal voltage source has zero series resistance (internal).

Current source: a practical current generator may be approximated as an ideal current generator with a shunt resistor in case of d.c. current source or a shunt impedance in case of a.c. current source.

5.12 Key terminology

Network – branch – circuit – linear network – non-linear network – Bilateral network – active network – passive network – inductance – capacitance – impedance – admittance – conductance – susceptance – reactance – phase – ideal voltage source – ideal current source – node – junction reference – ground – loop – mesh-Seimens.

5.13 Self assessment questions

1. Explain the nature of inductive and capacitive reactances.
2. Derive an ex precision for the effective impedance when an inductor and a capacitor connected (i) in series (ii) in parallel.
3. What is the effective resistance of the circuit shown in Fig. 5.10

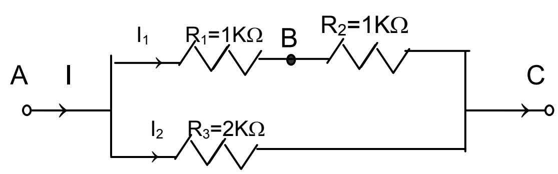


Fig.5.10

4. What is the current flowing through the resistance R_3 in Fig.5.10?
5. Calculate the voltage drop across points B and C shown in Fig.5.10.
6. Calculate the current flowing through various resistors in the circuit shown in Fig.5.10.
7. Define the terms: Ideal voltage source, Ideal current source, a node, a branch and admittance.

8.In the circuit shown in Fig 5.11, the current flowing through the 5Ω resistor is i(t)= 6 sin ωt .
Determine the current in the 15Ω and 10Ω resistors. Also find the voltages a to b and b to c. Compute the instantaneous and average powers consumed in each resistor.

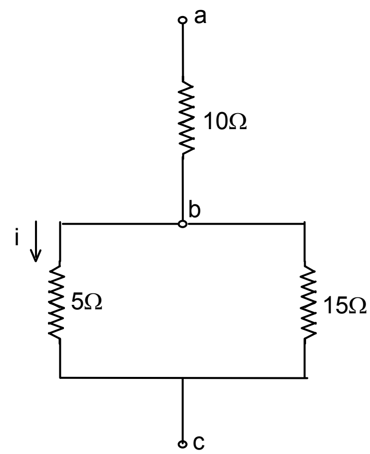


Fig 5.11

9. A voltage v(t) is applied across two inductances L_1 and L_2 in series as shown in Fig 5.12. Determine the equivalent inductance Le which can replace them to yield the same current.

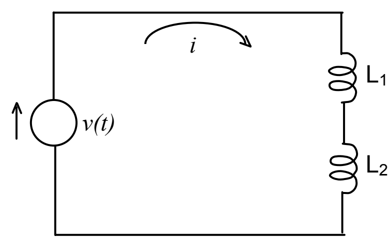


Fig 5.12

10. Find the equivant inductance L_e for the combination of inductances shown in Fig 5.13.

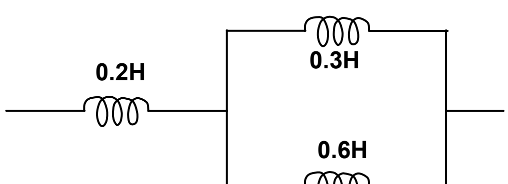


Fig 5.13

11. Find the equivalent capacitance C_e for the combination of capacitances shown in Fig 5.14

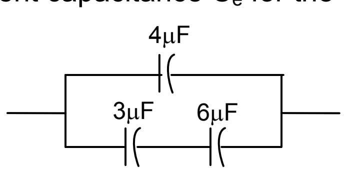


Fig 5.14

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UNIT II **LESSON 6**

NETWORK ANALYSIS - VARIOUS METHODS

Objectives

This lesson explains you the concepts of Kirchhoff's current and voltage laws through application to simple networks consisting of resistors, D.C. and A.C sources. Also explains how to obtain solutions to problems associated with single source and two source networks by different methods – Determinant method, Substitution method, loop current method and node voltage method.

Structure of the lesson

6.1 Kirchhoff's laws

- 6.1.a Kirchhoff's current law
- 6.1.b Kirchhoff's voltage law
- 6.1.c Sign convention

6.2 Maxwell's loop current method

6.3 Loop current method for an n-loop network having sources in two loops

6.4 Nodal analysis

6.5 Node voltage method for an n-loop network having sources in two loops

6.6 Method of determinants

6.7 Substitution method

6.8 Key terminology

6.9 Summary

6.10 Self assessment questions

6.11 References

UNIT 2 LESSON 6

NETWORK ANALYSIS

Using Ohm's law to analyze electric networks having many branches is often very difficult, particularly when it contains more than one energy source and when components are connected in a complex way. So, other methods like application of Kirchhoff's laws, Mesh method of analysis, Nodal method of analysis etc. were developed. In this lesson, we will learn the basic concepts of these methods with suitable examples.

6.1 Kirchhoff 's Laws

Stated in 1847 by the German physicist Gustav R. Kirchhoff, the two basic rules for voltage and current are

1. The algebraic sum of the currents meeting at a point zero.
2. The algebraic sum of the voltage sources and IR voltage drops must total zero around any closed path.

6.1.a Kirchhoff 's First law (or) Point law (or) Current law (or) Junction law

Statement: It states that in any electrical network, the algebraic sum of currents meeting at a point (or) junction is zero. In another form, it simply means that total current leaving a junction is equal to the total current entering that junction.

According to Kirchhoff 's First law we can write

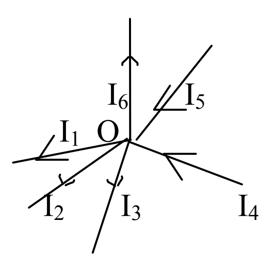


Fig 6.1.1

$I_1 + I_2 - I_5 + I_3 - I_4 + I_6 = 0$

i.e., $I_1 + I_2 + I_3 + I_6 = I_4 + I_5$

6.1.b Kirchhoff 's Second Law (or) Mesh Law (or) Voltage Law [K.V.L.] (or) Loop law

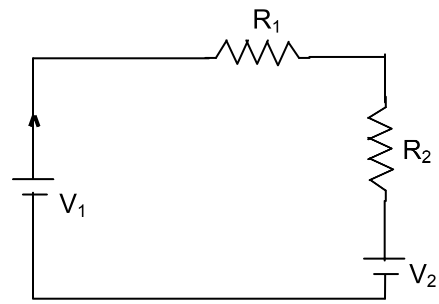


Fig 6.1.2

3. **Statement:** It states that the algebraic sum of the voltage sources and IR voltage drops must total zero around any closed path. Mathematically it is denoted as

$$\sum IR + \sum E.M.F = 0$$

Applying Kirchhoff's voltage law to the circuit in Fig.6.1.2, we can write

$$IR_1 + IR_2 + V_1 - V_2 = 0$$

Almost every circuit can be analyzed using these laws.

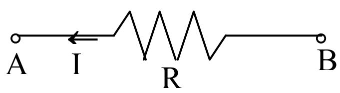
There are many applications for the KCL and KVL. Here two of the applications are discussed in detail. For KVL, the Maxwell loop current method and for KCL, Nodal analysis methods are discussed.

6.1.c Sign convention

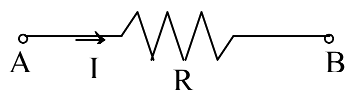
There are two sign conventions to be followed in applying KVL in any loop or mesh. They are (i) Sign of IR drop (ii) Sign of battery e.m.f.

(i) **Sign of IR drop:** If we travel in a direction same as that of current in a resistor, then the voltage drop should be given a ***Negative sign*** because there is fall in potential. On the other hand, if we travel in a direction opposite to that of current in that resistor, then the voltage drop should be given a ***Positive sign***

because there is a rise in potential. It is important to note that ***sign of IR drop*** is ***dependent on the direction of current*** through that branch (resistor).

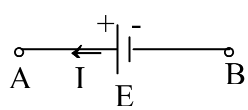


V_{AB}= +IR

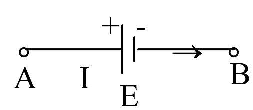


V_{AB}= -IR

(ii) **Sign of battery e.m.f:** If we travel from positive terminal of a battery to its negative terminal, then the emf of the battery should be given ***negative sign*** because there is a fall in potential. On the other hand, if we travel from negative terminal of a battery to its positive terminal, then the emf of the battery should be given ***positive sign*** because there is a rise in potential. It is important to note that ***sign of emf of battery*** is ***independent of direction of current*** through that branch in which the battery is connected.



V_{AB}= -E



V_{BA}= +E

Ex. 6.1.1: A battery of 5V e.m.f. and 0.5Ω internal resistance is joined in parallel with another battery of 15V e.m.f. and 1Ω internal resistance. The combination is used to send current through an external resistance of 12Ω as shown in Fig.6.1.3. Calculate, by application of Kirchhoff 's law, the current through each battery.

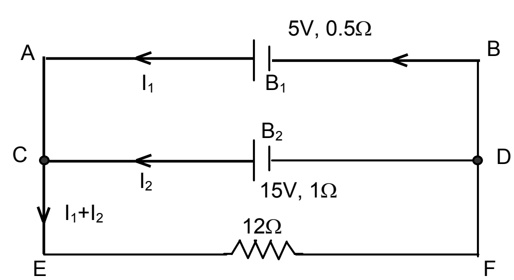


Fig 6.1.3.

Sol: Let the current through the batteries B_1 and B_2 be I_1 and I_2 .

Now by applying the Kirchhoff's current law to the point C, the current through the external resistance is (I_1+I_2).

Applying Kirchhoff's voltage law to the mesh ABFEA

$$I_1 \times 0.5 + (I_1+I_2) \times 12 = 5$$

or $$12.5I_1+12I_2 = 5 \qquad \text{---------(6.1.1)}$$

Similarly, applying Kirchhoff's voltage law to the mesh CEFDC,

$$I_2 x1+(I_1+I_2) \times 12 = 15$$

$$I_2+(I_1+I_2) x12 = 15$$

$$12I_1+13I_2 = 15 \qquad \text{---------(6.1.2)}$$

Now solving equations (6.1.1) and (6.1.2), we get

The current through each battery is

$$I_1 = - 6.216A$$

$$I_2 = 6.892A$$

6.2 Maxwell's Loop current method

This method is particularly well suited to circuits, which consists of many parallel branches. Basically this method consists of writing loop voltage equations by using Kirchhoff's voltage law in terms of unknown loop currents.

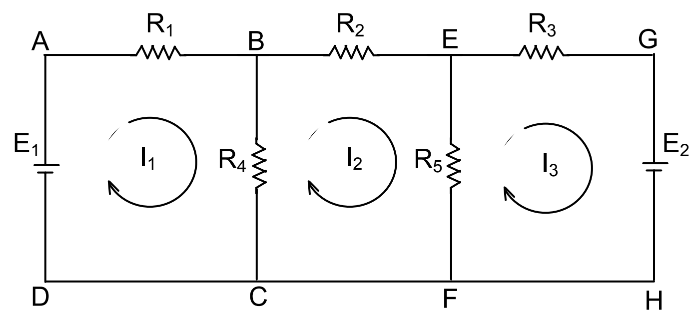


Fig 6.2.1.

In the above circuit, two batteries E_1 and E_2 are connected. The network consists of five resistors. The network can be considered as a three-mesh (loop) network. Let the loop currents through the three meshes be I_1, I_2 and I_3. It is obvious that current through R_4 is $(I_1 - I_2)$ and that through R_5 is $(I_2 - I_3)$. Similarly, when R_5 is considered as the part of third loop, current through it is $(I_3 - I_2)$.

Applying K.V.L. to each of the three loops, we get

For Loop (1), $V_1+I_1R_1+R_4[I_1-I_2] \times R_5=0$

For Loop (2), $I_2[R_2+R_4+R_5]+R_4[-I_2]+R_5[-I_3]=0$

For Loop (1), $I_3[R_3+R_5]+R_5[-I_2]-V_2=0$

The values of I_1, I_2 and I_3 are calculated by solving the three equations.

Ex 6.2.1: For the circuit shown in Fig.6.2.2, find the current flowing through the voltage source.

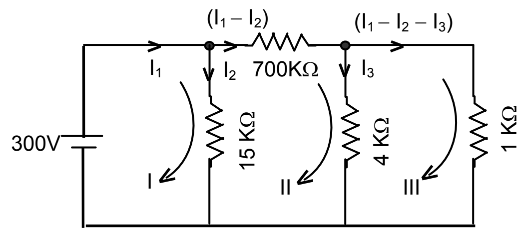


Fig 6.2.2.

Sol: Current drawn from supply is I_1. Current through 15KΩ resistor is I_2, Current through 4KΩ resistor is I_3. By applying KVL to the three meshes I, II and III, we get

$300 = 15 \times 10^3 I_2$ -------------------(6.2.1)

or $$I_2 = \frac{300}{15\times10^3} = 20\times10^{-3} A$$

$0= 700 (I_1 - I_2) + 4000 I_3 - 15000 I_2$

$0 = 700 I_1 - 15700 I_2 + 4000 I_3$ ---------------(6.2.2)

and $0 = (I_1 - I_2 - I_3) 1000 - 4000 I_3$

$0 = I_1 - I_2 - 5 I_3$

or $$I_3 = \frac{I_1 - I_2}{5}$$ ---------------(6.2.3)

Using equations (6.2.2) and (6.2.3), we can write

$$0 = 7I_1 - 157I_2 + 40\frac{(I_1 - I_2)}{5}$$

$0 = 15 I_1 - 165 I_2$

$$I_1 = \frac{165}{15} I_2 = 11\times 20\times10^{-3}(\text{using eq.}(6.2.1)) = 220\times10^{-3} A$$ = 220 mA.

6.2.2. For the circuit given in Fig.6.2.3, find the current through R_1 by the method of mesh currents.

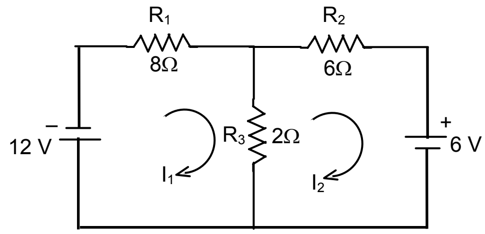


Fig 6.2.3.

Sol:- If the currents in the two meshes are I_1 and I_2 then applying KVL,

$-12V = I_1(8+2) - 2 \times I_2$ or $-12V = 10\,I_1 - 2\,I_2$ ---------------(6.2.4)

For second mesh

$-6 = (2+6)\,I_2 - 2 \times I_1$ or $-6 = 8\,I_2 - 2\,I_1$

$8\,I_2 = 2\,I_1 - 6$ ---------------(6.2.5)

Substituting this value, of I_2 in eq. (6.2.4) we get $I_1 = -1.421A$.

6.2.3. Determine the currents in the unbalanced bridge circuit shown in Fig.6.2.4. Also determine the potential difference across BD and the resistance from B to D.

Sol:- Consider the current directions as shown in the Fig.6.2.4

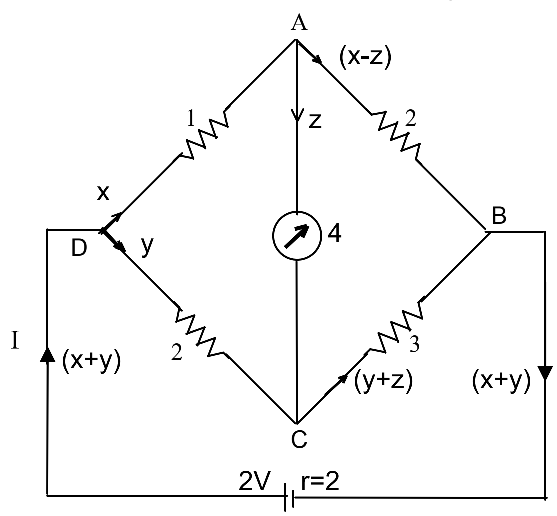


Fig 6.2.4.

Applying Kirchhoff 's voltage law to the mesh DACD, we get

$-x-4z+2y = 0$

or x-2y+4z=0 ----------------(6.2.6)

Applying Kirchhoff 's voltage law to the mesh ABCA, we get

2(x-z) + 3(y+z)+4z=0

2x - 3y - 9z = 0 -----------(6.2.7)

-x-2(x-z) -2 (x+y)+2 =0

5x+2y-2z = 2 --------------(6.2.8)

Multiplying (6.2.6) by 2 and subtracting eq.(6.2.7) from it, we get

-y+17z=0 ----------------(6.2.9)

Similarly, multiplying eq.(6.2.6) by 5 and subtracting eq.(6.2.8) from it, we get

-12y + 22z = -2

-6y + 11z= -1 -------------(6.2.10)

Eliminating y from eq.(6.2.9) and eq.(6.2.10), we get

91z= 1

z = 1/91 A

z= 0.0109 A

From eq.(6.2.9) y = 17/91 A

= 0.186.A

Now, keeping y and z values we get x = 30/91 A

x= 0.329 A.

Now, Current in branch DA = $x = \frac{30}{91} A$

Current in branch DC $= y = \frac{17}{31} A$

Current in branch AB = x-z =

$$\frac{30}{91} - \frac{1}{91} = \frac{29}{91}\text{A} = 0.318\text{A}$$

Current in branch CB = y + z=

$$\frac{30}{91} + \frac{17}{91} = \frac{47}{91}\text{A} \quad = 0.516\text{ A}$$

Current in branch AC = z $= \frac{1}{91} A$

Internal voltage drop in the cell = 2(x+y) $= 2 \times \frac{47}{91} = \frac{94}{91} = 1.032V$

Potential difference across points D and B $= 2 - \frac{94}{91} = \frac{88}{91} = 0.967V$.

Equivalent resistance of the bridge between points D and B.

$$\frac{\text{P.D between points B and D}}{\text{Current}} = \frac{88/91}{47/91} = \frac{88}{47} = 1.87\Omega$$

6.3 Nodal Analysis

The node equation method is based directly on Kirchhoff's current law. Like loop current method, Nodal method also has the advantage that minimum number of equations are needed to determine the unknown quantities. It is suited for networks having many parallel circuits with common ground connection. For the application of this method, every junction in the network where there are more branches meet is regard as a node. One of these is regarded as the Reference Node or Datum Node or '0' potential Node.

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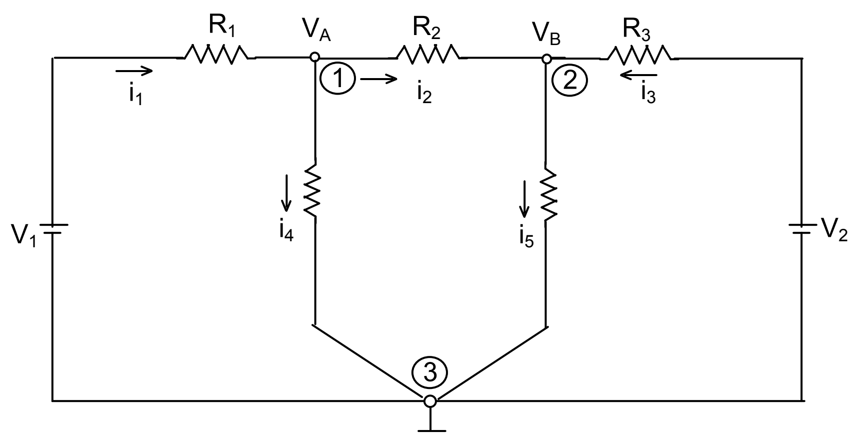


Fig 6.3.1.

In the Fig.6.3.1, the sum of the three currents i_1, i_2 and i_3 at Node1 must be zero according to Kirchhoff 's current law. Adding and rearranging the terms, we get eq.(6.3.1) . Likewise, application of Kirchhoff 's current law to Node2 gives the eq.(6.3.2).

$$V_A\left[\frac{1}{R_1} + \frac{1}{R_4} + \frac{1}{R_2}\right] - \frac{V_B}{R_2} - \frac{V_1}{R_1} = 0 \qquad \text{---------------------(6.3.1)}$$

$$V_B\left[\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_5}\right]-\frac{V_A}{R_2}-\frac{V_2}{R_3}=0 \qquad \text{---------------------(6.3.2)}$$

The values of V_A and V_B are calculated by solving the two equations. From these, all the branch currents can be calculated. We can solve for the currents by mesh method also in which case, we have to solve three mesh equations. For parallel networks, nodal method often proves to be advantageous when compared to mesh method.

6.4 Loop Current Method for an n-Loop Network having Sources in Two Loops

The analysis of the above section can be easily extended to an electrical network having any number n of loops. Thus, for an electrical network of 'n' meshes and in two meshes having sources of e.m.f.s E_1 and E_2, we can write the set of loop equations as,

$$\begin{aligned} E_1 &= I_1 z_{11} + I_2 z_{12} + \ldots + I_n z_{1n} \\ E_1 &= I_1 z_{21} + I_2 z_{22} + \ldots + I_n z_{2n} \\ 0 &= I_1 z_{31} + I_2 z_{32} + \ldots + I_n z_{3n} \\ 0 &= I_1 z_{n1} + I_2 z_{n2} + \ldots + I_n z_{nn} \end{aligned} \qquad \text{----- (6.4.1)}$$

where z_{11} represents the total impedance in the mesh1, z_{22} represents the total impedance in the mesh2, and so on. z_{ij} represents the impedance mutually between the meshes I and j and is given a positive or negative value according as the currents I_i and I_j flow in the same direction or in opposite directions through the impedance z_{ij}.

The set of above mesh equations can be solved simultaneously to yield the values of currents $I_1, I_2, \ldots, I_n$ in terms of known impedances and voltages.

6.5 Node-Voltage Method For an n-Loop Network Having Sources In two Loops

If the network consists of n-loops and if in two loops, there are current sources, say $\boldsymbol{I_1}$ and $\boldsymbol{I_2}$ and if the voltages of nodes 1, 2, 3 etc. are V_1, V_2, V_3 etc. respectively, then we can write the general form of nodal equations as,

$$
\begin{aligned}
Y_{11}V_1 - Y_{12}V_2 \ldots - Y_{1n}V_n &= I_1 \\
-Y_{21}V_1 - Y_{22}V_2 \ldots - Y_{2n}V_n &= I_2 \\
-Y_{31}V_1 - Y_{32}V_2 \ldots - Y_{3n}V_n &= 0 \\
-Y_{n1}V_1 - Y_{n2}V_2 \ldots - Y_{nn}V_n &= 0
\end{aligned}
\qquad \text{--- (6.5.1)}
$$

where Y_{11} is the total admittance referred to node1, Y_{22} that referred to the node2 and so on. Y_{ij} is the admittance between nodes i and j.

These equations can be solved simultaneously to find the values of V_1, V_2, …, V_n, from which the current through any element can be determined.

The loop equations or nodal equations can be solved by the method of systematic elimination or by the method of determinants.

6.6 Method of Determinants

We now generalize the two methods.

For a 3-mesh network, the mesh equations will be of the form

$Z_{11}I_l + Z_{12}I_2 + Z_{13}I_3 = V_1$

$Z_{21}I_! + Z_{22}I_2 + Z_{23}I_3 = V_2$ ----(6.6.1)

$Z_{31}I_1 + Z_{32}I_2 + Z_{33}I_3 = V_3$

The equations can be extended to ‘n’ mesh network.

Likewise for a 4-node network, the equations will be

$Y_{11}V_l + Y_{12}V_2 + Y_{13}V_3 = I_1$

$Y_{21}V_l + Y_{22}V_2 + Y_{23}V_3 = I_2$ --- (6.6.2)

$Y_{31}V_l + Y_{32}V_2 + Y_{33}V_3 = I_3$

The equations can be extended to (n+1) node network.

Whether we use mesh method or nodal method, we can put the governing equations in the general form

$$\begin{aligned} &a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n + k_1 = 0 \\ &a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n + k_2 = 0 \\ &\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\ &\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\ &a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n + k_n = 0 \end{aligned} \qquad \text{----(6.6.3)}$$

For solving the equations, the determinants method based on Cramer's rule of matrices method is used.

This method is based upon Cramer's rule and provides a simple method to solve network equations. If a set of equations containing variables x_1, x_2 ..., x_n be expressed in form eq (6.4.1), it can be shown that,

$$\frac{(-1)^n x_1}{\Delta_1} = \frac{x_2}{\Delta_2} = \frac{(-1)^n x_3}{\Delta_3} = \ldots = \frac{1}{\Delta_0}$$

when Δ_0 is the system determinant found by coefficients when we omit the column of the absolute terms, i.e.,

$$\Delta_0 = \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{vmatrix}$$

Δ_1 is the determinant found by the coefficients when we omit the column of the terms containing the variable x_1 and similarly for Δ_2, Δ_3, … subject to the condition that the columns follow one another in cyclic order. Thus,

$$\Delta_1 = \begin{vmatrix} a_{12} & a_{13} & a_{1n} & \ldots & z_1 \\ a_{22} & a_{23} & a_{2n} & \ldots & z_2 \\ & \vdots & & & \\ a_{n2} & a_{43} & a_{nn} & \ldots & z_n \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} a_{13} & a_{14} & \ldots & a_{1n} & z_1 & a_{11} \\ a_{23} & a_{24} & \ldots & a_{2n} & z_2 & a_{21} \\ & \vdots & & & & \\ a_{n3} & a_{44} & \ldots & a_{nn} & z_n & a_{n1} \end{vmatrix}$$

and similarly for Δ_3, Δ_4, ….

Ex. 6.6.1 In the network shown in Fig.6.6.1, find the currents in different meshes using the method of determinants.

Solution:

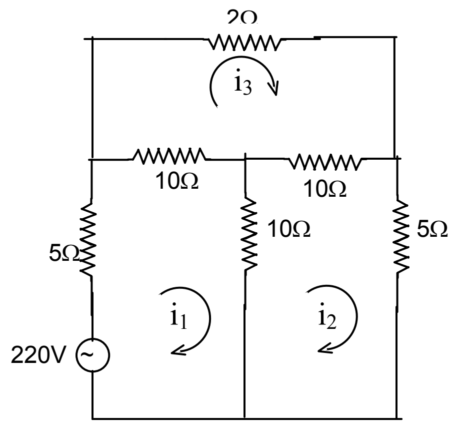


Fig 6.6.1

The loop equations in this case are,

$$25i_1 - 10i_2 - 10i_3 - 220 = 0$$
$$-10i_1 + 25i_2 - 10i_3 + 0 = 0$$
$$-10i_1 + 25i_2 - 10i_3 + 0 = 0$$

$$Thus, \qquad -\frac{i_1}{\Delta_1} = \frac{1}{\Delta_0}$$

$$where, \qquad \Delta_0 = \begin{vmatrix} 25 & -10 & -10 \\ -10 & 25 & -10 \\ -10 & +25 & -10 \end{vmatrix}$$

$$= 25(25 \times 22 - 10 \times 10)$$
$$+10(-10 \times 22 - 10 \times 10)$$
$$-10(-10 \times -10 + 10 \times 25)$$
$$= 25 \times 450 - 10 \times 320 - 10 \times 350 = 4550$$

$$\Delta_1 = \begin{vmatrix} -10 & -10 & -220 \\ 25 & -10 & 0 \\ -10 & 22 & 0 \end{vmatrix}$$

$$= -10(0-0) + 10(0-0) - 220(25 \times 22 - 10 \times 10) = -99000.$$

Thus, $$i_1 = -\frac{\Delta_1}{\Delta_0} = \frac{99000}{4550} 21.75A$$

$$\Delta_2 = \begin{vmatrix} -10 & -220 & 25 \\ -10 & 0 & -10 \\ 25 & 0 & -10 \end{vmatrix}$$

$$= -10(0-0) + 220(-10 \times -10 + 10 \times 22) \times 25(0-0) = 220 \times 320 = 70400$$

and $$i_2 = -\frac{\Delta_2}{\Delta_0} = \frac{70400}{4550} = 15.47A$$

$$\Delta_0 = \begin{vmatrix} -220 & 25 & -10 \\ 0 & -10 & 25 \\ 0 & -10 & -10 \end{vmatrix}$$

$$= -220(-10 \times -10 + 10 \times 25) - 25(0-0) - 10(0-0) = -220 \times 350 = -77000$$

and $$i_2 = -\frac{\Delta_3}{\Delta_0} = \frac{77000}{4550} = 16.92\ A$$

6.7 Substitution Method

This method is illustrated using an example given in Fig.6.7.1.

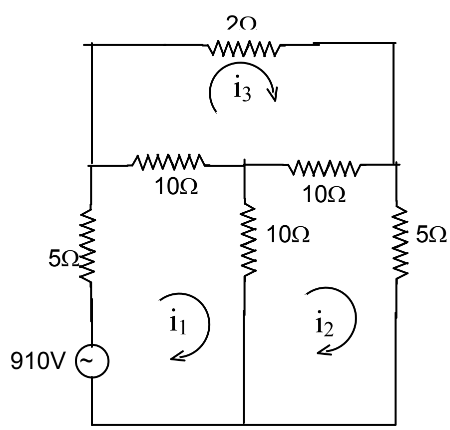


Fig 6.7.1

Consider the equilibrium equations of example 6.7.1 are written below.

$$\begin{aligned} 25i_1 - 10i_2 - 10i_3 &= 910 \\ -10i_1 + 25i_2 - 10i_3 &= 0 \\ -10i_1 - 10i_2 + 22i_3 &= 0 \end{aligned} \qquad \text{------------ (6.7.1)}$$

A simple and straightforward method of solving a set of simultaneous equations like eqs.(6.4.1) consists in systematically eliminating variables until we arrive at an equation containing a single variable. Having obtained the value of one variable, its value may be substituted in an equation containing two variables including this. This gives the value of the second variable. This process of substitution of values of variables in proper equations may be continued until all variables have been assessed.

Since only the numerical coefficients enter into the computation, we may omit symbols. i_1, i_2 etc. altogether and consider only the numerical matrix of (6.7.1).

$$\begin{bmatrix} 25 & -10 & -10 & 910 \\ -10 & 25 & -10 & 0 \\ -10 & -10 & 22 & 0 \end{bmatrix} \qquad \text{----------- (6.7.2)}$$

The steps involved in this systematic elimination method are given below:

(i) Eliminate i_1 from all but the first of these equations. With reference to matrix (6.7.2), this elimination amounts to eliminating second and third elements in the first column. The second element in the first column (symbolized by a_{21}) may be eliminated by adding to the elements of the second row, the respective multiplied elements 0 of the first row

where $$\alpha = -\frac{a_{21}}{a_{11}}, i.e., \alpha = \frac{-(-10)}{25} = \frac{2}{5}$$

The resulting new second row reads:

$$0 \quad 21 \quad -14 \quad 364 \qquad \text{------------ (6.7.3)}$$

Similarly, the new third row is formed by adding to the element or the row, the respective α-multiplied elements of the first row

where $$\alpha = -\frac{a_{31}}{a_{11}}, i.e., \alpha = \frac{-(-10)}{25} = \frac{2}{5}$$

The resulting new third row reads:

$$0 \quad 21 \quad -14 \quad 364 \qquad \text{----------- (6.7.4)}$$

$$\begin{bmatrix} 25 & -10 & -10 & 910 \\ 0 & 21 & -14 & 364 \\ -10 & -14 & 18 & 364 \end{bmatrix} \qquad \text{----------- (6.7.5)}$$

(ii) Eliminate i_2 from all but the first two equations corresponding to the new matrix (6.7.5). This may be done by adding to the third row, the respective α-multiplied elements of the second row

where $$\alpha = -\frac{a_{32}}{a_{22}}, i.e., \alpha = \frac{(-14)}{21} = \frac{2}{3}$$

The resulting new third row reads:

$$0 \quad 0 \quad \frac{26}{3} \quad \frac{1820}{3} \qquad \text{----------- (6.7.6)}$$

The new matrix now becomes:

$$\begin{bmatrix} 25 & -10 & -10 & 910 \\ 0 & 21 & -14 & 364 \\ 0 & 0 & \frac{26}{3} & \frac{1820}{3} \end{bmatrix} \qquad \text{----------- (6.7.7)}$$

The last row in matrix (6.7.7) represents the equation,

$$\frac{26}{3} i_3 = \frac{1820}{3} \qquad \text{---------- (6.7.8)}$$

Hence $$i_3 = \frac{1820}{26} \text{ A} \qquad \text{----------- (6.7.9)}$$

The second row in matrix (6.6.7) represents the equation:

$$21 i_2 - 14 i_3 = 364 \qquad \text{---------- (6.7.10)}$$

Substituting the value of i_3 from eq. (6.7.9) into eq.(6.7.10), we get

$$21 i_2 = 364 + 14 \times 70 = 1344$$

Hence $$i_2 = \frac{1344}{21} = 64 \text{ A} \qquad \text{---------- (6.7.11)}$$

Finally substituting the values of i_2 and i_3 from eq. (6.7.9) and eq.(6.7.11) into first of eqs. (6.7.1), we get

$$25i_1 - 10(64) - 10(70) = 910$$

Hence $i_1 = 90$ A ---------- (6.7.12)

This method of finding the variables through transformation of equations to eliminate variables involves a minimum loss of time. Hence it is the best method to apply in numerical problems.

6.8 Key Terminology

Current law – Node voltage – loop – mesh point – mesh current – network voltage law - loop equation – branch – determinant method- Cramer's rule – substitution method.

6.9 Short Answer Type Questions

1. State and explain Kirchhoff's laws for the distribution of current in a network of conductors.
2. What do you know about node voltage and loop current method to analyze electrical networks?
3. Explain briefly the determinant method of solving simultaneous equations.

Long Answer Type Questions

1. Explain loop current method for analyzing an electrical network. Obtain loop equations in a network containing n-meshes and if there is a source of e.m.f. in each mesh.
2. State Kirchhoff's voltage and current rules. Discuss the loop current and determinant methods of network analysis.
3. State and explain Kirchhoff's laws. Discuss node voltage method in a network containing three nodes and voltage sources by using Kirchhoff's laws.

6.10 Text and Reference Books

1. Basic Electronics, Grob/Schultz, 9th ed. Tata McGraw-Hill.
2. Engineering Circuit Analysis, William H. Hayat, J E Kemmerly & S M Durbin
3. Network Lines and Fields, John D. Ryder, 2nd ed. PHI.

4. Passive components and circuit analysis by VCM, RS & CVR, B.S. Publications
5. Pulse, Digital and Switching waveforms, Millman and Halkias, Tata McGraw-Hill
6. Electrical Technology, Vol.1. , B.L. Theraja, A.K. Theraja- S.Chand.&Co.
7. Circuit Theory, Umesh Sinha
8. Circuit Theory, K.A. Gangadhar, Khanna Publishers.
9. Electric Circuit Analysis, S.N Sivanandam, 2nd ed., Vikas
10. Theory and problems of electric circuits. Joseph A, Edminister (Schaum Publishing Co.)
11. Network analysis by G.K.Mittal & Ravi Mittal (Khanna publishers, Delhi))
12. Passive components and circuit analysis by Agarwal & Arora (A.S.Prakashan,Meerut)

UNIT II **LESSON 7**

NETWORK THEOREMS

The network theorems usually provide shorter methods of analyzing circuits. The reason is that the theorems enable us to convert the network into simple circuit, equivalent to the original. The equivalent circuit can then be analyzed by the rules of series and parallel circuits.

Objectives of the lesson: It introduces various principles involved in the analysis of complicated networks by replacing them with suitable equivalent circuits using the Network theorems like Superposition, Thevenin's theorem, Norton's theorem and Millman's theorem. Even though examples are worked out for d.c networks, these theorems are applicable to circuits consisting of alternating voltage and current sources and complex impedances.

Structure of the lesson

7.1 SUPER POSITION THEOREM

Statement: In any network containing bilateral linear impedances and energy sources, the current flowing in any element is the vector sum of currents that are separately caused to flow in that element by each energy source.

Proof:-

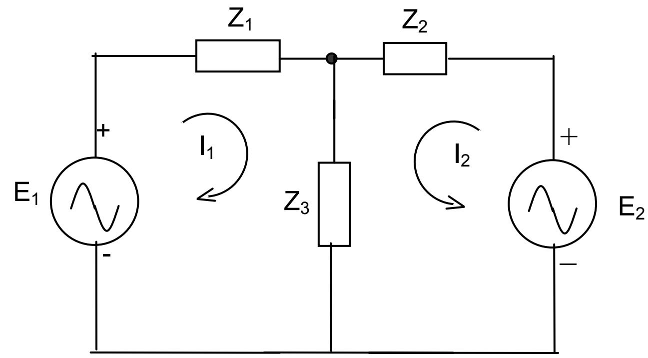


Fig.7.1a

To verify the theorem, consider the simple network with two generators of e.m.f.s E_1 and E_2.

Let the currents due to E_1 and E_2 acting together be I_1 and I_2. By applying K.V.L. to above circuit, we have

$$E_1 = I_1[Z_1 + Z_3] + I_2 Z_3 \qquad \text{--------------} \quad (7.1.1)$$

$$E_2 = I_2[Z_2 + Z_3] + I_1 Z_3 \qquad \text{--------------} \quad (7.1.2)$$

When E_1 is considered to act alone (E_2=0), the equations after applying KVL to the two loops of Fig.7.1b are

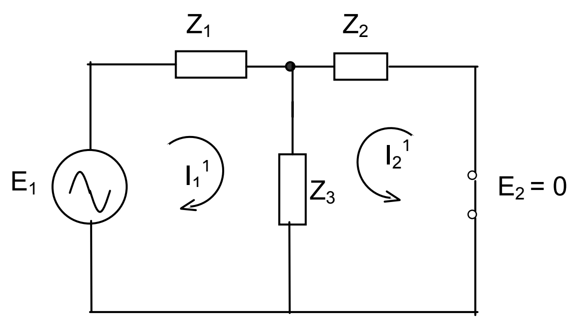


Fig.7.1b

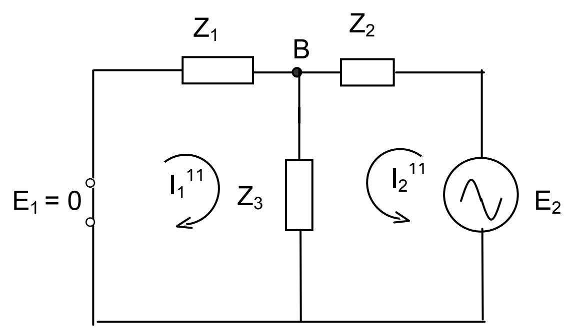


Fig.7.1c

$$E_1 = I_1^1[Z_1 + Z_3] + I_2^1 Z_3 \qquad \text{-------------}(7.1.3)$$

$$0 = I_2^1[Z_2 + Z_3] + I_1^1 Z_3 \qquad \text{-------------}(7.1.4)$$

When E_2 is considered to act alone (E_1=0), the equations after applying KVL to the two loops of Fig.7.1c are

$$0 = I_1^{11}\left[Z_1 + Z_3\right] + I_2^{11} Z_3 \qquad \text{---------------------} \quad (7.1.5)$$

$$E_2 = I_2^{11}\left[Z_2 + Z_3\right] + I_1^{11} Z_3 \qquad \text{---------------------} \quad (7.1.6)$$

Adding eq.(7.1.3) and (7.1.5), we get

$$E_1 = \left[I_1^1 + I_1^{11}\right]\left[Z_1 + Z_3\right] + \left[I_2^1 + I_2^{11}\right]\left[Z_3\right] \qquad \text{------------} \quad (7.1.7)$$

$$E_2 = \left[I_2^1 + I_2^{11}\right]\left[Z_2 + Z_3\right] + \left[I_1^1 + I_1^{11}\right]\left[Z_3\right] \qquad \text{------------} \quad (7.1.8)$$

Comparing coefficients of equations (7.1.7) and (7.1.1) we get

$$I_1 = I_1^1 + I_1^{11}$$

Comparing coefficients of equations (7.1.2) and (7.1.8) we get

$$I_1 = I_2^1 + I_2^{11}$$

This proves the super position theorem. In the circuit analysis, the super position theorem has many applications.

Ex7.1.1. Find the current I in the circuit given below using superposition theorem.

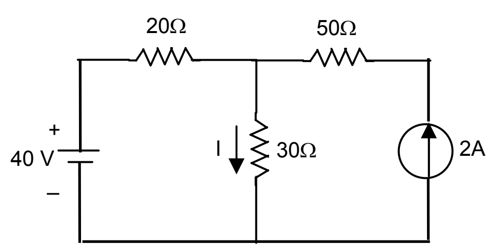


Fig 7.1.1a.

Sol:- Considering first the voltage source, then the circuit is reduced to Fig. 7.1.1b

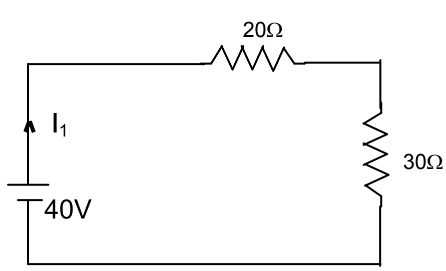


Fig 7.1.1b

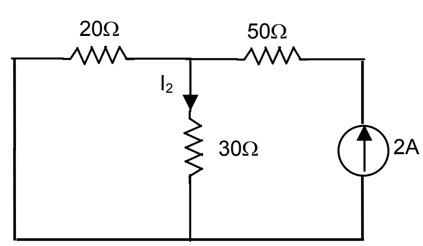


Fig 7.1.1c.

when the current source is open circuited then,

$$I_1 = \frac{40}{50} = 0.8A$$

Now, considering the current source alone then from Fig.7.1.1c,

The current flowing through 30Ω resistor,

$$I_2 = 2 \times \frac{20}{(20+30)} = 0.8A$$

According to superposition theorem, the total current I = I_1+I_2=0.8 + 0.8 = 1.6A

7.2 THEVENIN'S THEOREM

Thevenin's theorem provides a mathematical technique for replacing a given network, as viewed from two output terminals, by a single voltage source with a series resistance. The theorem named after the French Telegraphic Engineer M.L.Thevenin, who proposed this statement in 1893. With the application of this theorem, many complicated networks can be solved.

STATEMENT: Any two terminal linear network containing energy sources and impedances can be replaced with an equivalent circuit consisting of a voltage source E' in series with an impedance Z'. The value of ***E' is the open-circuit voltage*** between the terminals of the network and ***Z' is the impedance*** measured between the terminals with all energy sources replaced by their internal impedances if any.

Proof:-

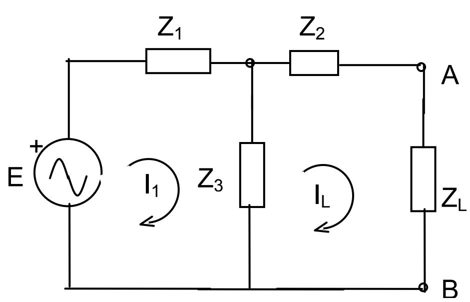


Fig.7.2a

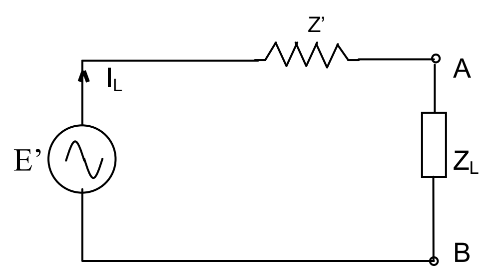


Fig 7.2b

Consider circuit of Fig.7.2a where E is the e.m.f of the source, I_1 is current supplied by the source and I_L is the current flowing through the load impedance. Then according to Thevenin's theorem, the circuit of Fig.7.2b with voltage source E^1 and Z^1 will be equivalent to that of

Fig.7.2a with identical voltage and current at Z_L. To find the expression for load current I_L, applying Kirchhoff's voltage law to the meshes in Fig.7.2a, we have

$$E = I_1(Z_1 + Z_3) - I_L Z_3 \qquad \text{----(7.2.1)}$$

$$0 = I_L(Z_2 + Z_3 + Z_L) - I_1 Z_3 \qquad \text{------(7.2.2)}$$

From eq.(7.2.2) we have

$$I_1 = \frac{I_L(Z_2 + Z_3 + Z_L)}{Z_3}$$

Substituting this value of I_1 in eq. (7.2.1), we get

$$E = \frac{I_L(Z_2 + Z_3 + Z_L)}{Z_3}(Z_1 + Z_3) - I_L Z_3$$

or $$I_L = \frac{E}{\frac{(Z_2 + Z_3 + Z_L)(Z_1 + Z_3)}{Z_3} - Z_3}$$

Or $$I_L = \frac{EZ_3}{Z_2(Z_1 + Z_3) + Z_1 Z_3 + Z_L(Z_1 + Z_3)}$$

By dividing numerator and denominator with $[Z_1 + Z_3]$ we get

$$I_L = \frac{\frac{EZ_3}{Z_1 + Z_3}}{Z_2 + \left[\frac{Z_1\, Z_3}{Z_1 + Z_3}\right] + Z_L}$$

This is the equation for load current of circuit in Fig.7.2a.

From Fig.7.2a, the open circuit voltage at terminals A and B, when load is disconnected is given by

$$E^1 = \frac{E \times Z_3}{Z_1 + Z_3} \qquad \text{---------------------------------(7.2.3)}$$

The impedance between terminals A and B is given by

$$Z^1 = \left[\frac{Z_1 \times Z_3}{Z_1 + Z_3}\right] + Z_2 \qquad \text{-------------------------------(7.2.4)}$$

Substituting the values of E^1 and Z^1 we can rewrite the equation for load current as

$$I_L = \frac{E^1}{Z^1 + Z_L}$$

This is same as the equation for load current of circuit in Fig.7.2b. Hence the Thevenin's Theorem is proved. Thus a simple network having a single voltage source E' in series with an impedance Z' can replace the entire given network.

7.2.1 PROCEDURE TO THEVENIZE A GIVEN CIRCUIT

To find Thevenin's equivalent circuit for a given network, the following steps are to be followed. Let A and B are the terminals across which load impedance Z_L is connected.

(1) Temporarily remove the load resistance Z_L .

(2) Find the open circuit voltage [V_{OC}] which appears across the two terminals A and B .

It is also called "Thevenin Voltage" V_{TH} [E'] .

(3) Compute the resistance of the network as looked into from these two terminals A and B after all voltage sources have been removed leaving behind their internal resistances [If any] and current sources have been replaced by open circuit i.e. Infinite resistance. It is also called Thevenin's Resistance R_{TH}[Z^1].

(4) Replace the entire network by a single Thevenin's voltage source whose voltage is V_{TH} in series with Thevenin's resistance R_{TH}.

(5) Connect the 'Z_L' to the other end of R_{TH} (A terminal) and to the generator(B terminal).

(6) Finally calculate the current flowing through 'Z_L' by using the equation

$$I_L = \frac{E^1}{Z^1 + Z_L} \quad \text{or} \quad I_L = \frac{V_{TH}}{R_{TH} + R_L}$$

Ex 7.2.2 : Find the current in 10Ω resistor shown in Fig.7.2.1a, using Thevenin's theorem.

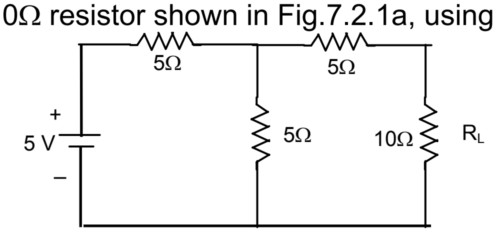


Fig 7.2.1a.

Sol:- The open circuit voltage or Thevenin voltage V_{th} after disconnecting the load.

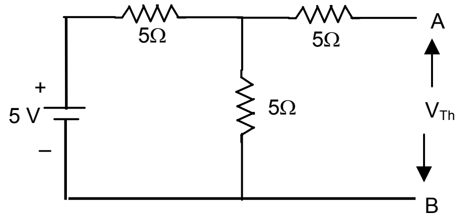


Fig 7.2.1b.

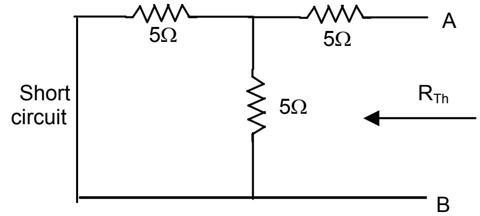


Fig 7.2.1c.

$$V_{Th} = 5 \times \frac{5}{5+5} = 2.5V$$

The Thevenin's resistance is found by removing load and short-circuiting voltage source as shown in Fig,7.2.1c

$$R_{Th} = \frac{5 \times 5}{5+5} + 5 = 7.5\Omega$$

According to Thevenin's theorem, the circuit can be replaced by the equivalent circuit shown in Fig.7.2.1d.

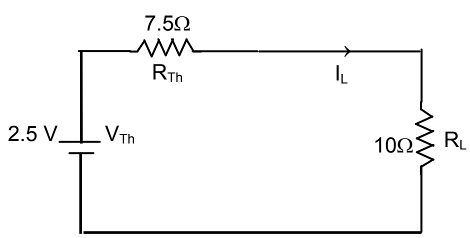


Fig 7.2.1d.

Hence current through the load is given by $I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{2.5}{10+7.5} = \frac{2.5}{17.5} = \frac{1}{7}A$

7. 3. NORTON'S THEOREM

Norton's theorem is an alternative to the Thevenin's theorem. The Thevenin's theorem reduces a two terminal active network of linear resistances and generators to an equivalent constant voltage source and equivalent series resistance, whereas Norton's theorem replaces the network by an equivalent constant current source and a parallel resistance.

Statement : Any two terminal linear network containing energy sources(generators) and impedances can be replaced with an equivalent circuit consisting of a current source I' in parallel with an admittance Y'. The value of **I'** is the ***short-circuit current*** measured between the terminals of the network and **Y'** is the ***admittance*** measured between the terminals with all energy sources replaced by their admittances if any.

Proof: :

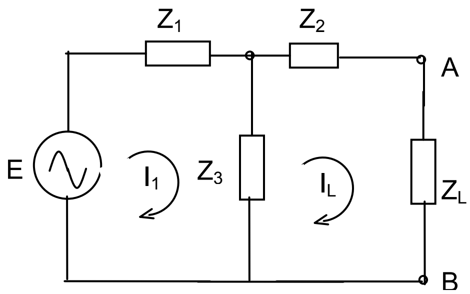


Fig.7.3a

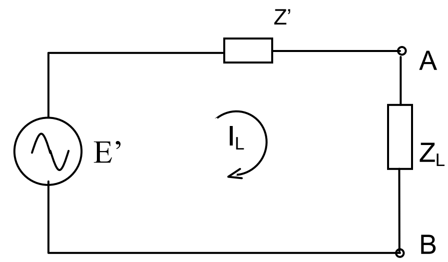


Fig 7.3b

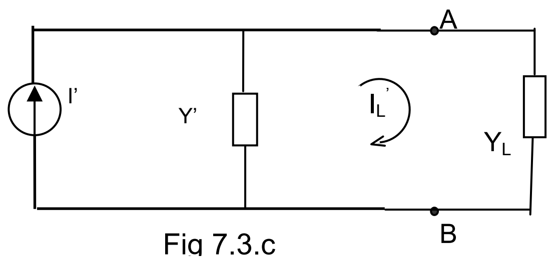


Fig 7.3.c

The circuit shown in Fig.7.3.a is the original T-network to which the Norton's theorem is applied. This theorem can be proved by considering Thevenin's equivalent circuit as shown in Fig. 7.3.b. Applying KVL to the Thevenin's equivalent circuit shown in Fig.7.3.b, we get

$$I_L = \frac{E'}{\left(Z' + Z_L\right)} = \frac{E'}{\left(\dfrac{1}{Y'} + \dfrac{1}{Y_L}\right)} = E'Y'\left[\frac{Y_L}{Y' + Y_L}\right] ----------------(7.3.1)$$

where Y' and Y_L are the reciprocals of Z' and Z_L respectively and are known as Admittances of the corresponding components.

Consider the circuit shown in Fig.7.3.c in which a constant current source I' is supplying current to a parallel network of admittances Y' and Y_L. Applying current division rule to Fig.7.3.c, we get

$$I_L' = I'\left[\frac{Y_L}{Y' + Y_L}\right] ----------(7.3.2)$$

bwhere I_L' is the load current and may be made equal to that of I_L in the circuit of Fig.7.3.a; if in eq.(7.3.1)

I' = E'Y' = E'/Z' -----------------(7.3.3) where I' is the short circuit current of the Thevenin's equivalent network of Fig.7.3.b. Eq.(7.3.3) reveals that circuits represented by Fig.7.3.b and Fig.7.3.c are equivalent. Thus the Norton's theorem is proved.

7.3.1 PROCEDURE TO NORTONIZE A CIRCUIT:- To Nortonize a given circuit, the following steps are to be followed.

(a) Remove the load resistance temporarily [if any] across the two given terminals and put a short circuit between them.

(b) Compute the short circuit current I_{SC} .

(c) Remove all voltage sources but retain their internal resistances, if any.

(d) Similarly remove all the current sources and replace them by open circuit i.e. by infinite resistance.

(e) Now estimate the resistance 'R_N' [Norton's equivalent resistance of the circuit] as looked back into the given terminals. It is exactly equal to R_{Th}.

(f) The current source I_{SC} joined in parallel across 'R_L or 'R_N' between two terminals gives Norton's equivalent circuit.

Ex.7.3.2 Find the current in load of Fig.7.3.1a using Norton's theorem

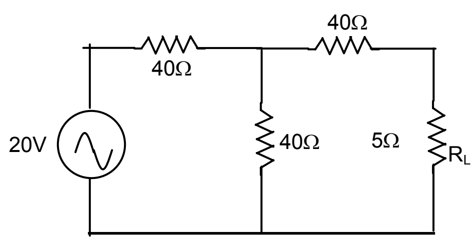


Fig 7.3.1a.

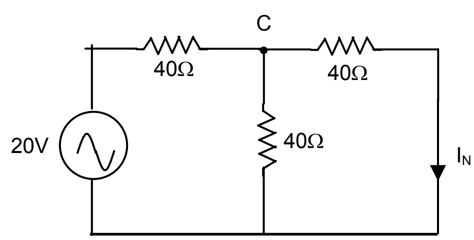


Fig 7.3.1b.

Sol:- Removing the load and short circuiting the terminals, the circuit reduces to Fig.7.3.1b

$$I = \frac{20}{40 + (40//40)} = \frac{20}{60}$$

$$I = \frac{20}{40 + (40 \amalg 40)} = \frac{1}{3} A$$

$$I_N = I \times \frac{40}{40 + 40} = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6} A$$

To find R_N , load is removed and voltage source is short circuited

$$R_N = (40 \,||\, 40) + 40$$
$$= (20) + 40$$
$$= 60\Omega$$

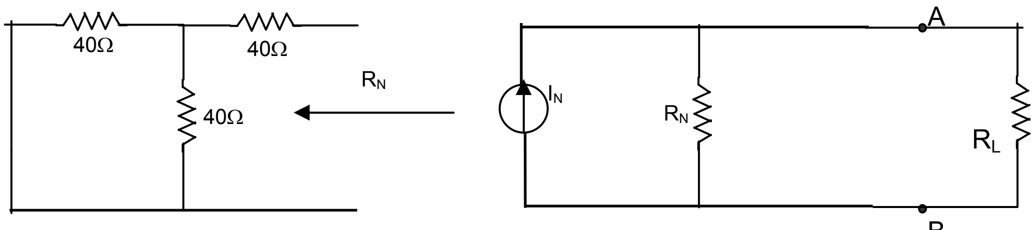


Fig 7.3.1c. Fig 7.3.1d

According to Norton's equivalent circuit shown in Fig.7.3.1d, load current I_L is given by

$$I_L = \frac{R_N}{R_N + R_L} \times I_N = \frac{60}{60+5} \times \frac{1}{6} A = \frac{2}{13} A$$

7.4. RECIPROCITY THEOREM

Statement:- In any linear network, containing bilateral linear impedances and energy sources, the ratio of a voltage **E** introduced in one mesh to the current **I** in any second mesh is the same as the ratio obtained if the positions of **E** and **I** are interchanged other e.m.f.s being removed.

Proof:-

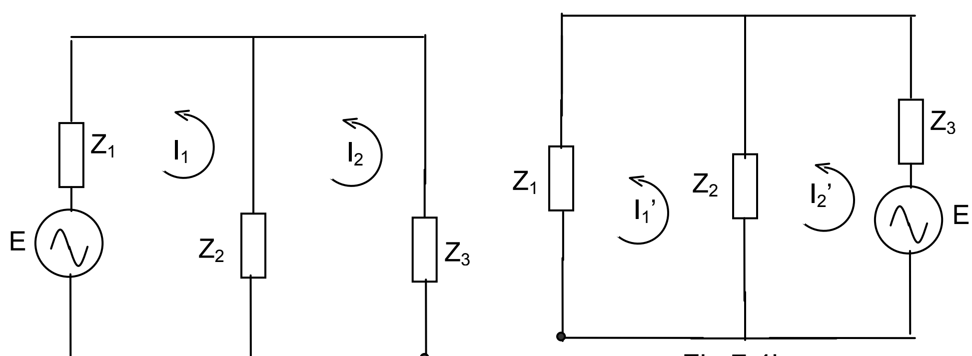


Fig.7.4a Fig.7.4b

It is obvious that the E.M.F source is in first mesh, let the current in 1st and 2nd meshes be 'I_1' and 'I_2' respectively. By applying K.V.L. to two meshes of Fig.7.4a, we get

$$E = I_1 Z_1 + I_1 Z_2 - I_2 Z_2$$

$$E = I_1 [Z_1 + Z_2] - I_2 Z_2 \rightarrow (7.4.1)$$

$$0 = I_2 [Z_2 + Z_3] - I_1 Z_2 \rightarrow (7.4.2)$$

From Eq. (7.4.2)

$$I_1 Z_2 = I_2 [Z_2 + Z_3]$$

$$I_1 = \frac{I_2 [Z_2 + Z_3]}{Z_2} \rightarrow \quad (7.4.3)$$

Substituting

$$I_2 \left[\frac{Z_1 + Z_2}{Z_2} [Z_2 + Z_3] \right] - I_2 Z_2 = E$$

$$\therefore I_2 \left[\frac{(Z_1 + Z_2)(Z_2 + Z_3) - Z_2^2}{Z_2} \right] = E$$

$$\therefore I_2 = \frac{E Z_2}{(Z_1 + Z_2)(Z_2 + Z_3) - Z_2^2} \qquad (7.4.4)$$

Again consider the network given in Fig.7.4b where the voltage source E is in 2nd mesh and currents are $I_1^{'}$ & $I_2^{'}$

Applying K.V.L. to the two meshes, we get

$$0 = I_1^1 [Z_1 + Z_2] - I_2^1 Z_2 \rightarrow (7.4.5)$$

$$E = I_2^1 [Z_2 + Z_3] - I_1^1 Z_2 \rightarrow (7.4.6)$$

From Eq.(7.4.5)

$$I_2^1 Z_2 = I_1^1 [Z_1 + Z_2]$$

$$I_2^1 = \frac{I_1^1 [Z_1 + Z_2]}{Z_2} \rightarrow (7.4.7)$$

Substituting eq.(7.4.7) in to eq.(7.4.6) we get.

$$E = I_1^1 \frac{[(Z_2 + Z_3)(Z_1 + Z_2)]}{R_2} - I_1^1 Z_2$$

$$E = I_1^1 \frac{[(Z_2 + Z_3)(Z_1 + Z_2)] - Z_2^2}{Z_2}$$

$$I_1^1 = \frac{EZ_2}{[(Z_2 + R_3)(Z_1 + Z_2)] - Z_2^2} \rightarrow (7.4.8)$$

From eq (7.4.4) and (7.4.8) $\qquad I_2 = I_1^1$

i.e., as per the reciprocity theorem, the load and voltage source can be interchanged.

Ex 7.4.1 In the network, find

a) Ammeter current when battery is at A and ammeter at B (b) when battery at B and ammeter is at point A. Values of various resistances are shown in the circuits. Also calculate the transfer resistance.

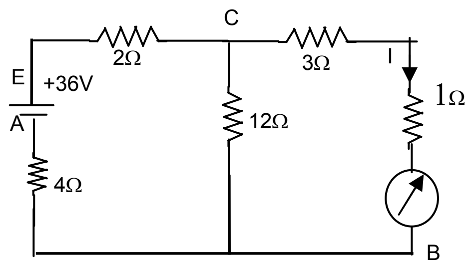


Fig 7.4.1a

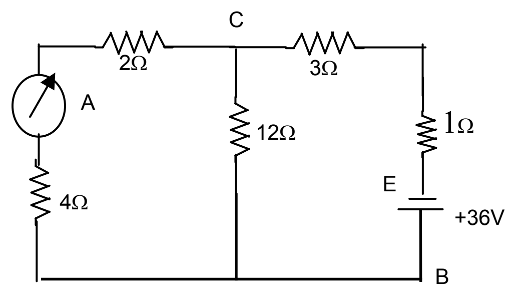


Fig 7.4.1b

Solution: Equivalent resistance between points C and B is

$$= 12 \times \frac{4}{16} = 3\Omega$$

Total circuit resistance = 2+3+4= 9Ω

$$\text{Ammeter current } = 4 \times \frac{12}{16} = 3\text{A}$$

$$\therefore \text{Battery current } = \frac{36}{9} = 4\text{A}$$

Equivalent resistance between points C and D is =12 x 6/18 = 4Ω

Total circuit resistance = 4+3+1=8Ω

Battery current = 36/8 = 4.5A

Ammeter current = 4.5 x 12/18 = 3A

Hence Ammeter current in both cases is the same

Transfer resistance = 36/3 = 12Ω

7.5 MAXIMUM POWER TRANSFER THEOREM

Maximum power transfer theorem is useful for analyzing communication networks.

Statement:- A resistive load will abstract maximum power from a network, when the load resistance is equal to the resistance of the network as viewed from the output terminals, with all energy sources removed, leaving behind their internal resistances.

Proof:- Consider the following network in which the load resistance R_L is connected across the terminals A and B. The network consists of a generator of e.m.f 'E' of internal resistance R_g and a series resistance 'R' .

Let R_i=R_g+ R = Internal resistance from A and B. According to the theorem, R_L will abstract maximum power from network when R_L=R_i.
The current I of the circuit in Fig.7.5 is

$$I = \frac{E}{R_i + R_L}$$

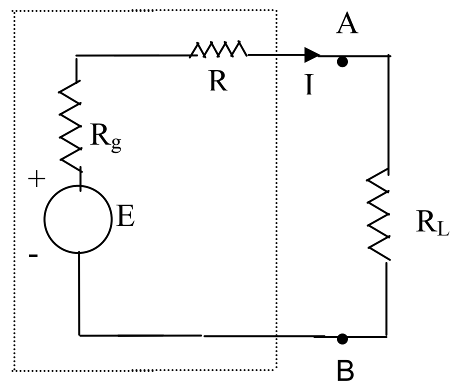


Fig 7.5a: Circuit with voltage source

Power consumed by load

$$P_L = I^2 R_L$$

$$P_L = \left[\frac{E}{R_i + R_L}\right]^2 R_L \rightarrow (7.5.1)$$

For the power of load P_L to be maximum, derivative of load power with respect to load resistance should equal to zero. I.e.

$$\frac{dP_L}{dR_L} = 0$$

By differentiating eq.(7.5.1) with respect to R_L, we get

$$E^2 = \frac{R_L}{[R_i + R_L]^2}$$

$$\frac{dP_L}{dR_L} = E^2 \left\{\frac{1}{[R_i + R_L]^2} - \frac{2R_L}{[R_L + R_i]^3}\right\}$$

$$O = E^2 \left\{\frac{1}{[R_i + R_L]^2} - \frac{2R_L}{[R_L + R_i]^3}\right\}$$

Multiplying both sides by $(R_i + R_L)^3$

$$R_i + R_L = 2R_L$$

or , R_i - R_L=0 or R_i = R_L

It is to be observed that under this condition the load will abstract maximum power. By substituting the above in eq. (7.5.1), we get

$$P_{max} = \frac{E^2}{4R_i} \text{ or } \frac{E^2}{4R_L} \text{ watts}$$

The maximum power transfer theorem has great importance in electronics .To send or receive data information, from or to any load or input device, it is necessary to match the resistance of both sender and receiving circuits to get maximum power. Whenever both resistances (or) impedances become equal; then maximum power will be received by the receiver .So in communications at transmitting antenna, impedance matching networks are used in the receiving stages balun transformers used.

Ex 7.5.1: Find the value of R_L for maximum power in the circuit shown in 7.5.1.1a.

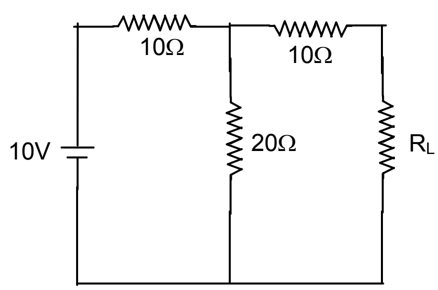


Fig 7.5.1.a

Sol: Maximum power will be delivered when $R_L = R_{Eq}$. R_{Eq} is obtained by removing load and short circuiting voltage source. (Voltage source is assumed to be ideal). Thus

$$R_{Eq} = (10 \,||\, 20)+10 = (20/3) + 10 = 16.67\Omega$$

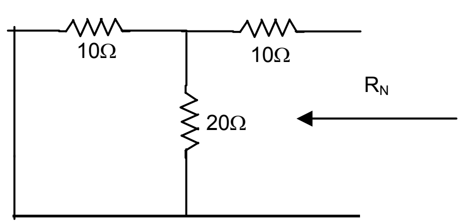


Fig 7.5.1b

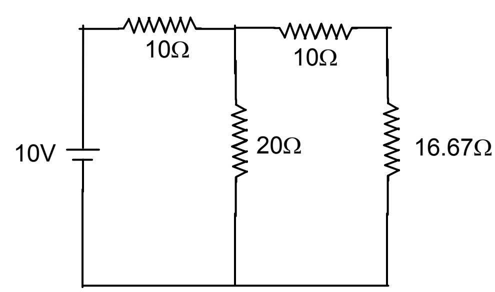


Fig 7.5.1.c

Maximum power will be delivered when $R_L = R_{Eq} = 16.67\Omega$

$$I = \frac{V}{R} = \frac{10}{\{(16.67 + 10) \text{II} 20\} + 10} = 0.47A$$

Load Current = $I_L = I \times \frac{20}{20 + 26.67} = 0.2A$

Maximum power = $I_L^2 R = (0.2)^2 \times 16.67 = 0.67$ Watts.

7-6. MILLMAN'S THEOREM

This theorem provides a short cut for finding the common voltage across any number of parallel branches with different voltage sources.

Statement I:- If a number of constant voltage generators say n, of e.m.f.s E_1, E_2............E_n and internal impedance Z_1, Z_2, ...Z_n respectively are connected in parallel then the resultant potential difference across the out put terminals is given by

$$E = \frac{\frac{E_1}{Z_1} + \frac{E_2}{Z_2} - - - - - - - - + \frac{E_N}{Z_n}}{\frac{1}{Z_1} + \frac{1}{Z_2} - - - - - - - - + \frac{1}{Z_n}}$$

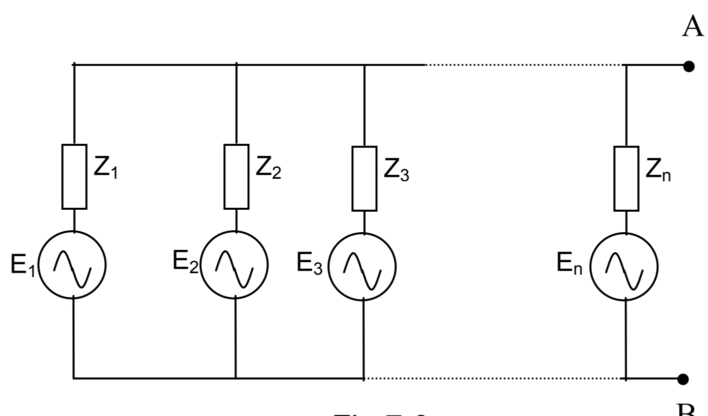


Fig 7.6

where

$$Y_1 = \frac{1}{Z_1}, Y_2 = \frac{1}{Z_2}, - - - - - - - Y_n = \frac{1}{Z_n}$$

are the admittances

Proof:- If I_1, I_2 ....I_n are the currents through generators of e.m.f.s E_1, E_2.......E_n respectively, then the P.D across output terminals A and B is

$E = E_1 - I_1Z_1 = E_2 - I_2Z_2 ...... = E_n - I_nZ_n$

(Because, in parallel connection voltage drops are equal).

$$\frac{E}{Z_1} = \frac{E_1}{Z_1} - I_1 \quad \text{[By dividing with } Z_1 \text{ on both sides]}$$

$$\frac{E}{Z_2} = \frac{E_2}{Z_2} - I_2$$

$$\frac{E}{Z_n} = \frac{E_n}{Z_n} - I_n$$

By adding these equations we get

$$E\left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} - - - - - \frac{1}{Z_n}\right] = \frac{E_1}{Z_1} + \frac{E_2}{Z_2} + - - - - - \frac{E_n}{Z_n} - [I_1 + I_2 + - - - - - I_n]$$

According to K.C.L.

$$\sum I = I_1 + I_2 + I_3 .... I_n = 0$$

$$\text{we are left with } E\left[\frac{1}{Z_1} + \frac{1}{Z_2} + - - - - - - \frac{1}{Z_N}\right] = \frac{E_1}{Z_1} + \frac{E_2}{Z_2} + - - - - - \frac{E_n}{Z_n}$$

$$\therefore E = \frac{\frac{E_1}{Z_1} + \frac{E_2}{Z_2} + - - - - - - - \frac{E_n}{Z_n}}{\frac{1}{Z_1} + \frac{1}{Z_2} + - - - - - - - \frac{1}{Z_n}} \qquad (7.6.1)$$

which proves Millman's theorem for voltage sources connected in parallel.

7.6.1 Calculate the current I in the Fig.7.6.1 using Millman's theorem.

Solution: In the Fig.7.6.1, a parallel circuit with three branches connected between two nodes is given. We have to calculate the current flowing through branch$_2$ using Millman's theorem. So let the voltage source of 33V in branch$_1$ be taken as V_1. As there is no physical voltage source V_2 in branch$_2$, we assume that there is a voltage generator of 0V in series with resistance 3Ω. Let the voltage source of 22V in branch$_3$ be taken as V_3.

Following the eq (7.6.1), we can express the equivalent voltage source E_m as

$$E_m = \frac{\frac{33}{1} + \frac{0}{3} + \frac{22}{2}}{\frac{1}{1} + \frac{1}{3} + \frac{1}{2}} = \frac{44}{1.833} = 24V$$

$$R_m = \frac{6}{11} = 0.5454\Omega$$

Current I through 3Ω resistor = 24/3 = 8A

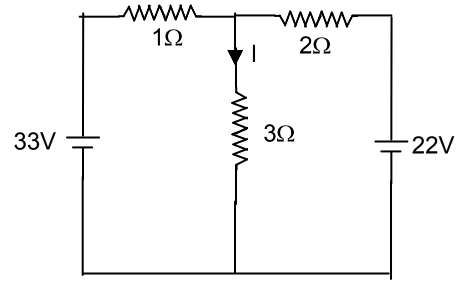


Fig 7.6.1

7.7 Key words

Internal resistance, superposition, transfer impedance, open-circuited voltage, short - circuit current.

7.8 Summary

1. **Superposition theorem:-** In any network containing bilateral linear impedances and energy sources, the current flowing in any element is the vector sum of currents that are separately caused to flow in that element by each energy source , when considered separately.
2. **Thevenin's theorem:-** Any two terminal linear network containing energy sources and impedances can be replaced with an equivalent circuit consisting of a voltage source E' in series with an impedance Z'. The value of ***E' is the open-circuit voltage*** between the terminals of the network and ***Z' is the impedance*** measured between the terminals with all energy sources replaced by their internal impedances if any.
3. **Norton's theorem:-** Any two terminal linear network containing energy sources and impedances can be replaced with an equivalent circuit consisting of a current source I' in parallel with an admittance Y'. The value of **I'** is the ***short-circuit current*** measured between the terminals of the network and **Y'** is the ***admittance*** measured between the terminals with all energy sources replaced by their internal admittance if any.
4. **Reciprocity theorem:-** In any linear network, containing bilateral linear impedances and energy sources, the ratio of a voltage **E** introduced in one mesh to the current **I** in any second mesh is the same as the ratio obtained if the positions of **E** and **I** are interchanged other generators being removed.

5. Maximum power transfer theorem:- A resistive load will abstract a maximum power from a network, when the load resistance is equal to the resistance of the network as viewed from the output terminals, with all energy sources replaced by their internal resistances.

7.9 Self Assessment Questions

Short answer type

1. Define the following terms
   a) Network b) Node c) Branch
2. What type of elements are called non linear elements? Explain.
3. Give statements of
   a) Thevenin's theorem b) Norton's theorem
4. State and explain the principle of superposition theorem.
5. State and prove Norton' Theorem
6. State and prove Reciprocity theorem
7. State and explain Maximum power transfer theorem
8. State and explain Millman's theorem
9. State and prove Thevenin's theorem.

Long answer questions

1. State and prove maximum power transfer theorem and obtain expression for the maximum power delivered to the load.
2. State and prove superposition theorem.
3. State and prove reciprocity theorem.
4. State and explain Norton's theorem. How is Norton's equivalent circuit related with the Thevenin's equivalent circuit?
5. State and prove Millman's theorem.
6. State and prove Thevenins' theorem.
7. Define and compare Thevenin's and Norton's theorems.
8. What do you understand by the terms node, mesh, branches, linear and bilateral as applied to networks?

Numerical Problems: Candidate is supposed to work out numerical problems. So he is advised to workout problems given in various books suggested under 'Text and reference books'

7.10 Text and Reference Books

1. Basic Electronics, Grob/Schultz, 9th ed. Tata McGraw-Hill.
2. Engineering Circuit Analysis, William H. Hayat, J E Kemmerly & S M Durbin
3. Networks, Lines and Fields, John D. Ryder, 2nd ed. PHI.
4. Passive components and circuit analysis by VCM, RS & CVR, B.S. Publications

5. Pulse, Digital and Switching waveforms - Millman and Halkias, Tata McGraw-Hill

6. Electrical Technology, Vol.1. , B.L.Theraja, A.K.Theraja- S.Chand.&Co.

7. Circuit Theory, Umesh Sinha

8. Circuit Theory, K.A. Gangadhar, Khanna Publishers.

9. Electric Circuit Analysis, S.N Sivanandam, 2nd ed., Vikas

10.Theory and problems of electric circuits. Joseph A, Edminister (Schaum Publishing Co.)

11. Network Analysis ,G.K.Mittal & Ravi Mittal (Khanna Publishers, Delhi))

12. Passive Components and Circuit Analysis, Agarwal & Arora (A.S.Prakashan, Meerut)

UNIT – III LESSON 8

RC AND RL CIRCUITS - I

Objectives of the lesson

In this lesson, we study the transient behavior of LR and CR circuits to various types of signals like step, ramp, impulse and ac.

Structure of the lesson

TRANSIENT RESPONSE

Resistance has only opposition to current, there is no reaction to a change, because 'R' has no concentrated magnetic field to oppose change in current, like inductance, and no electric field to store charge that opposes a change in voltage, like capacitance.

In any circuit consisting of elements such as inductance, capacitance etc. the steady current or steady value is not attained immediately. There should be a transient time before it gains the steady state. This condition is called **Transient Condition**. After this transition interval called the **Transient**, the circuit attains a steady state. So, the complete response of a circuit is a combination of transient and steady state responses. The transient response, which occurs because of single small duration excitations, is often termed ***natural response***. However, it depends on the initial conditions of charge, voltage and current prevailing in the circuit. When the excitation repeats with a definite period, the response of the circuit is called **Forced response**. Forced response is often called **Steady state response**.

The complete response of a circuit can be analyzed by means of the time constant for RC and RL circuits for various types of excitations.

The transients are produced whenever

a) An apparatus or circuit is suddenly connected to (or) disconnected from the supply.
b) A circuit is shorted
c) There is a sudden change in the applied voltage from one finite value to another.
d) There is a change in the value of any circuit component.

8.1 Types of Transients

There are single energy transients and double energy transients.

Single energy transients are those in which only one form of energy is involved as in RL and RC circuits. In RC circuit only electrical energy; in LR circuit only magnetic energy is stored in reactive components. Double energy transients are those in which both the energies are involved as in RLC-circuit.

The transient behavior of a circuit may take the system to different working modes. For example, a circuit designed for an amplifier may some times break into oscillations (works as an oscillator), or a circuit designed for oscillations may be damped. Some times, a circuit may work unpredictably. So, studying the transient behaviour of circuits is very essential. The transient behaviour is tested using standard types of input signals. These are: step, ramp, impulse, square wave and sinusoidal signals of various frequencies. In the second part of the lesson we study the use of RC and RL circuits as filters. When output is taken across a capacitor, a simple RC-circuit connected as in Fig 8.2.1c behaves as a low pass filter. We refer this as RC-circuit. When output is taken across a resistor as in Fig 8.2.1c, it behaves as a high pass filter. We refer this as CR-circuit. Because of this reason, the RC and CR circuits for distinction sake are termed as Low pass RC-circuit and High pass RC-circuit. You will find this type of nomenclature in various textbooks.

Further, in lesson on inductors and capacitors we learnt that the voltage across an inductor is linearly related to first order derivative of current and in a capacitor the current in it is proportional to the first order derivative of voltage. Because of this in RC, RL and RLC circuits, the current and voltage variables are related by integral and differential equations. A circuit for which the differential equation is of first order, is called a **First order circuit** (for example first order filter). In the case where the differential equation is a second order, it is called a **Second order filter** and so on.

Some frequently encountered time functions:

1. Step function:

A step potential occurring at t=0 is mathematically represented as

$$f(t) = V \text{ for } t > 0 \qquad \text{----------} \quad (8.1)$$
$$= 0 \text{ for } t < 0.$$

A step function of unit amplitude is called unit step function. A step function occurring at time t_0 is defined as

$$u(t-t_0) = 0 \text{ for } t-t_0 < 0 \quad t<t_0 \qquad \text{-----------} \quad (8.1a)$$
$$= 1 \text{ for } t-t_0 > 0 \quad t>t_0$$

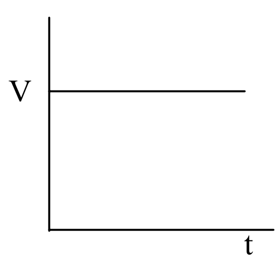


Fig 8.1.1.a step function

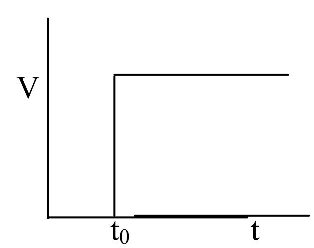


Fig 8.1.1.b Delayed step function

2. Ramp function

If the signal generated varies linearly with time, then the waveform of the signal is called a **ramp**, A unit ramp function can be represented mathematically as a product of time and unit step function u(t) i.e.

$$r(t) = t\,u(t) \qquad \text{------------ (8.2)}$$

More generally, a ramp of slope α that begins at time $t= t_0$ is defined by the expression

$$f(t)=(t-t_0)\,\alpha u(t-t_0) \qquad \text{---------- (8.2a)}$$

where $u(t-t_0)$ is unit step function.

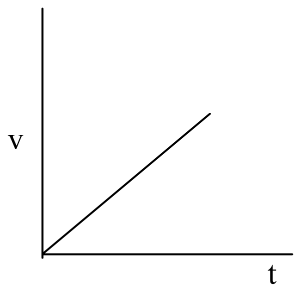


Fig 8.1.2.a Ramp function.

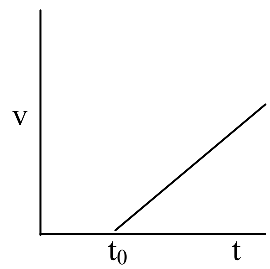


Fig 8.1.2.b Delayed ramp function

A ramp function may be considered as an integral of step function.

3. Impulse function

An impulse function can be considered as a square wave with zero pulse width and infinite amplitude but with the product of pulse width and amplitude is definite. It is usually represented by $\delta(t)$. It is defined as derivative of step function. Impulse function of non-unity value may be thought of as representing the derivative of step functions of non-unity value. Thus in general we may write

$$k\delta(t-t_0) = k\frac{d}{dt}u(t-t_0) \qquad \text{------------ (8.3)}$$

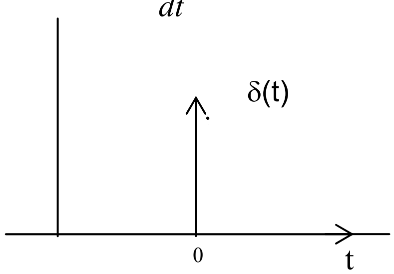


Fig 8.1.3.a Impulse function

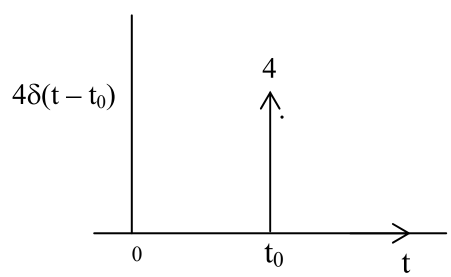


Fig 8.1.3.b Delayed impulse function

A symbol frequently used to represent impulse function is shown in Fig. 8.2.4b. In Fig.8.2.4b, a delayed impulse function in shown. A number written along side arrow head refer to the area or strength of the impulse.

4. Pulse function:

A simple pulse rises abruptly from some steady level (say V=0) and remains at that level $V=V_f$ for some time and falls abruptly to the steady level (V=0). It is usually referred to as rectangular pulse because of its shape. Pulses may take complex shape, but we confine to rectangular pulses. A simple pulse f(t) shown in Fig.8.1.4.a, may be constructed from two step functions. A unit step applied at t=0, as shown in Fig.8.1.4.b, and a step function applied at t=2, as shown in Fig.8.1.4.c. Thus we may write

$$f(t) = u(t)-u(t-2)$$

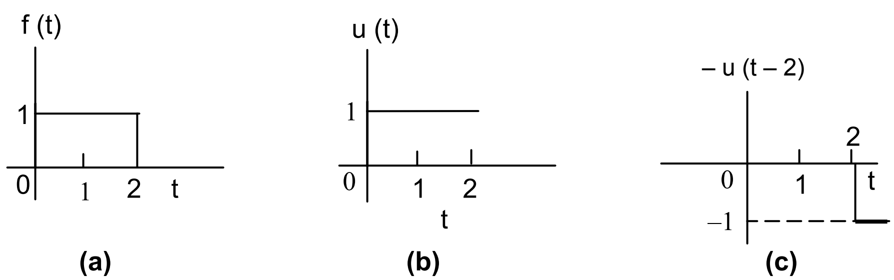


Fig.8.1.4 Rectangular pulses and its components

8.2.1a Transient response of parallel CR-circuit for DC-source (Step excitation)

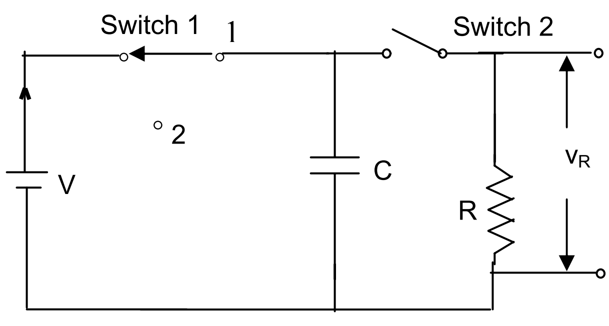


Fig 8.2.1.a

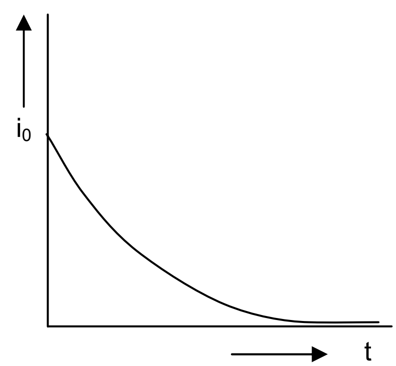


Fig 8.2.1.b Current flow in the circuit as a function of time

The behaviour of the circuit given in Fig 8.2.1a can be studied under four cases.

Case 1: Switch1 is at position 1 with switch 2 open;

Case 2: In switch1, contact is shifted from position 1 to position 2, with switch 2 open.

Case 3: Switch 1 is at position 2 and switch 2 is closed.

Case 4: Switch 1 is at position 1 and switch 2 is closed.

Case 1: Assuming that the initial condition of $v_C = 0$, the condenser charges to the battery potential exponentially. As the switch 2 is open there is no discharge path.

Case 2: When the switch 1 is moved to position 2, battery gets disconnected from the circuit and as the switch 2 is open, condenser potential remains at the maximum value of $v_C = V$.

Case 3: As we move from case 2 to case 3, condenser discharges through R.

Case 4: When both the switches are closed, assuming $v_C = 0$, the condenser charges to battery potential slowly, as there is a discharge path.

Circuit with a resistance and a capacitor connected in series have more applications. Hereafter we concentrate more on the transient behavior of these circuits.

8.2.1b Transient response of series CR-circuit for DC-source (Step excitation)

Consider the circuit as shown in Fig.8.2.1.c. When battery is switched on, then there is flow of charge through the resistance 'R' and condenser is gradually charged to the battery voltage.

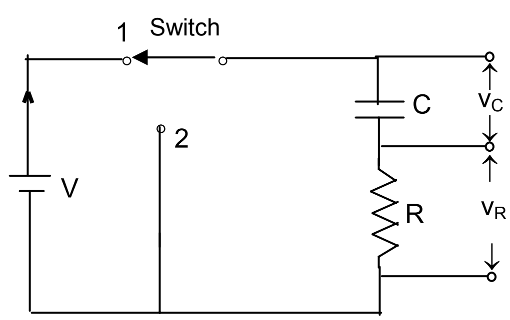


Fig 8.2.1(c)

Switching on the battery can be considered as applying a step potential to the RC-circuit.

The response of the circuit can be understood as follows. Let us suppose that after time 't' during the process of charging, the charge on the condenser is 'q' and current through the resistance 'r' is 'i'. Applying KVL to the above closed circuit, the equation of e.m.f. is

$$Ri + \frac{q}{C} = V$$

But $i = \frac{dq}{dt}$

$$R\frac{dq}{dt} + \frac{q}{C} = V$$

$$R\frac{dq}{dt} = V - \frac{q}{C}$$

$$dt = \frac{Rdq}{V - \frac{q}{C}}$$

$$\frac{-CR\left[\frac{-1}{C}\right]dq}{V - \frac{q}{C}}$$ [by multiplying and dividing with – C]

Integrating on both sides, we get

$$t = RC\log\left[V - \frac{q}{C}\right] + A$$ (Since A is constant of integration)

From initial conditions, i.e. t = 0, q = 0

$0= -RC \log_e V + A$

$A = RC \log_e V$

Substituting the value of A in the above relation, we get

$$t = RC\log_e\left[V - \frac{q}{C}\right] + RC\log_e V = RC\log_e\left[\frac{V - \frac{q}{C}}{V}\right];$$

$$v_C = V(1-e^{-t/RC}) \qquad \text{------------ (8.4)}$$

where q = CV

CV = maximum value of charge = q_0 [say]

$$q = q_0[1-e^{-t/RC}] \qquad \text{----------- (8.5)}$$

The factor RC is called the ***time constant*** τ of the circuit. The second term $q = q_0\ e^{-t/RC}$ represents the transient response whereas the first term q_0 represents the steady state response. Putting RC = τ = t in above equation, we get

$$q = 0.638\, q_0 \qquad \text{------------ (8.6)}$$

Thus the time constant RC of R-C circuit is defined as "the time in which the charge of condenser reaches to 0.638 times its final value"

$$\frac{dq}{dt} = \frac{d}{dt}\left[q_0\left[1 - e^{-t/RC}\right]\right] = \frac{\frac{dq_0}{dt}\cdot e^{-t/RC}}{RC} = \frac{V}{R}\cdot e^{-t/RC} \quad [\therefore q = cv]$$

$$\therefore i = i_0 e^{-t/RC}$$

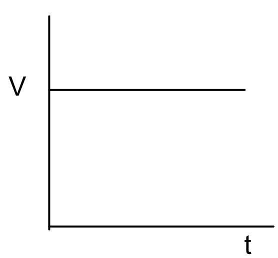


Fig 8.2.1(d) Step function.

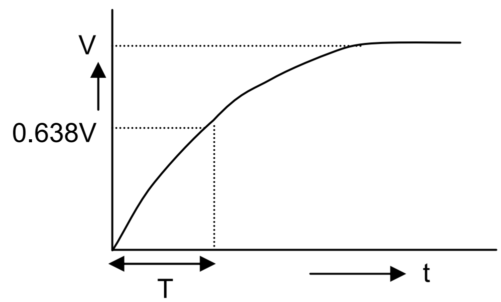


Fig 8.2.1(e) Response to step input

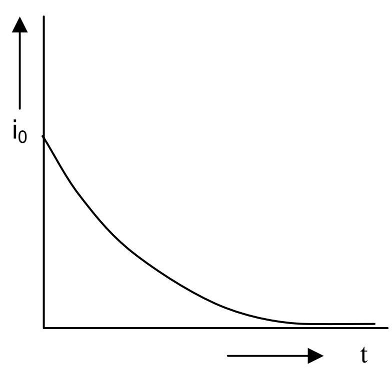


Fig 8.2.1(f) Current flow in the circuit as a function of time

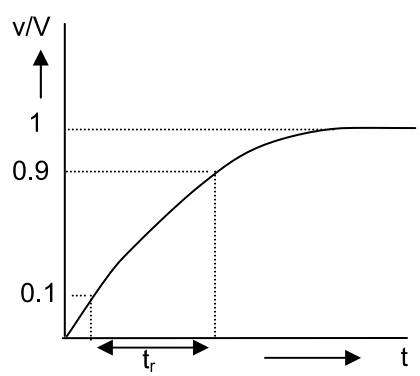


Fig 8.2.1(g) Illustration of rise time.

The graph shown in Fig 8.2.1(e) is a charge graph in which the time is taken on X-axis and voltage on Y-axis. The shape of the graph is exponential. To get the time constant of the circuit, intersect the graph at the value of 0.638V, extend the line on to X-axis to get the time constant.

Rise time: The rise time is defined as the time it takes for the output voltage to rise from 0.1 to 0.9 of its final value. It indicates the quickness with which the circuit can respond to sudden changes in signal levels.

Expression for rise time: Let t_1 and t_2 be the instants at which the output voltage is 0.1 and 0.9 of its final value. From eqn. (8.3) we evaluate $v_C = v_o$ at t_1 and t_2.

$$V_{ot1} = 0.1V = V(1-e^{-t1/RC})$$

$$V_{ot2} = 0.9V = V(1-e^{-t2/RC})$$

From these t_1 and t_2. Rise time t_r is $t_2 - t_1$. So,

$$t_r = RC \ \ln 9 = 2.2RC \qquad \text{------------ (8.7)}$$

Discharging of condenser:- The series RC-circuit shown in Fig 8.2.1(a) has the switch in position 1 for sufficient time to establish the steady state and at t = 0, the switch is moved to position 2 as in Fig 8.2.1(h).

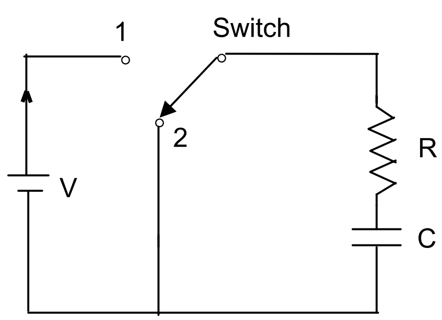


Fig 8.2.1(h)

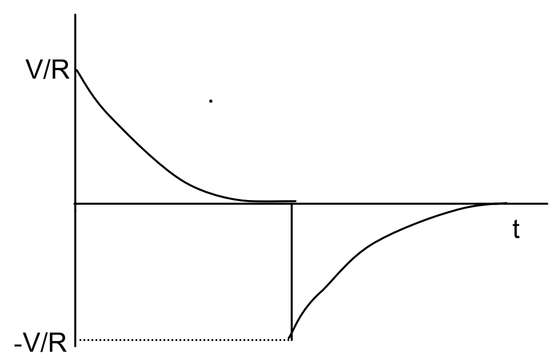


Fig 8.2.1(i)

Suppose after time ‘t’ during the process of discharging, the charge of condenser is ‘q’ and current through resistance is ‘i’. By applying K.V.L. to the loop, when battery is switched off

$$R_i + \frac{q}{C} = 0$$

$$R\frac{dq}{dt} + \frac{q}{C} = 0 \Rightarrow R\frac{dq}{dt} + \frac{-q}{C}$$

$$\frac{dq}{q} = -\frac{1}{RC}$$

Integrating on both sides

$$\log_e q = \frac{-t}{RC} + B \quad [\therefore \text{B = constant of integration}]$$

From initial conditions at t = 0, q = q_0, where q_0 is the maximum charge on the capacitor.

$$\log_e q_0 = B$$

Substituting the value of 'B' in above equation

$$\log_e q_0 = \frac{-t}{RC} + \log_e q_0$$

$$\log_e \frac{q}{q_0} = \frac{-t}{RC}$$

By canceling log we get $\quad \frac{q}{q_0} = e^{-t/RC}$

If time constant t = RC, then

q = q_0[0.362] = 0.362q_0

v = v_0[e^{-1}] or v = v_0(1/e) ------------ (8.8)

Therefore time constant of RC-circuit may be defined as "the time in which the charge of the condenser falls to 1/e or [= 0.362] times its initial value during the process of discharging".

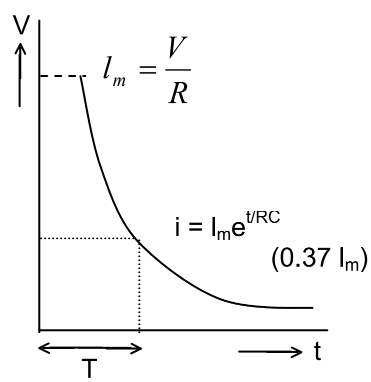


Fig 8.2.1(j) Discharging.

So far, we discussed the transient response of RC circuit with output taken across capacitor or we may say we studied the transient response of RC low pass filter. We can study the transient response of RC circuit with output taken across R, that is, the transient response of RC high pass filter, which is equally important.

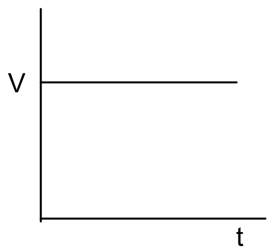


Fig 8.2.1(k) Step function.

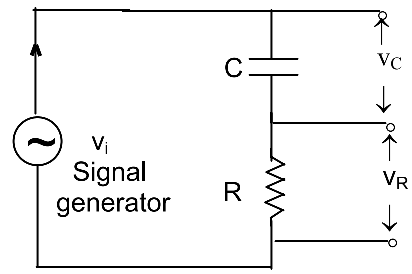


Fig 8.2.1(l)

The capacitor blocks the d.c component of the input. Since the input is constant for t>0, the steady state final output voltage is zero. Immediately on switching on the circuit i.e at t= 0, the input voltage changes discontinuously by an amount V. The capacitor offers zero reactance to such transient changes and the entire voltage appears across R, which in turn decreases to zero in an exponential manner because of the charging up of the condenser.

Therefore $v_o = Ve^{-t/RC}$

For most of the applications, steady state is reached after t = 5RC.

Example:

A capacitor of capacity 0.5μF and resistance 10MΩ is charged to a potential difference of 10V. Find the time constant and the maximum charge stored.

Solution:

The time constant is

$$CR = (10.5\times10^{-6} \text{ Farad}) \times (10\times10^{6}\ \Omega) = 5 \text{ seconds.}$$

The maximum charge stored is,

$$Q_0 = CE_0 = 0.5\mu F \times 10\ V = 5\ \mu\ C.$$

Example:

A capacitor is being charged form a d.c. source through a resistance of 2MΩ. If it takes 0.5 seconds for the charge to reach three-quarters of its final value, find the capacity of the capacitor?

Solution:

The equation of charging a capacitor is,

$$q = q_0\left(1 - e^{\frac{t}{CR}}\right)$$

Here, $\frac{q}{q_0} = \frac{3}{4}$, t = 0.5 sec. and R = 2 × 10^6 Ω

Thus, $\frac{3}{4} = 1 - e^{-0.5/(C\times 2\times 10^6)}$ (or) $e^{-5/(C\times 2\times 10^7)} = 1 - \frac{3}{4} = \frac{1}{4}$ (or) $e^{5/(C\times 2\times 10^7)} = 4$

or $\frac{5}{2\times 10^7 C} = \log_e^4 = 2.3026\ \log_{10}^4$.

Thus, $\frac{5}{2\times 10^7 \times 2.3026 \times 0.602} = 0.18\mu F$

8.2.2 DC-Transient Response of RL-circuit (Step input)

In this circuit, as soon as battery is switched on, current begins to flow and a magnetic flux is linked with the coil, as a result self induced e.m.f is set up in the coil which opposes the raise of current in the circuit. Hence the current does not attain final steady value i_0 instantaneously but grows at a rate depending upon the inductance and resistance of the circuit. During the variable state of growth of current, if it is instantaneous current in the circuit, the e.m.f induced in the inductor L is given by -L (di/dt).

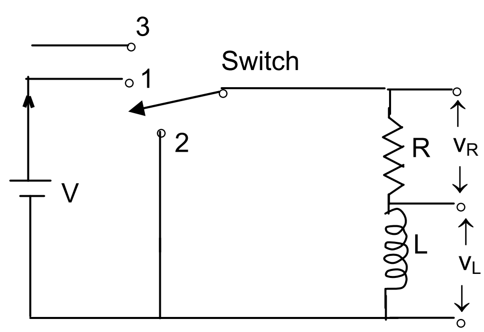


Fig 8.2.2(a) LR circuit

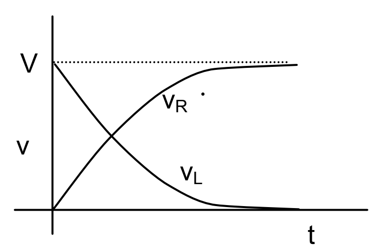


Fig 8.2.2(b)

Hence the effective e.m.f. in the circuit can be calculated by applying K.V.L to the above mesh.

$$V - \frac{Ldi}{dt} = Ri$$

$$-L\frac{di}{dt} = Ri - V$$

$$L\frac{di}{dt} = -Ri + V$$

$$\frac{di}{V - Ri} = \frac{dt}{L}$$

Integrating on both the sides

$$-\frac{1}{R}\log_e(V - Ri) = \frac{t}{L} + A \quad [\because \text{A = Integration constant}]$$

From the boundary conditions, t = 0 & i = 0, we will get the value of A from the above equation as

$$\because A = -\frac{1}{R}\log_e V$$

Substituting A in above relation

$$-\frac{1}{R}\log_e(V - Ri) = \frac{t}{L} - \frac{1}{R}\log_e V$$

$$\frac{V - Ri}{V} = e^{-\frac{R}{L}\times t}$$

$$V_o = V(1 - e^{-Rt/L}) \qquad \text{----------- (8.9)}$$

$$i = \frac{V}{R}\left[1 - e^{-\frac{R}{L}\times t}\right]$$

$$\left[\therefore \frac{V}{R} = i_0 \text{ is the maximum current stored in inductor}\right]$$

$$i = i_0\left[1 - e^{-\frac{R}{L}\times t}\right] \qquad \text{----------- (8.10)}$$

In the above equation, first term i_0 give steady state value and second term $i_0\left[e^{-\frac{R}{L}\times t}\right]$ gives transient response at that particular instant. The quantity L/R is known as **Time constant** of LR circuit. After a time interval of L/R seconds, the current in the circuit is given by

$$i = i_0[1-e^{-1}] = 0.638\ i_0$$

Time constant of LR-circuit may be defined as “the time, which the current raises to nearly 2/3rd of its maximum steady value”.

We can define rise time for this circuit also and following the procedure given for RC circuit, one can show that rise time for RL circuit is

$T_r = 2.2$ (R/L)

Decay of current

When the applied e.m.f 'V' is suddenly closed then transient response of the circuit can be calculated as follows. For discharging, throw the switch S to position 2. During the variable state of decay of current if i is the instantaneous current in the circuit, the e.m.f. induced in the circuit is given by $-L\frac{di}{dt} = e$

$\therefore$ By Ohm's Law, we have $-L\frac{di}{dt} = Ri = e$

$$\frac{di}{i} = \frac{-R}{L}dt$$

Integrating on both sides we have

$\log_e i = \frac{-R}{L} \times t + A$ [A = Constant of Integration]

when t = 0, $i = i_0$ we have $\log_e i_0 = -\frac{R}{L} \times 0 + A$ (or) $A = \log_e i_0$

Substituting A value in above relation, and rearranging the terms, we get $i = i_0\left[e^{-\frac{R}{L}\times t}\right]$

When after time $\frac{L}{R}$, the current in the circuit is given by i.e., when $t = \frac{L}{R}$.

$$i = i_0\left[e^{\frac{R}{L}\times\frac{L}{R}}\right];\ i = i_0\left[e^{-1}\right];\ i = 0.362 i_0$$

Therefore time constant L/R of RL- circuit may also be defined as " the time in which the current in the circuit falls to 1/e of its maximum value, when external source of e.m.f. is removed". At time t= 5(L/R) the value of I will be almost zero and the transient is considered terminated.

In an inductor, changes in current must be continuous. If switch in Fig 8.2.2(a) is shifted after keeping the switch in position 1 for some time, we expect the current in inductor to become zero producing an impulse of voltage across the inductor. As we open the switch, an arc will be observed across the switch contacts. It is because of the large voltage induced in the inductor. The voltage is large enough to provide an ionized path, which permits the current to flow until the gap between the contacts becomes so large that the arc is extinguished. The entire process takes place very quickly, but the arc effect is that the current flow is stopped gradually. Thus the continuity condition is not violated.

Example:

A coil of resistance 20Ω and inductance 0.5H is switched to direct current 200V supply. Calculate the rate of increase of current.

(i) At the instant of closing the switch, and

(ii) At t = L/R sec after the switch is closed. Also

(iii) Find the steady state value of the current in the circuit, and

(iv) The transient current after a time t = L/R.

Solution:

The current in a R – L circuit is,

$$i = i_0 \left[1 - e^{-(R/L)t}\right]$$

The rate of growth of current is

$$\frac{di}{dt} = i_0 \frac{R}{L} e^{-(R/L)t}$$

The maximum steady value of current is,

$$i_0 = \frac{E}{R} = \frac{200V}{20\Omega} = 10A$$

The transient current is given by,

$$i_{transient} = i_0 e^{-(R/L)t}$$

At $t = \frac{L}{R}$, $i_{transient} = i_0 e^{-(R/L)\bullet(L/R)} = i_0 e^{-1} = \frac{10}{2.72} = 3.6A$

(i) The rate of increase of current at the instant of closing the switch i.e., at t = 0, is given by

$$\left(\frac{di}{dt}\right)_{t=0} = i_0 \frac{R}{L} = (10A) \cdot \frac{(20\Omega)}{(0.5H)} = 400A/\sec.$$

(ii) The rate of increase of current at t = L/R second after the switch is closed, is given by

$$\left(\frac{di}{dt}\right)_{t=0} = i_0 \frac{R}{L} e^{-(R/L)\cdot(L/R)} = i_0 \frac{R}{L} e^{-1}$$

$$= (10A) \times \left(\frac{20\Omega}{0.5H}\right) e^{-1} = 400e^{-1} = 400 \times (2.72)^{-1}$$

$$= 400 \times 0.3676 = 147.04 A/\sec.$$

8.3.1 Transient response of RC-circuit to ramp input

Circuit given in Fig.8.2.1(h) is usually written as in Fig.8.3.1(b). where the battery is replaced with a signal generator.

If the signal generated is varied linearly with time, the wave form of the signal is called a ***ramp,*** it can be represented mathematically as

$$v_i = \alpha t = \frac{q}{C} + v_R$$

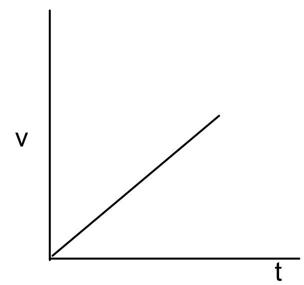


Fig 8.3.1(a) Ramp function

Here q is charge on the capacitor.

Differentiating the above equation, we get

$$\frac{dv_i}{dt} = \alpha = \frac{1}{C}\frac{dq}{dt} + \frac{dv_R}{dt}$$

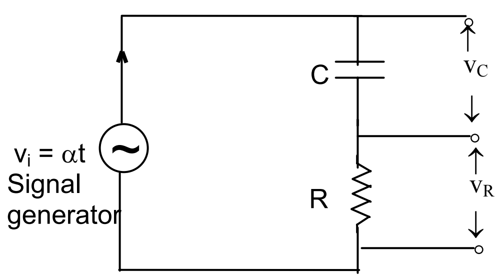


Fig 8.3.1(b)

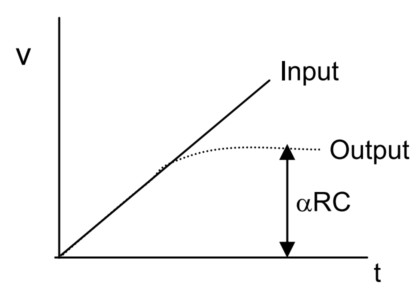


Fig 8.3.1(c) Response of CR circuit to ramp signal.

$$\frac{dv_i}{dt} = \frac{i}{C} + \frac{dv_R}{dt}$$

Sine $v_R = iR$

$$\frac{dv_i}{dt} = \alpha = \frac{v_0}{RC} + \frac{dv_R}{dt}$$

The standard solution for the differential equation

$$v_R = \alpha RC\left(1 - e^{-\frac{t}{RC}}\right)$$

$$v_C = \alpha t - \alpha RC\left(1 - e^{-\frac{t}{RC}}\right)$$

If $\frac{t}{RC} << 1$ $v_R = \alpha RC\left(1 - e^{-\frac{t}{RC}}\right)$, $\frac{t}{RC}$ becomes very small

Expanding the exponential up to quadratic term we have

$$v_R = \alpha RC\left(1 - \left(1 - \frac{t}{RC} + \frac{t^2}{2R^2C^2}\right)\right) = \alpha t - \frac{\alpha t^2}{2RC}$$

$$v_C = \alpha t - \alpha RC\left(1 - e^{-\frac{t}{RC}}\right) = \alpha t - \alpha RC\left(1 - \left(1 - \frac{t}{RC} + \frac{t^2}{2R^2C^2}\right)\right) = \alpha t + \alpha t - \frac{\alpha t^2}{2RC}$$

If $\frac{t}{RC} >> 1$ $v_R = \alpha RC$

$$v_C = \alpha t - \alpha RC$$

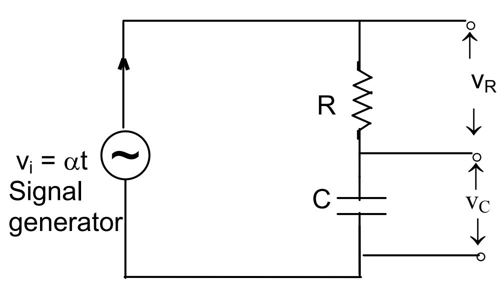


Fig 8.3.1(d)

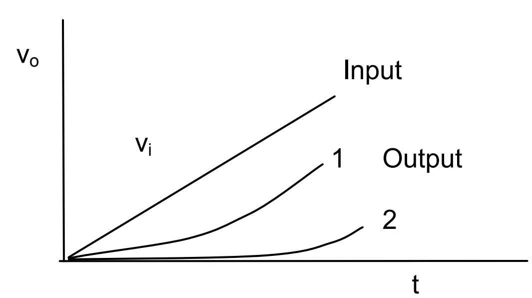


Fig 8.3.1(e) Response of CR circuit to ramp signal with output across C

In Fig 8.3.1(e), graph1 corresponds to $t > RC$ and graph 2 corresponds to $t < RC$

$$v_0 = \alpha t - \alpha RC$$

For $\frac{t}{RC} >> 1$, transients are negligible. For $\frac{t}{RC} << 1$ transient behaviour predominates.

$$v_0 = \alpha t - \alpha RC\left(1 - e^{-\frac{t}{RC}}\right)$$

8.3.2 Response of RL-circuit to ramp input:

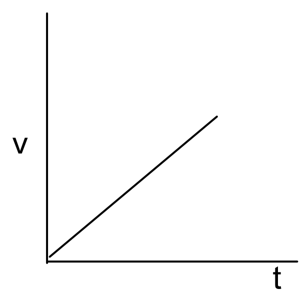
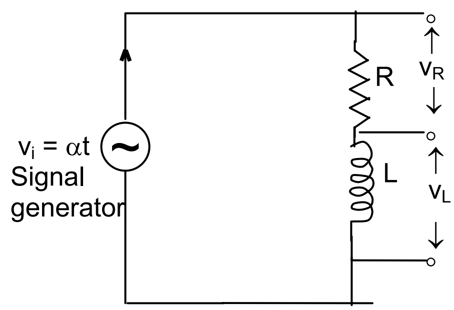


Fig 8.3.2(a) Ramp function



8.3.2(b) RL circuit with Ramp input.

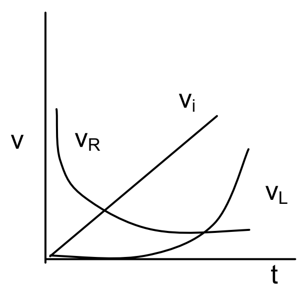


Fig 8.3.2(c) Response of RL circuit to ramp function.

8.4.1 Transient response of RC-circuit (with output taken across capacitor) to pulse input

If v_i is a pulse of rectangular shape, the mathematical function of it can be represented as

U(t)=V for the interval $0<t<t_P$.

=0 for $t<0$ and $t>=t_P$.

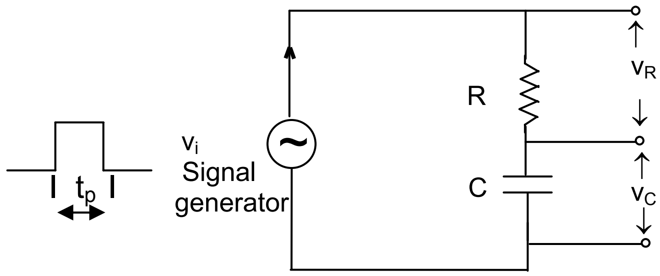


Fig 8.4.1(a)

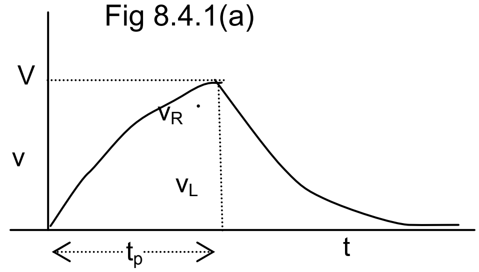


Fig 8.4.1(b)

For time intervals less than pulse width t_p , the response is the same as that for the step input. At the end of the pulse, the output voltage is V_p . Finally the output decreases to zero from V_p in an exponential manner as shown in Fig 8.2.5b, the rate of decrease depending on time constant.

8.4.2 Transient response of RC-circuit (with output taken across resistor) to pulse input

As is already mentioned when in an RC circuit output is taken across R, it behaves like a high pass filter under steady state conditions.

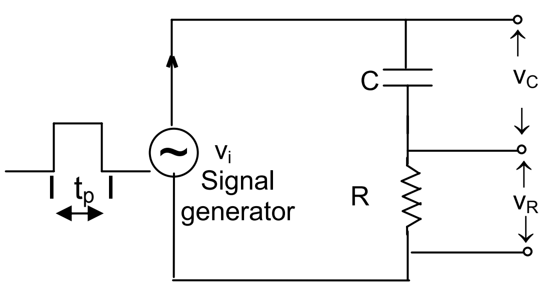


Fig 8.4.2(a)

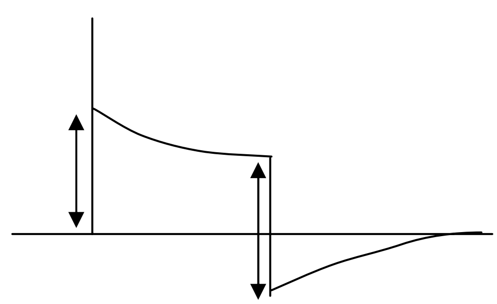


Fig 8.4.2(b)

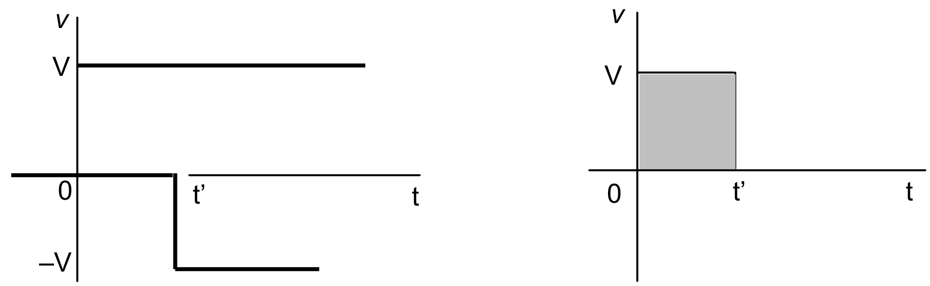


Fig 8.4.2(c) A rectangular pulse formed from two steps.

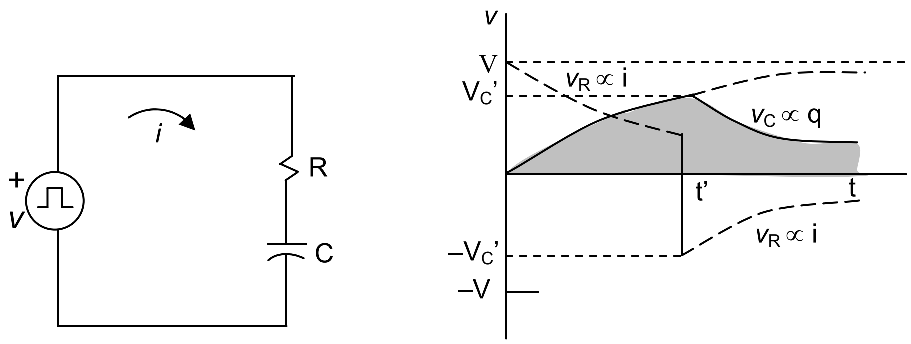


Fig 8.4.2(d) The pulse response of an RC circuit.

The response of the RC circuit (Fig 8.3.3b) to a rectangular pulse can be obtained as the response to two voltage steps (see Fig 8.2.5). There is no forced response because $Z_C = \propto$ and the complete response to first step is

$$i = i_n = \frac{V}{R}e^{-t/RC} \qquad 0 < t < t' \qquad \text{--------------- (8.11)}$$

Assuming the capacitor as uncharged, the capacitor voltage at time t is

$$v_C = \frac{1}{C}\int_0^t \frac{V}{R}e^{-t/RC}dt = \frac{1}{C}\frac{V}{R}(-RC)e^{-t/RC}\Big]_0^t$$

$$= -Ve^{-t/RC}\Big]_0^t = V\left(1 - e^{-t/T}\right) \qquad \text{--------------- (8.12)}$$

where T=RC is the time constant.

If a negative step is applied at t = t' the current response has the form of eqn. (8.11)

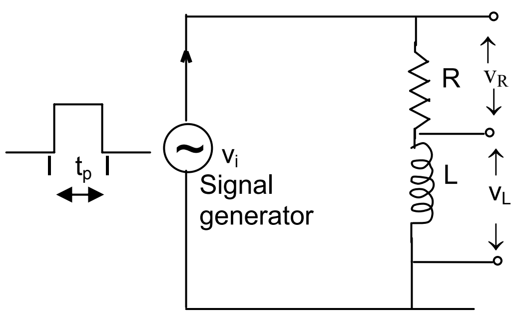
Since the capacitance voltage cannot change suddenly, at t = t', $V_C = V_C^{'}$ as evaluated from eqn (8.12) and shown in Fig 8.4.2(d). The current reverses as the capacitance discharges and

$$i^{'} = i_n^{'} = -\frac{V_C^{'}}{R} e^{-\left(t-t^{'}\right)/RC} \quad t^{'} < t < \infty$$

Since I is proportional to v_R at all times, a separate current curve is not shown.. The charge on the capacitor is equal to Cv_C and varies as v_C.

8.4.3 Response of RL-circuit to pulse input

We have seen that the derivation for the Pulse of RC circuit is quite involved. An alternate method of handling the responses of circuit to complicated input waveforms is to use Laplace Transform method. This method reduces differential equations to algebraic equations and thus enables us to solve the problem more simply. The inverse Laplace transformation gives us the needed result as a function of time. Standard Laplace Transforms and the corresponding inverse Laplace Transforms were compiled and are available in standard text books.



8.4.3(a) RL circuit with pulse input.

Considering the rectangular pulse as a superposition of two step functions as is done in the case of RC network, following the method Laplace transform method for a LR circuit with inductance 1H and R=1 ohm we get the expression for i(t) as

$$i(t) = \left(1 - e^{-t}\right)u(t) - \left[1 - e^{-\left(t-t_p\right)}\right]u(t - t_p)$$

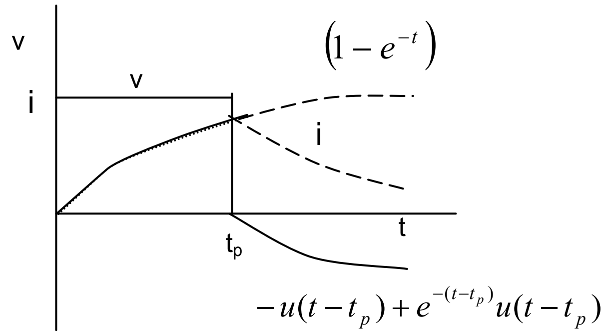


Fig 8.4.3(b) Response curve to pulse input.

8.5.1 Response of an RC circuit to impulse function

Applying Kirchhoff's current law, we obtain the relation,

$$C\frac{dv}{dt}+\frac{v(t)}{R}=i(t)=K\delta(t) \qquad \text{-------------(8.5.1)}$$

where $K\delta(t)$ is an impulse of strength K applied at t=0.Let us assume that there is no charge present on the capacitor for t<0. Integrating the equation (8.5.1), we obtain

$$Cv(0+)+\frac{1}{R}\int_{-\infty}^{0+}(t)dt=\int_{-\infty}^{0+}i(t)dt=K$$

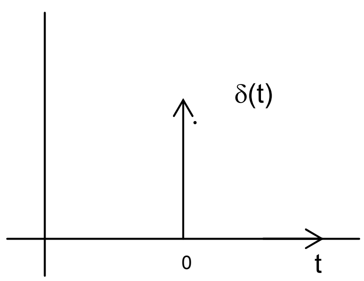


Fig 8.5.1(a) Impulse function

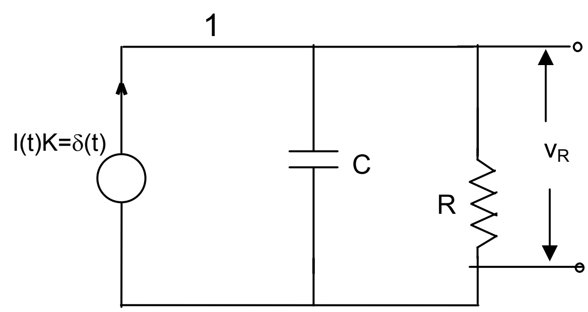


Fig 8.5.1(b)

The integral term in the left side of this equation must be zero as no excitation is applied before t=0. Therefore we see that

$$v(0+)=\frac{K}{C}$$

We conclude that the impulse of current applied by the current source has established an initial condition on the previously uncharged capacitor. Or simply we can say that the effect of impulse is to store charger or establish a voltage on a capacitor The response of the capacitor voltage to the application of such impulse of current at time t = 0 is thus easily found and is given by

$$v(t) = \frac{K}{C} e^{-t/RC} \quad t>0$$

8.5.2 Response of RL-circuit to impulse input

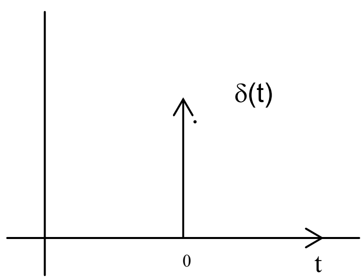
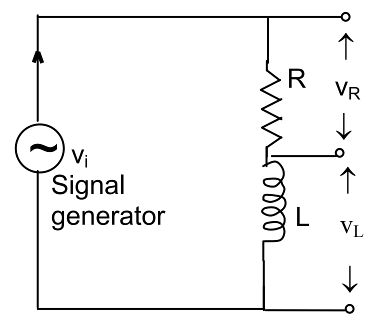


Fig 8.5.2(a) Impulse function



8.5.2(b) RL circuit with impulse input.

If an impulse of voltage of strength K is applied to the circuit by the voltage source at t=0. We assume the initial condition at t =0- is zero. From Kirchhoff's voltage law, we obtain

$$L\frac{di}{dt} + Ri(t) = v(t) = K\delta(t)$$

Integrating from – to 0+ , we obtain

$$Li(0+) + R\int_{-\infty}^{0+} i(\tau)d\tau = \int_{-\infty}^{0+} v(\tau)d\tau = K$$

As before, the integral in the left member of this equation is zero, and we see that the impulse of voltage has produced an initial condition of current in the circuit, which has the value

$$i(0+) = \frac{K}{L}$$

Thus the solution for i(t) is

$$i(0+) = \frac{K}{L} e^{-Rt/L} \qquad t > 0$$

It should be noted that in both the examples given above, the use of impulses has violated the continuity condition for the energy-storage element concerned. We may summarize the results given in the examples above as follows:

The use of impulses to generate initial conditions: An impulse of current applied by current source in parallel with a capacitor will generate an initial condition of voltage on the capacitor. An impulse of voltage applied by a voltage source in series with an inductor will generate an initial condition of current in the inductor. i.e It establishes a current. Since there is no forced response to an impulse, the complete response is the natural response of the circuit.

8.4.1 A.C. Transient response of RC-circuit

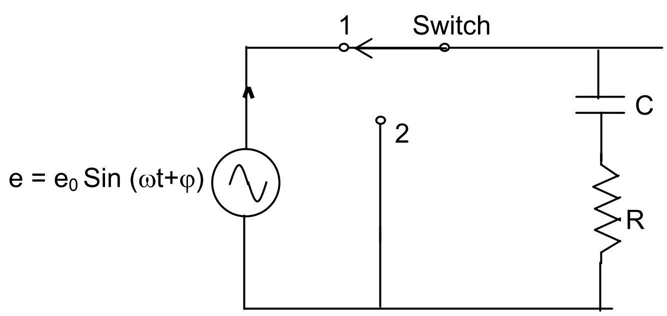


Fig 8.2.2(c)

In the case, the applied voltage is sinusoidal [similar to sine wave]. The solution of differential equation for current consists of complementary function and a particular integral representing the transient state and study state solution respectively. The voltage drops across resistance and capacitors are.

$$E = V_R + V_C$$

$$E_0 \sin \omega t = Ri + \frac{1}{c}\int idt$$

Where *i* is instantaneous value of current at time 't'. Differentiating w.r.t 't'

$$E_0 \omega \cos \omega t = R\frac{di}{dt} + \frac{1}{c}$$

$$\frac{di}{dt} + \frac{1}{RC} i = E_0 \omega \cos \omega t$$

The complete solution is $i = \dfrac{E_0}{\sqrt{R^2 + \dfrac{1}{C^2\omega^2}}} \times \sin[\omega t + \alpha] + Ae^{-t/RC}$

where $\alpha = \dfrac{1}{\omega CR}$ and A is constant of integration.

$$i = i_0 \sin[\omega t + \alpha] + Ae^{-t/RC}$$

The value of constant A can be obtained from knowledge of initial condition the first part i.e., $i = i_0 \sin[\omega t + \alpha]$ gives the study state component while the second part gives transient response if circuit is closed at t = t^1 so that i = 0 then

$$\phi = Ae^{-t/RC}$$

$$0 = i_0 \sin(\omega t^1 + \alpha) + Ae^{-t^1/RC}$$

$$\therefore A = \frac{i_0 \sin(\omega t^1 + \alpha)}{e^{-t^1/RC}}$$

Substituting A in above eq. we get

$$i = \sin(\omega t^1 + \alpha) - \frac{i_0 \sin(\omega t^1 + \alpha)}{e^{-t^1/RC}}$$

$$i = i_0 \sin(\omega t^1 + \alpha) - i_0 \sin(\omega t^1 + \alpha) e^{-t^1/RC}$$

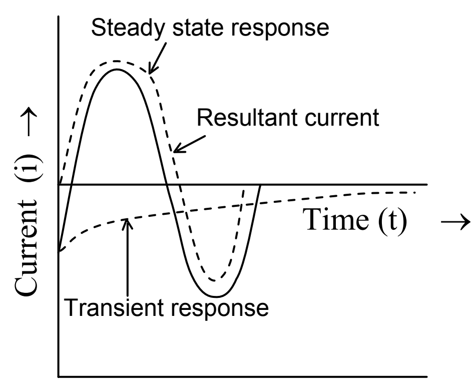


Fig 8.2.2(d) A.C. Response curve

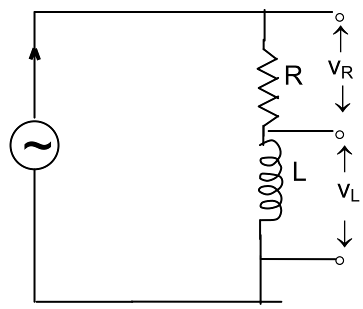
The effect of transient response to cause dissymmetric in the first few cycles. This soon disappears and the current acquires the normal sinusoidal wave from.

8.4.2 Transient response of RL-circuit with AC source

First method:

Let a voltage given by e = E_0 Sinωt be suddenly applied across an RL - circuit at a time t = 0, the resultant current is given by

$$i = i_s + i_t$$



8.3.2(b) RL circuit with Ramp input.

where i_s is steady state current and i_t is transient current. The value of steady state current is found by normal circuit theory. The peak steady state current is given by

$$i_m = \frac{V_0}{\sqrt{R^2 + X_1^2}} = \frac{V_0}{Z}$$

where $\sqrt{R^2 + X_1^2}$ is impedance of the circuit. This current lags behind applied voltage by an angle ϕ such that $\tan\phi = \frac{X_1}{R}$

$$\therefore \phi = \tan^{-1}\left[\frac{X_1}{R}\right]$$

Hence the equation for instantaneous value of the steady state current becomes

$$i_s = i_m \sin[\omega t + \psi - \phi] \qquad \text{--------------} \quad (8.4.2)$$

As before, the transient current is

$$i_t = i_0 e^{-\frac{RT}{L}} \qquad \text{-------------- (8.4.3)}$$

Adding equations (8.4.2) and (8.4.3) gives

$$i = i_m \sin[\omega t + \psi - \phi] + i_0 e^{-\frac{Rt}{L}} \qquad \text{-------------- (8.4.4)}$$

Now when *t* = 0, *i* = 0 then the above equation becomes

$$0 = i_m \sin \psi - \phi + i_0$$

$$\therefore i_0 = -i_m \sin[\psi - \phi]$$

sub in (3) $i = i_m \sin[\omega t + \psi - \phi] - i_m \sin[\psi - \phi] e^{-\frac{RT}{L}}$ -------------- (8.4.5)

From the above equation it is seen that the value of i_O, and hence the size of transient current depend on i.e. it depends on the instant in the cycle at which the circuit is closed.

Second method

Let an a.c. voltage E = E_0 sin ωt be impressed across R – L circuit, shown in Fig.8.3.2(b), then

$$E = V_R + V_L$$

or $$E_0 \sin \omega t = Ri + L\frac{di}{dt}$$

where *i* is the instantaneous value of current at time t. Rearranging the above equation, we have,

$$\frac{di}{dt} + \frac{R}{L} i = \frac{E_0}{L} \sin \omega t$$

Its complete solution is, $i = \frac{E_0}{L} \cdot e^{-\frac{R}{L}\cdot t} \int e^{\frac{R}{L}\cdot t} \cdot \sin \omega t dt + A e^{-\frac{R}{L}\cdot t}$

where A is constant of integration.

The above equation may be solved to give,

$$i = \frac{E_0}{L} \cdot e^{-\frac{R}{L}\cdot t} \left[\frac{e^{\frac{R}{L}\cdot t}\left(\frac{R}{L}\sin \omega t - \omega \cos \omega t\right)}{\frac{R^2}{L^2} + \omega^2} \right] + A e^{-\frac{R}{L}\cdot t}$$

$$= \frac{E_0}{L\left[\frac{R^2}{L^2} + \omega^2\right]} \cdot \frac{R\sin\omega t - \omega L\cos\omega t}{L} + Ae^{-\frac{R}{L}\cdot t}$$

$$= \frac{E_0}{\left(R^2 + \omega^2 L^2\right)} \cdot \sqrt{(R^2 + \omega^2 L^2)} \cdot \left[\frac{R}{\sqrt{(R^2 + \omega^2 L^2)}}\sin\omega t - \frac{\omega L}{\sqrt{(R^2 + \omega^2 L^2)}}\cos\omega t\right] + Ae^{-\frac{R}{L}\cdot t}$$

$$= \frac{E_0}{\sqrt{(R^2 + \omega^2 L^2)}}\sin(\omega t - \propto) + Ae^{-\frac{R}{L}\cdot t} = i_0 \sin(\omega t - \propto) + Ae^{-\frac{R}{L}\cdot t} \quad \text{------------ (8.4.6)}$$

where, $\propto = \tan^{-1}\left(\frac{\omega L}{R}\right)$

and $i_0 = \frac{E_0}{\sqrt{\left(R^2 + \omega^2 L^2\right)}}$

The constant of integration can be obtained from a knowledge of initial conditions.

The first part in equation (8.4.6) [= i_0 sin (ωt - ∝)] represents the steady state component, while the second part [=$Ae^{-(R/L).t}$] represents the transient part. The transient part causes dissymmetry for first few cycles of the current wave and after that current approaches its normal (steady state) value.

If the current I = 0 at t = t′ then equation (8.4.6) becomes

$$0 = i_0(\omega t' - \propto) + Ae^{-\frac{R}{L}\cdot t'}$$

$$A = -i_0 \sin(\omega t' - \propto) + e^{\frac{R}{L}\cdot t'}$$

substituting this value of A in equation (8.4.6), we get,

$$i = i_0 \sin(\omega t - \propto) - i_0 e^{\frac{R}{L}\cdot(t' - t)} \cdot \sin(\omega t' - \propto) \quad \text{------------ (8.4.7)}$$

Here again the first represents the steady state component and the second part represents the transient component.

Fig.8.3.2(c) gives the wave shapes of steady state and transient currents for sinusoidal impressed voltage. Now let us consider some special cases.

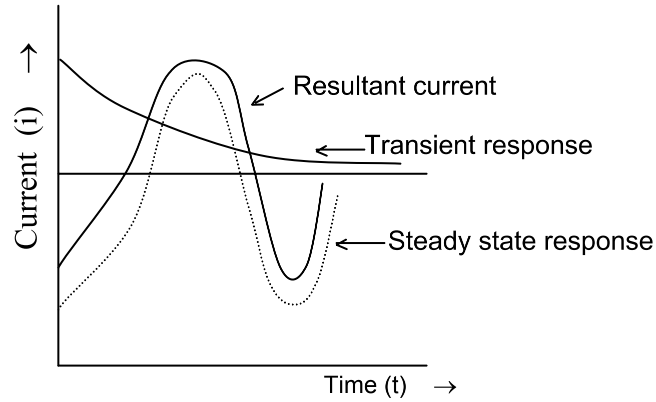


Fig.8.3.2(c) A.C. Response curve of LR Circuit

If R is small compared to L, i.e., R<<L, then

$$\propto = \frac{\pi}{2},$$

and equation (2) becomes

$$i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right) - i_0 e^{\frac{R}{L}\cdot(t'-t)} \cdot \sin\left(\omega t' - \frac{\pi}{2}\right)$$

(A) If the switch is closed at the instant when *E* is passing through its maximum value, i.e.,

$$\omega t' = \frac{\pi}{2}$$

then above equation gives, $i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right)$

i.e., the total current is sinusoidal and contains no transients.

(B) If switch is closed at the instant when *E* is passing through zero,

therefore, $t' = 0$ or $\omega t' = 0$

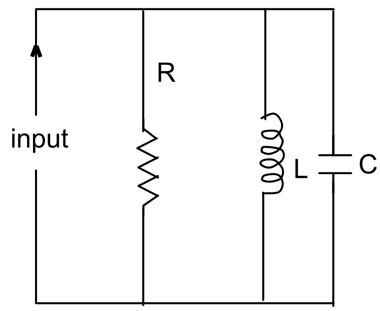
Hence $i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right) - i_0 e^{-\frac{R}{L}\cdot t} \sin\left(-\frac{\pi}{2}\right) = i_0 \sin\left(\omega t - \frac{\pi}{2}\right) + i_0 e^{-\frac{R}{L}\cdot t}$

The current thus contains both the steady and transient components. During the early period after switching $e^{-(R/L).t}$ does not decay rapidly from the value of unity and the current is, therefore, approximately gives by,

$$i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right) + i_0.$$

The maximum value of i is $2i_0$ and is reached at $\omega t = \pi/2$. This phenomenon is known as doubling.

8.6. The ringing circuit for pulse generation:



8.3.2(d) Parallel RLC Circuit.

This circuit is used typically to generate sequence of pulses spaced regularly in time. The ringing circuit employs a parallel RLC combination as in Fig.8.3.2(d) with the resistance of L included in R. If the current in L at t = 0 is i_0, then using Kirchhoff's law.

$$i_L + i_R i_C + i_0 = 0$$

Or $$\frac{1}{L}\int v\,dt + \frac{v}{R} + C\frac{dv}{dt} + i_0 = 0$$

On differentiating and dividing by C

or $$p^2 v + \frac{1}{RC}pv + \frac{1}{LC}v = 0$$

Where $p \equiv \frac{d}{dt}, p^2 \equiv \frac{d^2}{dt^2}$

The roots of this equation are

$$p = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{4R^2C^2} - \frac{1}{LC}\right)}$$

The case of interest is that of under damping, where the radical becomes

$$\sqrt{\left(\frac{1}{4R^2C^2} - \frac{1}{LC}\right)} = j\sqrt{\left(\frac{1}{LC} - \frac{1}{4R^2C^2}\right)} = j\omega$$

The solution of the differential equation is,

$$v = \left(K_1 e^{j\omega t} + K_2 e^{-j\omega t}\right) e^{-\frac{t}{2RC}}$$

Nothing that at t = 0, v = 0 and $\dfrac{dv}{dt} = \dfrac{i_0}{C}$, the solution to the original equation is,

$$v = \frac{i_0}{\omega C} e^{-\frac{t}{RC}} \cdot \sin \omega t$$

The result is oscillatory, at a frequency,

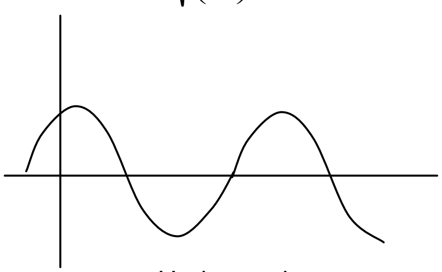
$$f = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC} - \frac{1}{4R^2C^2}\right)}$$

And the circuit voltage is damped by the factor $e^{-t/2RC}$. If the losses are small, as required for the under damped case, then the maximum energy stored in the inductor is transferred substantially without loss to the capacitor during the first fraction of a cycle. Thus,

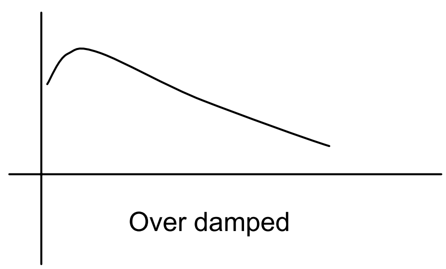
$$\frac{Cv_{\max}^2}{2} = \frac{1}{2Li_0^2}$$

and the peak voltage during the first half cycle will be

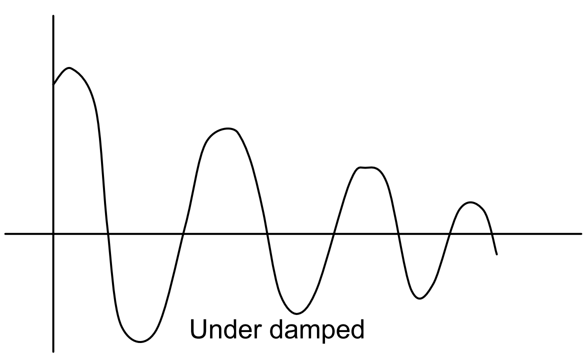
$$v_{\max} = -i_0\sqrt{\left(\frac{L}{C}\right)}.$$



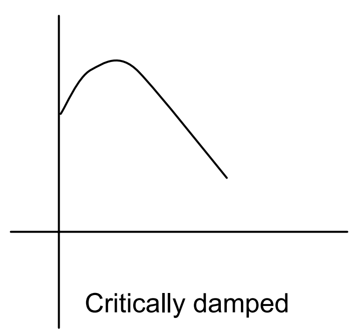
Undamped



Over damped



Under damped



Critically damped

The above mentioned discussion holds good for series LCR circuit and the derivation is left to the student as an exercise.

If the RLC circuit excited impulsively, it will oscillate or ring at its own natural frequency. The RLC circuit is generally connected into the output of an electronic circuit and a high amplitude square wave signal is applied into the input of this electronic circuit. During the positive half circuit of the input signal, if the output current is maximum say i_0, the output circuit resistance will be minimum and because of this damping the RLC circuit cannot oscillate. During negative half cycle of the square wave, the output current of the electronic circuit will be zero, the RLC circuit is effectively disconnected and left free to oscillate or ring at its own natural frequency.

8.7 Summary

1. The Transient response of an inductive circuit with non-sinusoidal current is indicated by the time constant L/R.
2. The transient response of a capacitive circuit with non sinusoidal voltage is indicated by the time constant 'RC'
3. With a.d.c. applied across an R-L circuit, the transient current is given by $i_{transient} = i_0\left(1 - e^{-(R/L)t}\right)$
4. Transient current for decay in R-L circuit is given by $i_{transient} = i_0 e^{-(R/L)t}$
5. With a. d.c. is applied across R-C circuit, the transient current is given by $i_{transient} = i_0 e^{-(t/RC)}$ and charge is given by $\upsilon_{transient} = \upsilon_0\left(1 - e^{-t/RC}\right)$
6. The transient part causes dissymmetry of the current in the beginning then it becomes vanishingly small and after that current approaches its normal (steady state) value.

8.8 Key terminology

Transient condition – Time constant – Step function – Ramp function – Impulse function – Ringing – Damping.

8.9 Self-assessment questions

Short answer questions

1. What is transient response?
2. Draw the transient response graph of RL- circuit?
3. Draw the transient response graph of RC- circuit?
4. What is meant by Ringing in LCR- circuit?

Long answer questions

1. What is Transient response? What are different types of transients?
2. Discuss the transient response of RC- circuit containing DC source.
3. Discuss the transient response of RL- circuit containing DC source.
   (Or) Derive equations of growth and decay of current in a circuit containing inductance and resistance. What is meant by time constant of the circuit?
4. Discuss transient response of RL-circuit to an AC signal.
5. Discuss transient response of RC-circuit to an AC signal.
6. Discuss the transient response of RLC-circuit. Why this circuit is called double energy transients?
7. Derive an expression for the current flowing in a series LCR-circuit for an impulse input. Explain with diagrams, various types of damping conditions that may occur in a resonant circuit.

Numerical Problems

1. A condenser of capacity 0.1 nF is first charged and then discharged through a resistance of 10 mega-ohms. Find the time in which the potential will fall to half its original value (log,, 2 = 0.6931). (***Ans:*** 0.6931 sec)
2. A condenser is being charged from a d.c. source through a resistance of 2 mega-ohms. If it takes 0.5 sec., for the charge to reach three-quarters of its final value, what is the capacity of the condenser? (***Ans:*** 0.18 μF)

3. A $4\mu F$ condenser is connected to a $250V$ supply through a 0.25 MΩ resistor. Calculate the potential difference between the terminals of the condenser 0.1 sec after the application of voltage. Also calculate the initial charging current. (***Ans:*** 24.88V, 10"3 amp.)
4. An inductor of self inductance 500mH and resistance 5Ω is connected to a battery of negligible internal resistance. Calculate the time in which the current will attain half of its steady value. (***Ans:*** 0.0693 sec).
5. A direct voltage of 120V is applied to a circuit having an inductance of 10H and a separate resistance of 5Ω. If the resistance of the inductance coil is 5Ω, calculate
   (i) The current 1 sec. after switching and
   (ii) The p.d. across the inductance. (***Ans:*** (i) 7.598 amp (ii) 37.495 *V*)
6. A 2V battery of negligible internal resistance is connected to a coil of inductance 1H and of resistance 1Ω. Calculate the time required by the current to attain a value half of that in the steady state. (***Ans:*** 0.6931 sec.)
7. A charged condenser of capacity $4\mu F$ is shunted by high resistance. If half the charge leaks through in 50 seconds, calculate its resistance. (***Ans:*** 18.MΩ)
8. A capacitance of I00 μF is connected in series with a reactance of 8000Ω. Estimate the time constant of the circuit. (***Ans:*** 0.8 sec.)
9. A charged capacitor of 4.μF is shunted by a high resistance. If half the charge leaks through in 50 seconds, calculate the voltage of the resistance. (***Ans:*** 36 MΩ)
10. Capacitor of capacitance 0.5 μF is discharged through a resistance of 10 M£2. Find the time taken for half the charge on the capacitor to escape. (***Ans:*** 3.46 sec.)
11. A constant e.m.f. is applied to a circuit containing a resistance R and capacitance C in series. Find the time in which the capacitor acquires 90% of the total charge. (***Ans:*** 2.3026 RC sec.)
12. A solenoid having a resistance of *5 0hms,* and self-inductance of 4H is connected to a battery of e.m.f. 10V and negligible resistance. After how long will the current in it rise to 1 amp. ? (***Ans:*** 0.55 sec.)
13. A potential difference of 100V is applied to a circuit consisting of a resistance of 50Ω and an inductance of 5H. Find the current in the circuit 0.1 sec. after the application of voltage. (***Ans:*** 1.264A)

14. A coil having a resistance of 15Ω and an inductance of 10H is connected to 90V supply. Determine the value of current (i) after 0.67 second and (ii) after 2 second. (***Ans:*** (i) 3.83 A (ii) 5.7A)
15. A damped oscillation has the equation $I = 50\ e^{-10\ t} \sin 628\ t$. Find the number of the oscillations which will occur before the amplitude of the oscillations decays to 1/10th of its undamped value. (***Ans:*** 23)
16. A 5F capacitor is discharged suddenly through a coil having an inductance of 2H and a resistance of 200Ω. The capacitor is initially charged to a voltage of 10V. Find (a) an expression for the current and (b) the additional resistance required to give critical damping. (***Ans:*** (a) $i = 0.016\ e^{-50t} \sin 312.3t$ (b) 1065 ohms.)

8.10 References

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UNIT – III LESSON-9

RC AND RL CIRCUITS–II

Objectives of the lesson

In the first part of the lesson, we learnt about the transient behaviour of LR and RC circuits. We study the steady state behaviour and applications of these networks in the present lesson.

Structure of the lesson

9.1 Introduction

In the lesson 8, we had seen that the complete solution of differential equation governing the circuit consists of steady state and transient parts. In this lesson, we concentrate on steady state behavior of these circuits. The performance of a network over a range of driving frequencies is called the ***Frequency Response*** of the network.

9.2 Steady state frequency response of RC-network

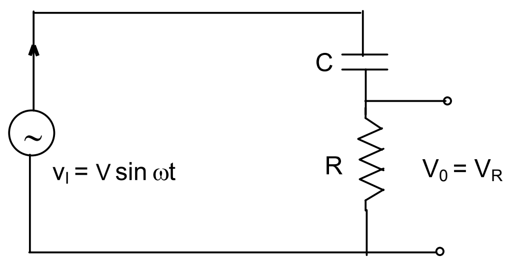


Fig 9.2(a)

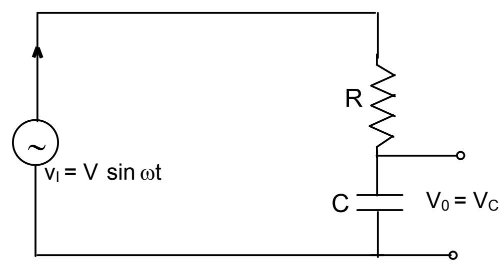


Fig 9.2(b)

The expression for current in an RC-circuit

The complete solution is $$i = \frac{V}{\sqrt{R^2 + \frac{1}{C^2\omega^2}}} \times \sin[\omega t + \alpha] + Ae^{\frac{-t}{RC}} \qquad (9.1)$$

where ω is angular frequency given by $\omega = 2\pi f$ and $\alpha = \frac{1}{\omega CR}$

The effect of transient response is to cause dissymmetry in the first few cycles. This soon disappears and the current acquires the normal sinusoidal waveform. Either the voltage across the capacitor or resistor can be taken as output voltage as shown in Figs 9.2(a) and 9.2(b). Let us consider the first circuit in Fig 9.2(a).

The total impedance of the circuit is $Z = R + \frac{1}{j\omega C}$

As we increase the frequency of the generator, the impedance decreases as the capacitive term decreases. Correspondingly, the current drawn from the generator increases. The output voltage v_0 which is equal to Ri also increases accordingly. At a frequency f_1 , the value of R equals the value of X_C.

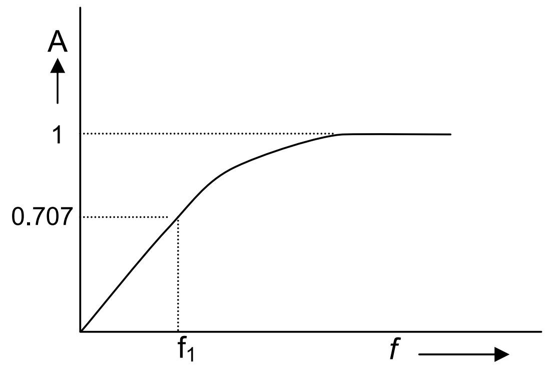


Fig 9.2(c) Frequency response of RC-circuit

An expression can be derived for f_1 and it can be shown to be $\frac{1}{2\pi CR}$. The ratio of output voltage to input voltage becomes $\frac{1}{\sqrt{2}}$. It is called conventionally ***Cut-off frequency***. Below cut off frequency, the output amplitude will be small. Above f_1 , the output voltage increases and equals input signal in value. This situation prevails until the hitherto negligible shunt capacitances start offering low shunt capacitance. This will reduce the effective output impedance and thereby output voltage stats decreasing as frequency increases. In ideal case, no shunt capacitances exist and output remains maximum after f_0 and remains constant for all frequencies above f_1**.** Out of all the frequencies generated by the generator, we find that output contains only those frequencies, which are above or higher than f_1. The frequency region which does not appear at the output is said to be ***attenuated*** and this frequency band is called ***attenuation band***. The band of frequencies, which appear in the output whose amplitude does not suffer attenuation, is called ***Pass band***. In the present case, the pass band consists of frequencies above f_1. If a signal, which is combination of several frequencies, is applied to this circuit, the frequencies below f_1 will not appear at the output. These are removed or filtered out. Hence this circuit which filters low frequencies (frequencies below f_1) and allows all high frequencies (above f_1) is called a ***High pass filter***.

Expression for cut-off frequency:

$$\frac{V_0}{V_i} = A = \frac{R}{\left(R + \frac{1}{j\omega C}\right)} = \frac{1}{\left(1 + \frac{1}{j2\pi fCR}\right)}$$

Let $f_1 = \dfrac{1}{2\pi CR}$

$$A = \frac{1}{\left(1 + \frac{f_1}{jf}\right)} = \frac{1}{\left(1 - \frac{jf_1}{f}\right)}$$

The gain is complex. Its magnitude is given by

$$|A| = \frac{1}{\sqrt{\left(1 + \frac{f_1}{f}\right)^2}} \qquad \text{---------(9.2)}$$

and phase angle is given by

tan $\phi = \dfrac{f_1}{f}$ or ϕ = arc tan $\dfrac{f_1}{f}$. ------------(9.3)

9.2 Steady state A.C frequency response of RC-circuit

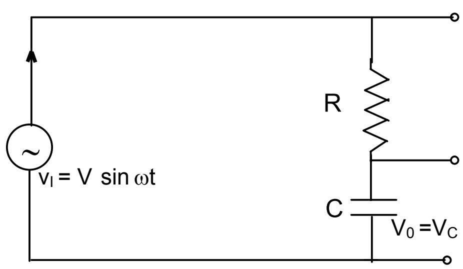


Fig 9.2(d)

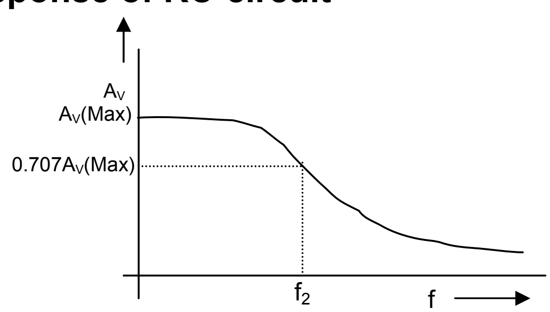


Fig 9.2(e) Steady state A.C response of RC-circuit.

If the input signal is written as $v_i = V \sin \omega t$

Then output voltage $V_o = \dfrac{V \sin \omega t \cdot \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$

$$Gain\ A = \frac{V_0}{V_i} = \frac{1}{1 + j\omega CR}$$

Let $f_2 = \dfrac{1}{2\pi RC}$

$$|A| = \frac{1}{\sqrt{\left\{1 + \left(\frac{f}{f_2}\right)^2\right\}}}$$ and phase angle $\theta = \tan^{-1}\left(\dfrac{f}{f_2}\right)$

The frequency f_2 is called upper 3-db frequency.

9.3 Steady state frequency response of LR- and RL-circuits

Steady state frequency response of LR-and RL-circuits can be worked out on similar grounds as is done in the case of CR-and RC-networks. The derivation is left as an exercise for students.

9.4 FILTERS

Filter is an electronic circuit as a reactive network, that freely pass desired range of frequency while almost suppressing other range of frequency. The concept of filters was given by G.A. Campbell & O.J. Zobel of Bell telephone laboratories.

Types of Filters

Filters may be active filters or passive filters depending upon the active and passive components present in them.

Depending upon the range of frequencies allowed or disallowed, filters may be of the following types :

i) Low Pass filters ii) High Pass filters iii) Band Pass filters iv) Band Elimination filters.

The filters, if they are passive, may contain resistors, capacitors and inductors. Hence there are RC, LC and RL – filters. The filters can be used as differentiating and integrating circuits.

9.4.1 RL – filters

An RL – filter can be either a low pass or a high pass-filter.

RL – Low pass filter

A low pass filter will allow a range of frequencies below the cut off frequency very easily, but disallows the range of frequencies greater than f_c with much more attenuation. A series combination of resistor R and an inductor L can act as a low pass filter if the output is taken across the resistance R as shown in Fig 9.4.1(a).

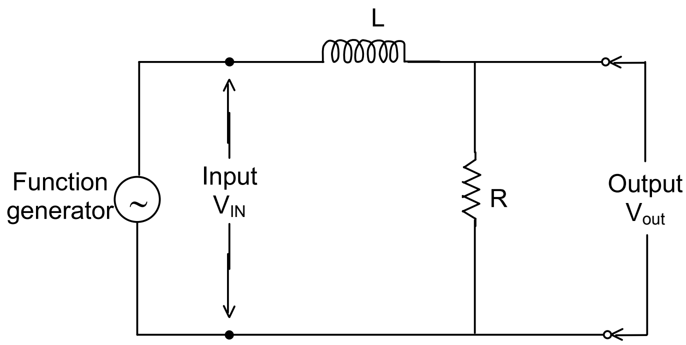
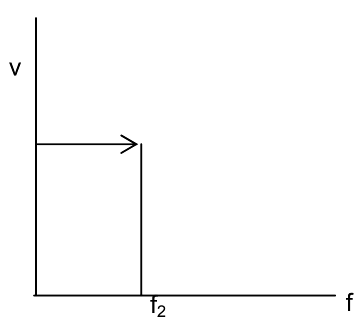


Fig 9.4.1(a) RL – Low pass filter

Working

Since the reactance of an inductor increases with increasing frequency (because $X_L = 2\Pi f L$) , as a series element, it allows the sine waves with frequencies below f_C to reach the output easily and disallow high frequency signals (i.e., above f_C , inductor acts almost as an open-circuit because of its high reactance) to reach the output. Thus the series RL-circuit, with output taken across the resistor, can act as a low pass filter.

Frequency response curve



9.4.1(b) Ideal RL-Low pass filter

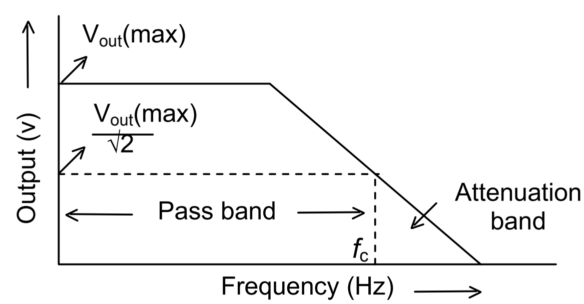


Fig 9.4.1(c)

Since the reactance of the inductor increases with increasing frequency, the RL-low pass filter allows low frequency signals readily but attenuates the high frequency signals.

Analysis

The R.M.S. value of output voltage $V_{out} = \frac{R.I_{\max}}{\sqrt{2}}$

The R.M.S. value of input voltage $V_{IN} = I_{RMS}\sqrt{R^2 + \varpi^2 L^2} = \frac{I_{nax}}{\sqrt{2}}\sqrt{R^2 + \varpi^2 L^2}$

$$\therefore \frac{V_{OUT}}{V_{IN}} = \frac{\frac{R \cdot I_{\max}}{\sqrt{2}}}{\frac{\sqrt{R^2 + \omega^2 L^2} \cdot I_{\max}}{\sqrt{2}}} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\Rightarrow \frac{V_{OUT}^2}{V_{IN}^2} = \frac{R}{R^2 + 4\pi^2 f^2 L^2}$$

$$(\because \omega = 2\pi f)$$

At cut off frequency, $f = f_c$ then

$$\Rightarrow \frac{V_{OUT}}{V_{IN}} = \frac{1}{\sqrt{2}} \Rightarrow \frac{V_{OUT}^2}{V_{IN}^2} = \frac{1}{2}$$

From above equations

$$\frac{1}{2} = \frac{R}{R^2 + 4\pi^2 f_C^2 L^2}$$

$$\Rightarrow R^2 + 4\pi^2 f_C^2 L^2 = 2R^2$$

$$\Rightarrow 4\pi^2 f_C^2 L^2 = R^2 \Rightarrow 2\pi f_C L = R$$

$$\Rightarrow f_C = \frac{R}{2\pi L}\ Hz \qquad \text{------------- (9.4)}$$

From eq.(9.4), it is clear that the cutoff frequency depends on R and L values. The frequency response curve of RL-Low pass filter is shown in Fig. 9.4.1(c)

. This curve gives us an idea of how the output voltage or gain varies with frequency of input signal.

R L- High pass filter

A high pass filter will allow range of frequencies above the cutoff frequency very easily but disallows range of frequencies less than f_C with much more attenuation. A series combination of inductor L and resistor R can act as high pass filter if the output is taken across the inductance L.

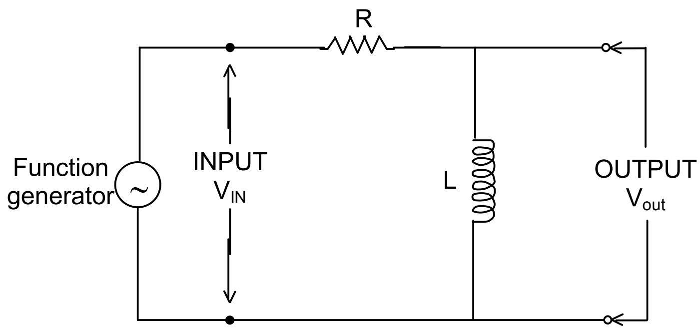


Fig 9.4.1(d) RL – High pass filter

Analysis:

The RMS value of output voltage $V_{OUT} = I_{RMS} \cdot X_L = \frac{I_{nax}}{\sqrt{2}} \cdot \omega L$

The RMS value of input voltage $V_{IN} = I_{RMS}\sqrt{R^2 + \varpi^2 L^2} = \frac{I_{nax}}{\sqrt{2}}\sqrt{R^2 + \varpi^2 L^2}$

$$\frac{V_{OUT}}{V_{IN}} = \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}}$$

At cut off frequency, $f = f_c$ then $\frac{V_{OUT}}{V_{IN}} = \frac{1}{\sqrt{2}}$

From the above equations we get

$$\frac{1}{\sqrt{2}} = \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}} \Rightarrow \frac{1}{2} = \frac{\omega^2 L^2}{R^2 + \omega^2 L^2}$$

$$\Rightarrow R^2 = \omega^2 L^2 \Rightarrow R = \omega L \Rightarrow \omega = \frac{R}{L}$$

$$\Rightarrow 2\pi f_C = \frac{R}{L} \Rightarrow f_C = \frac{R}{2\pi L} \text{ Hz} \qquad \text{------------- (9.5)}$$

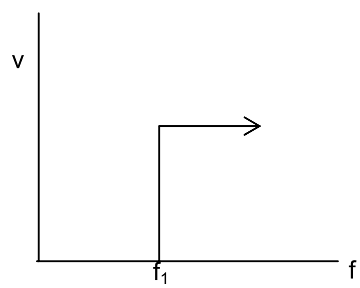
From eq.(9.5), it is clear that the cutoff frequency depends upon R and L values.

Working

Since the reactance of an inductor increases with increasing frequency (because $X_L = 2\Pi f L$), as a shunt element, it allows the sine waves with frequencies above f_C to reach the output easily and disallow low frequency signals (i.e., above f_C , inductor acts almost as a short circuit because of its low reactance) to reach the output. Thus the series RL-circuit, with output taken across the inductor, can act as a high pass filter.

Frequency response curve

The frequency response curve of RL-high pass filter is shown in Fig.9.4.1(f). This curve gives us an idea of how the output voltage or gain varies with frequency of the input signal.



9.4.1(e) Ideal High-pass filter

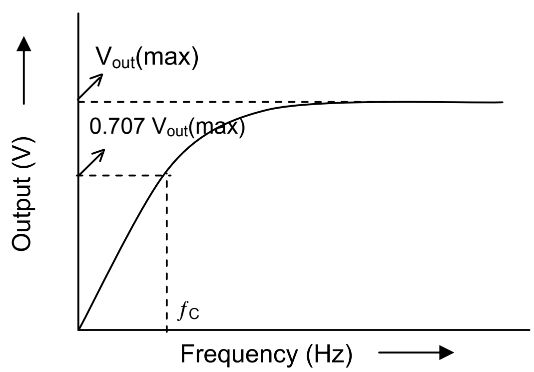


Fig 9.4.1(f)

Since the reactance of inductor increases with increasing frequency, the RL-high pass filter allows high frequency signals readily but attenuates the low frequency signals.

9.4.2 RC-filters

A filter can also be constructed by using passive components like resistor and capacitor. A series combination of R, C can act as a filter.

RC-filter may be either low pass or high pass.

RC – Low pass filter

A low pass filter will allow a range of frequencies below the cut-off frequency very easily but disallows range of frequencies greater than f_C with much more attenuation.

A series combination of RC can act as a low pass filter if the output is taken across 'C' as shown in Fig.9.4.2 (a). Figure shows the circuit of an RC-low pass filter. Since the reactance of a capacitor decreases with increasing frequency, it acts almost as a short circuit at high frequencies. Thus at low frequencies, the input appears at the output due to high impedance or reactance of shunt capacitor. At low frequencies, the capacitor acts as an open circuit and the circuit acts as a low pass filter.

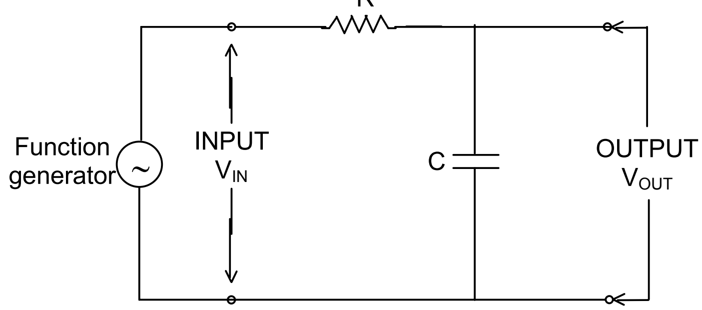


Fig 9.4.2(a) RC – Low pass filter

Analysis:

Let the RMS value of the output voltage be V_{OUT} and let the RMS value of the input voltage be V_{IN}. I_{max} be the max value of the input current.

RMS value of output voltage $V_{OUT} = \frac{I_{\max}}{\sqrt{2}} \cdot X_C$

RMS value of input voltage $V_{IN} = \frac{I_{\max}}{\sqrt{2}}\sqrt{R^2 + X_C^2} = \frac{I_{\max}}{\sqrt{2}}\sqrt{R^2 + \frac{1}{\omega^2 C^2}}$

$$\therefore \frac{V_{OUT}}{V_{IN}} = \frac{\frac{I_{\max}}{\sqrt{2}} \cdot \frac{1}{2\pi f C}}{\frac{I_{\max}}{\sqrt{2}}\sqrt{R^2 + \frac{1}{4\pi^2 f^2 C^2}}}$$

at cut off frequency, $f = f_c$ then, $\frac{V_{OUT}}{V_{IN}} = \frac{1}{\sqrt{2}}$

$$\frac{1}{2}=\frac{\left(\frac{1}{2\pi f_C C}\right)^2}{R^2+\frac{1}{4\pi^2 f_C^2 C^2}}=\frac{1}{4R^2\pi^2 f_C^2 C^2+1}$$

$$4R^2\pi^2 f_C^2 C^2+1=2$$

$$\text{or} \quad 4R^2\pi^2 f_C^2 C^2=1$$

$$\text{or} \quad f_C^2=\frac{1}{4R^2\pi^2 C^2} \Rightarrow f_C=\frac{1}{2\pi\, \mathrm{R}C} \qquad \text{-------- (9.6)}$$

The cutoff frequency f_C depends on the values of R and C.

Frequency response curve

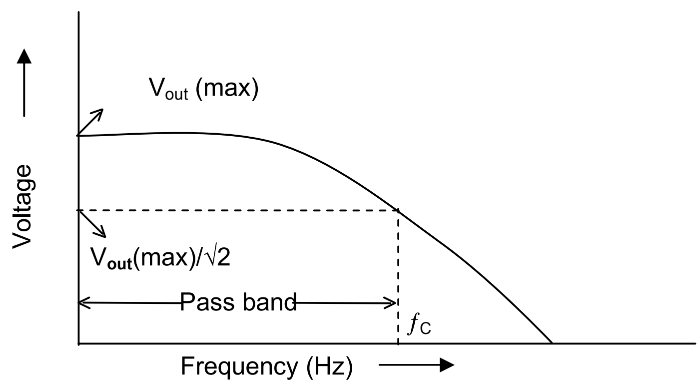


Fig 9.4.2(b)

The above figure shows the frequency response curve of RC-low pass filter. It is a curve drawn between frequencies and output voltage. This gives us an idea about how the output voltage or voltage gain varies with frequency.

RC – high pass filter

Def: A high pass filter will allow a range of frequencies above the cutoff frequency very easily but disallows the range of frequencies below the cutoff frequency with much more attenuation.

A series combination of RC can acts as a high pass filter if the output is taken across resistor, R as shown in Fig.9.4.2(c)

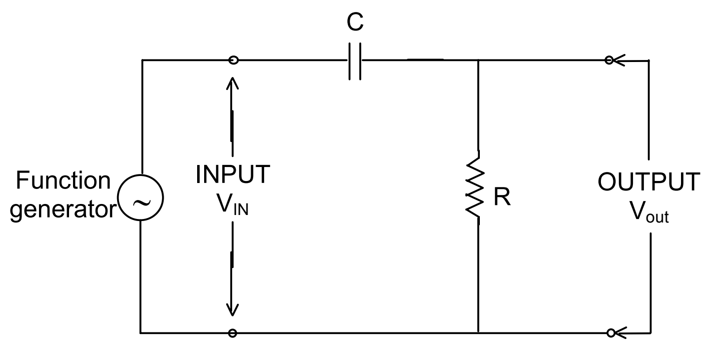


Fig 9.4.2(c) RC – High pass filter.

Working:

Figure 9.4.2(c) shows the circuit of RC–high pass filter. Since the reactance of a capacitor decreases with increase in frequency, it acts almost as a short circuit at high frequencies. Thus at low frequencies, the input will not appear at the output due to high impedance or reactance of the series capacitor.

Analysis:

Let the RMS value of output voltage be V_{OUT} and R.M.S. value of input voltage be V_{IN}.

Let I_{max} be the maximum value of the input current.

RMS value of output voltage $V_{OUT} = \dfrac{I_{\max}}{\sqrt{2}} \cdot R$

RMS value of input voltage $V_{IN} = \dfrac{I_{\max}}{\sqrt{2}}\sqrt{R^2 + X_C^2} = \dfrac{I_{\max}}{\sqrt{2}}\sqrt{R^2 + \dfrac{1}{\omega^2 C^2}}$

$$\therefore \frac{V_{OUT}}{V_{IN}} = \frac{\dfrac{I_{\max}}{\sqrt{2}}R}{\dfrac{I_{\max}}{\sqrt{2}}\sqrt{R^2 + \dfrac{1}{\omega^2 C^2}}}$$

At cut off frequency, $f = f_c$ then $\dfrac{V_{OUT}}{V_{IN}} = \dfrac{1}{\sqrt{2}}$.

Hence above equation becomes

$$\frac{1}{2}=\frac{R^2}{R^2+\frac{1}{\omega^2C^2}}=\frac{R^2\omega^2C^2}{R^2\omega^2C^2+1}$$

$$2R^2\omega^2C^2 = R^2\omega^2C^2+1$$

$$R^2\omega^2C^2 = 1$$

$$2\pi f_C = \frac{1}{RC} \quad \Rightarrow f_C = \frac{1}{2\pi RC} \qquad \text{-------(9.7)}$$

The cutoff frequency f_C depends on the values of R and C.

The Fig.9.4.2(d) shows the frequency response curve of RC – high pass filter. It is a curve drawn between frequencies and voltage gains. This gives us an idea about how the voltage gain varies with frequency.

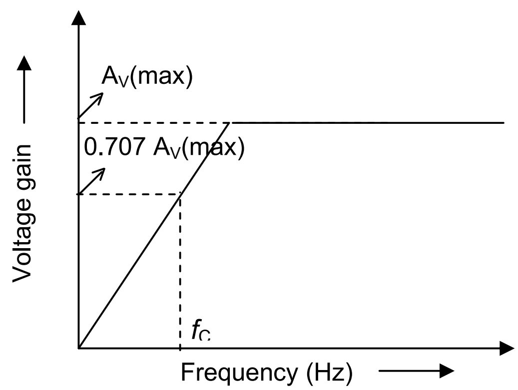


Fig 9.4.2(d)

From the above study on RC and RL circuits, we observe that the frequency response study and analysis of RC and RL filters is almost identical.

Application of filter circuits

One of the important use of filter circuits is in Rectifier circuits. Rectifier circuits provide pure D.C. voltages and currents by rectifying the alternating voltages. However, their output contains D.C. component and A.C components. The A.C components are called ***ripple*** and have frequency components at the input A.C frequency (in India the power line frequency is 50Hz) and its harmonics. As such the unregulated rectifier is not fit for using it as a D.C. power supply.

The unwanted ripple must be removed. A capacitance placed across the rectifier circuit comes in shunt with the input resistance of the device to which the rectifier circuit is connected to power it. The input resistance of the device and the capacitance connected across it form an RC filter. The value of capacitance is selected such that its reactances at the fundamental frequency of the input A.C. source is zero. The A.C ripple gets filtered out and only D.C. voltage appears across the load resistance. An inductance placed in series with load resistance form an LR-filter. A large value of inductance (in Henries) is used for this. The reactance of the inductor is so large such that ripple voltage drops across it and will not appear at the output. As the cost of inductor is high, sometime in low cost circuits, a resistor is used instead of an inductor. However, a resistor drops both A.C and D.C voltages. The series resistor drops some D.C. voltage and hence the D.C. voltage will be less than the required value for load. The heat generated may affect the performance of other components in the circuit. The RC and LC-filters are used to remove these A.C components. A resistance or inductance is used in series and the capacitance will be in shunt. These series inductance or resistance value is so selected such that it offers maximum resistance to A.C or ripple component. The shunt capacitance is so selected such that it offers zero resistance to A.C. components, so that the ripple component is bypassed and it will not appear at the output. The series resistance (or inductance) and ***shunt capacitance filter*** is called ***an L-section filter*** because of the appearance like English alphabet "L",

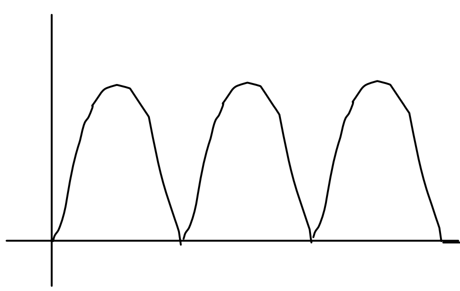


Fig:9.4.2.(e) Full wave rectifier output

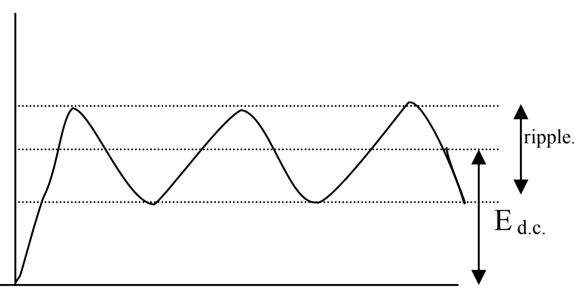


Fig:9.4.2.(f) The effect of capacitance on full wave rectifier output

. Two L-sections are connected to form a π- section filter. It can be shown that the π- section filters can more effectively remove ripple than single L-section filters. The performance of a filter to reduce ripple is measured by ripple factor, which is the ratio of the ripple output voltage to the D.C. voltage. LC filters are costlier to fabricate than RC filters, however very low D.C. resistance of inductances helps us in avoiding large D.C. voltage drops which otherwise occur if we use RC filter. Further, it can be shown that unlike RC-filter, an LC-filter ripple is independent of load resistance. When we use LR and LC-filters, the surge voltages produced during on and off of the circuit will be large and we have to use components which can withstand these voltages. The iron core inductors are not suitable for higher frequencies, as losses will be more. As load current increases, the D.C. output voltage of the filter decreases. The variation is more in capacitance input filters (Π-filters) when compared to Choke input filters (LC-filters). The steadiness of output D.C.voltage is expressed in terms of a parameter called ***voltage regulation*** which is defined as the ratio of the change in voltage with respect to no load voltage to the output voltage at maximum current drawn.

Resistances and capacitances can be connected in a complicated way to affectively remove a band of unwanted frequencies. For example, in bridge circuits, we balance a bridge at a frequency and obtain null voltage. They are nothing but filters, which eliminate a particular frequency. Twin-T filter, Wien bridge filters are examples of somewhat complicated RC-filters. We study about these in detail in bridge networks lesson. So, we do not discuss them here.

9.5 Differentiating & Integrating circuits

Introduction

The differentiator and integrator are the two wave shaping circuits for the input signal applied at the input terminals. RC and RL-filters can act as wave shaping circuits.

9.5.1 RC Differentiating circuit

A circuit in which the output voltage is directly proportional to the derivative of input voltage is known as a ***Differentiating circuit***. A high pass RC-circuit can work as a differentiating circuit (the output is taken across R).

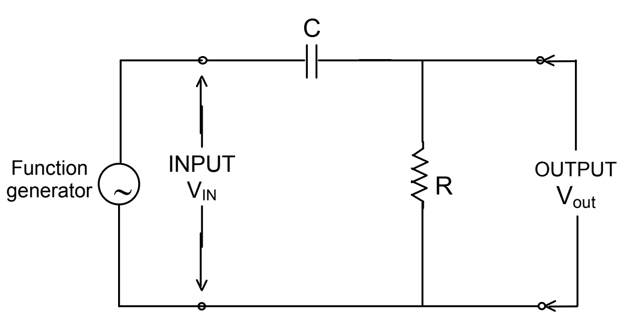


Fig 9.5.1(a) High pass RC differentiator.

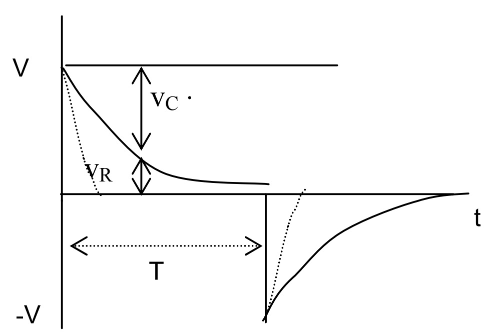


Fig 9.5.1(b) Performance of RC differentiating circuit

Condition: An RC-circuit can work as a differentiator only when the time constant RC is far less than the time period of the wave (i.e., **RC<<T**).

Analysis:

Let V_{IN} be the input voltage and V_{OUT} be the output voltage and i be the current flowing in the combination, we can write

$$V_{IN} = \frac{q}{C} + iR$$

$$V_{IN} = \frac{1}{C}\int idt + iR \qquad \text{-----------(9.8)}$$

If RC<<T, then the condenser will charge and discharge rapidly. This means that out of two terms in eq (9.8), the first term will predominate. Then the above equation becomes

$$V_{IN} \approx \frac{1}{C}\int idt \quad \text{or} \quad i = C\frac{d}{dt}(V_{IN})$$

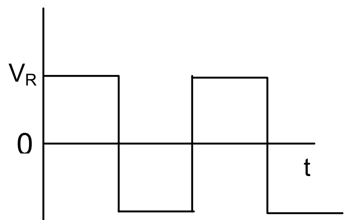
But V_{OUT} = iR . So $V_{OUT} = iR = C\frac{d}{dt}(V_{1N}) \cdot R = RC\frac{d}{dt}(V_{IN})$

$$V_{OUT} \;\alpha\; \frac{d}{dt}(V_{1N}) \qquad \text{-------------- (9.9)}$$

where the proportionality constant 'RC' is termed as the time constant of the circuit and is often represented by the symbol τ.

From eq.(9.9), it is clear that the output voltage is proportional to differential of input voltage. Hence the name differentiating circuit. When RC is less than T, the period of the applied pulse train, the output will be described by the dotted curve shown in Fig 9.5.1(b) which starts resembling the derivative for values of T>RC. Thus differentiation is erroneous near the origin i.e. at T<<RC, the error being maximum at t=0. It only provides the correct answer for T>>RC. That is why it is called pseudo differentiator.

The resistance R or capacitance C may be chosen sufficiently small to keep RC<<T. In practice, we cannot reduce R indefinitely. The reason for this is that a practical voltage source will have some source resistance though it may not be small. We have neglected it in our treatment as the introduction of source resistance Rs, in series will simply be added in R making time constant to C(R+R_s). Regarding output voltage, our previous conclusion will be true so long as R_s<<R. If R_s becomes comparable to R, the output voltage will be attenuated by a factor R/(R+R_s). It is for this reason R cannot be reduced indefinitely in practice.



9.5.1c

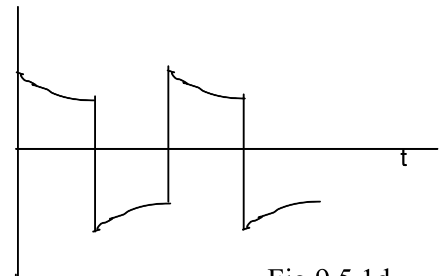


Fig 9.5.1d

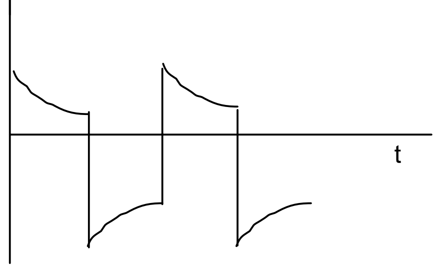


Fig 9.5.1e

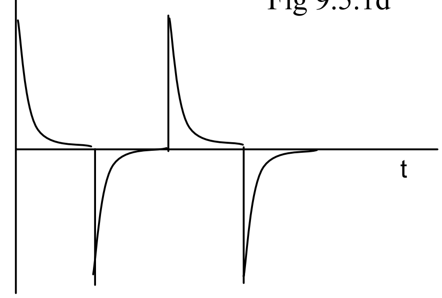


Fig.9.5.1f

Different Input & Output wave forms of a differentiator:

A differentiating circuit will produce a spike output corresponding to square wave input. In the same way it produces a square wave output corresponding to triangular input. Similarly it produces a cosine wave output corresponding to sine wave input.

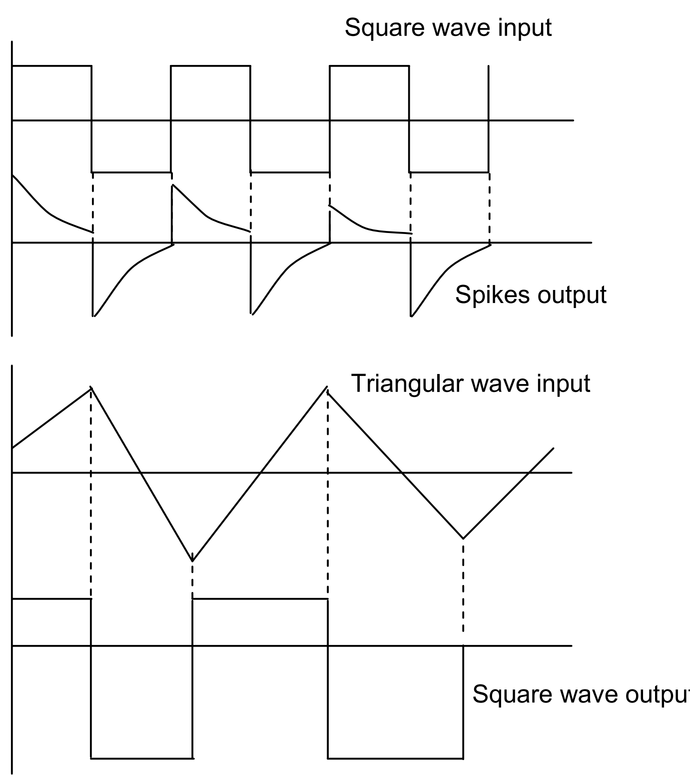


Fig.9.5.1(g)

9.5.2 RC-Integrator

A circuit in which the output voltage is directly proportional to the integral of input voltage is known as an integrating circuit or an ***Integrator.***

Condition: A low pass RC-circuit can work as an integrating circuit (output is taken across C). The circuit is shown in Fig.9.5.2(a) and it can work as an integrator only when the time constant RC of the circuit is far greater than the time period of the wave form (**RC>>T**).

Analysis

Let V_{IN} be the instantaneous input voltage, V_{OUT} be the output voltage and i be the current flowing in the combination.

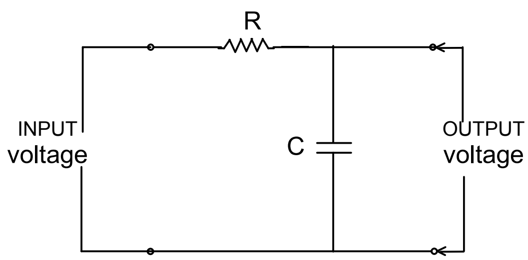


Fig 9.5.2 (a) RC-Integrator

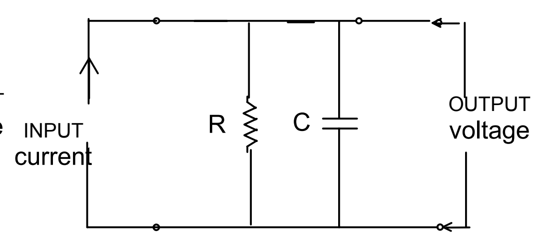


Fig. 9.5.2(b) RC-Integrator

Then we can write

$$V_{IN} = iR + \frac{1}{C}\int idt$$

or $$i = \frac{V_{IN}}{R} - \frac{1}{RC}\int idt$$

If the time constant (T) is very large as compared to the time period of the input signal, then the second term of the above equation will be very small and hence $i = \frac{V_{IN}}{R}$

$$V_{OUT} = \frac{1}{C}\int idt = \frac{1}{C}\int \frac{V_{IN}}{R} dt = \frac{1}{RC}\int V_{IN} dt$$

$$V_{OUT} \quad \alpha \quad \int V_{IN} dt \qquad \text{-------------- (9.9)}$$

Hence the circuit can act as an integrator if current forms the input variable and voltage across the capacitance is the output variable. As t becomes comparable to RC, the error in the output increases. Assume that $R \to \infty$, the current source now becomes an ideal current source (Fig

9.5.2.(b)) and the voltage across the capacitor becomes proportional to the true integral of the current. An RC-circuit with finite R behaves as a true integrator only when T<<RC in the asymptotic sense, it is called a ***Psuedo Integrator***.

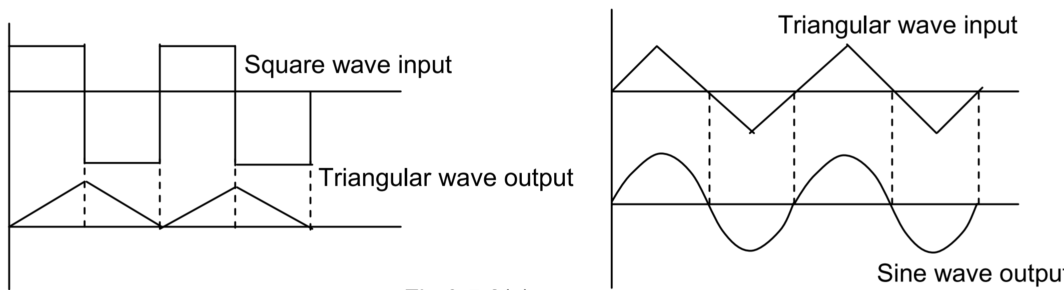


Fig.9.5.2(c)

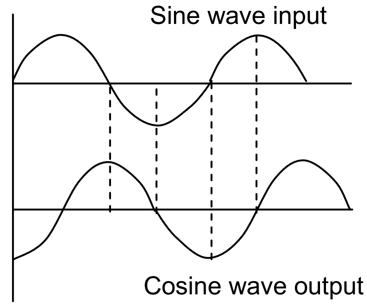


Fig 9.5.2(d) Integrator output for different input waveforms

An RC-integrator circuit can produce a triangular wave output corresponding to a square wave input. In the same way, it produces a sine wave output corresponding to a triangular wave input and similarly a cosine wave output corresponding to a sine wave input.

<u>RL – Circuits</u>

A series combination of RL-circuit can act as a differentiator or integrator depending upon whether the output is taken across 'L' or 'R'.

9.5.3 RL – Differentiator

A series combination of RL-circuit, working as a high pass filter, can act as a differentiator. In this differentiator, the output voltage is proportional to differential of input voltage. This circuit will act as a differentiator, if the time constant $\left(\tau = \frac{L}{R}\right)$ of the circuit is very much less than the time period of the input signal.

Circuit diagram:

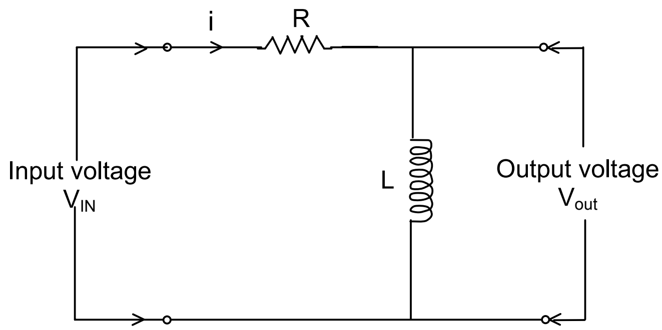


Fig. 9.5.3

Analysis:

Let V_{IN} be the input voltage, V_{OUT} be the output voltage, and i be the current flowing in the circuit.

From the Fig.9.5.3, we can write

$$V_{IN} = iR + L\frac{di}{dt}$$

or $$\frac{V_{IN}}{R} = i + \frac{L}{R}\frac{di}{dt}$$

Since $\frac{L}{R} << T$; the term $\frac{L}{R}\frac{di}{dt}$ is negligible as compared to i. Hence the above equation becomes

$$i = \frac{V_{IN}}{R}$$

But $$V_{OUT} = L\frac{di}{dt} = L\frac{d\left(\frac{V_{IN}}{R}\right)}{dt}$$

or $$V_{OUT} = \frac{L}{R}\frac{d(V_{IN})}{dt} \qquad \text{---------(9.10)}$$

or $$V_{OUT} \; \alpha \; \frac{d}{dt}(V_{IN})$$

From the above equation, it is clear that the output is proportional to differential of the input voltage and hence the circuit can act as a differentiator. The proportionality constant $\frac{L}{R}$ is known as the Time Constant of the circuit (τ).

9.5.4 RL – Integrator

A series combination of RL-circuit, working as a low pass filter, can act as an integrator. In this integrator, the output voltage is proportional to the integral of input voltage. This circuit will act as an integrator, if the time constant $\left(\tau = \frac{L}{R}\right)$ of the circuit is very much greater than the time period of the input signal.

Circuit diagram:

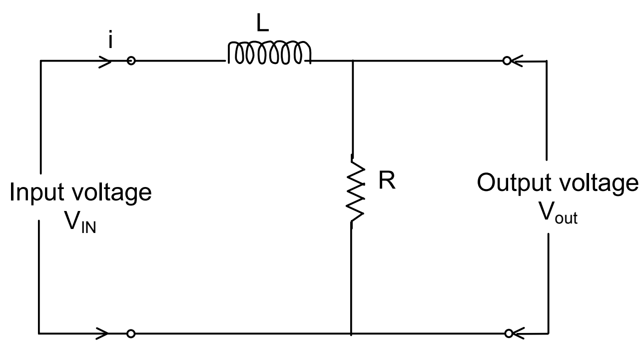


Fig. 9.5.4

Analysis:

Let V_{IN} be the input voltage, V_{OUT} be the output voltage, and i be the current flowing in the circuit.

For the input section of Fig.9.5.3, applying KVL we get

$$V_{IN} = iR + L\frac{di}{dt}$$

or $$\frac{V_{IN}}{R} = i + \frac{L}{R}\frac{di}{dt}$$

Since $\frac{L}{R} \rangle\rangle T$; the term 'i' is neglected as compared to $\frac{L}{R}\frac{di}{dt}$. Hence the above equation becomes

$$\frac{V_{IN}}{R} = \frac{L}{R}\frac{di}{dt}$$

or $\int di = \frac{1}{L}\int V_{IN}\,dt$

or $i = \frac{1}{L}\int V_{IN}\,dt$

But $V_{OUT} = iR$

Hence $V_{OUT} = R \times \frac{1}{L}\int V_{IN}\,dt = \frac{R}{L}\int V_{IN}\,dt$

or $V_{OUT} \; \alpha \int V_{IN}\,dt$

From the above equation, it is clear that the output is proportional to integral of the input voltage and hence the circuit can act as an integrator. Thus the wave is said to be integrated; since it develops another wave, the amplitude of which is proportional to the integral of input wave. The proportionality constant $\frac{L}{R}$ is known as the Time Constant of the circuit (τ).

9.6 Summary

1. The cut off frequency of a high pass filter is f_1 and it can be shown to be $\frac{1}{2\pi CR}$.

   Above f_1, the input sinusoidal signal appears at the output without attenuation.

   The frequency region of unattenuation is called **Pass band**. The frequency region where the signal suffers attenuation is called **Attenuation band** or stop band.

2. The cut-off frequency of a low pass filter is given by f_2=1/2π RC. The pass band lies below f_2.

3. Low-pass and high pass filters can be formed using LR-circuits also. The time constant of LR circuit is given by R/L.

4. The capacitance input and inductance input filter find use in rectifier filter circuits. Here the unwanted A.C. ripple is removed and the variations in D.C. output voltage are smoothened.

5. RC and RL circuits act as integrators and differentiators also. An RC circuit can work as a differentiator only when the time constant RC is far less than the time period of the

wave (i.e., RC<<T). A differentiating circuit will produce a spike output corresponding to square wave input. In the same way it produces a square wave output corresponding to triangular input. Similarly it produces a cosine wave output corresponding to sine wave input.

6. A circuit in which the output voltage is directly proportional to the integral of input voltage is known as an integrating circuit or an integrator. A low pass RC circuit can work as an integrating circuit (output is taken across C).and it can work as an integrator when the time constant RC of the circuit is greater than the time period of the wave form (RC>>T). An RC-integrator circuit can produce a triangular wave output corresponding to a square wave input; in the same way it produces a sine wave output corresponding to a triangular wave input and similarly a cosine wave output corresponding to a sine wave input.

9.7 Key terminology: Low-pass filter – high pass filter – pass band – attenuation band – cut off frequency – time constant.

9.8 Self-Assessment Questions

Short Answer Questions

1. What is meant by time constant of a CR-circuit?
2. Draw the frequency response curve of RC low-pass filter.
3. What is meant by time constant of an LR circuit?
4. Draw a circuit diagram of RC-integrator.
5. Draw the circuit diagram of RC-differentiator.
6. Draw the frequency response curve of a differentiator to square wave input.
7. Draw the frequency response curve of a high pass filter.
8. Draw the frequency response curve of an integrator to square wave input.

Long Answer Questions

1. Explain the frequency response of RC or RL-low pass and high pass filters.
2. What is a differentiating circuit?. Show that the output form a differentiating circuit is derivative of the input.Explain the working of differentiator with RC combination.
3. What is an integrating circuit? Show that the output form an integrating circuit is integral of the input.
4. Describe the response of RC circuit to square wave input.

5. What are L-section and π-section filters? Describe their functions in rectifier circuits.

6. Derive an expression for the output pulse if $V_i(t) = V_P\left(1 - e^{-t/\tau}\right)$ is applied to an RC differentiating circuit.

Numerical Problems

1. In Figure shown below, if L=100mH and R=1KΩ, Calculate the cut-off frequency. What type of filter it is?.

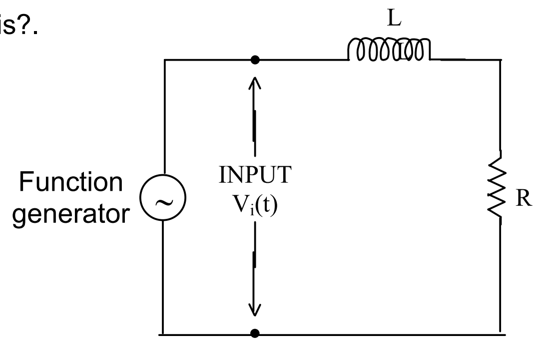


Fig.9.5.5

2. In Fig.9.5.5 shown in Q.1 if output is a triangular wave, what type of input has to be applied?

9.9 Reference Books

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2. Passive components and circuit analysis , VCM, RS & CVR, B.S. Publications
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4. Theory and problems of electric circuits. Joseph A. Edminister (Schaum Publishing Co.)
5. Network analysis by G.K.Mittal & Ravi Mittal (Khanna publishers, Delhi))
6. Passive components and circuit analysis by Agarwal & Arora (A.S.Prakashan, Meerut)
7. Text Book of Electronics – I , S.V.Subrahmanyam, K.Malakondaiah & Y.Narasimha Reddy, HiTech Publishers
8. Basic circuit theory by Lawrence P.Huelsman, (PHI)
9. Circuits devices and systems , Ralph J.Smith, (John Wiley& Sons)

UNIT III Lesson 10

RESONANCE

Objectives of the lesson

In this lesson, we learn about the resonance condition and resonance behavior of series and parallel LCR circuits. We study the importance of Q-factor, its relation to bandwidth and selectivity. We also learn how a Quartz crystal behaves as a resonant circuit.

Structure of the lesson

10.1 Introduction

When the applied voltage and the resulting current are in phase with each other, then the circuit is said to be in ***Resonance***. Inductive reactance (X_L) increases as the frequency increases, but capacitive reactance (X_c) decreases with increase in frequency. The phase of these reactances differ by 180^0. Because of these opposite characteristics, for any LC combination, there must be frequency at which the X_L equals the X_C. Then the reactive impedance becomes zero and the total impedance of RLC-circuit becomes purely resistive. Such a condition is called "***Resonance***" and an RLC circuit, which is operated at or near resonance condition, is known as a ***Resonant circuit***. An RLC-circuit formed with a single inductance and a single capacitance resonates at a single frequency f_0 given by the expression $f_0 = \frac{1}{2\pi\sqrt{LC}}$

If a circuit has two inductances and a capacitance, it can have two resonant frequencies. If the operating frequency is so high, it is quite likely that the stray capacitances and lead inductances form an LCR-circuit and the circuit oscillates at those frequencies determined by the stray capacitances and lead inductances. As the dimensions of the signal wavelength (at Microwave frequencies) and the dimensions of the components match, resonance occurs (example is Microwave cavity resonator)

The main application of resonance is in Radio Frequency (RF) circuits. These circuits are tuned to a desired frequency. Radio and Television receivers, transmitters are some examples that use resonance circuits.

10.2. Series Resonance in RLC - circuits

The circuit in which an inductor, a capacitor and a resistor connected in series with the input, generally from a function generator, is termed as ***Series LCR*** circuit. Typical circuit is shown in the Fig.10.2.1 (a). Because of series connection of L, C and R, the total impedance of the circuit is given by

$$Z = R + X_L + X_C$$
$$Z = R + J\omega L + \frac{1}{j\omega C}$$

where jωL is the inductive reactance X_L. $\frac{1}{j\omega C}$ is capacitive reactance X_c and $\omega = 2\pi f$ is the angular frequency.

$$= R + j\omega L - \frac{j}{\omega C}$$

$$= R + j\left[\omega L - \frac{1}{\omega C}\right]$$

In electrical circuits, the condition to get resonance is that both inductive reactance and capacitive reactance should equal and opposite to each other.

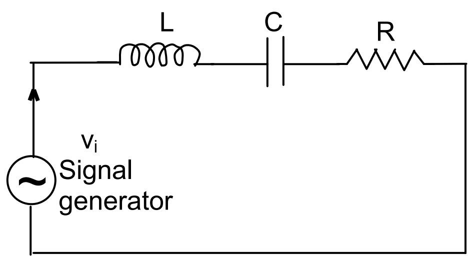

Fig. 10.2.1(a) Series LCR circuit.

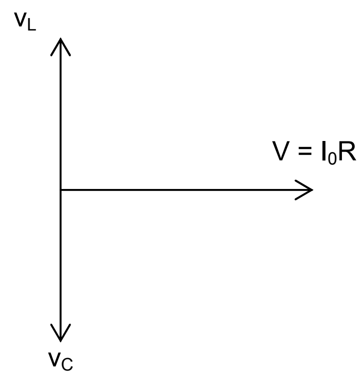

Fig 10.2.1(b) Relationship between voltages across L and C with respect to generator voltage.

$$i.e., X_L = X_C$$

$$\Rightarrow j\omega L = \frac{1}{j\omega C}$$

$$\therefore \omega L. - \frac{1}{\omega C} = 0$$

$$\omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}} \quad [\because \omega = 2\pi f]$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$2\pi f = \frac{1}{\sqrt{LC}}$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore f_r = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

This gives the resonance frequency of RLC-series circuit. Thus under this condition, the impedance of the circuit is equal to the ohmic resistance R. So, current is maximum and is limited by value of R alone. The magnitude of impedance Z is given by

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Here we can observe three conditions.

Case-I: At frequencies lower than the resonant frequency, the capacitive reactance of the circuit is more as compared to inductive reactance. $\frac{1}{\omega C} > \omega L$. The phase angle of the impedance is negative. As ω approaches zero the angle of Z approaches -90^0

Case - II: At frequencies higher than the resonant frequency, inductive reactance is high as compared to capacitive reactance $\frac{1}{\omega C} < \omega L$. The phase angle of the impedance is positive and approaches $+90^0$ as $\omega >> \omega_0$.

Case – III : At resonance, both inductive and capacitive reactances are same. $\frac{1}{\omega C} = \omega L$

At resonance, the current in the circuit is given by $I_R = \frac{E}{R}$

From resonant frequency, it is obvious that the resonance may be obtained either varying the frequency of applied voltage or by varying inductor or capacitor value.

Example:

1. A 2μF capacitor, 100Ω resistor and 8H inductor are connected in series with an AC source. What should be the frequency of the source for which the current drawn in the circuit is maximum? If the peak value of e.m.f of source is 200V, find the maximum current. Also

i) The inductive and capacitive reactance

ii) Total impedance of the circuit

iii) Peak value of current in the circuit

iv) Phase relation between voltage across inductor and resistor

v) Phase difference between voltage across inductor and capacitor

Sol: The current in the circuit becomes maximum at resonant frequency given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\times 3.14\sqrt{8\times 2\times 10^{-6}}} = 79.58Hz$$

(i) At resonant frequency (Maximum current in the circuit)

$$X_L = X_C = 2\pi f_0 L = 2\pi \times 79.58 \times 8 = 4000\Omega$$

(ii) At resonance, total impedance of the circuit

$$Z = R = 100\Omega$$

(iii) Peak value of current

$$I_0 = \frac{E_0}{Z} = \frac{E_0}{R} = \frac{200}{100} = 2A$$

(iv) The phase difference in voltage across inductor and resistor = 90^0

(v) Phase difference in voltage across inductor and capacitor = 180^0

10.2.1 Graphical Representation of Resonance

The impedance of the Series LCR circuit is minimum at resonance and is purely resistive. See Fig.10.2.1(c). This curve between current and frequency is known as Resonance curve, shown in Fig.10.2.1(d). The shape of the curve varies as the value of resistance changes.

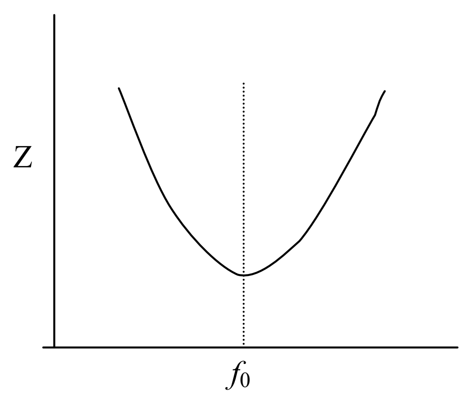

Fig 10.2.1(c)

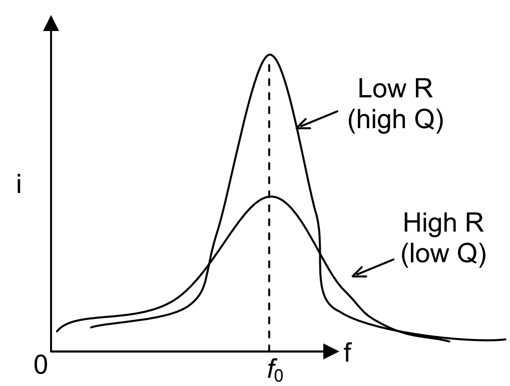

Fig 10.2.1(d)

10.2.2. Voltage in series Resonance Circuit

If a voltage E is applied to a series circuit, then the voltage across capacitor or inductor may be calculated from knowledge of total current i at resonance, given by

$$i = \frac{E}{R}$$

Hence the voltage across inductor is given by

$$V_L = i\omega_0 L = \frac{E}{R}\omega_0 L = E\left[\frac{\omega_0 L}{R}\right]$$

$$V_L = EQ$$

where $$Q = \frac{\omega_0 L}{R}$$

Voltage across capacitor $V_C = i\dfrac{1}{\omega_0 C} = \dfrac{E}{R}\dfrac{1}{\omega_0 C} = EQ$

Thus the voltage across the inductor or a capacitor is Q-times the applied voltage. In other words, the series circuit exhibits a voltage magnification of Q times. In series LCR circuit, Q-factor is also called **Voltage magnification factor**.

Example:

1. A capacitor, a 15Ω resistor and 101.5 mH inductor are placed in series with a 50 Hz AC source. Calculate the capacitance of the capacitor, if the current is observed in phase with the voltage.

Solution: The current and voltage are in phase with each other in LCR circuit at resonance, when

$$\omega L = \frac{1}{\omega C}$$

$$\therefore C = \frac{1}{\omega^2 C} = \frac{1}{(2\pi f)^2 L}$$

$$= \frac{1}{(2 \times 3.14 \times 50)^2 \times 101.5 \times 10^{-3}} = 100\mu F$$

2. A series circuit has an inductance of 200μH, A capacitance 0.0005μF and a resistance of 10Ω find a) resonant frequency b) voltage magnification.

Solution: Resonant frequency $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2 \times 3.14\sqrt{200 \times 10^{-6} \times 0.0005 \times 10^{-6}}}$$

$$= \frac{10^6}{6.28\sqrt{0.1}} = 502{,}000\ Hz = 502 KHz$$

Now voltage magnification at resonance

$$Q = \frac{\omega_0 L}{R} = \frac{2\pi f_0 L}{R}$$

$$= \frac{2 \times 3.14 \times 502 \times 10^3 \times 200 \times 10^{-6}}{10} = 63.2$$

10.2.3 Bandwidth and Selectivity

The frequency response curve shown in Fig.10.2.3(a) is more flat with high resistance whereas it is sharply peaked at low value of resistance. The ability of a resonant circuit to discriminate between one particular frequency and all other is called the **Selectivity**"

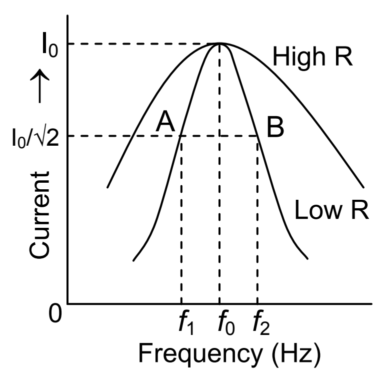

Fig 10.2.3(a)

Selectivity of different resonance circuits is compared in terms of their bandwidths. The band width of a circuit is given by "The band of frequencies which lie between two points on either side of resonant frequency where current falls to $\frac{1}{\sqrt{2}}$ of it maximum value at resonance."

Narrower bandwidth, selectivity is high. Therefore, from the graph shown in Fig.10.2.3(a), the bandwidth is given by

$$\Delta f = f_2 - f_1 \text{ (or) } \delta\omega = \omega_2 - \omega_1$$

The current at resonant frequency $I_0 = \frac{E}{R}$

The power is given by the relation, $I^2 R$

The power at two points at A and B is $\left[\frac{I}{\sqrt{2}}\right]^2 R$, because at these points, the current falls to $\frac{1}{\sqrt{2}}$ of maximum value at resonance.

$$\left[\frac{I_0}{\sqrt{2}}\right]^2 R = \frac{I_0^2 R}{2} = \frac{1}{2} \text{ power at resonance}$$

10.2.4. Finding values of half power points

At resonance $I_0 = \frac{E}{R}$ and at all other frequencies, current $I = \frac{E}{|Z|}$ at points A and B which are at non-resonant frequencies.

$$I = \frac{I_0}{\sqrt{2}} = \frac{1}{\sqrt{2}}\frac{E}{R}$$

Thus it should be equal to $\frac{E}{|Z|}$

$$\therefore \frac{E}{|Z|} = \frac{E}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\frac{1}{\sqrt{2}}.\frac{E}{R} = \frac{E}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

On squaring both sides and by inverting the expressions we get

$$2R^2 = R^2 + \left[\omega L - \frac{1}{\omega C}\right]^2$$

$$\therefore R = \pm\left[\omega L - \frac{1}{\omega C}\right]$$

Multiplying both sides by ω, we get

$$\omega R = \pm\omega^2 L - \frac{1}{C}$$

$$\omega^2 L \pm \omega R - \frac{1}{C} = 0$$

Dividing both sides by L, we get $\omega^2 \pm \frac{\omega R}{L} - \frac{1}{LC} = 0$

This equation is in the form of quadratic equation of second order and the roots of this equation are found using the relation $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\omega = \pm \frac{\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} + \frac{4}{LC}}}{2} = \frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

$$\because \frac{R^2}{L^2} \text{ is much less than } \frac{1}{\sqrt{LC}}$$

$$\therefore \omega = \pm \frac{R}{2L} \pm \frac{1}{\sqrt{LC}}$$

$$\Rightarrow \pm \frac{R}{2L} + \omega_0 = \omega$$

$\therefore$ Only positive values of ω_0 are considered

$$\therefore \omega = \omega_0 + \frac{R}{2L}$$

$$\therefore \omega_1 = \omega_0 - \frac{R}{2L}$$

The frequency ω_1 is called **lower cutoff frequency.**

$$\therefore \omega_2 = \omega_0 + \frac{R}{2L}$$

The frequency ω_2 is called **higher cutoff frequency.**

Bandwidth is the difference between higher and lower cutoff frequencies.

ie., $\omega_2 - \omega_1$ and is denoted by $\Delta\omega$

$$\Delta\omega = \frac{R}{L} \text{ Radians /sec}$$

Since $\omega = 2\pi f$, from above relation we get

$$\Delta f = f_2 - f_1 = \frac{R}{2\pi L} \text{ Hz}$$

$$\therefore f_1 = f_0 - \frac{R}{4\pi L} \text{ Hz}$$

$$\therefore f_2 = f_0 + \frac{R}{4\pi L} \text{ Hz}$$

f_1 is called lower cut off frequency

f_2 is called upper cut off frequency, and the difference between them is called **band width** and is given by $$\Delta f = f_2 - f_1 = \frac{R}{2\pi L} \text{ Hz}$$

Example

1. In a series RLC circuit R= 100 Ω, L=0.5H and C=40μF. Calculate the resonant frequency, lower and upper half power frequencies

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\times 3.14\sqrt{0.5\times 40\times 10^{-6}}}$$

Resonant frequency = $\frac{1}{6.28\sqrt{20\times 10^{-6}}} = \frac{1}{6.28\times 10^{-3}\times 4.47} = 35.6 \text{ Hz}$

Lower half power frequency

$$f_1 = \frac{\omega_1}{2\pi} = \frac{1}{2\pi}\left(\omega_0 - \frac{R}{2L}\right) = f_0 - \frac{R}{4\pi L}$$

$$= 35.6 - \frac{100}{4\times 3.14\times 0.5} = 35.6 - 15.9 = 19.7 Hz$$

Upper half power frequency

$$f_2 = f_0 + \frac{R}{4\pi L} = 35.6 + 15.9 = 51.5 Hz$$

10.2.5 Quality factor or Q-Factor

One measure of the sharpness of resonance peak in a band-pass circuit is the Quality-factor designated as Q. This may be defined as

$$Q = \frac{resonance\ frequency}{bandwidth} = \frac{\omega_0}{B}$$

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0}{R/L} = \frac{\omega_0 L}{R}$$

(or)

$$\omega_2 - \omega_1 = \Delta\omega = \frac{\omega_0}{Q}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q = \frac{L}{R}.\frac{1}{\sqrt{LC}} = \frac{1}{R}.\sqrt{\frac{L}{C}}$$

Thus the Q factor of the resonance circuit can be calculated by using this relation.

Quality factor is also defined as the ratio of energy stored in an inductor or capacitor to the energy lost per cycle in it.

$$Q = 2\pi \frac{Energy\ stored\ \ per\ cycle}{Energy\ lost\ \ per\ cycle} = \frac{I^2 \varpi L}{I^2 R} = \frac{\omega L}{R} = \frac{I^2 / \omega C}{I^2 R} = \frac{1}{\omega CR}$$

R is the d.c resistance of the component. In an LCR circuit, R represents the total dc resistance of the circuit. Hence Q in a resonant circuit is sometimes called circuit Q-factor or effective Q-factor.

Example:2. A series LCR circuit has Q = 120 at resonance, a capacitance 200PF connected in series with an inductance of 150 μH. Calculate its bandwidth.

Solution:

Resonant frequency $f_0 = \frac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{6.28\sqrt{(150\times10^{-6}\times200\times10^{-12})}} = 9.19\times10^5\,Hz$$

Now $$Q = \frac{f_0}{f_2 - f_1} = \frac{f_0}{\Delta f}$$

or Band width

$$\Delta f = \frac{f_0}{Q} = \frac{9.19\times10^5}{120} = 7.66 KHz$$

10.3.1 Parallel Resonance in RLC-Circuit

The circuit shown in Fig. 10.3.1(a) is called ***ideal parallel resonance circuit***. In this circuit, inductor, capacitor and resistor are connected in parallel to each other and to the output of function generator. At resonance, the only circuit element having impact in the circuit is the resistor only. Because at resonance frequency, the inductive and capacitive reactances are equal and opposite, hence cancel out. This circuit can be considered as a dual of series LCR circuit. The resonance frequency of this circuit is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$ and is independent of R.

Because of considerable d.c. resistance of the coil, an inductor has to be represented as a pure inductance L in series with a resistor R. So, a simple parallel LC circuit has to be considered as a parallel LCR circuit as shown in Fig 10.3.1(b), we analyze this circuit in this lesson. The combined impedance 'Z' of the two parallel branches is given by

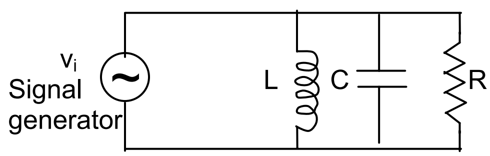

Fig 10.3.1(a) Ideal parallel LCR circuit

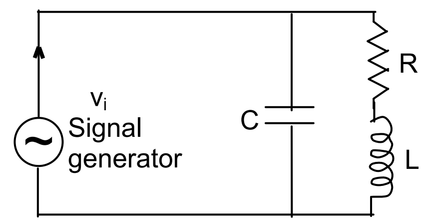

Fig 10.3.1(b) parallel LCR circuit.

$$\frac{1}{Z} = \frac{1}{R + j\omega L} + \frac{1}{\frac{1}{j\omega C}}$$

$$\frac{1}{Z} = \frac{1}{R + j\omega L} + \frac{j\omega C}{1}$$

$$\frac{1}{Z} = \frac{R - j\omega L}{(R + j\omega L)(R - j\omega L)} + j\omega C$$

$$\frac{1}{Z} = \frac{R - j\omega L}{R - (j\omega L)^2} + j\omega C$$

$$= \frac{R}{R - (j\omega L)^2} - \frac{j\omega L}{R - (j\omega L)^2} + j\omega C$$

$$= \frac{R}{R^2 + \omega^2 L^2} + j\omega\left[C - \frac{L}{R^2 + \omega^2 L^2}\right] \quad \text{------------ (10.3.1)}$$

At resonance, reactive components of impedance is zero. i.e., the imaginary parts of above equation will be zero. Hence resonance occurs when.

$$j\omega\left[C - \frac{L}{R^2 + \omega^2 L^2}\right] = 0$$

$$C = \frac{L}{R^2 + \omega^2 L^2}$$

$$R^2 + \omega^2 L^2 = \frac{L}{C} \qquad \text{------------} \quad (10.3.2)$$

$$\omega^2 L^2 = \frac{L}{C} - R^2$$

Dividing both the sides by L^2, we get

$$\omega^2 = \frac{L}{L^2 C} - \frac{R^2}{L^2}$$

$$\therefore \omega = \sqrt{\frac{L}{L^2 C} - \frac{R^2}{L^2}}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{L}{L^2 C} - \frac{R^2}{L^2}}$$

As $\frac{R^2}{L^2}$ is small when compared to $\frac{1}{LC}$ then at resonant frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

The impedance of the circuit at resonance can be calculated using equations (10.3.1) and (10.3.2)

From eq.(10.3.1) $\qquad \frac{1}{Z} = \frac{R}{R^2 + \omega^2 L^2}$

From eq.(10.3.2) $\qquad \frac{1}{Z} = \frac{R}{L/C}$

$$\therefore Z = \frac{L}{CR}$$

This impedance at resonance is known as dynamic resistance.

10.3.2 Q Factor of parallel Resonance circuit

It is defined as the ratio of the current circulating between its two branches to the line current drawn from the supply or simply as the "current magnification".

Hence $\qquad$ Q–Factor = $\frac{I_C}{I}$

$$I_C = \frac{V}{X_C} = \frac{V}{\frac{1}{\omega C}} = \omega CV$$

and $$I = \frac{V}{Z} = \frac{V}{\frac{L}{CR}} = \frac{CRV}{L}$$

$$\therefore \text{ Quality factor } = \frac{\frac{\omega CV}{CRV}}{L} = \frac{\omega L}{R}$$

Thus Q factors of both series and parallel circuit are the same.

The half power points of parallel circuit can be calculated in similar manner of series resonance circuit. Parallel resonant circuits with large Q are used as tunable high impedance loads in amplifiers. When such a circuit is excited by a small signal pulse, whose repetition frequency is equal to the resonant frequency of the tuned circuit, the resonant circuit sustains oscillations thereby giving continuous output for intermittent input.

10.3.3. Graphical Representation

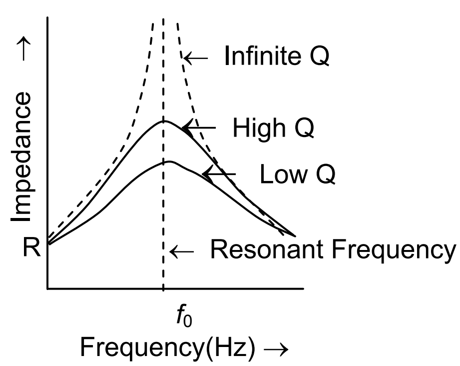

Fig. 10.3.2(a)

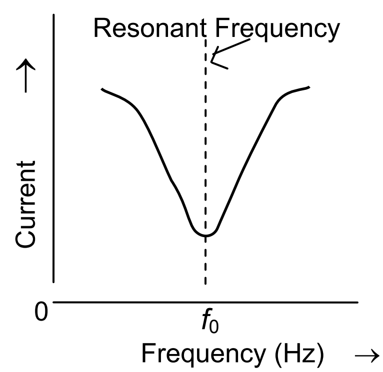

Fig.10.3.2(b)

Fig.10.3.2(a) is drawn by taking impedance on Y-axis and frequency on X-axis whereas Fig.10.3.2(b) is drawn by taking current on Y-axis and frequency on X-axis.

Comparison between series and parallel resonant circuits

Item	Series resonance	Parallel Resonance
Impedance at resonance	Minimum	Maximum
Current at resonance	Maximum	Minimum
Effective impedance	R	L/CR
Amplifies	Q times generator voltage	Q times generator current
Resonant frequency	$f_0 = \frac{1}{2\pi\sqrt{LC}}$	$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$

Applications

The main applications of a parallel resonant circuit is to act as load in the output circuit of radio frequency amplifiers and these are very much used as tuners in communication systems like television to receive the channels, in radio sets to receive the signal. In every system, where wireless communication is adopted, the resonance circuits are employed to tune the desired frequency.

10.4 Quartz crystal Resonator

When a high degree of frequency stability particularly over long periods of time is required, the usual resonant tuned circuit of an oscillator is replaced by a quartz crystal. The principle of operation of a crystal oscillator depends on the "Piezo electric effect". Certain crystals like quartz, tourmaline etc exhibit this effect. They have two sets of axes. One set of axes are called mechanical axes (say Y-axis) and another set axes are called electrical axes (say X-axis). When a Quartz crystal is subjected to mechanical stress along Y-axis, electrical charges appears on the faces of the crystal parallel to X-axis and vice-versa. This property by which electrical and mechanical properties are interconnected in a crystal is called the **"Piezo Electric Effect".** A piezo electric crystal can be considered as a resonant circuit with exceptionally high Q-factor of the order of 10^6. Because of this a mechanical stress of short duration induces electrical oscillations for a long time and oscillations are sustained in it. Alternately it can be driven by a.c. signal applied along electrical axes.

Whenever applied A.C frequency is equal to crystals natural frequency, the amplitudes of vibrations become high.

In order to make electrical connections to the crystal, it is generally mounted horizontally between flat metal plates. The metal plates touch the crystal and are connected to the external circuits through soft springs so as to allow for the required amount of mechanical vibrations. Depending upon the geometrical cut used to produce slices of piezoelectric crystals, they may have a specific resonance frequency. Temperature coefficient of oscillator frequency also depends on the crystal cut and dimensions of the crystal. The frequency of a piezoelectric oscillator is very stable compared to conventional oscillators. The second figure is the equivalent circuit for quartz circuit C_1 is the capacity of mounting plates.

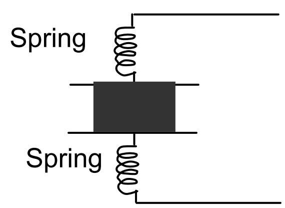

Fig 10.4(a) Piezo electric crystal placed between two metal plates with spring contact.

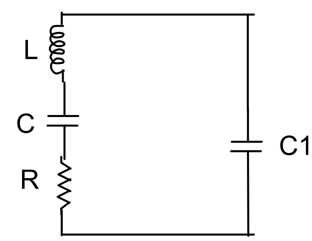

Fig 10.4(b) Equivalent circuit of Piezo electric crystal.

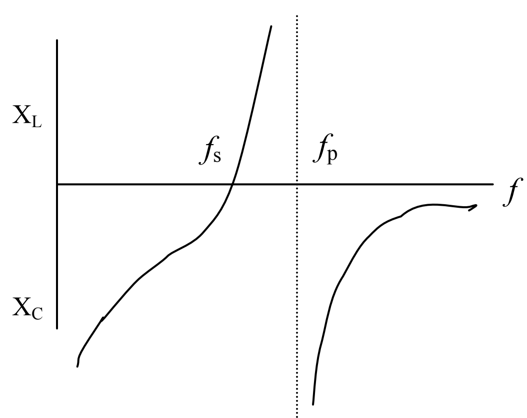

Fig:10.4(c) Variation of crystal terminal reactance with frequency

To change the frequency of the crystal, the crystals are to be cut in different sizes. In the early days, crystals were available in frequency range from 15 KHz to 10 MHz. As the frequency of operation is increased, thinner crystals are required, and this sets an upper limit of about 50MHz. The piezo electric crystals are fragile and hence can only be used in low power circuits. The quartz crystal oscillators are used extensively as master oscillators in computers, in transmitters of radios and T.Vs., time signal receivers, test equipment, harmonic generators, air craft communication operators, in clocks and in wrist watches etc.

Example

3. A series LCR circuit the resonant frequency is 800Hz. The half power points are obtained at frequencies 745 and 855 Hz. Calculate the value of Q.

Solution:

Given, lower half power frequency = 745 Hz

Upper half power frequency = 855 Hz

Resonant frequency = 800 Hz

$$Q = \frac{f_0}{\Delta f} = \frac{f_0}{f_2 - f_1} = \frac{800}{855-745} = \frac{800}{110} = 7.3.$$

4. An RLC series circuit is excited by a constant amplitude but variable frequency source. It is desired to have a current maximum at 1000 Hz with a bandwidth of 100 Hz. If C = 0.1μF, specify the values of L and R.

Solution:

Since current maximum is obtained at resonance, therefore,

$$f_0 = 1000 Hz$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore L = \frac{1}{4\pi^2 f_0^2 C} = \frac{1}{4\times(3.14)^2\times(1000)^2\times 0.1\times 10^{-6}} = 0.254H$$

Also $Q = \frac{f_0}{\Delta f} = \frac{1000}{100} = 10.$

But $Q = \frac{\omega_0 L}{R} = \frac{2\pi f_0 L}{R}$

$$\therefore R = \frac{2\pi f_0 L}{Q} = \frac{2\times 3.14\times 1000\times 0.254}{10} = 159.5\Omega$$

5. A coil of 10Ω resistance and 0.1H inductance is connected in parallel with a capacitor of 100μF capacitance. Calculate the frequency at which the circuit will act as a non-inductive resistance. Calculate the value of this resistance.

Solution:

The circuit acts as non-inductive resistance at resonant frequency given by

$$f_0 = \frac{1}{2\pi\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}} = \frac{1}{2 \times 3.14\sqrt{\frac{1}{0.1 \times 100 \times 10^{-6}} - \frac{10^2}{(0.1)^2}}}$$

$$= \frac{1}{6.28\sqrt{10^5 - 10^4}} = 47.7\,Hz$$

The impedance of the circuit at resonant frequency called dynamic resistance, is given by

$$\frac{L}{CR} = \frac{0.1}{100 \times 10^{-6} \times 10} = 100\Omega$$

10.5 Summary

(1) In LC circuits, whenever X_L equal to X_C, the resonance is said to be occurred.

(2) Series Resonance: Any circuit which has LC and R in series with each other will be termed as Series circuit. The main characteristics of the circuit are

(i) The circuit offers minimum impedance Z_{min}=R

(ii) Circuit behaves like a pure resistive

(iii) The power factor is unity.

(iv) Resonate frequency is given by $f_0 = \frac{1}{2\pi\sqrt{LC}}$ Hz

(v) Voltage across inductance or capacitance is Q times the applied voltage.

Thus $V_C = V_L = Q \times E$

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

(vi) Bandwidth $\Delta f = \frac{R}{2\pi L} = \frac{f_0}{Q}$

(3) Parallel resonant circuit

Any circuit, which has L, C and R in parallel with source, is termed as parallel resonant circuit.

(i) Maximum impedance at resonance $Z_{\max} = \frac{L}{CR}$

(ii) Resonance frequency is given by $f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$ Hz

(4) The property by which electrical and mechanical properties are interconnected in a crystal is

called the piezo electric effect. Quartz crystal oscillators employ piezo electric effect to produce high frequency waves, when the natural frequency of crystal is equal to LCR resonance circuit connected in parallel to the crystal.

10.6 Key terminology

Tuning – Series resonance – Parallel resonance – Bandwidth – Q - of resonant circuit – tank circuit – half-power frequencies - Crystal resonator – Piezo electric effect.

10.7 Self assessment Questions

a) Long Answer Questions

1. Explain the phenomenon of resonance.
2. Derive the expression for resonant frequency, Q-factor, bandwidth of series RLC circuit. Draw its frequency response curve.
3. Explain the resonance in a parallel RLC circuit and obtain expression for maximum impedance, current and resonant frequency. Explain its frequency response curve.
4. Explain bandwidth and Q-factor of a parallel resonant circuit.
5. Distinguish between series and parallel resonance circuits.
6. How Q is related to L/C, R and the shape of the response curve? How does the L/C ratio and R affect the response curve?
7. What is piezoelectric effect? Draw the equivalent circuit of a quartz crystal and explain. Describe the working of quartz crystal oscillator.

b) Short Answer Type Questions

1. Distinguish between series resonance and parallel resonance.
2. Discuss series and parallel resonance in RLC circuits.
3. Define resonance in AC circuits. How does it differ from the general definition of resonance?
4. Define electrical resonance.
5. Show that in series circuit the impedance at resonance is minimum and is equal to *R.*
6. Why a series circuit is called ‘acceptor circuit’?
7. Show that the resonant frequency does not depend upon circuit resistance.
8. Discuss the effect of variation of resistance on resonance current.

9. What will be the effect on
    (i) Resonant current
    (ii) Frequency of a series circuit of replacing *R* by 2R ?
10. Explain the term quality factor.
11. How does the sharpness of resonance of a series circuit depend upon the circuit Q ?
12. Why a parallel resonant circuit is called a ‘rejector circuit’?
13. Write briefly on the quality factor and bandwidth of series resonant circuits.
14. How do you express quality factor in terms of half power points.
15. What is quality factor of resonant circuit? How it is related to bandwidth of the circuit?
16. Write briefly on quartz crystal oscillator.

c) Numerical problems

1. Obtain the resonant frequency and *Q* factor for a series *LCR* circuit with L = 3.0 H, C = 27 nF and R = 7.4 Ω **(*Ans:*** 111.1rad/s, 45.05**)**
2. In an *LCR* circuit, a variable frequency 230V source is connected to L = 5.0 H, C = 80nF”, R = 40 Ω.
    i. Determine the source frequency, which drives the circuit in resonance.
    ii. Obtain the impedance of the circuit and the amplitude of the current at the resonant frequency.
    iii. Determine the r.m.s. potential drop across the three elements of the circuit and also across the *LC* combination.

    (*Ans:* a) 50 rad/s b) 40Ω, 5.75A c) V_R =230V, V_L= 1437.5V, V_C=1437.5V,V_{LC}=0)
3. Find the capacitive reactance of a 15pF capacitor at 2000 KHz. Calculate the inductance required to produce series resonance with the capacitor at this frequency.

    (*Ans* 5.31, 4.2 x 10^{-4} H)
4. A radio can tune over the frequency range of 800 kHz to 1200 kHz. If its tuned circuit has a effective inductance of 200 μH, what must be the range of its variable capacitor?

    (*Ans:* 87.8 to 197.7 PF)
5. An *RLC* resonant circuit has a resonant frequency of 2 MHz and a *Q* factor of 100. Calculate
    a. Bandwidth of the circuit,
    b. Lower and upper half power frequencies,
    c. Sharpness of resonance. **(*Ans:*** (i) 20 KHz (ii) 1990 KHz, 2010 KHz (iii) 1/100)

6. A coil of resistance 5Ω is connected in series with an inductance of 100μH and a capacitor of 100 PF. Calculate (i) resonant frequency (ii) *Q* factor (iii) bandwidth.
   (***Ans:*** (i) 1.59 MHz (ii) 200 (iii) 7.95 KHz.)
7. Capacity of 200 km long telegraph wire is 0.014μF per km. If a current of 5 KHz frequency is flowing through it, then find the value of that inductance which when connected in series will give minimum impedance. [Hint: Impedance is minimum at resonance where ωL = 1/ωC] (***Ans:*** 0.36 mH)
8. 360meter long waves are transmitted from a transmitter. What inductance should be connected with a 1.20 μF capacitor in resonant circuit to receive them?(***Ans:*** L = 3 x 10^{-8} H.)
   $$\left[\text{Hint } f = \frac{c}{\lambda} = \frac{1}{2\pi\sqrt{LC}}\right]$$
9. An alternating potential of 100V and 50Hz is applied across a series circuit having an inductance of 5H, a resistance of 100Ω and a variable capacitor. At what value of the capacitance will the current in the circuit be in phase with the applied voltage? Calculate the current in this condition. What will be the potential difference across the resistance, inductance and capacitance (***Ans:*** 2 μF, 1 A, 100V, 1570V, 1570V)
10. A resistance of 10Ω is joined in series with an inductance of 0.5H. What capacity should be put in series with the combination to obtain maximum current? What will be the p.d. across each of these elements? The current is being supplied by 200V and 50 Hz mains.
    (***Ans:*** 20 μ*F,* 20 A, 200 V, 3140 V, 3140 V.)
11. A series *LCR* circuit has *L* = 1*mH, C*=0.1μF and *R* =10Ω*.* Calculate the resonant frequency of the circuit. What is the separation between two half power points when R= 10Ω and when R = 0.1Ω ? What is the power factor at the resonant frequency?
    (***Ans:*** 10^5 rad/sec, 10^4, 10^2)
12. Calculate the impedance of an inductance of 2.0H in parallel with a capacitor of 100μF at resonance. The resistance of the inductance is 10Ω. (***Ans:*** 2000Ω)
13. A resistance *R* = 48Ω is connected is series with inductance *L* = 450 *mH* and a capacitance *C* = 9nF. What is the resonant frequency? What is the current in the circuit when it is supplied from a 120V source operating at resonant frequency? (***Ans:*** 79.1 Hz, 2.5 A.)
14. A series circuit contains a resistance of 4 ohms, an inductance of 0.5 H and a variable capacitor across a 100V, 50Hz supply. Find (i) capacitance for getting resonance (ii) *Q* factor. (***Ans:*** (i) 20.3μ*F* (ii) 39.2)

15. A coil of 0.1H with $R = 8\Omega$ is connected in series with $200\mu F$ capacitor. Find resonant frequency (***Ans:*** 35.6 Hz)
16. A 10H inductance coil is connected in series to a 100Ω resistor and a variable capacitor and this series combination is connected to a 100V, 60Hz source. Find the values of capacity C at resonance. How much power is dissipated in the circuit? (***Ans:*** $0.7\mu F$, 100 W)
17. An *RLC* series circuit consists of a resistance of 1000Ω., an inductance of 100*mH* and a capacitance of 10*nF.* If a voltage of 100V is applied across the combination, calculate
    i. The resonant frequency
    ii. Q factor of the circuit
    iii. The half power points.

    (***Ans:*** (i) 159.2 KHz (ii) 100 (iii) 158.2 KHz, 159.8 KHz)
18. A resistor $R = 5\Omega$, an inductor $L = 4mH$ and a capacitor C = 0.1pF are connected in series and a variable frequency voltage signal of constant amplitude 20V *(rms)* is applied across the circuit. Find the resonant frequency and the voltage across inductor and capacitor.
    (***Ans:*** 7.96 KHz, 800V, 800V)
19. An AC voltage of 10V is applied to a series combination of $50\mu F$ capacitor,0.2H inductor and of 0.1Ω resistor. Calculate resonant frequency and the current in the circuit at resonant frequency. (***Ans:*** 50.3 Hz. 100A)
20. A coil of 10 *mH* inductance and 10Ω resistance is connected in parallel to a capacitor of $0.1\mu F$. Calculate the impedance of the circuit at resonance. (***Ans:*** $10^4\Omega$)
21. A 2mH inductance coil and a resistance of 15Ω are connected in parallel to a 0.001 μF capacitor. Calculate (i) Frequency at which current from an a.c. source is minimum, (ii) peak value of make up current when peak value of AC supply is 2V. (***Ans:*** 112.6 KHz, $15\mu A$.)
22. A parallel circuit consists of a $2.5\mu F$ capacitor and a coil whose resistance and inductance are 15Ω and 260*mH* respectively. Calculate
    i. Resonant frequency
    ii. Q-factor of the circuit
    iii. Dynamic impedance. (***Ans:*** (i) 197 Hz (ii) 21.5 (iii) 6933 Ω)
23. Determine Q and resonant frequency of the following circuit shown in Fig.10.6.
    (***Ans:*** 223, 71 KHz)

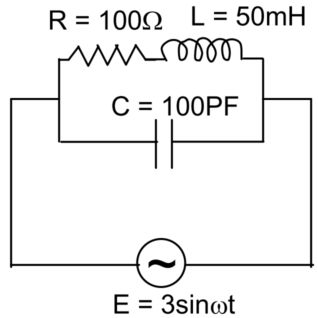

Fig 10.6 Parallel LCR circuit.

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UNIT – IV LESSON – 11

TWO PORT NETWORKS

Objectives

This lesson explains the concept of

- 1. Ports
- 2. Network parameters (Z, Y, h, ABCD)
- 3. Relationship between parameters (inter conversions)
- 4. T-Network, Π- Network and their inter conversions
- 5. Bridge-T and Twin-T Networks

Structure of the lesson

Introduction

An electrical or electronic network as the name implies consists of electric or electronic components interconnected. We come across networks in electronic communications, electronic control systems, transmission and distribution systems etc. To study the performance of a given network, it becomes necessary to give input signal and see the response of the circuit at the output.

The input and output signals may be in the form of voltage or current etc. Input signal will be given across a pair of terminals called ***Input Terminals*** or simply ***Input Port***. One of the input terminals will be called a ***Reference Terminal*** or normally ***Ground Terminal***. Similarly output signal is taken across a pair of terminals called ***Output Terminals*** or ***Output Port***. One of the output terminals also will be a reference or ground terminal. If the reference leads are at different levels of potentials, such network is called a ***4 terminal network***. When the reference terminals are at common ground potential, they can be tied together and the network is called a ***3 terminal network***. A circuit with one input port and one output port is called a ***2 port network***. In between the two ports, there can be a complex network. Often it will be sufficient if we know the input signal nature i.e. voltage or current and the impedance or admittance against which signal enters into the circuit. Likewise, at the output we will be interested in knowing the nature of output signal against which it works to drive the load. In effect, very often we need not know the intrinsic details of the network. So, a network is generally represented by a rectangular box. It is called ***black box representation*** of a circuit. In fact, any process whose input and output status is only of importance to us can be represented by a black box.

11.1 One port and two port networks

One port network

Any network having only a pair of terminals at which a signal may enter or leave is called a single-port or one-port network. It is shown is Fig 11.1. Here 1 and 1^1 are the terminals. In network analysis, the voltage is represented by the symbol V or v and current with I or i. This type of approach appears in various text books. Hence same notation is adopted here also.

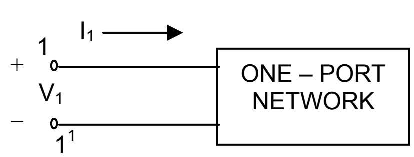

Fig 11.1

Input impedance (Driving Point Impedance): $Z_{in} = \frac{V_1}{I_1} = \frac{1}{Y_{in}}$

Two port network

Any network which is having two ports is called a ***Two-port network***. In a two-port network, there are four variables V_1, I_1; V_2, I_2. The driving force is connected to port-1 i.e. input port and port 2 (called the output port) is connected to the load; It is shown in Fig.11.2.

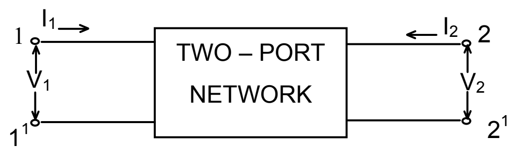

Fig 11.2

The output port may also be called as load port. The two-port network has two driving point functions and two transfer functions. These functions are expressed as open-circuit and short-circuit parameters.

11.2 Differences between one port and two port networks

One-port network	Two-port network
1. It has one pair of terminals. 2. One voltage V_1 and corresponding current I_1 are defined. 3. No conditions are to be specified. 4. No isolation between input and output. 5. Examples are capacitor, resistor, inductor.	1. It has two pairs of terminals. 2. Two voltages V_1, V_2 and corresponding currents I_1, I_2 are defined. 3. Output condition must be specified either as short-circuit or open-circuit. 4. There is isolation between input and output ports. 5. Examples are transistor, triode, transformer.

The behavior of a two-port network may be expressed in terms of the four electrical quantities – input and output currents I_1, I_2 and input and output voltages V_1, V_2. The functional relationship between these electrical quantities may be expressed in different ways.

A two-port network can be characterized by parameters in terms of voltage and current variables. Consider a two-port network with two ports 1-2 and 3-4 as shown in Fig.11.3.

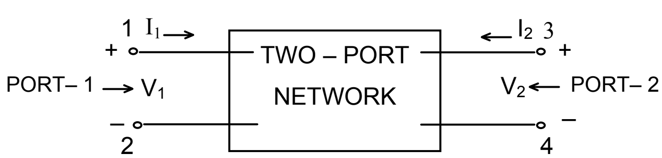

Fig 11.3

The port-1 is normally connected to the input while the port-2 is connected to the output (the load). We can describe the dependence of the two of the four variables on the other two, in a number of ways with the help of input/output parameters. Four types of such parameters are

1. Z – parameters or Impedance parameters.
2. Y–parameters or Admittance parameters.
3. h - parameters or Hybrid parameters.
4. A, B, C, D - parameters or Transmission parameters.

11.3 Z – parameters

In some circuits, input voltage and output voltage are functions of input and output currents. The functional dependence may be written

$V_1 = f_1(I_1,I_2)$; $V_2 = f_2(I_1,I_2)$

If the variations in V_1 and V_2 around their quiescent values are small, they can be written as

$$\Delta V_1 = \frac{\partial f_1(I_1,I_2)}{\partial I_1}\Delta I_1 + \frac{\partial f_1(I_1,I_2)}{\partial I_2}\Delta I_2$$

$$\Delta V_2 = \frac{\partial f_2(I_1,I_2)}{\partial I_1}\Delta I_1 + \frac{\partial f_2(I_1,I_2)}{\partial I_2}\Delta I_2$$

The coefficients of $\Delta I_1, \Delta I_2$ are the partial derivatives and they have the dimensions of impedance. For sufficiently small changes, we can replace the changes in voltages and currents with continuous variables. The equations can now be rewritten as

$$v_1 = Z_{11}i_1 + Z_{12}i_2 \qquad \text{----------} \quad (11.1)$$

$$v_2 = Z_{21}i_1 + Z_{22}i_2 \qquad \text{----------} \quad (11.2)$$

The Z- parameter equivalent circuit of a two port network can be drawn as shown in Fig 11.4

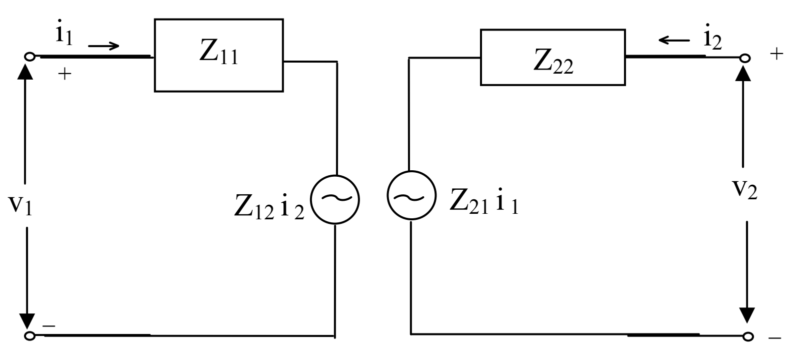

Fig 11.4 Z-Parameter equivalent circuit

The coefficients Z_{11}, Z_{12}, Z_{21} and Z_{22} are called the *Z*- parameters or Impedance parameters. They are defined as follows.

$Z_i = Z_{11} = \left.\frac{v_1}{i_1}\right|_{i_2=0}$ – Input impedance when output is open circuited.

$Z_o = Z_{22} = \left.\frac{v_2}{i_2}\right|_{i_1=0}$ – Output impedance when input is open circuited.

$Z_r = Z_{12} = \left.\frac{v_1}{i_2}\right|_{i_1=0}$ – Mutual or reverse transfer impedance when input is open circuited.

$Z_f = Z_{21} = \left.\frac{v_2}{i_1}\right|_{i_2=0}$ – Mutual or transfer impedance when output is open circuited.

Z - Parameters are also called ***Open – circuited parameters*** because they are defined with either input or output open-circuited. These parameters possess the unit of ohms.

If in a network $Z_{12} = Z_{21}$, then it is called a ***Bilateral Network***. Some authors prefer lower case letters to represent Z parameters, i.e. they write z_{11} instead of Z_{11}. Only for certain circuits, Z-parameters can be determined experimentally with needed accuracy. In other cases we have to try another type of representation.

11.4 Y – parameters

In some networks, input and output voltages are independent and input current and output current are dependent on them i.e. their functional relation can be expressed as

$I_1 = f_1(V_1, V_2)$; $I_2 = f_2(V_1, V_2)$

If the variations in I_1 and I_2 around their quiescent values are small, they can be written as

$$\Delta I_1 = \frac{\partial f_1(V_1, V_2)}{\partial V_1}\Delta V_1 + \frac{\partial f_1(V_1, V_2)}{\partial V_2}\Delta V_2$$

$$\Delta I_2 = \frac{\partial f_2(V_1, V_2)}{\partial V_1}\Delta V_1 + \frac{\partial f_2(V_1, V_2)}{\partial V_2}\Delta V_2$$

The coefficients of $\Delta V_1, \Delta V_2$ are the partial derivatives and they have the dimensions of admittance. For sufficiently small changes, we can replace the changes in voltages and currents with continuous variables. The equations can now be rewritten as

$$i_1 = Y_{11}v_1 + Y_{12}v_2 \qquad \text{--------------} \quad (11.3)$$

$$i_2 = Y_{21}v_1 + Y_{22}v_2 \qquad \text{--------------} \quad (11.4)$$

The coefficients Y_{11}, Y_{12}, Y_{21} and Y_{22} are called the Y-parameters or Admittance parameters.

They are defined as follows.

$Y_i = Y_{11} = \left.\dfrac{i_1}{v_1}\right|_{v_2=0}$ – Input admittance when output is short–circuited.

$Y_o = Y_{22} = \left.\dfrac{i_2}{v_2}\right|_{v_1=0}$ – Output admittance when input is short–circuited.

$Y_r = Y_{12} = \left.\dfrac{i_1}{v_2}\right|_{v_1=0}$ – Mutual or reverse transfer admittance when input is short–circuited.

$Y_f = Y_{21} = \left.\dfrac{i_2}{v_1}\right|_{v_2=0}$ – Mutual or forward transfer admittance when output is short–circuited.

Y - Parameters are also called ***Short–circuited parameters*** because they are defined with either input or output short–circuited. These parameters possess the unit of mhos.

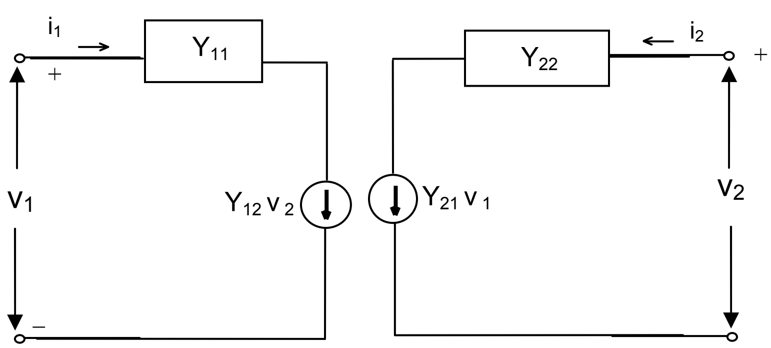

Fig 11.5 Y-Parameter equivalent circuit

11.5 Hybrid parameters

Some networks cannot be represented satisfactorily either by Z-parameters or by Y-parameters, as all the parameters cannot be determined experimentally with sufficient accuracy.

For example, in a transistor input current and output voltage are independent parameters. In such cases, all the parameters cannot have the same dimensions. These parameters are called ***Hybrid parameters***. Here we consider two types of hybrid parameters because of their importance. The first type is simply called h-parameters and the other type is called ABCD parameters

11.5(a) h – parameters

In some networks input current and output voltage are independent; and input voltage and output current are dependent on them i.e. their functional relation can be expressed as

$V_1 = f_1(I_1, V_2)$; $I_2 = f_2(I_1, V_2)$

If the variations in V_1 and I_2 around their quiescent values are small, they can be written as

$$\Delta V_1 = \frac{\partial f_1(I_1, V_2)}{\partial I_1}\Delta I_1 + \frac{\partial f_1(I_1, V_2)}{\partial V_2}\Delta V_2$$

$$\Delta I_2 = \frac{\partial f_2(I_1, V_2)}{\partial I_1}\Delta I_1 + \frac{\partial f_2(I_1, V_2)}{\partial V_2}\Delta V_2$$

The coefficients of $\Delta I_1, \Delta V_2$ are the partial derivatives and they have different dimensions. For sufficiently small changes, we can replace the changes in voltages and currents with continuous variables. The equations can now be rewritten as

$$v_1 = h_{11}i_1 + h_{12}v_2 \qquad \text{--------------} \quad (11.5)$$

$$i_2 = h_{21}i_1 + h_{22}v_2 \qquad \text{--------------} \quad (11.6)$$

The coefficients h_{11}, h_{12}, h_{21} and h_{22} are called the *h*-parameters.

They are defined as follows.

$h_i = h_{11} = \left.\frac{v_1}{i_1}\right|_{v_2=0}$ – Input impedance when output is short–circuited.

$h_o = h_{22} = \left.\frac{i_2}{v_2}\right|_{i_1=0}$ – Output admittance when input is open–circuited.

$h_r = h_{12} = \left.\frac{v_1}{v_2}\right|_{i_1=0}$ – Reverse voltage gain when input is open–circuited.

$h_f = h_{21} = \left.\frac{i_2}{i_1}\right|_{v_2=0}$ – Forward current gain when output is short–circuited.

h - Parameters are also called ***Hybrid Parameters*** or ***Mixed Parameters*** because they are not alike dimensionally. h_{11} possess the unit of ohms, h_{12} possess the unit of mhos, h_{12} and h_{21} possess no units.

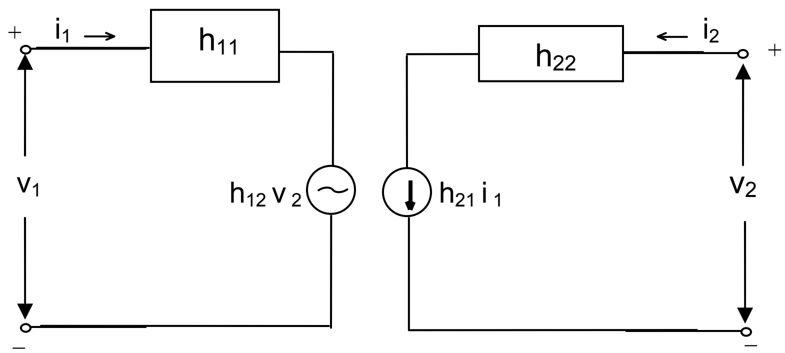

Fig 11.6 h-Parameter equivalent circuit

11.6 ABCD – parameters

For transmission networks, input current and input voltage are functions of output current and output voltage. We have another set of hybrid parameters and these are called ***Transmission Network Parameters*** or simply ABCD – parameters.

In this system of parameters, the voltage and current at port–1 are expressed in terms of voltage and current at port–2. Referring to Fig.11.3, we can write

$$v_1 = Av_2 - Bi_2 \qquad \text{----------} \quad (11.7)$$

$$i_1 = Cv_2 - Di_2 \qquad \text{----------} \quad (11.8)$$

The negative sign arises from the fact that reference direction of i_2 is opposite to that in power transmission problems. The coefficients are called A, B, C, D - parameters or transmission parameters. They are defined as follows:

$A = \left.\dfrac{v_1}{v_2}\right|_{i_2=0}$ – Ratio of input voltage to output voltage when output is open–circuited.

(or open-circuit reverse voltage gain)

$B = \left.\dfrac{v_1}{-v_2}\right|_{v_2=0}$ – Ratio of input voltage to output current when output is short–circuited.

(or short-circuit transfer impedance)

$C = \left.\dfrac{i_1}{v_2}\right|_{i_2=0}$ – Ratio of input current to output voltage when input is open–circuited.

(or open-circuit transfer admittance)

$D = \left.\dfrac{i_1}{-i_2}\right|_{v_2=0}$ – Ratio of input current to output current when output is short–circuited.

(or short–circuit reverse current gain)

11.7 Relation between Z- and Y – parameters

(a) Z – parameters in terms of Y – parameters

We know that for a two-port network,

$$v_1 = Z_{11}i_1 + Z_{12}i_2 \qquad \text{----------} \quad (11.9)$$

$$v_2 = Z_{21}i_1 + Z_{22}i_2 \qquad \text{----------} \quad (11.10)$$

also

$$i_1 = Y_{11}v_1 + Y_{12}v_2 \qquad \text{----------} \quad (11.11)$$

$$i_2 = Y_{21}v_1 + Y_{22}v_2 \qquad \text{----------} \quad (11.12)$$

From eq.(11.11)

$$v_1 = \left(\frac{1}{Y_{11}}\right)i_1 - \left(\frac{Y_{12}}{Y_{11}}\right)v_2 \qquad \text{----------} \quad (11.13)$$

from eq.(11.12)

$$v_2 = \left(\frac{1}{Y_{22}}\right)i_2 - \left(\frac{Y_{21}}{Y_{22}}\right)v_1 \qquad \text{----------} \quad (11.14)$$

Substituting the value of v_2 in to eq.(11.13) we get,

$$v_1 = \frac{1}{Y_{11}}i_1 - \frac{Y_{12}}{Y_{11}}\left[\frac{i_2}{Y_{22}} - \frac{Y_{21}}{Y_{22}}v_1\right] = \left(\frac{1}{Y_{11}}\right)i_1 + \left(\frac{-Y_{12}}{Y_{11}Y_{22}}\right)i_2 + \frac{Y_{12}Y_{21}}{Y_{11}Y_{22}}v_1$$

or

$$v_1\left[\frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{11}Y_{22}}\right] = \left(\frac{1}{Y_{11}}\right)i_1 + \left(\frac{-Y_{12}}{Y_{11}Y_{22}}\right)i_2$$

or

$$v_1 = \left(\frac{Y_{22}}{\Delta}\right)i_1 + \left(\frac{-Y_{12}}{\Delta}\right)i_2 \qquad \text{----------} \quad (11.15)$$

Comparing eq. (11.15) with eq.(11.9), we get

$$Z_{11} = \left(\frac{Y_{22}}{\Delta}\right) \;;\quad Z_{12} = \left(\frac{-Y_{12}}{\Delta}\right) \text{ where } \Delta = (Y_{11}Y_{22} - Y_{12}Y_{21})$$

Similarly, from eq.(11.12),

$$v_2 = \left(\frac{1}{Y_{22}}\right)i_2 + \left(\frac{-Y_{21}}{Y_{22}}\right)v_1 \qquad \text{----------} \quad (11.16)$$

From eq.(11.11),

$$v_1 = \left(\frac{1}{Y_{11}}\right)i_1 + \left(\frac{-Y_{12}}{Y_{11}}\right)v_2 \quad \text{----------} \quad (11.17)$$

Substituting the value of v_1 into eq.(10.16), we get

$$v_2 = \left(\frac{1}{Y_{22}}\right)\, i_2 + \left(\frac{-Y_{21}}{Y_{22}}\right)\left[\frac{1}{Y_{11}} i_1 + \left(\frac{-Y_{12}}{Y_{11}}\right) v_2\right]$$

$$= \left(\frac{1}{Y_{22}}\right)\, i_2 + \left(\frac{-Y_{21}}{Y_{22}Y_{11}}\right) i_1 + \frac{Y_{12}Y_{21}}{Y_{11}Y_{22}} v_2$$

or $$v_2\left[\frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{11}Y_{22}}\right] = \left(\frac{1}{Y_{22}}\right)\, i_2 + \left(\frac{-Y_{21}}{Y_{22}Y_{11}}\right)\, i_1$$

or $$v_2 = \left(\frac{Y_{11}}{\Delta}\right)\, i_2 + \left(\frac{-Y_{21}}{\Delta}\right)\, i_1 \quad \text{----------} \quad (11.18)$$

Comparing eq. (11.18) with eq.(11.10), we get

$$Z_{22} = \left(\frac{Y_{11}}{\Delta}\right) \;;\quad Z_{21} = \left(\frac{-Y_{21}}{\Delta}\right)$$

b. Y-parameters in terms of Z-parameters

We know that for a two-port network,

$$i_1 = Y_{11}v_1 + Y_{12}v_2 \quad \text{----------} \quad (11.19)$$

$$i_2 = Y_{21}v_1 + Y_{22}v_2 \quad \text{----------} \quad (11.20)$$

also $$v_1 = Z_{11}i_1 + Z_{12}i_2 \quad \text{----------} \quad (11.21)$$

$$v_2 = Z_{21}i_1 + Z_{22}i_2 \quad \text{----------} \quad (11.22)$$

from eq (11.21),

$$i_1 = \left(\frac{1}{Z_{11}}\right)\, v_1 + \left(-\frac{Z_{12}}{Z_{11}}\right)\, i_2 \quad \text{----------} \quad (11.23)$$

from eq (11.22)

$$i_2 = \left(\frac{1}{Z_{22}}\right)\, v_2 + \left(\frac{-Z_{21}}{Z_{22}}\right)\, i_1 \quad \text{----------} \quad (11.24)$$

Substituting the value of i_2 in to eq.(11.23) we get,

$$i_1 = \left(\frac{1}{Z_{11}}\right)\, v_1 + \left(\frac{-Z_{12}}{Z_{11}}\right)\left[\left(\frac{1}{Z_{22}}\right)\, v_2 + \left(\frac{-Z_{21}}{Z_{22}}\right)\, i_1\right]$$

or $$i_1 = \left(\frac{1}{Z_{11}}\right) v_1 + \left(\frac{-Z_{12}}{Z_{11}Z_{22}}\right) v_2 + \frac{Z_{12}Z_{21}}{Z_{11}Z_{22}}\ i_1$$

or $$\left[\frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{11}Z_{22}}\right] i_1 = \frac{v_1}{Z_{11}} + \left(\frac{-Z_{12}}{Z_{11}Z_{22}}\right) v_2$$

or $$i_1 = \left(\frac{Z_{22}}{\Delta}\right) v_1 + \left(\frac{-Z_{12}}{\Delta}\right) v_2 \qquad \text{----------} \quad (11.25)$$

Comparing eq. (11.25) with eq. (11.19) we get

$$Y_{11} = \left(\frac{Z_{22}}{\Delta}\right) \ ; \quad Y_{12} = \left(\frac{-Z_{12}}{\Delta}\right) \text{ where } \Delta = (Z_{11}Z_{22} - Z_{12}Z_{21})$$

Similarly, from eq.(11.22),

$$i_2 = \left(\frac{1}{Z_{22}}\right) v_2 + \left(\frac{-Z_{21}}{Z_{22}}\right) i_1 \qquad \text{----------} \quad (11.26)$$

From eq.(11.21),

$$i_1 = \left(\frac{1}{Z_{11}}\right) v_1 + \left(\frac{-Z_{12}}{Z_{11}}\right) i_2 \qquad \text{----------} \quad (11.27)$$

Substituting the value of i_1 into eq. (11.26), we get

$$i_2 = \left(\frac{1}{Z_{22}}\right) v_2 + \left(\frac{-Z_{21}}{Z_{22}}\right)\left[\frac{1}{Z_{11}} v_1 + \left(\frac{-Z_{12}}{Z_{11}}\right) i_2\right]$$

$$= \left(\frac{1}{Z_{22}}\right) v_2 + \left(\frac{-Z_{21}}{Z_{22}Z_{11}}\right) v_1 + \frac{Z_{12}Z_{21}}{Z_{11}Z_{22}}\ i_2$$

or $$\left[\frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{11}Z_{22}}\right] i_2 = \left(\frac{-Z_{21}}{Z_{11}Z_{22}}\right) v_1 + \left(\frac{1}{Z_{22}}\right) v_2$$

or $$i_2 = \left(\frac{-Z_{21}}{\Delta}\right) v_1 + \left(\frac{Z_{11}}{\Delta}\right) v_2 \qquad \text{----------} \quad (11.28)$$

Comparing eq.(11.28) with eq.(11.20), we get

$$Y_{21} = \left(\frac{-Z_{21}}{\Delta}\right) \ ; \quad Y_{22} = \left(\frac{Z_{11}}{\Delta}\right)$$

11.8 T-Network and Π- Network

Another approach to network analysis is to represent a network by means of a simple T- or Π-networks as shown in Fig.11.7 and Fig.11.8. Cascaded networks are analyzed by this approach. The two types of equivalent circuits are used depending upon the convenience. Input impedance is measured with output short circuited (Z_{1sc}) and output open circuited (Z_{2oc}). When networks are cascaded, transmission loss in power has to be minimized. For this, the output impedance of first stage must match with the input impedance of the next stage. Likewise the source impedance of the generator must match with the input impedance of first stage. Also the output impedance of final stage must match with load impedance. Two impedances are defined for cascaded networks. The first one is called Characteristic Impedance and the other is called Image Impedance.

(i)T-Network

Any two-port network may be represented by an equivalent T-Network as shows in Fig 11.7. The name T –network is given because of its shape similar to English alphabet 'T'. It may also be called ***Y-network*** or **Star network**. The impedances Z_1 and Z_2 are known as Series arm impedances. It is often convenient to represent T impedances in terms of network parameters.

a) Representing T- impedances in terms of any of Z-parameters

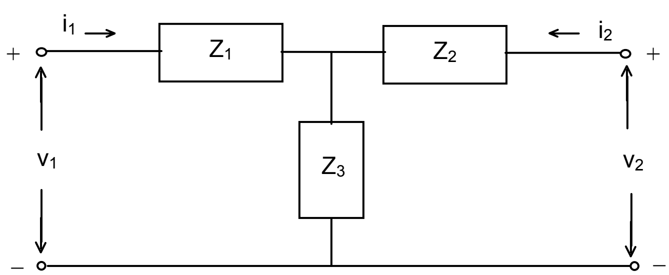

Fig 11.7 T-Network

$$Z_{11} = \left|\frac{v_1}{i_1}\right|_{i_2=0}$$ = Input impedance with output open circuited.

$$= Z_1 + Z_3 \qquad \text{---------- (11.29)}$$

$$Z_{21} = \left|\frac{v_2}{i_1}\right|_{i_2=0}$$ = Forward transfer impedance with output open circuited.

$$= Z_3 \qquad \text{---------- (11.30)}$$

$$Z_{22} = \left|\frac{v_2}{i_2}\right|_{i_1=0}$$ = Output impedance with input open circuited.

$$=Z_2+Z_3 \qquad \text{----------} \quad (11.31)$$

$$Z_{12} = \left|\frac{v_1}{i_2}\right|_{i_1=0}$$ = Reverse transfer impedance with input open circuited.

$$= Z_3 \qquad \text{----------} \quad (11.32)$$

On solving eq.(11.29) to (11.32), we get

$$\left.\begin{aligned} Z_1 &= Z_{11} - Z_{12} = Z_{11} - Z_{21} \\ Z_2 &= Z_{22} - Z_{12} = Z_{22} - Z_{21} \\ Z_3 &= Z_{12} = Z_{21} \end{aligned}\right\} \qquad \text{----------} \quad (11.33)$$

(b) Representing T- impedances in terms of any of ABCD parameters

$$A = \left|\frac{v_1}{v_2}\right|_{i_2=0}$$

Applying KVL to the input and output circuits of Fig.11.7, we get,

$$v_1 = i_1(Z_1 + Z_3); \quad \mathrm{v}_2 = i_1 Z_3$$

Hence $A = \dfrac{v_1}{v_2} = \dfrac{I_1(Z_1+Z_3)}{I_1 Z_3} = \dfrac{Z_1}{Z_3} + 1$

or $$A = 1 + \frac{Z_1}{Z_3} \qquad \text{----------} \quad (11.34)$$

$$B = \left|\frac{v_1}{-i_2}\right|_{v_2=0}$$

Applying KVL to the input and output circuits of Fig.11.7, we get,

$$v_1 = i_1(Z_1 + Z_3) + \mathrm{i}_2 Z_3$$

$$0 = I_1 Z_3 + I_2(Z_2 + Z_3)$$

Hence $i_1 = -\dfrac{(Z_2+Z_3)}{Z_3} \cdot i_2$ ---------- (11.35)

$$v_1 = -\frac{(Z_2+Z_3)}{Z_3}(Z_1+Z_3)i_2 + i_2 Z_3$$

or $$B = \frac{v_1}{-i_2} = \frac{(Z_2+Z_3)(Z_1+Z_2) - Z_3^2}{Z_3} \qquad \text{----------} \quad (11.36)$$

$$C = \left|\frac{i_1}{v_2}\right|_{i_2=0} = \frac{i_1}{i_1 Z_3} = \frac{1}{Z_3} = Y_3 \qquad \text{----------} \quad (11.37)$$

$$D = \left|\frac{i_1}{-i_2}\right|_{v_2=0} = \frac{Z_2 + Z_3}{Z_3} = 1 + \frac{Z_2}{Z_3} \qquad \text{----------} \quad (11.38)$$

[using eq.(11.35)];

On solving eq.(11.34) to eq. (11.38), we get

$$\left.\begin{aligned} Z_1 &= \frac{A-1}{C} \\ Z_2 &= \frac{D-1}{C} \\ Z_3 &= \frac{1}{C} \end{aligned}\right\} \qquad \text{----------} \quad (11.39)$$

(ii) The Π- Network

Any two-port network may be represented by an equivalent Π- Network as shown in Fig.11.8. The name is given because of its shape similar to Greek alphabet 'Π'. It may also be called ***Δ-Network***. The impedances Z_B and Z_C are known as Shunt arm impedances. The elements Z_A, Z_B and Z_C of this network may be represented in terms of any of the four types of parameters of the two-port network.

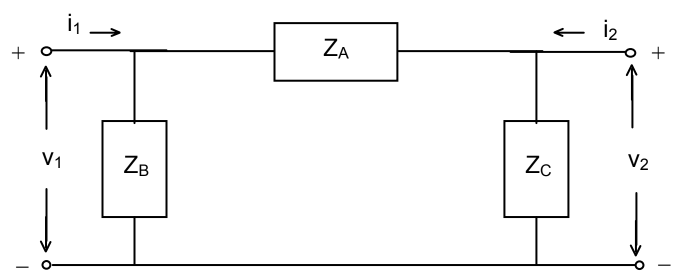

Fig. 11.8 Π-Network

(a) Representing Π- impedances in terms of Y-parameters

$$Y_{11} = \left|\frac{i_1}{v_1}\right|_{v_2=0}$$ **=** Input admittance with output short-circuited.

Applying KVL to the input and output circuits we get.

$v_1 = i_1[Z_B \| Z_A]$ [Because Z_C is shorted as per the definition]

$$v_1 = i_1 \left[\frac{Z_A Z_B}{Z_A + Z_B} \right]$$

or $$Y_{11} = \frac{i_1}{v_1} = \frac{Z_A + Z_B}{Z_A \cdot Z_B} = Y_A + Y_B \qquad \text{---------- (11.40)}$$

Here $Y_A = \frac{1}{Z_A}; \;\; Y_B = \frac{1}{Z_B}$

$Y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0}$ **=** Forward transfer admittance with output short-circuited.

Applying KCL to the junction at the output, we get

$$i_2 = -i_1 \frac{Z_B}{Z_A + Z_B}$$

Hence $$Y_{21} = \frac{i_2}{v_1} = \frac{-i_1 Z_B /(Z_A + Z_B)}{i_1 \cdot Z_A Z_B /(Z_A + Z_B)} = \frac{-1}{Z_A}$$

or $$Y_{21} = -\frac{1}{Z_A} = -Y_A \qquad \text{---------- (11.41)}$$

$Y_{22} = \left|\frac{i_2}{v_2}\right|_{v_1=0}$ **=** Output admittance with input short-circuited.

Under this condition, Z_A, Z_C become parallel, so applying KVL to the output port, we get

$$v_2 = i_2 \left[\frac{Z_A Z_C}{Z_A + Z_C} \right]$$

$$Y_{22} = \frac{i_2}{v_2} = \frac{Z_A + Z_C}{Z_A \cdot Z_C} = Y_A + Y_C \qquad \text{---------- (11.42)}$$

$Y_{12} = \left|\frac{i_1}{v_2}\right|_{v_1=0}$ **=** Reverse transfer admittance with input short-circuited.

Under this condition, Z_A and Z_C become parallel. Applying KCL to the junction at the input, we get

$$i_1 = -i_2 \frac{Z_C}{Z_A + Z_C}$$

$$Y_{12} = \frac{i_1}{v_2} = \frac{-i_2 Z_C /(Z_A + Z_C)}{i_2 \cdot Z_A Z_C /(Z_A + Z_C)} = \frac{-1}{Z_A} = -Y_A$$

Hence $$Y_{12} = -\frac{1}{Z_A} = -Y_A \qquad \text{-------- (11.43)}$$

On solving equations (11.40) to (11.43), we get

$$\left.\begin{aligned} Y_A &= -Y_{12} = -Y_{21};\; Y_B = Y_{11} + Y_{21} \\ Y_C &= Y_{22} + Y_{21} \end{aligned}\right\} \qquad \text{---------- (11.44)}$$

Using ABCD parameters

$$A = \left|\frac{v_1}{v_2}\right|_{i_2=0} = \frac{v_1}{v_1 \cdot \dfrac{Z_C}{Z_A + Z_C}} = \frac{Z_A + Z_C}{Z_C} = \frac{Y_C + Y_A}{Y_A} \qquad \text{---------- (11.45)}$$

$$B = \left|\frac{v_1}{-i_2}\right|_{v_2=0} = \frac{v_1}{\dfrac{v_1}{Z_A}} = Z_A \qquad \text{---------- (11.46)}$$

$$C = \left|\frac{i_1}{v_2}\right|_{i_2=0} = \frac{\dfrac{v_1}{Z_B \cdot (Z_A + Z_C)/(Z_A + Z_B + Z_C)}}{v_1 \cdot Z_C/(Z_A + Z_C)}$$

$$= \frac{Z_A + Z_B + Z_C}{Z_B Z_C} = \frac{Z_A}{Z_B Z_C} + \frac{1}{Z_C} + \frac{1}{Z_B} \qquad \text{---------- (11.47)}$$

$$D = \left|\frac{i_1}{-i_2}\right|_{v_2=0} = \frac{i_1}{i_1 Z_B/(Z_A + Z_B)} = \frac{Z_A + Z_B}{Z_B} = 1 + \frac{Z_A}{Z_B} = 1 + \frac{Y_B}{Y_A} \qquad \text{--------- (11.48)}$$

On solving equations (11.45) to (11.48) we get

$$Y_A = \frac{1}{Z_A} = \frac{1}{B};\; Y_B = \frac{D-1}{B};\; Y_C = \frac{1}{Z_C} = \frac{(A-1)}{B} \qquad \text{---------- (11.49)}$$

10.9 T - Π Inter conversions or Star – Delta Transformations

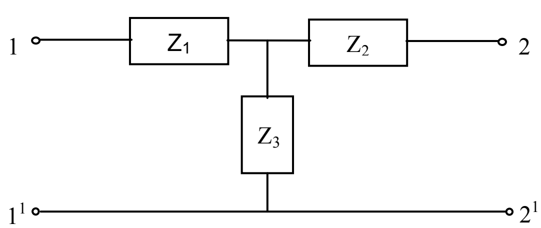

Fig 11.9 T-Network

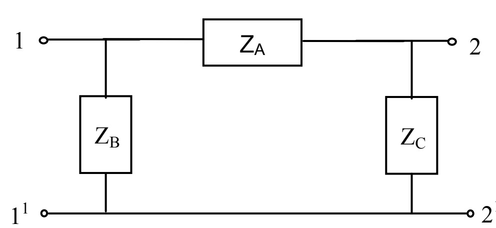

Fig 11.10 Π -Network

Let

$Z_{1OC} \rightarrow$ Impedance measured across the 1, 1^1 terminals with the 2, 2^1 terminals open – circuited.

Z_{1SC} → Impedance measured across the 1, 1^1 terminals with the 2, 2^1 terminals short – circuited.

Z_{2OC} → Impedance measured across the 2, 2^1 terminals with the 1, 1^1 terminals open – circuited.

Now from Fig.11.9 and 1Fig.11.10, we can write the above values as

T – Network

$$Z_{1OC} = Z_1 + Z_3 \quad \text{----------} \quad (11.50)$$

$$Z_{2OC} = Z_2 + Z_3 \quad \text{----------} \quad (11.51)$$

$$Z_{1SC} = Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3} \quad \text{----------} \quad (11.52)$$

Π - Network

$$Z_{1OC} = \frac{Z_B(Z_A + Z_C)}{(Z_A + Z_B + Z_C)} \quad \text{--------} \quad (11.53)$$

$$Z_{2OC} = \frac{Z_C(Z_A + Z_B)}{(Z_A + Z_B + Z_C)} \quad \text{--------} \quad (11.54)$$

$$Z_{1SC} = \frac{Z_A Z_B}{Z_A + Z_B} \quad \text{--------} \quad (11.55)$$

(a) Π - T Conversion (or) Delta – Star Conversion

If T and Π networks are equivalent, then external networks must be equivalent. This means that Equation (11.50), equation (11.53) represent the same. Similarly equation (11.51), equation (11.54); and equation (11.52) and equation (11.55). Hence we can write

$$Z_1 + Z_3 = \frac{Z_B(Z_A + Z_C)}{(Z_A + Z_B + Z_C)} \quad \text{----------} \quad (11.56)$$

$$Z_2 + Z_3 = \frac{Z_C(Z_A + Z_B)}{(Z_A + Z_B + Z_C)} \quad \text{----------} \quad (11.57)$$

$$Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3} = \frac{Z_A Z_B}{Z_A + Z_B} \quad \text{----------} \quad (11.58)$$

Subtracting eq.(11.56) from eq.(11.58), we get

$$Z_3 - \frac{Z_2 Z_3}{Z_2 + Z_3} = \frac{Z_B(Z_A + Z_C)}{(Z_A + Z_B + Z_C)} - \frac{Z_A Z_B}{(Z_A + Z_B)}$$

or $$\frac{Z_3^2}{(Z_2+Z_3)} = \frac{Z_B^2 Z_C}{(Z_A+Z_B+Z_C)(Z_A+Z_B)} \quad \text{---------- (11.59)}$$

Substituting value of (Z_2+Z_3) from eq.(11.57) into eq.(11.59) we get

$$\frac{Z_3^2}{\left[\dfrac{Z_C(Z_A+Z_B)}{(Z_A+Z_B+Z_C)}\right]} = \frac{Z_B^2 Z_C}{(Z_A+Z_B+Z_C)(Z_A+Z_B)}$$

or $$Z_3^2 = \frac{Z_B^2 Z_C^2}{(Z_A+Z_B+Z_C)^2}$$

or $$Z_3 = \frac{Z_B Z_C}{(Z_A+Z_B+Z_C)} \quad \text{---------- (11.60)}$$

Similarly

$$Z_1 = \frac{Z_B Z_A}{(Z_A+Z_B+Z_C)} \quad \text{---------- (11.61)}$$

and $$Z_2 = \frac{Z_A Z_C}{(Z_A+Z_B+Z_C)} \quad \text{---------- (11.62)}$$

(b) T to Π Conversion (or) Star – Delta Conversion

Multiplying equations (11.57) and (11.58), we get

$$Z_1Z_2 + Z_2Z_3 + Z_3Z_1 = \frac{Z_A Z_B Z_C}{(Z_A+Z_B+Z_C)}$$

Replacing $\dfrac{Z_A Z_B}{(Z_A+Z_B+Z_C)}$ with Z_1 (according to eq.11.61), we get

$$Z_1Z_2 + Z_2Z_3 + Z_3Z_1 = Z_1 \cdot Z_C$$

or $$Z_C = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_1} \quad \text{---------- (11.63)}$$

Similarly $$Z_B = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_2} \quad \text{---------- (11.64)}$$

$$Z_A = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3} \quad \text{---------- (11.65)}$$

Note: To remember T to Π or Π to T conversion, draw the following Fig.11.11. Here Z_A, Z_B, Z_C forms Π-network (Delta) Z_1, Z_2, Z_3 forms T-network (or star)

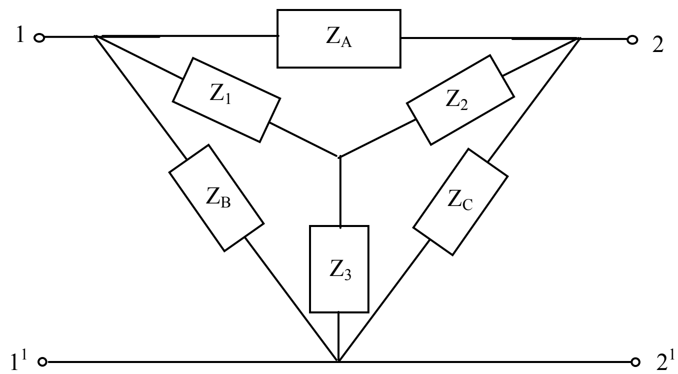

Fig 11.11

$$Z_{star} = \frac{\text{Product of two adjacent } Zs \text{ in Delta}}{\Sigma \text{ all } Zs \text{ in Delta}}$$

$$Z_{Delta} = \frac{\Sigma \text{ all cross products of } Zs \text{ in Star}}{\text{Oppsite } \ Z \text{ in Star}}$$

11.10 Bridged T- Network

A symmetrical T-network in which the two series impedances are identical is a simplified representation of general network. However, in some networks we have to consider the feedback between input and output. In those circumstances, bridge impedance is added between input and output. Such a T- network is called ***Bridged T-Network***.

(In a T-network, when the two series arms impedances are equal, the network is said to be ***Symmetrical***. To an ordinary T-network, when another impedance is added that “Bridges” the input to the output, than resulting circuit is called Bridge-T network.) It is widely used in filters. It is shown in Fig.11.12. The characteristic impedance of a symmetrical network is defined as its input impedance when such networks are connected together. A point to note is that at low frequencies the inductive component will be very small. It is neglected when compared to the purely resistive part. The capacitive component which is represented in shunt with the purely resistive component will be so large at these frequencies that the effective impedance is equal to the resistive component. So, impedance is usually shown with the symbol of resistance. If the label is Z, it is an impedance. If the label is R, it is a resistance.

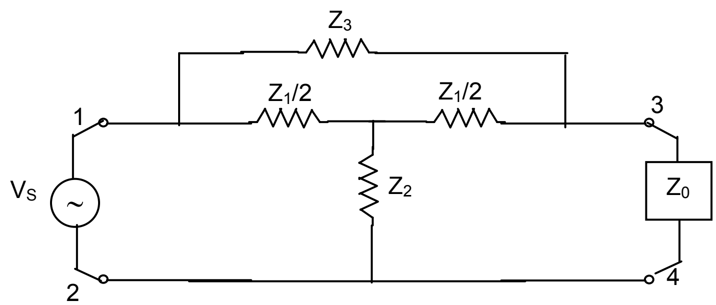

Fig 11.12.

Characteristic impedance $Z_O = \sqrt{Z_{OC} \times Z_{SC}}$ ----------- (11.66)

We can calculate Z_{OC}, open-circuit impedance between the terminals 1, 2 by open-circuiting the terminals 3, 4 as shown in Fig.11.13 (a).

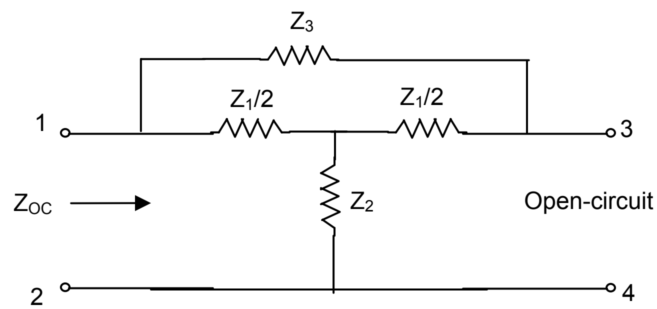

Fig 11.13(a)

$$Z_{OC} = Z_2 + \frac{\frac{Z_1}{2}\left(\frac{Z_1}{2} + Z_3\right)}{\frac{Z_1}{2} + \frac{Z_1}{2} + Z_3} = \frac{Z_2 + \frac{Z_1^2}{4} + \frac{Z_3}{2}}{(Z_1 + Z_3)}$$

or $$Z_{OC} = \frac{Z_1 Z_2 + Z_2 Z_3 + \frac{Z_1^2}{4} + \frac{Z_3 Z_3}{2}}{(Z_1 + Z_3)}$$ ----------- (11.67)

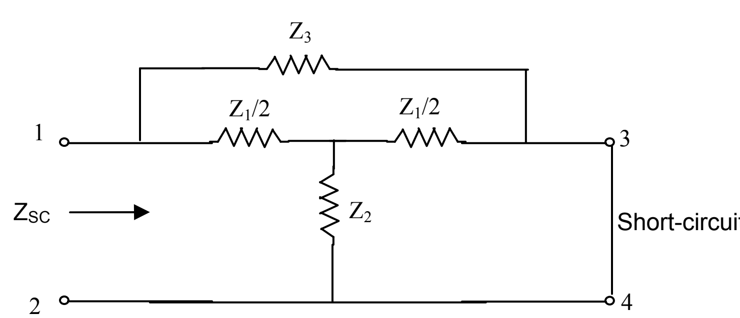

Fig 11.13(b)

$$Z_{SC} = \frac{Z_3\left[\frac{Z_1}{2} + \frac{\frac{Z_1}{2}+Z_2}{\frac{Z_1}{2}+Z_2}\right]}{\left[Z_3 + \frac{Z_1}{2} + \frac{\frac{Z_1}{2}Z_2}{\frac{Z_1}{2}+Z_2}\right]} = \frac{\left[\frac{Z_1^2 Z_3}{4} + \frac{Z_1Z_2Z_3}{2} + \frac{Z_1Z_2Z_3}{2}\right]}{\left[\frac{Z_1Z_3}{2} + Z_2Z_3 + \frac{Z_1^2}{4} + \frac{Z_1Z_2}{2} + \frac{Z_1Z_2}{2}\right]}$$

$$Z_{SC} = \frac{\frac{Z_1^2Z_3}{4} + Z_1Z_2Z_3}{\frac{Z_1Z_3}{2} + Z_2Z_3 + \frac{Z_1^2}{4} + Z_1Z_2} \qquad \text{-----------} \quad (11.68)$$

Hence, $$Z_O = \sqrt{Z_{OC} \times Z_{SC}} = \sqrt{\frac{\frac{Z_1^2Z_3}{4} + Z_1Z_2Z_3}{(Z_1+Z_3)}}$$

$$Z_O = \sqrt{Z_1Z_3\left[\frac{(Z_1+4Z_2)}{4(Z_1+Z_3)}\right]} \qquad \text{-----------} \quad (11.69)$$

11.11 Twin – T or Parallel T-Network

When two symmetrical T-networks are connected in parallel, the resulting structure is termed as Twin–T or parallel –T network, shown in Fig.11.14. They are widely used as null network.

The selectivity curve of a twin T network resembles very closely that of the anti-resonant circuit and it may be used to reject a particular frequency completely. Hence it is called as a ***Notch Filter***. The selectivity can be achieved without the use of inductance and effective Q value is much greater than that of the usual anti- resonant circuit. The twin T- network has the advantage over Wien bridge because it has a common input and output terminal and hence it can be connected into a practical system, However, a Wien bridge needs only two elements to be ganged for its tuning whereas a twin-T filter needs three elements to be ganged. Therefore, Wien bridge is preferred for a tunable notch filter while twin T- network is preferred for a fixed frequency notch filter.

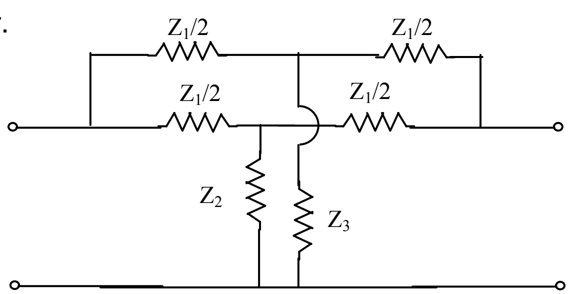

Fig 11.14

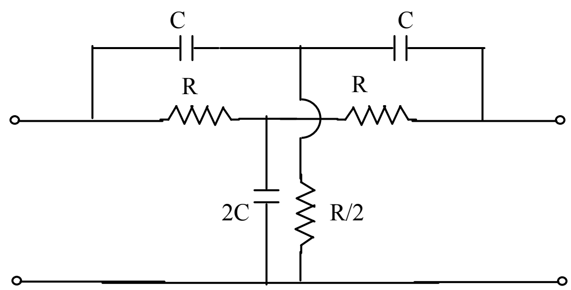

Fig 11.14a

Derivation of the null frequency

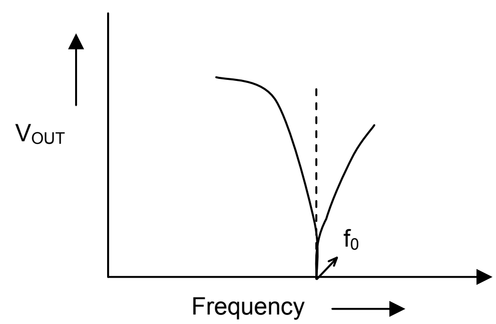

Fig 11.15 Frequency response curve

Convert C, C, R/2 T- network into Π-network. Let the equivalent Π impedances be Z_{1A}, Z_{1B} and Z_{1C}, Convert R, R and 2C T-network into Π-network. Let the equivalent Π impedances be Z_{2A}, Z_{2B} and Z_{2C}. The two Π-networks will be in parallel such that the equivalent Π-network of Twin-T will be

$Z_A = Z_{1A}*Z_{2A}/(Z_{1A}+Z_{2A})$,etc. and the simplified expressions are

$Z_A = Z_C = 0.5*(R-j/WC)$ and $Z_B= 2jwCR/(w^2C^2R^2-1)$

$$\frac{v_o}{v_i} = \frac{\frac{1}{2}\left(R - \frac{j}{\omega C}\right)}{\frac{1}{2}R - j\left(\frac{1}{2\omega C} + \frac{2\omega CR}{\omega^2 C^2 R^2 - 1}\right)}$$

Separate real and imaginary parts and equate the imaginary part to zero, as at resonance, network impedance is purely resistive.

The null frequency will be $\omega = \dfrac{1}{\sqrt{5}CR}$

SOLVED PROBLEMS

Example 1: Convert the following Π-network into T-network.

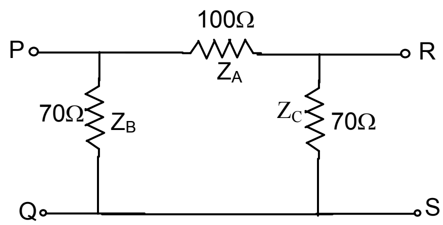

Fig 11.16

Solution: The equivalent T-network can be drawn as shown below; along with the given Π-network.

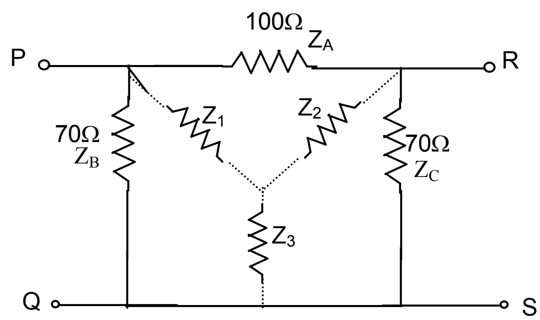

Fig 11.17

We know that,

$$Z_{star} = \frac{\text{Product of two adjacent } Zs \text{ in } '\Pi'}{\Sigma \text{ all } Zs \quad \text{in } '\Pi'}$$

Hence $Z_1 = \dfrac{Z_A Z_B}{Z_A + Z_B + Z_C}$; $Z_2 = \dfrac{Z_B Z_C}{Z_A + Z_B + Z_C}$; $Z_3 = \dfrac{Z_A Z_C}{Z_A + Z_B + Z_C}$;

So, the network impedances are

$$Z_1 = \frac{100 \times 70}{100 + 70 + 70} = \frac{7000}{240} = \frac{175}{6}\Omega$$

$$Z_2 = \frac{100 \times 70}{240} = \frac{7000}{240} = \frac{175}{6}\Omega$$

$$Z_1 = \frac{70 \times 70}{240} = \frac{490}{24} = \frac{245}{12}\Omega$$

Example 2: Convert the following T-network into Π-network.

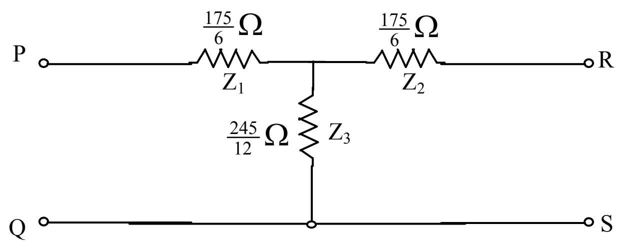

Fig 11.18

Solution: The equivalent Π-network can be drawn as shown below; along with the given T-network.

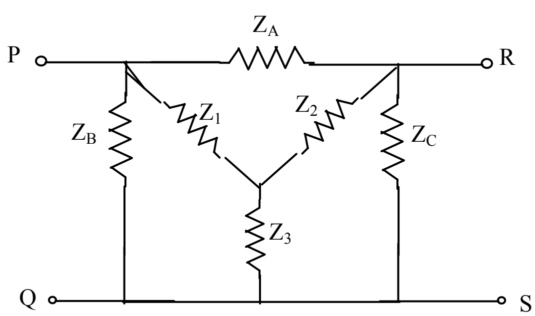

Fig.11.19

We know that,

$$Z_{Delta} = \frac{\Sigma \text{ All cross products of } Zs \text{ in Star}}{\text{Oppsite } Z \text{ in Star}}$$

$$Z_C = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_1}$$

$$Z_B = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_2}$$

$$Z_A = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3}$$

Hence, the equivalent Π-network impedances are

$$Z_A = \frac{\frac{175}{6}\cdot\frac{175}{6} + \frac{175}{6}\cdot\frac{245}{12} + \frac{245}{12}\cdot\frac{175}{6}}{\frac{245}{12}} = 100\Omega$$

$$Z_B = \frac{\frac{175}{36} \times \frac{840}{12}}{\frac{175}{6}} = \frac{175}{36} \times \frac{840}{12} \times \frac{16}{175} = 70\Omega$$

$$Z_C = \frac{\frac{175}{36} \times \frac{840}{2}}{\frac{175}{6}} = 70\Omega$$

Example 3: Find the short-circuit admittance parameters for the resistive Π-network shown in Fig.11.20.

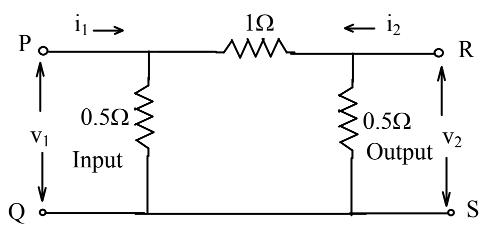

Fig 11.20

Solution: We know that

$$Y_{11} = \left| \frac{i_1}{v_1} \right|_{v_2=0}$$

Applying KVL to the input section, we get (with $v_2 = 0$)

$$v_1 = i_1 \left[\frac{0.5 \times 1}{0.5 + 1} \right]$$

or $$Y_{11} = \frac{i_1}{v_1} = \frac{1.5}{0.5} = 3\Omega^{-1}$$

$$Y_{21} = \left| \frac{i_2}{v_1} \right|_{v_2=0} = \frac{-v_1/1}{v_1} = -1\Omega^{-1}$$

$$Y_{22} = \left| \frac{i_2}{v_2} \right|_{v_1=0}$$

Applying KVL to the output section, we get

(with v_1=0)

$$v_2 = i_2 \left[\frac{0.5 \times 1}{0.5 + 1} \right] = i_2 \left[\frac{0.5}{1.5} \right] = \frac{i_2}{3}$$

so $Y_{22} = \frac{i_2}{v_2} = 3\Omega^{-1}$

$$Y_{12} = \left|\frac{i_1}{v_2}\right|_{v_1=0} = \frac{-v_2/1}{v_2} = -1\Omega^{-1}$$

So, the Y-parameters are

$Y_{11} = 3\Omega^{-1}$ $\quad$ $Y_{21} = -1\Omega^{-1}$

$Y_{12} = -1\Omega^{-1}$ $\quad$ $Y_{22} = 3\Omega^{-1}$

Example 4: Find the h-parameters of the circuit shown below in Fig.11.21.

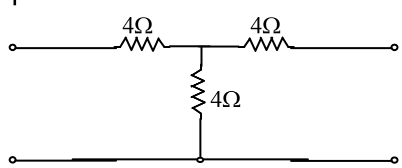

Fig 11.21

Solution: h-parameters are defined as

$$h_{11} = \left(\frac{v_1}{i_1}\right)_{v_2=0}$$

$$h_{21} = \left(\frac{i_2}{i_1}\right)_{v_2=0}$$

$$h_{12} = \left(\frac{v_1}{v_2}\right)_{i_1=0}$$

$$h_{22} = \left(\frac{i_2}{v_2}\right)_{i_1=0}$$

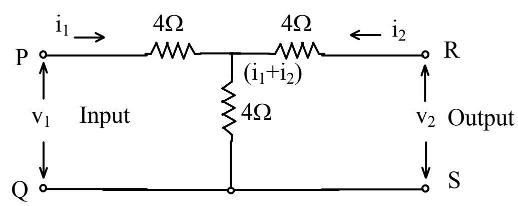

Fig 11.22

Applying KVL to the input section of Fig.11.22 we get

$$v_1 = 4i_1 + i_1\left[\frac{4\times 4}{4+4}\right] = 4i_1 + 2i_1 = 6i_1$$

or $h_{11} = \frac{v_1}{i_1} = 6\Omega$

Applying KVL to the output section we get

(with $v_2 = 0$) (short R, S)

$4i_2+4(i_1 + i_2) = 0$

or $8i_2 + 4i_1 = 0$ or $2i_2 + i_1 = 0$ or $i_2 = -i_1/2$

So $$h_{21} = \frac{i_2}{i_1} = \frac{-i_1/2}{i_1} = \frac{-1}{2}$$

When $i_1 = 0$, the p.d. across 4Ω resistor in the parallel arm becomes v_1. Hence

$$v_1 = \frac{4}{4+4} \times v_2 = \frac{4}{8} v_2 = \frac{1}{2} v_2 \qquad \text{Hence} \qquad h_{12} = \frac{v_1}{v_2} = \frac{1}{2}$$

Also $v_2 = i_2[4 + 4]$ Hence $$h_{22} = \frac{i_2}{v_2} = \frac{1}{8}\,\text{mho.}$$

The h-parameters are

$$h_{11} = 6\Omega;\ h_{12} = \frac{1}{2};\ h_{21} = \frac{1}{8};\ h_{22} = \frac{1}{8}\,\text{mho;}$$

Example 5: The Z-parameters of a two-port network are $Z_{11} = 20\Omega$; $Z_{12} = Z_{21} = 10\Omega$; $Z_{22} = 25\Omega$. Find the equivalent T-network and Π-network. (Ref: Article 11.8(a))

Solution:

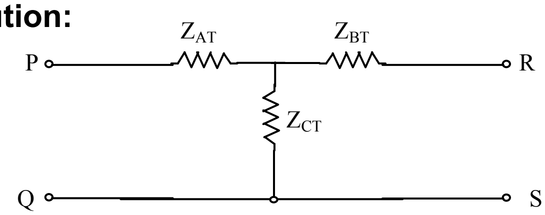

Fig 11.23

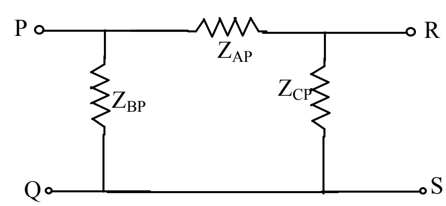

Fig 11.24

$Z_{AT} = Z_{11} - Z_{11} = 20 - 10 = 10\Omega$

$Z_{BT} = Z_{22} - Z_{12} = 25 - 10 = 15\Omega$

$Z_{CT} = Z_{12} = Z_{21} = 10\Omega$

Hence,

$$Z_{AP} = \frac{Z_{AT} \cdot Z_{BT} + Z_{BT} \cdot Z_{CT} + Z_{CT} \cdot Z_{AT}}{Z_{CT}}$$

$$= \frac{10 \times 15 + 15 \times 10 + 10 \times 10}{10} = 40\Omega$$

$$Z_{BP} = \frac{400}{Z_{BT}} = \frac{400}{15} = \frac{80}{3}\Omega \ ; \quad Z_{CP} = \frac{400}{Z_{AT}} = \frac{400}{10} = 40\Omega$$

11.12 Summary

In circuit analysis, we come across one-port and two-port networks. Frequently encountered networks are two-port networks. They possess two ports namely input and output. At each port, there are two variables voltage and current. The dependence of these variables on each another can be expressed in 4 different ways. In the four ways, four sets of parameters are defined. They are Z, Y, h and ABCD parameters. Each of the individual parameters are defined with some conditions either open or short-circuit. The frequently used networks are of the shape T (Star) or Π (Delta). These networks can be converted one into the other using the suitable relations. Other similar networks are bridge-T and twin-T networks. Though these are not true bridges but possess the properties of bridge networks. They can be used in filters.

11.13 Key terminology

Terminals – Single port – Two port – T-network – Π-network – parameters.

11.14 Self Assessment Questions

A. Long Answer Questions

1. Discuss the method to convert a given T-network into Π-network and vice versa.
2. Give the difference between single-port and two-port networks. Define Y-and Z-parameters of a two-port network and find the relation between them.
3. Differentiate between single-port and two-port networks. Define Z, Y, h and ABCD parameters of a two-port network.
4. Draw the circuit diagram of a bridged T-network. Derive an expression for its characteristic impedance.

B. Short Answer Questions

1. Define h-parameters of a four terminal network.
2. Obtain expressions for Z-parameters of a two-port network.
3. Define Z-parameters of a four-terminal network.
4. Define Y-parameters of a four-terminal network.
5. Define ABCD parameters of a four-terminal network.
6. Explain bridge–T and twin–T networks.

C. Numerical Problems

1. Find the h-parameters of the circuit shown in Fig.11.25

   Ans: $h_{11}=150\Omega$; $h_{21} = 0.5$; $h_{12} = 1$; $h_{22} = 1/2090\ \Omega^{-1}$

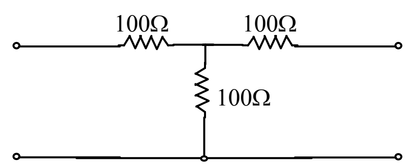

Fig 11.25

2. The Z-parameters of a two-port network are $Z_{11} = 20\Omega$; $Z_{12} = Z_{21} = 15\Omega$; $Z_{22} = 30\Omega$. Find the equivalent T, Π-networks.

   (Ans: $Z_{AT} = 5$; $Z_{BT} = 15$; $Z_{CT} = 15$
   $Z_{AP} = 25$; $Z_{BP} = 25$; $Z_{CP} = 75$)

3. Find the elements of equivalent Π-network for the given T-network with $Z_1 = 6\Omega$; Z2 = 10Ω; $Z_3 = 5\Omega$;

   Ans: $Z_{AP} = 28\Omega$; $Z_{BP} = 14\Omega$; $Z_{CP} = 70/3\Omega$

10.15 Reference Books

1. Networks, Lines and Fields

   ---- John D.Ryder (PHI).

2. Circuit Analysis

   ---- Umesh Sinha (Umesh Publications)

3. Transmission Lines and Networks

   ---- Umesh Sinha.

UNIT – IV LESSON – 12

AC BRIDGES

OBJECTIVES

This lesson explains you the concept of

1. AC Bridges and analysis
2. AC Wheatstone's Bridge
3. Maxwell's Inductance Bridge
4. Maxwell's L/C Bridge
5. Anderson's Bridge
6. Desauty's Bridge
7. Schering's Bridge
8. Wien's Series Bridge
9. Wien's Parallel Bridge
10. Bridge sensitivity and balancing of bridges

Structure of the Lesson

Introduction

It often becomes necessary to measure inductance, capacitance, effective resistance, 'Q' factor, dielectric loss and frequency very accurately. A convenient method is to compare the unknown values with standard values to the extent of accuracy required. Bridge circuits adopted from the standard Wheatstone's bridge method are developed for the purpose. These bridges are driven with alternating current sources and hence these bridges are called AC bridges. The operating frequency may be at 50 Hz from a power transformer or it may be an audio frequency or a high frequency from a signal generator. The null detector must be an AC instrument like headphones, CRO or electronic galvanometer.

In an AC bridge, there are two components to be adjusted to obtain balance conditions. One component is first adjusted for the best available null and then the other component is adjusted to further improve the null. This process is repeated until the best possible null is obtained.

Bridge Sensitivity

The bridge sensitivity may be defined in terms of the smallest change in the measured quantity that causes the detector to deviate from zero. Bridge sensitivity can be improved by using a more sensitive detector and/or by increasing the level of supply voltage. It is usually desired that the sensitivity be as large as possible.

12.1 AC Wheatstone's Bridge

An AC Wheatstone's bridge consists of four arms, a source of excitation and a balance detector. In this bridge, each of the four arms is an impedance and the battery and galvanometer of the Wheatstone's bridge are replaced respectively by an AC source and a detector, sensitive to small alternating potential differences.

<u>Analysis</u>: Fig 12.1 shows the basic AC Wheatstone's bridge. Z_1, Z_2, Z_3 and Z_4 are impedances in four arms. The condition for bridge balance is that the potential difference between points B and D should be zero. This means that the voltage drop from A to B must be equal to voltage drop from A to D, both in magnitude and phase. In complex notation,

$$E_1 = E_2$$

$$I_1 Z_1 = I_2 Z_2 \qquad \text{----------- (12.1)}$$

At balance,

$$I_1 = I_3 = \frac{E}{Z_1 + Z_3} \qquad \text{------------ (12.2)}$$

and $$I_2 = I_4 = \frac{E}{Z_2 + Z_4} \qquad \text{------------ (12.3)}$$

Hence equation (12.1) becomes

$$\frac{\cancel{E}}{Z_1 + Z_3} \,.\, Z_1 = \frac{\cancel{E}}{Z_2 + Z_4} \,.\, Z_2$$

or $$Z_1 Z_2 + Z_1 Z_4 = Z_1 Z_2 + Z_2 Z_3$$

or $$Z_1 Z_4 = Z_2 Z_3 \qquad \text{------------ (12.4)}$$

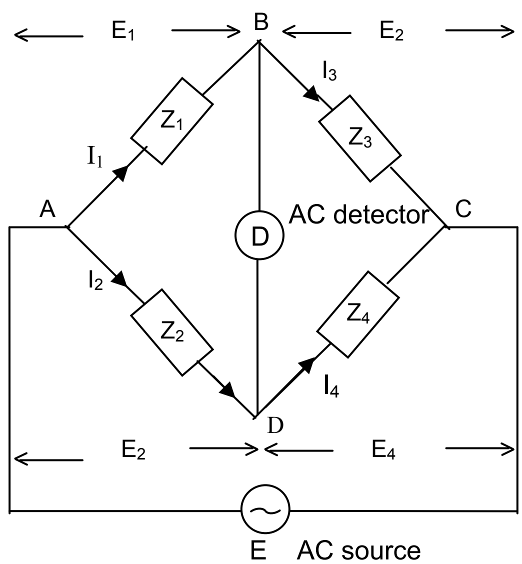

Fig. 12.1 AC Wheatstone's Bridge

Equation (12.4) is the basic equation for the balance of an AC bridge. It states that product of impedances of one pair of opposite arms must equal the product of impedances of the other pair of opposite arms expressed in complex notation. Rewriting equation (12.4) in polar form (i.e. $\overline{Z} = Z\angle\theta$, where Z represents the magnitude and 'θ' represents the phase angle of the complex impedance), we get

$$(Z_1\angle\theta_1)\,(Z_4\angle\theta_4) = (Z_2\angle\theta_2)\,(Z_3\angle\theta_3)$$

or $$Z_1 Z_4 \angle\theta_1 + \theta_4 = Z_2 Z_3 \angle\theta_2 + \theta_3 \qquad \text{------------ (12.5)}$$

Hence to balance the AC Bridge, the following two conditions are to be satisfied:

(i) $$\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3 \qquad \text{------------ (12.6)}$$

(ii) $$Z_1 Z_4 = Z_2 Z_3 \qquad \text{------------ (12.7)}$$

The bridge must be balanced with respect to both resistances and phase angles.

11.2 Maxwell's Inductance Bridge

This bridge is used to determine inductance of an unknown inductor by comparing with another inductance of known value. Fig.11.2 shows the circuit of Maxwell's inductance bridge.

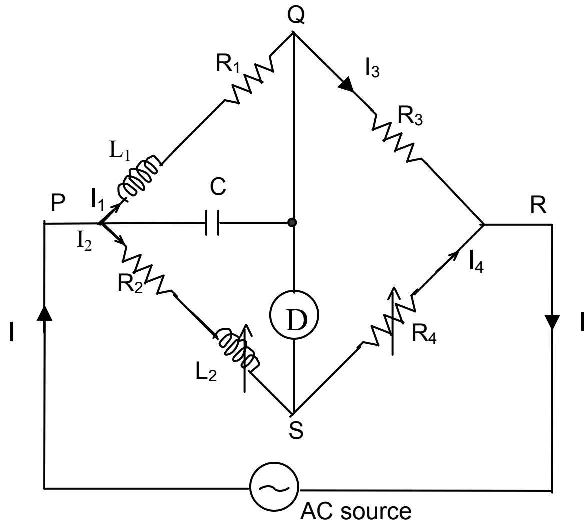

Fig. 12.2 Maxwell's Inductance Bridge

In the Fig.12.2 L_2 - Known inductance of resistance R_2.

L_1 - Unknown inductance of resistance R_1.

R_3, R_4 - Non-inductance resistances.

D - AC detector.

To balance the bridge, L_2 is varied and simultaneously R_4 is changed till the detector shows least current.

Analysis: At balance $I_1=I_3$; $I_2=I_4$.

or Potential difference across PQ = Potential difference across PS

and Potential difference across QR = Potential difference across RS

(or) $$\frac{V_{PQ}}{V_{QR}} = \frac{V_{PS}}{V_{SR}} \quad \text{------------} \quad (12.8)$$

(or) $$\frac{(R_1 + j\omega L_1)I_1}{I_3 R_3} = \frac{(R_2 + j\omega L_2)I_2}{I_4 R_4}$$

(or) $$R_1 R_4 + j\omega L_1 R_4 = R_2 R_3 + j\omega L_2 R_3 \quad \text{----------} \quad (12.9)$$

Equating real parts, we get

$$R_1 R_4 = R_2 R_3$$

Equating imaginary parts we get $L_1 R_4 = L_2 R_3$

or $\frac{R_1}{R_2} = \frac{R_3}{R_4}$ and $\frac{L_1}{L_2} = \frac{R_3}{R_4}$

or $\frac{R_1}{R_2} = \frac{R_3}{R_4} = \frac{L_1}{L_2}$ ------------ (12.10)

Knowing the value of L_2, value of L_1 may be calculated using equation (12.10).

11.3 Maxwell's L/C Bridge (or) Maxwell-Wien's Bridge

This bridge is used to measure inductance by comparison with standard variable capacitance. In this bridge, the positive phase angle of inductive impedance may be compensated by the negative phase angle of capacitive impedance put in the opposite arm, as shown in Fig.12.3. In this bridge, L_3 is unknown inductance, C_1 is variable capacitance, R_1, R_2, R_4 are resistances and R_3 is a variable resistance.

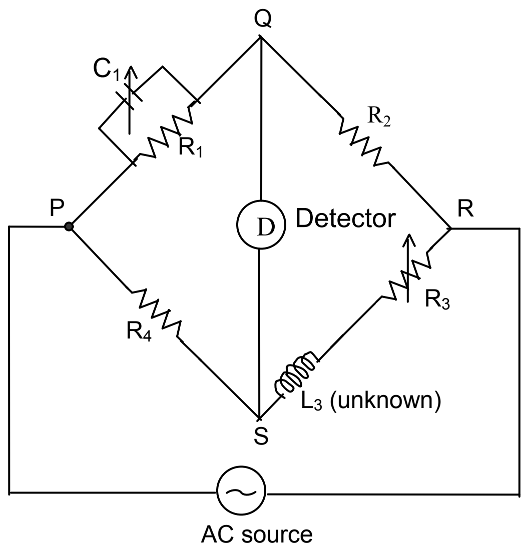

Fig. 12.3

Analysis:

Impedance of PQ arm is

$$\frac{1}{Z_1} = \frac{1}{R_1} + \frac{1}{\frac{1}{j\omega C_1}} = \frac{1}{R_1} + j\omega C_1 = \frac{j\omega C_1 R_1 + 1}{R_1} \Rightarrow Z_1 = \frac{R_1}{j\omega C_1 R_1 + 1}$$

Impedance of QR arm = $Z_2 = R_2$

Impedance of RS arm = $Z_3 = R_3 + j\omega L_3$

Impedance of SP arm = $Z_4 = R_4$

Bridge balance condition is $Z_1 Z_3 = Z_2 Z_4$ ------------ (12.11)

Substituting the values of Z_1, Z_2, Z_3 and Z_4 into equation (12.11), we get

$$\frac{R_1}{(j\omega C_1 R_1 + 1)}.(R_3 + j\omega L_3) = R_2 R_4$$

on cross multiplication, we get

$$R_2 R_4 + j\omega C_1 R_1 R_2 R_4 = R_1 R_3 + j\omega L_3 R_1$$

Equating the real parts, we get

$$R_2 R_4 = R_1 R_3;$$

$$R_3 = \frac{R_2 R_4}{R_1} \qquad \text{-----------} \quad (12.12)$$

Equating the imaginary parts, we get

$$j\omega L_3 R_1 = j\omega C_1 R_1 R_2 R_4$$

or $$L_3 = C_1 R_2 R_4 \qquad \text{--------} \quad (12.13)$$

Using the relation (12.13), we can estimate the value of unknown inductance of an inductor. Relation (12.12) is the balancing condition.

12.4 Anderson's Bridge

Anderson's bridge method is capable of precise measurements of inductances over a wide range of values from a few micro Henrys to several Henrys. In this bridge, the unknown inductance is measured in terms of a known capacitance and resistance.

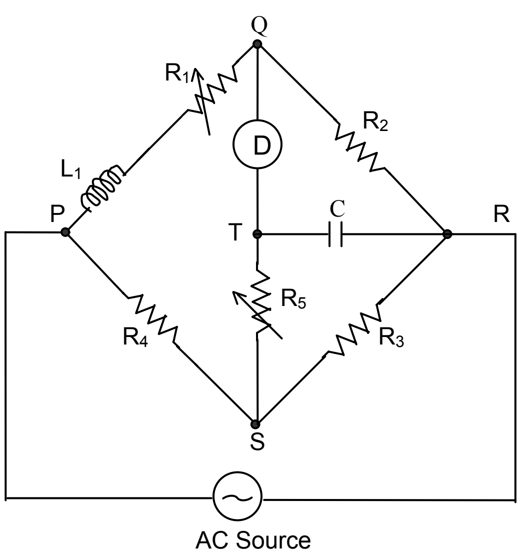

Fig. 12.4(a)

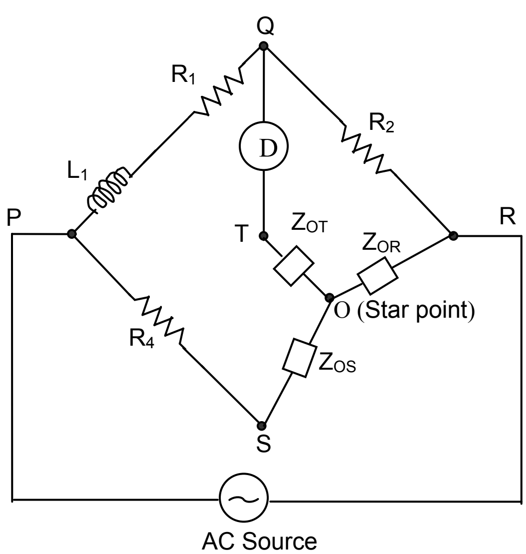

Fig. 12.4(b)

Analysis:

In Fig.11.4 (a), L_1 is unknown inductance, R_1, R_5 are variable resistances, R_2, R_3, R_4 are fixed resistances; C is a standard capacitance. The balance condition for this bridge is obtained by converting the mesh of impedances C, R_5 and R_3 to an equivalent star network with star point 'O' by Delta to Star transformation. From Fig.12.4 (b),

$$Z_{OS} = \frac{R_3 R_5}{R_3 + R_5 + \frac{1}{j\omega C}}; \qquad Z_{OR} = \frac{R_3\left(\frac{1}{j\omega C}\right)}{R_3 + R_5 + \frac{1}{j\omega C}};$$

$$Z_{OT} = \frac{R_5\left(\frac{1}{j\omega C}\right)}{R_3 + R_5 + \frac{1}{j\omega C}};$$

Hence in the modified bridge network PQROSP, shown in Fig 11.4b, we can write

Impedance of PQ arm = $Z_1 = R_1 + j\omega L_1$

Impedance of QR arm = $Z_2 = R_2$

Impedance of RO arm = $Z_3 = Z_{OR}$

Impedance of RO arm = $Z_4 = Z_{OS} + Z_{PS}$

$= Z_{OS} + R_4$

For bridge balance, $Z_1 Z_3 = Z_2 Z_4$ ------------ (12.14)

On substituting the values of Z_1, Z_2, Z_3 and Z_4 into equation (12.14), we get

$$[(R_1 + j\omega L_1).Z_{OR}] = [R_2 (R_4 + Z_{OS})]$$

$$\text{or} \quad (R_1 + j\omega L_1)\frac{\left(\frac{R_3}{j\omega C}\right)}{\left(R_3 + R_5 + \frac{1}{j\omega C}\right)} = R_2\left[R_4 + \frac{R_3 R_5}{\left(R_3 + R_5 + \frac{1}{j\omega C}\right)}\right]$$

$$\text{or} \quad \frac{R_1 R_3}{j\omega C} + \frac{R_3 L_1}{C} = R_2 R_3 R_4 + R_2 R_4 R_5 + \frac{R_2 R_4}{j\omega C} + R_2 R_3 R_5$$

Equating real parts, we get

$$R_2 R_3 R_4 + R_2 R_4 R_5 + R_2 R_3 R_5 = \frac{R_3 L_1}{C}$$

$$\text{or} \quad L_1 = CR_2\left[R_4 + R_5 + \frac{R_4 R_5}{R_3}\right] \quad \text{-----------} \quad (12.15)$$

Equating imaginary parts, we get

$$\frac{R_2 R_4}{j\omega C} = \frac{R_1 R_3}{j\omega C} \quad \text{(or)} \quad R_1 = \frac{R_2 R_4}{R_3} \quad \text{-----------} \quad (12.16)$$

Determination of unknown inductance using Anderson's Bridge

The circuit is to be connected as shown in Fig.12.4(a). The unknown inductance is to be introduced in place of L_1. Now by adjusting the resistance value in place of R_1, first D.C. balance, given by equation (12.16), is to be achieved. Then the A.C. balance is to be achieved by adjusting the resistance in place of R_5. A perfect balance is to be obtained by varying the resistances R_1 and R_5 alternately till the AC detector shows minimum current. The value of L_1 is then to be calculated by using the relation

$$L_1 = CR_2\left(R_4 + R_5 + \frac{R_4 R_5}{R_3}\right)$$

For more accuracy, the procedure is to be repeated with different resistances in place of R_2. Finally, the average value of L_1 is to be estimated.

12.5 DeSauty's Bridge

DeSauty's Bridge is used to determine the capacitance of an unknown capacitor by comparing it with standard known capacitor. Fig.11.5 shows the circuit of DeSauty's bridge. This consists of two resistive arms and two capacitive arms.

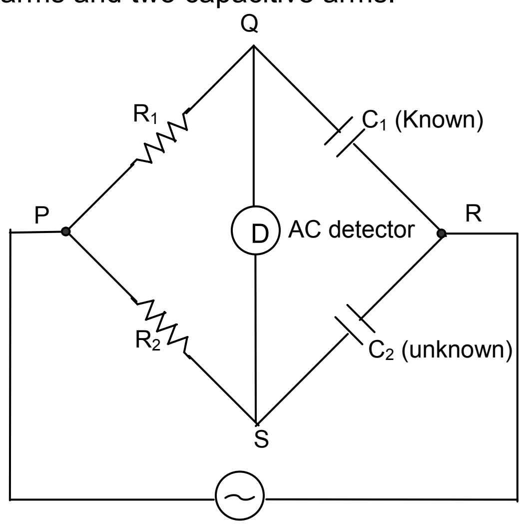

Fig. 12.5 DeSauty's Bridge

Analysis: Impedance of PQ arm = $Z_1 = R_1$

Impedance of QR arm = $Z_2 = \dfrac{1}{j\omega C_1}$

Impedance of RO arm = $Z_3 = \dfrac{1}{j\omega C_2}$

Impedance of RO arm = $Z_4 = R_2$

When the bridge is balanced,

$$Z_1 Z_3 = Z_2 Z_4 \qquad \text{---------------- (12.17)}$$

On substituting the values of Z_1, Z_2, Z_3 and Z_4 into equation (11.17), we get

$$R_1 \cdot \frac{1}{j\omega C_2} = \frac{1}{j\omega C_1} \cdot R_2$$

$$\text{or} \quad \frac{R_1}{C_2} = \frac{R_2}{C_1}$$

$$\text{or} \quad C_2 = \frac{R_1}{R_2} C_1 \qquad \text{------------ (12.18)}$$

Determination of unknown capacitance using DeSauty's Bridge

To determine the unknown capacitance, the circuit is to be connected as shown in Fig.12.5. The unknown capacitance is to be introduced in place of C_2 and a known capacitance is to be introduced in place of C_1. A selected value of resistance is to be introduced in place of R_2. Now the resistance in place of R_1 is to be adjusted (using a variable resistance), to get least possible current through AC detector. Using the relation $C_2 = \frac{R_1}{R_2} C_1$, the value of unknown capacitance is to be determined. For more accuracy, the above procedure is to be repeated with other selected values of resistances and finally the average value is to be determined.

12.6 Schering's Bridge: Schering bridge is the most accurate bridge for determining the capacitance of an unknown capacitor, particularly of small capacitance value.

Analysis: The circuit of Schering bridge is shown in Fig.12.6. In figure, C_1 is the unknown capacitor, C_2 is a variable air capacitor and R_3 is a variable resistance.

Impedance of PQ arm = $Z_1 = R_1 + \dfrac{1}{j\omega C_1}$

Impedance of QR arm = $Z_2 = R_2$

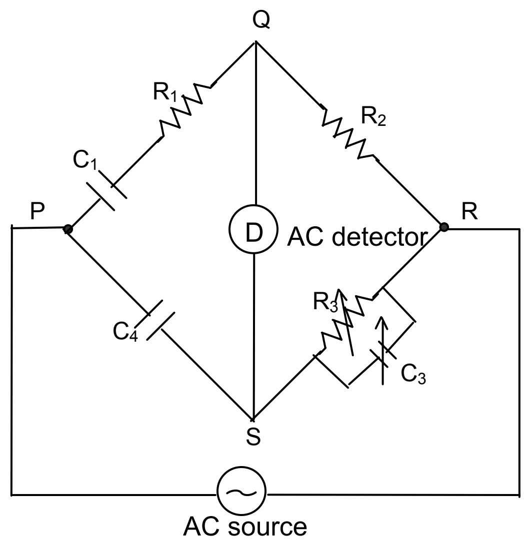

Fig. 12.6 Schering's Bridge

Impedance of RS arm = $Z_3 = \dfrac{R_3}{j\omega R_3 C_3 + 1}$

$$\left[\frac{1}{Z_3} = \frac{1}{R_3} + \frac{1}{\dfrac{1}{j\omega C_3}} = \frac{1}{R_3} + j\omega C_3 = \frac{j\omega C_3 R_3 + 1}{R_3}\right]$$

Impedance of SP arm = $Z_4 = R_4$

Under bridge balance condition,

$$Z_1 Z_3 = Z_2 Z_4 \qquad \text{-------------- (12.19)}$$

Substituting the values of Z_1, Z_2, Z_3 and Z_4 into equation (12.19), we get

$$\left(R_1 + \frac{1}{j\omega C_1}\right)\left(\frac{1}{\dfrac{1}{R_3} + j\omega C_3}\right) = R_2 \frac{1}{j\omega C_4}$$

on cross multiplication, we get

$$j\omega C_4 R_1 + \frac{C_4}{C_1} = \frac{R_2}{R_3} + j\omega R_2 C_3$$

Equating the real parts, we get

$$\frac{C_4}{C_1} = \frac{R_2}{R_3} \text{ or } C_1 = \left(\frac{R_3}{R_2}\right) C_4 \qquad \text{------------- (12.20)}$$

Equating the imaginary parts, we get

$$j\phi C_4 R_1 = j\phi R_2 C_3$$

or $$\frac{R_1}{R_2} = \frac{C_3}{C_4}$$ -------------- (12.21)

Equations (12.20), (12.21) give the two conditions of balance, which can be achieved independently by varying R_3 and C_3 alternately until the detector shows zero current.

12.7 Wein's Series Bridge

This is a simple ratio bridge and is used for audio frequency measurement of capacitance over a wide range. The circuit is shown in Fig. 12.7

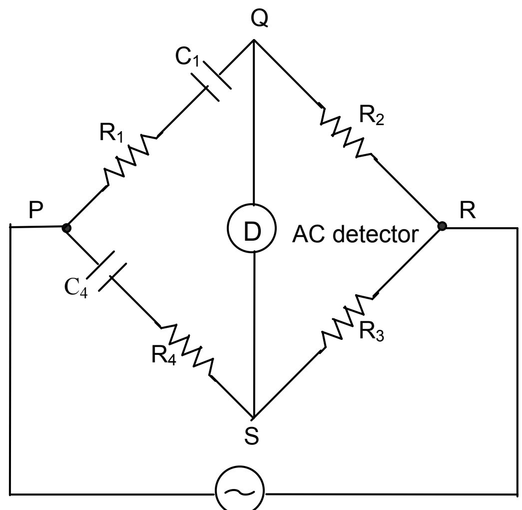

Fig. 12.7 Wien's Series Bridge

Analysis: Impedance of PQ arm = $Z_1 = R_1 + \frac{1}{j\omega C_1}$

Impedance of QR arm = $Z_2 = R_2$

Impedance of RS arm = $Z_3 = R_3$

Impedance of SP arm = $Z_4 = R_4 + \frac{1}{j\omega C_4}$

Under bridge balance condition,

$$Z_1 Z_3 = Z_2 Z_4$$ ------------ (12.22)

Substituting the values of Z_1, Z_2, Z_3 and Z_4 into equation (12.22), we get

$$\left(R_1 + \frac{1}{j\omega C_1}\right).R_3 = R_2\left(R_4 + \frac{1}{j\omega C_4}\right)$$

Equating real parts, we get

$$R_1 R_3 = R_2 R_4 \qquad \text{-------------} \quad (12.23)$$

Equating imaginary parts, we get

$$\frac{R_3}{j\omega C_1} = \frac{R_2}{j\omega C_4} \quad \text{or} \quad \frac{R_3}{C_1} = \frac{R_2}{C_4}$$

$$\text{or} \qquad C_1 = \frac{R_3}{R_2} C_4 \qquad \text{-------------} \quad (12.24)$$

Relation (12.23) is the bridge balance condition and relation (12.24) is used to estimate the capacitance of an unknown capacitor.

12.8 Wien's Parallel Bridge

A bridge that has a parallel combination in one arm and a series combination in an adjacent arm is known as a Wien's Bridge. The Wien bridge circuit is shown in Fig.12.8. It is used (i) to measure the frequency (ii) in harmonic distortion analysis (iii) in audio and high frequency oscillators as the frequency-determining element.

Analysis: Impedance of PQ arm $Z_1 = \dfrac{R_1}{1 + j\omega C_1 R_1}$

$$\left[\frac{1}{Z_1} = \frac{1}{R_1} + \frac{1}{\frac{1}{j\omega C_1}} = \frac{1}{R_1} + j\omega C_1 = \frac{j\omega C_1 R_1 + 1}{R_1} \right]$$

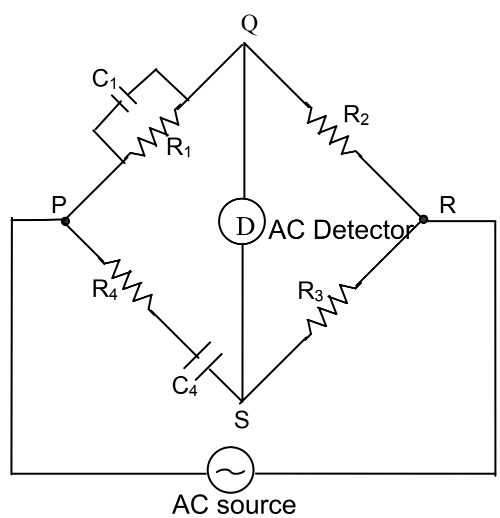

Fig. 12.8 Wien's Parallel Bridge

Impedance of QR arm $Z_2 = R_2$

Impedance of RS arm $Z_3 = R_3$

Impedance of SP arm $Z_4 \; = \; R_4 + \dfrac{1}{j\omega C_4}$

$$Z_1 Z_3 \; = \; Z_2 Z_4 \qquad \text{--------------} \quad (12.25)$$

Substituting the values of Z_1,Z_2,Z_3 and Z_4 into equation (12.25), we get

$$\left(\frac{R_1}{1 + j\omega C_1 R_1}\right) . R_3 \; = \; R_2 . \left(R_4 + \frac{1}{j\omega C_4}\right)$$

or $\dfrac{R_1 R_3}{R_2} \; = \; \left(1 + j\omega C_1 R_1\right)\left(R_4 + \dfrac{1}{j\omega C_4}\right)$

$$\frac{R_1 R_3}{R_2} \; = \; R_4 + j\omega C_1 R_4 R_1 + \frac{1}{j\omega C_4} + \frac{C_1 R_1}{C_4}$$

Equating real parts, we get

$$\frac{R_1 R_3}{R_2} \; = \; R_4 + \frac{C_1 R_1}{C_4}$$

or $\dfrac{R_3}{R_2} \; = \; \dfrac{R_4}{R_1} + \dfrac{C_1}{C_4}$ -------------- (12.26)

Equating imaginary parts, we get

$j\omega C_1 R_4 R_1 + \dfrac{1}{j\omega C_4} \; = \; 0$ or $-\omega^2 R_4 R_1 C_1 C_4 + 1 \; = \; 0$

or $\omega^2 = \dfrac{1}{R_1 R_4 C_1 C_4}$

or $\omega \; = \; \dfrac{1}{\sqrt{R_1 R_4 C_1 C_4}}$

or $f \; = \; \dfrac{1}{2\pi\sqrt{R_1 R_4 C_1 C_4}}$ -------------- (12.27)

From equation (12.27), it is clear that by adjusting two of the components for balance, the bridge is capable of determining the frequency of a sine wave source. It is also useful as a frequency selective network.

11.9 Summary

AC bridges are adoptions of Wheatstone's bridge. They are used for measuring unknown resistance, capacitance, inductance, frequency, in harmonic distortion analysis as frequency determining elements, dielectric loss, and 'Q' factor. AC bridge consists of four arms with impedance in each arm, AC source and an AC detector. The AC detector may be a CRO, sensitive galvanometer, AC micro ammeter and headphones. Impedance in each arm may be a pure resistor, a capacitor, an inductor or a combination of these. In order to determine the unknown component value, introduced in any arm, first the bridge is to be balanced by satisfying the bridge balance condition. Then by using the balancing component values, the unknown component value is to be estimated.

12.10 Key Terminology

Bridge balance – AC detector – Balancing condition.

Solved problems

Example.1

The arms of an AC Maxwell bridge are arranged as follows. AB is a non-inductive resistance of 1000Ω in parallel with a capacitor of capacitance 0.5μF, BC is a non-inductive resistance of 600Ω, CD is an inductive impedance (unknown) and DA is a non-inductive resistance of 400Ω. If balance is obtained under these conditions, find the value of resistance and inductance of the branch CD.

Solution: Balance condition

$$R_3 = \frac{R_2 R_4}{R_1} = \frac{600 \times 400}{1000}$$

$$= 240\ \Omega$$

$$L_3 = C R_2 R_4$$

$$= 0.5 \times 10^{-6} \times 600 \times 400$$

$$= 12 \times 10^{-2}$$

$$= 0.12\text{H}$$

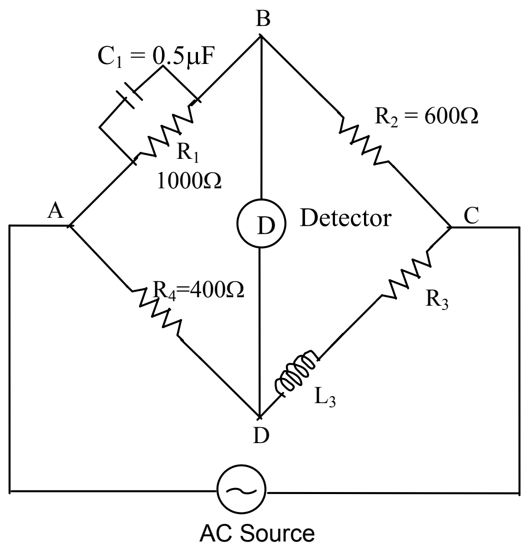

Fig 12.9

Example.2

Balance is obtained in Wien's bridge of Fig.12.10 with values R_1=1KΩ, R_4=1KΩ, $C_1 = C_4 = 2\mu$F. What is the frequency of the source?

Solution:

Frequency of AC source

$$f = \frac{1}{2\pi\sqrt{R_1 R_4 C_1 C_4}}$$

$$= \frac{7}{44\sqrt{1\times10^3 \times 1\times10^3 \times 2\times10^{-6} \times 2\times10^{-6}}}$$

$$= \frac{7}{44\sqrt{4\times10^{-6}}} = \frac{7\times1000}{88} = 79.6\ Hz$$

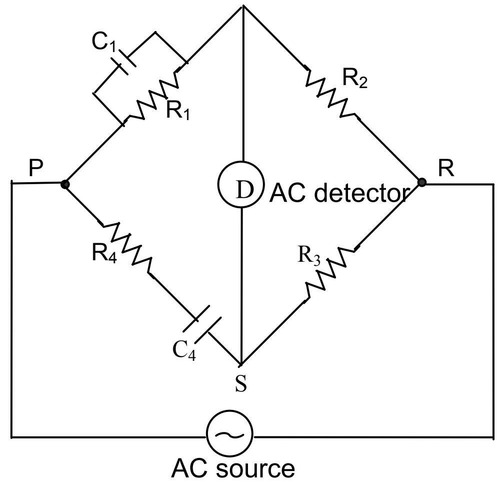

Fig. 12.10

Example.3

The components introduced in a DeSauty's bridge PQRS shown in Fig.12.11 are R_1 = 1000Ω; R_2 = 2000Ω; C_1 = 0.33μF; Find the unknown capacitance C_2 when the bridge is balanced?

Solution: When the bridge is balanced,

$$C_1 R_1 = C_2 R_2$$

or $$C_2 = \left(\frac{R_1}{R_2}\right) C_1$$

$$= \frac{1000}{2000}(0.33\times10^{-6})$$

$$= \left(\frac{0.33}{2}\right)\mu F$$

$$C_2 = 0.165\,\mu F$$

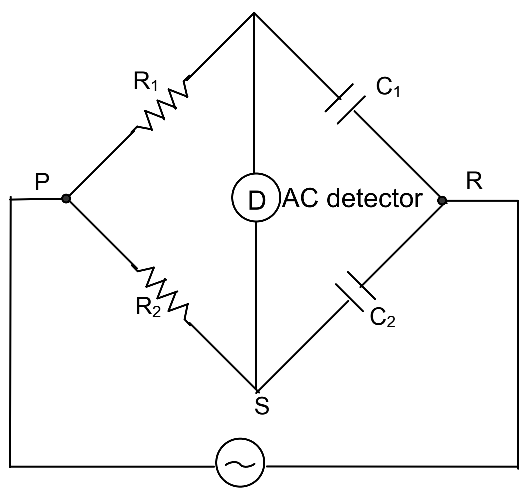

Fig 12.11

11.11 Self Assessment questions

A. Long Answer Questions

1) Explain how inductance of an inductor can be measured using Anderson's bridge. Give its theory.
2) What is an AC bridge? How capacitance can be measured by using DeSauty's bridge? Give its analysis.

3) What is an AC bridge? What is the principle of working of AC bridge? Explain the measurement of capacitance by Wien's bridge.
4) Explain how an AC Wheatstone's bridge is different from DC Wheatstone's bridge? Derive the formula for the frequency of a parallel Wien's bridge. Mention the applications of Wien's bridge.
5) Describe with theory, Schering bridge for the determination of capacitance.

B. Short Answer Questions

1) Draw the circuit diagram of AC Wheatstone's bridge and explain its working. What is bridge sensitivity?
2) Explain the working principle of Anderson's bridge for measuring the self- inductance.
3) Explain the measurement of capacitance by Desauty's bridge.

C. Numerical problems

1. The arms of a four-arm bridge ABCD, supplied with a sinusoidal voltage have the following values AB = 200Ω resistance in parallel with 2μF capacitor BC = 400Ω resistance; DA = Resistance R in series with 1μF capacitor. CD = 1000Ω resistance. Determine (i) the value of R and (ii) the supply frequency at which the bridge will be balanced. ***[Ans - R = 100Ω; f = 796Hz]***

11.12 References

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----- John D. Ryder (PHI)

UNIT-IV **LESSON-13**

CATHODE RAY OSCILLOSCOPE (CRO)

Objectives

This lesson explains you the concept of

1. Cathode Ray Tube (CRT)
2. Electric deflection
3. Magnetic deflection
4. Fluorescent screen
5. CRO Block diagram
6. Applications

Structure of the lesson

Introduction

CRO stands for **C**athode **R**ay **O**scilloscope. The CRO works as an 'EYE' for the Electronic engineer in testing and repairing of sophisticated electronic instruments. It is used for display, measurement and analysis of waveforms and other phenomenon in electric and electronic circuits. It presents visual representation of a wide variety of dynamic phenomenon.

CRO consists of a Cathode Ray Tube (CRT) and some additional circuitry to operate the CRT.

13.1 CRT and its working

The main parts of the CRT are:

i) Electron Gun Assembly
ii) Deflection Plate Assembly
iii) Fluorescent Screen
iv) Base

The parts of a CRT are shown in Fig 13.1

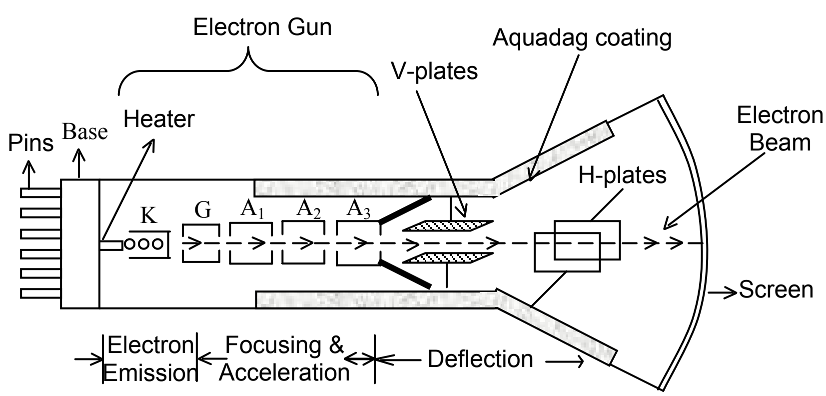

Fig. 13.1 Cathode Ray Tube

K – Cathode G – Control Grid A_1 – Pre Accelerating Anode

A_2 – Focusing Anode A_3 – Accelerating Anode.

The electron gun consists of a heater, cathode (K), control gird (G), pre-accelerating anode (A_1), focusing anode (A_2) and an accelerating anode (A_3). Electron gun is the source of focused and accelerated electron beam. It emits electrons and forms them into a beam.

Electrons are emitted from the indirectly heated cathode (in most of CRTs). These electrons pass through a small hole in the control grid. The grid is charged negatively with respect to cathode by the power supply. This grid concentrates these electrons. Electrons after passing though the grid are bunched together because of repulsion between the grid potential and electrons.

The electrons that are passing through the control grid are accelerated by the high positive potential which is applied to the pre-accelerating and accelerating anodes. The electron beam is focused by the focusing anode. After coming out of the focusing anode, the electron beam passes through the vertical and horizontal deflection plates (V-and H-plates), where it suffers vertical or horizontal or both deflections. The electron beam then goes on to the fluorescent screen, for producing a spot, trace, or a waveform.

13.2 Electrostatic Focusing System in a CRT

The focusing system in a CRT behaves like a lens and is known as ***Electron Lens***. The electrostatic focusing system arrangement is shown in the Fig 13.2.

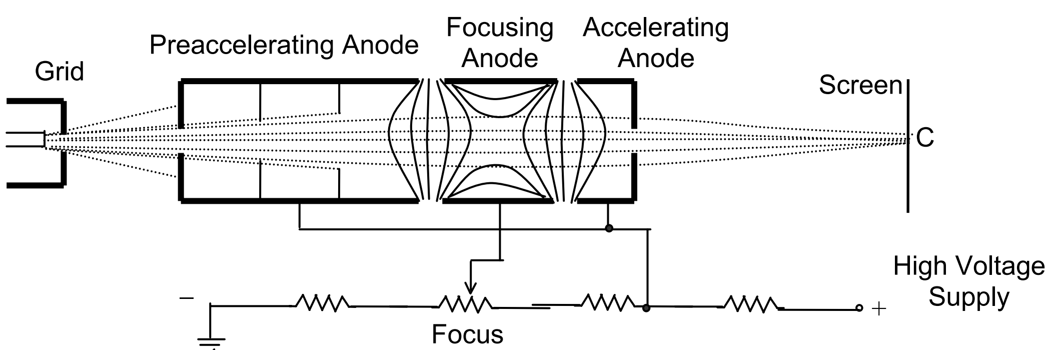

Fig 13.2 Focusing system of CRT

The pre-accelerating, focusing and accelerating anodes and their potentials play a vital role in focusing electrons on CRO screen. The pre-accelerating anode and the accelerating anodes are connected to the same potential. But the focusing anode is connected to a lower potential. As a result of difference of potential between focusing anode and two accelerating anodes, a non-uniform field exists on each of two ends of the focusing anode. The equipotential surfaces form a '***double concave lens***'.

The electron beam entering the field at angles other than the normal to the equipotential surfaces, will be deflected towards the normal. Thus the beam is focused towards the center of the tube axis. The electron lens refractive index can be changed by changing the focusing anode voltage. The change in voltage is brought about by potentiometer variation. This control is brought to the front panel of CRO and is marked ***FOCUS***.

13.3 Electrostatic Deflection

The deflection of the electron beam when an electric field is applied on it is called the Electric Deflection or Electrostatic Deflection.

In Electrostatic Deflection, two pairs of deflecting plates at right angles to each other are provided; the position of the spot (electron beam) on the fluorescent screen can be controlled by voltages applied between these plates. The deflection obtained is proportional to the deflecting voltage and inversely proportional to the beam voltage according to the relation.

$$D = \frac{lL}{2dV_a} V_d \quad \text{m/V}$$

where
- D $\rightarrow$ deflection of the beam on the screen.
- ℓ $\rightarrow$ length of the deflecting plates.
- L $\rightarrow$ distance between screen and centre of the deflecting plates.
- d $\rightarrow$ distance between the plates
- V_a $\rightarrow$ accelerating voltage.
- V_d $\rightarrow$ deflecting voltage applied between the plates.

Derivation of Electric (Electrostatic) Deflection expression

The deflection suffered by the electron beam due to the electric field applied between the plates is shown below in Fig 13.3.

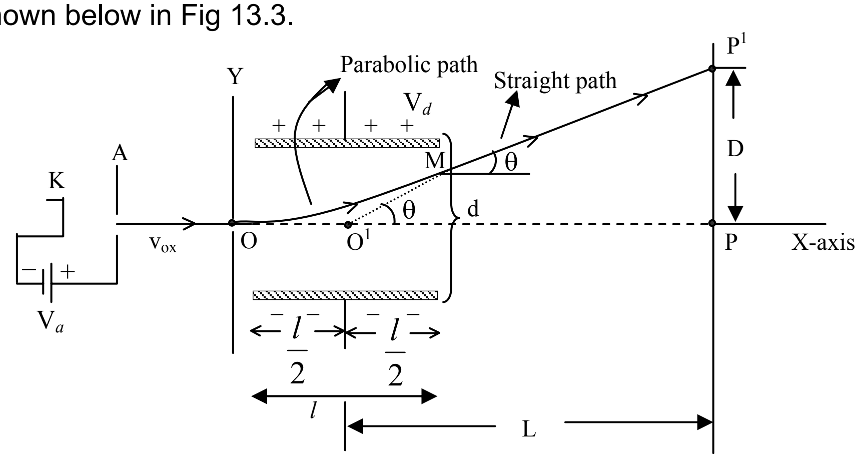

Fig. 13.3 Electric deflection system

The hot cathode K emits electron which are accelerated toward the anode by potential V_a. The electrons which are not collected by the anode pass through the tiny hole in the anode and strike the end of the glass envelope. The screen produces a visible spot. The displacement D of the electrons is determined by the potential V_d applied between the deflecting plates.

Initial velocity of the electrons which enter the area between the plates = v_{ox}

But kinetic energy = $\frac{1}{2}mv_{ox}^2$

= $-eV_a$

$$v_{ox} = \sqrt{\frac{-2eV_a}{m}} \qquad \text{-----------} \qquad (13.1)$$

where, e → charge of the electron

m → mass of the electron.

Force of attraction F, suffered by the electron in the electrostatic field towards the positive plate is given by

$$F = -eE \qquad \text{where } E = \frac{V_d}{d}$$

$$\text{or} \quad m\frac{d^2y}{dt^2} = \frac{-eV_d}{d}$$

$$\text{Hence} \quad \text{Acceleration} = \frac{d^2y}{dt^2} = \frac{-e}{md}.V_d \qquad \text{-----------} \qquad (13.2)$$

With the help of eq.(13.2), the velocity and displacement of the electron can be calculated, provided the initial boundary conditions are known.

At the point 'O', x = 0, y = 0 at t = 0 and

$$v_{oy} = 0; \quad v_{ox} = \sqrt{\frac{-2eV_a}{m}}$$

On integrating eq. (13.2), we get

$$\frac{dy}{dt} = v_y = \frac{-eV_d}{md}.t + C$$

where 'C' is constant of integration.

When t = 0; $v_y = 0$ So C = 0.

$$\text{Hence} \quad \frac{dy}{dt} = \frac{-eV_d}{md}.t \qquad \text{-----------} \qquad (13.3)$$

On integrating eq. (13.3), we get

$$y = \frac{-eV_d}{2md}.t^2 + K$$

But K = 0, as y = 0 when t = 0 so that

$$y = \frac{-eV_d}{2md}.t^2 \qquad \text{-----------} \quad (13.4)$$

Eq. (13.4) gives the displacement of electron along Y-axis at any instant.

Distance traveled by electron in time 't' along X-axis is,

x = velocity * time = v_{ox}.t

$$x = \sqrt{\frac{-2eV_a}{m}}.t$$

or

$$t = \sqrt{\left(\frac{-m}{2eV_a}\right)}.\ x \qquad \text{-----------} \quad (13.5)$$

Hence eq.(13.4) becomes

$$y = \frac{-eV_d}{2md}.\frac{-m}{2eV_a}.\ x^2 \quad \text{or} \quad y = \left(\frac{V_d}{4dV_a}\right).\ x^2 \qquad \text{-----------} \quad (13.6)$$

This is the equation of a ***parabola*** and hence the electron moves in a ***parabolic path*** between the plates. After the point M in Fig.13.3, it follows a straight line path and strikes the screen at a point P^1. Let the tangent P^1M makes an angle 'θ' with the X-axis.

From Fig 13.3,

$$\tan\theta = \frac{D}{L} \qquad \text{-----------} \quad (13.7)$$

$$\text{But} \quad \tan\theta = \left.\frac{dy}{dx}\right|_{x=l}$$

Using eq.(13.6), we get

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=l} = \left.\frac{V_d}{2dV_a}\cdot x\right|_{x=l} = \frac{V_d \cdot l}{2dV_a} \qquad \text{------------} \quad (13.8)$$

From eqs. (13.7) and (13.8),

$$\frac{D}{L} = \frac{V_d \cdot l}{2dV_a} \quad \text{or} \quad D = \left(\frac{lL}{2dV_a}\right)\cdot V_d \qquad \text{-----------} \quad (13.9)$$

Electrostatic Deflection Sensitivity

The Electrostatic Deflection Sensitivity is defined as the deflection (in meters) on the screen per volt of deflecting voltage.

Deflection sensitivity $= S = \dfrac{D}{V_d} = \dfrac{lL}{2dV_a}$ ------------- (13.10)

13.4 Magnetic Deflection

The deflection of the electron beam when a magnetic field is applied on it is called the ***Magnetic Deflection.***

Magnetic deflection is obtained by applying a magnetic field in such a way that the flux lines are perpendicular to the beam and also at right angles to the direction of the desired deflection. The magnetic field deflects the electron beam perpendicular to its direction of motion. The electron follows a ***circular path*** in the field. The deflection obtained is proportional to the magnetic flux density B according to the relation

$$D = \frac{lL}{\sqrt{2V_a}} \cdot \sqrt{\frac{e}{m}} . B \text{ meter}$$

where l → field length

L → distance between screen and filed

V_a → anode voltage.

e → charge of electron.

m → mass of electron

B → magnetic flux density. (Wb/m^2)

Derivation of Magnetic Deflection expression

The deflection suffered by the electron beam due to the magnetic field applied is shown in Fig 13.4. The electron path in the presence of magnetic field is a circle of radius 'R' meters. Let its velocity be 'v' m/s, its mass be 'm' Kg.

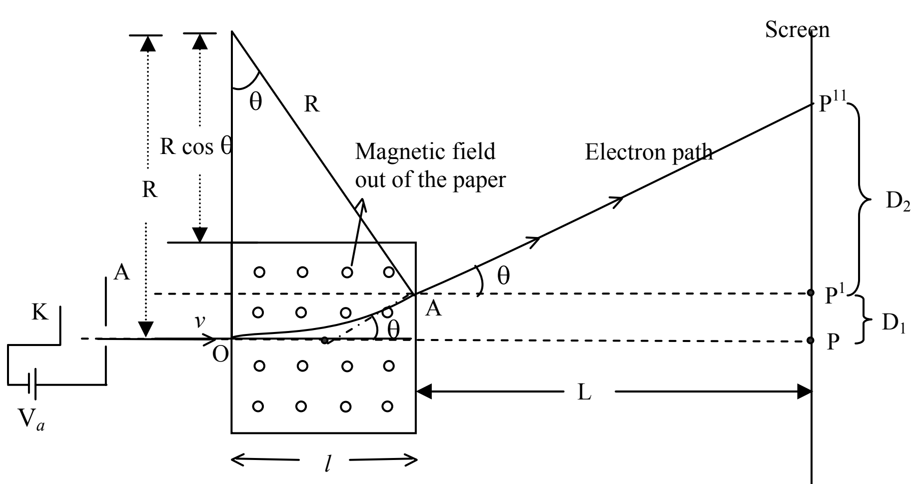

Fig.13.4 Magnetic deflection system.

The centrifugal force on electron = $\frac{1}{2}$ mv^2

The force exerted on the electron by the magnetic field $F = Bev\ sin\theta$ ------------ (13.11)

where 'B' is the magnetic flux density (Wb/m^2)

'θ' is the angle, which the electron path makes with the direction of magnetic flux, e the charge on the electron.

If $\theta = 90°$, $F = Bev$ ------------ (13.12)

Hence $Bev = \frac{mv^2}{R}$ (or) $R = \frac{mv}{Be}$ ------------ (13.13)

After coming out of the filed, the electron follows a straight-line path as show in Fig 13.4.

Total displacement $D = D_1 + D_2$

From Fig 13.4, $sin\ \theta = \frac{l}{R}$

Hence $\cos\theta = \sqrt{1 - \frac{l^2}{R^2}}$

Hence $D_1 = R - R\cos\theta = R - R\sqrt{1 - \frac{\ell^2}{R^2}}$ ------------ (13.14)

From ΔAP^1P^{11},

$$\tan\theta = \frac{D_2}{L} \quad \text{(or)} \quad D_2 = L\ \tan\theta$$

$$= L\frac{\sin\theta}{\cos\theta} = L\frac{\frac{l}{R}}{\sqrt{1 - \frac{l^2}{R^2}}}$$

or $D_2 = \frac{l\,L}{\sqrt{R^2 - l^2}}$ ------------ (13.15)

$$D = D_1 + D_2 = \left[R - R\sqrt{1 - \frac{l^2}{R^2}}\right] + \frac{l\ L}{\sqrt{R^2 - l^2}}$$

If $l << R$, then l and $\frac{l^2}{R^2}$ can be neglected.

$\therefore D = R - R + \frac{lL}{R\sqrt{1 - \frac{l^2}{R^2}}} = \frac{lL}{R}$ ------------ (13.16)

Now,

$$D = \frac{lL}{R} = \frac{lLBe}{mv} = \frac{lLBe}{m\sqrt{\frac{2eV_a}{m}}} \qquad (\because \frac{1}{2}mv^2 = eV_a \ \ or \ \ v = \sqrt{\frac{2eV_a}{m}})$$

$$\therefore D = \frac{lL \cdot B}{\sqrt{2V_a}}\sqrt{\frac{e}{m}} \qquad \text{------------} \quad (13.17)$$

Hence the magnetic deflection ‘D’ varies inversely with the square root of the anode voltage V_a.

Magnetic Deflection Sensitivity

The deflection produced on the screen due to unit magnetic filed applied is known as the ***Magnetic Deflection Sensitivity ‘S’***

$$\therefore S = \frac{D}{B} = \frac{lL}{\sqrt{2V_a}}\sqrt{\frac{e}{m}} \quad meter/weber$$

The deflection sensitivity changes from one CRO to another CRO.

Solved Problems

Problem .1

The deflection sensitivity of a CRT is 0.01mm/V. Find the shift produced in the spot when 400V voltage is applied to the vertical plates.

Solution: Spot deflection **=** Deflection Sensitivity x Applied voltage .

Spot shift = 0.01mm/V x 400V

= 4mm.

Problem .2

The deflection sensitivity of a CRT is 0.03mm/V. If an unknown voltage is voltage to the horizontal plates, the spot shifts 3mm horizontally. Find the value of unknown voltage.

Solution:

Spot shift = Sensitivity x applied voltage

Hence Applied voltage = 3mm / 0.03mm/V

= 100V

13.5 Fluorescent Screen

The property of the material to continue light emission even after the source of excitation (the electron beam) is cutoff is called ***Fluorescence***.

The Fluorescent Screen gives the visible display of dynamic phenomenon. The whole of the end inner surface of the glass tube is coated with a suitable fluorescent material called

Phosphor. This material absorbs the kinetic energy of the bombarding electrons and reemits it in the form of a bright spot. The bombarding electrons, striking the screen, release secondary emission electrons. These secondary electrons are collected by a conducting graphite layer coating, called Aquadag, which is connected to the third anode (A_3). This layer performs two functions:

(i) It accelerates the electron beam after it passes between the deflecting plates.

(ii) It collects the secondary emission electrons and hence it prevents the formation of negative charge on the screen. In other words, it maintains the state of electrical equilibrium of the screen.

13.6 CRO Block Diagram

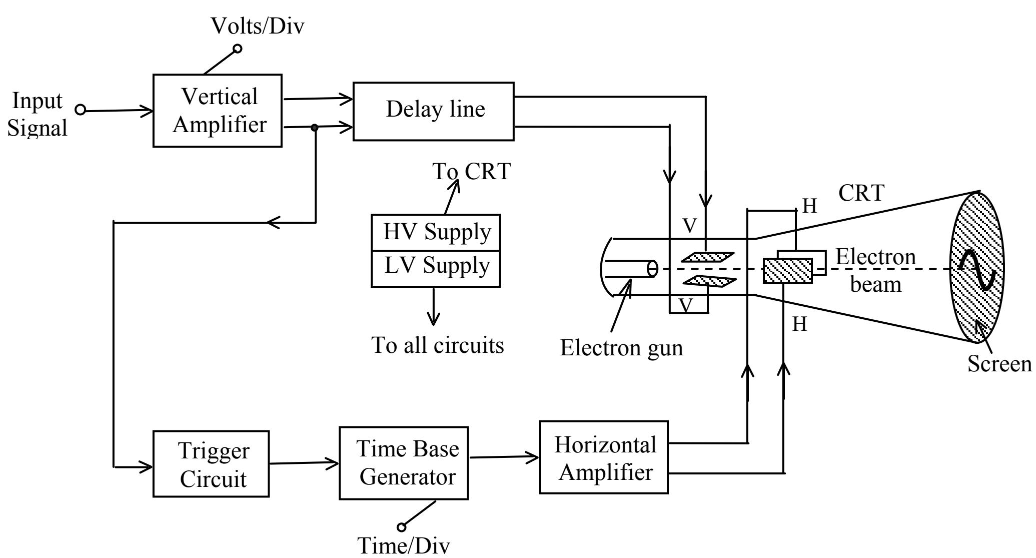

Fig 13.5 CRO Block diagram

The block diagram of CRO consists of the following major subsystems.

i) Cathode Ray Tube.

ii) Vertical Amplifier.

iii) Horizontal Amplifier.

iv) Sweep Generator. (Saw tooth Generator)

v) Trigger Circuit.

vi) Power Supply.

vii) Delay Line.

(i) Cathode Ray Tube

It is the heart of the CRO. It produces a concentrated beam of high velocity electrons and deflects the beam to create an image on a fluorescent screen.

(ii) Vertical Amplifier & Horizontal Amplifier

These amplifiers are connected between the input terminals and the deflection plates. The function of these amplifiers is to increase the deflection sensitivity for weak input voltages.

(iii) Sweep Generator (Time Base Generator)

This is connected to horizontal deflection plates. Its purpose is to get the waveform of applied voltage on CRO screen. The sweep generator produces a saw tooth voltage waveform. This voltage waveform is periodic in nature and increases linearly with time and suddenly falls to zero.

(iv) Trigger Circuit

This circuit produces trigger pulses to start horizontal sweep. To synchronize the horizontal deflection with the vertical input, such that the horizontal deflection starts at the same point of the input vertical signal each time it sweeps, a triggering circuit is used.

(vi) Power Supply

This section consists of a low voltage and high voltage power supplies. A low voltage power supply produces the D.C. potentials needed for vertical, horizontal amplifiers and time base circuits.

A high voltage supply produces high D.C. voltages needed for accelerating anodes (range 500-1500V D.C.).

(vii) Delay Line

To observe the leading edge of the signal waveform, the signal, which is used to drive the vertical CRT plates, is to be delayed by some amount of time. This is the function of vertical delay line.

13.7. Time-Base Generator (or) Saw Tooth Generator

Time-base generator is used to produce saw tooth voltage waveform, which is utilized to display the waveform of the applied AC voltage on CRO screen. It is connected internally to horizontal deflection plates. It is shown in Fig.13.6. The circuit consists of a resistor R, a capacitor C and a neon bulb. Here the capacitor ‘C’ can be charged by D.C. supply voltage through the resistance ‘R’.

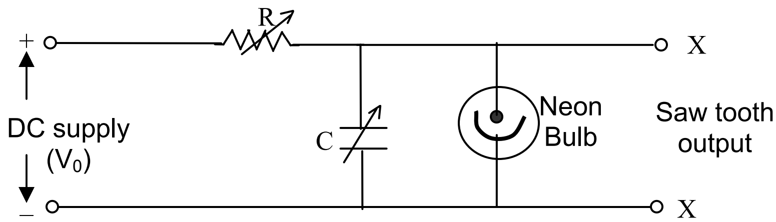

Fig 13.6(a) Neon Time Base Circuit

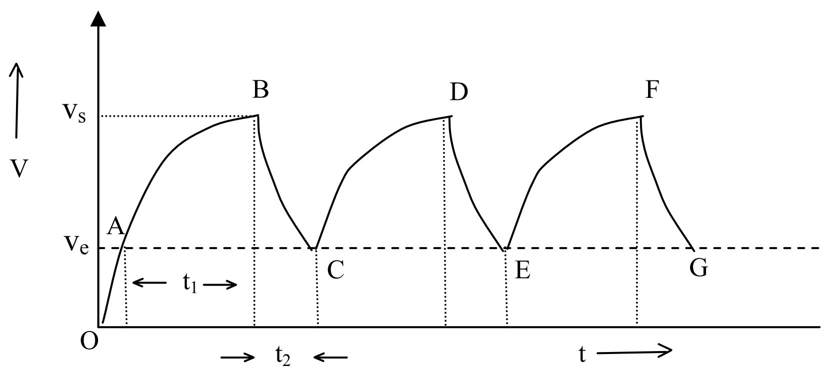

Fig. 13.6(b) Saw tooth waveform

Let V_0 be the D.C. supply voltage, V_s be the striking voltage of the electric discharge through the lamp and V_e be the extinguishing voltage at which lamps stops conducting the discharge (for small Neon bulb $V_s \cong 170$v, $V_e = 130$V). When the capacitor is charged through the resistance '*R*', the voltage across the capacitor rises from zero along OAB. When the voltage across the capacitor becomes equal to V_s, the neon bulb begins to glow. When the lamp starts conducting, its resistance falls to a low value. Hence the capacitor discharges through the low resistance along BC and this process continues until the voltage across the capacitor becomes equal to V_e. When the voltage across the capacitor fall to V_e, the neon bulb ceases to glow and behaves as an open-circuit. Consequently the capacitor is charged again. In this way, the alternate periodic charge and discharge of the capacitor is repeated as long as the dc supply voltage V_0 is applied across it. The voltage across the capacitor is plotted against time in Fig. 13.6(b). By varying the magnitude of *C* and *R*, the periodic time '*T*' (= $t_1 + t_2$) can be changed.

<u>Need for time base voltage</u>:

In a cathode ray oscilloscope having electric deflection system, the spot on the screen simply moves in one direction. The displacement of the spot is given by the multiplication of the deflection sensitivity and the applied deflection voltage. If, on the other hand, an alternating voltage (say sinusoidal in nature) is applied to the deflection plates, the spot has a reciprocatory motion in a straight line, the length of which is given by the deflection sensitivity multiplied by the peak-to-peak amplitude of the ac voltage. Due to the persistence of vision of the eye and persistence property of the fluorescent material, the path traced by the electron in its reciprocatory motion appears to be a continuous straight line. This line does not reveal the nature of the applied voltage waveform. In order to have a visual display of the waveform of applied voltage, it is necessary to apply this voltage to one set of deflection plates (say Y-plates) and to apply simultaneously to the other set of deflection plates (say X-plates) a voltage called "***sweep or time-base voltage***" of the form shown in Fig. 13.6(b). When this voltage is applied to the X-plates of a CRO, the spot moves at a uniform rate from one side to another (say from left to right) depending on the polarity of the voltage.

APPLICATIONS OF CRO

13.8 Measurement of Amplitude (AC)

To measure the amplitude of an unknown AC signal, the signal is to be applied to vertical deflection plates as shown in Fig.13.7. No voltage waveform is to be applied to horizontal deflection plates externally. Now, various controls of CRO are to be adjusted to get a clear sine wave as shown in Fig.13.8. on CRO screen. The height of the positive or negative peak, above or below the reference line is to be noted in cms. This height is to be multiplied with the amplitude/division switch reading to get the amplitude of the wave applied in volts.

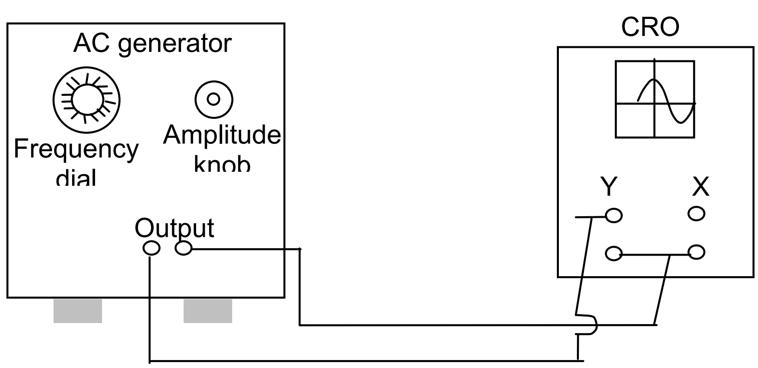

Fig. 13.7 Amplitude measurement

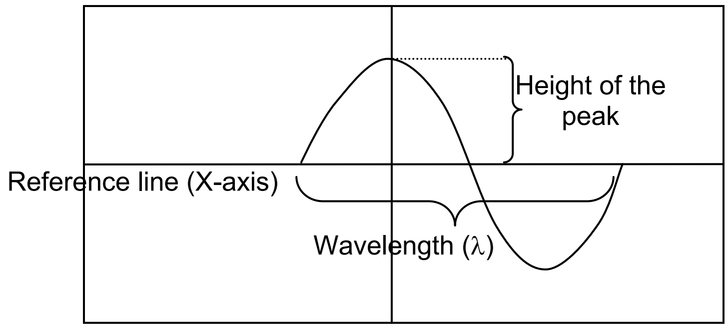

Fig. 13.8 AC waveform

Measurement of DC Voltage

To measure the magnitude of an unknown DC **v**oltage, the DC voltage from DCRPS is to be applied to vertical deflection plates and no voltage is to be applied to horizontal deflection plates. Now, by adjusting various controls, we can notice the shift of horizontal trace along Y-axis. This shift is to be noted in cms. This shift is to be multiplied with the amplitude/division switch reading, to get the magnitude of unknown DC voltage applied. As DC possesses no waveform, only horizontal line is seen on CRO screen.

13.9 Measurement of Frequency

To measure the frequency of an unknown AC signal, the signal is to be applied to vertical deflection plates as shown in Fig.13.7. No voltage waveform is to be applied to horizontal deflection plates externally. Now, adjust the various controls of CRO to get a clear sine wave on CRO screen. The wavelength (λ) of the waveform (along X-axis) is to be noted in cms. This wavelength is to the multiplied with Time/Division switch reading to get the time period of the applied waveform. The reciprocal of the time period will give us the frequency of the waveform.

13.10 Measurement of Phase

The phase or phase difference between two AC voltage waveforms can be measured using the circuit shown in Fig.13.9. For this purpose, two waveforms with phase difference can be generated by using a series RC-circuit.

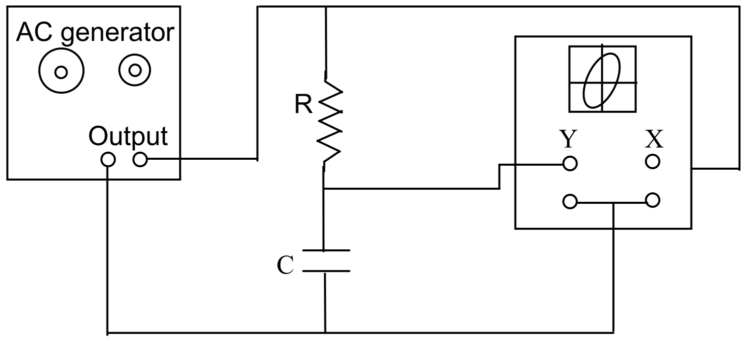

Fig. 13.9 Phase measurement circuit

To measure the phase difference, one AC voltage waveform is to be applied horizontal deflection plates and the other voltage waveform (developed across the capacitor ‘C’) is to be applied to vertical deflection plates. The various controls of CRO are to be adjusted to get a clear ellipse, which is to be centered as shown in Fig.13.10.

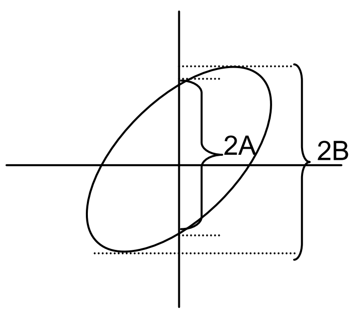

Fig 13.10 Ellipse for measuring phase

Now the values of 2A and 2B are to be measured. Finally the phase difference ‘θ’ can be estimated with the help of the following formula

$$\theta = \sin^{-1}\left(\frac{2A}{2B}\right)$$

13.11 Summary

The CRO is useful for displaying the AC voltage waveform, DC voltages and is used for checking complicated electronic circuits. It mainly consists of Cathode Ray Tube (CRT) and some other additional circuitry to control the waveform. In general, a CRO is used to measure the amplitude, frequency, phase, for radio servicing, and locating faults in various electronic equipment. It works as an eye for an electronic engineer. There are varieties of CROs available. But the mostly used is a general purpose CRO, which can display two waveforms at a time.

13.12 Key Terminology

Electron Gun – CRO – Fluorescent Screen – Deflection Plates – Electric Deflection – Magnetic Deflection.

13.13. Self Assessment Questions

A. Long Answer Questions

1). Draw the block diagram of CRO and explain the function of each block.

2). Explain how a CRO is used to measure amplitude, frequency, and phase.

3). Derive an expression for electrostatic deflection.

4). Derive an expression for magnetic deflection.

5). Draw the diagram of CRT and explain the function of each part in it.

B. Short Answer Questions

1) Explain fluorescent screen.

2) Give an idea about the necessity of time base generator.

3) Give a brief idea about electric deflection.

4) Give a brief idea about magnetic deflection.

5) Explain the electrostatic focusing system in a CRT.

6) Describe the working of time base generator.

C. Numerical problems

1. Calculate the deflection sensitivity of a Cathode Ray Tube in which deflecting plates are 2cm long, 0.5cm apart, the fluorescent screen is placed at a distance of 40cm form the centre of the plates and the anode voltage is 1000V.

   [Ans - 0.08cm/Volt]

2. When using a CRO to measure frequency of an alternating waveform, the time base is switched to the 5 millisec/cm. In the steady pattern on the screen, the horizontal distance for one cycle is 2.4cm. Calculate the frequency of the signal. ***[Ans - 83.3Hz]***

3. Find the deflection sensitivity of a CRO, if the spot on the screen deflects 27mm vertically. The voltage applied is 90V. ***[Ans - 0.3mm/Volt]***

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