THERMODYNAMICS AND OPTICS

(DSPHY21)

(BSC, PHYSICS - II)

ACHARYA NAGARJUNA UNIVERSITY

CENTRE FOR DISTANCE EDUCATION

NAGARJUNA NAGAR,

GUNTUR

ANDHRA PRADESH

ACHARYA NAGARJUNA UNIVERSITY:: NAGARJUNA NAGAR

PAPER -II Thermodymaics and optics (for Maths combination)

(PHYSICS)

Part – A : THERMODYNAMICS

UNIT -I

1. **KINETIC THEORY OF GASES:** Deduction of Maxwell's law of distribution of molecular speeds — Transport phenomena Viscosity, thermal conduction and diffusion of gases.

2. **THEROMODYNMICS:** Heat and work Internal energy – Indicator Diagrams work done in Isothermal and adiabatic processes - First law of thermodynamics- significance and applications of first law of thermodynamics- Reversible and irreversible process - carnot's Theorem Carnot's engine, efficiency- Clausius - Clapelyron equation - Second law of thermodynamics, different statements - Thermodynamic scale of temperature - Entropy concept - Entropy and disorder measurement of entropy changes in reversible and irreversible processes - Entropy of universe - Entropy – temperature diagrams.

3. **THERMODYNAMIC POTENTIAL AND MAXWELLS EQUATIONS:**

Thermodynamic potentials – Derivation of Maxwells thermodynamic relations - specific heats- Derivations for ratio and difference of two specific heats -Joule - Kelvin effect - expression for Joule- Kelvin coefficient.

UNIT -II

4. LOW TEMPERATURE PHYSICS:

Liquefaction of gases using Joule-Kelvin effect - porous plug experimentDistinction between Joules expansion. Adiabatic expansion -and Joule-Thomson's expansion – expression for Joule-Thomson cooling – liquefaction of Helium Kapitza's method- Adiabatic demagnetization production of low temperatures, Principle of Refrigeration – Vapour compression type.

5. QUANTUM THEORY OF RADIATION:

Block body Fery's black body – distribution of energy in the spectrum of a black body - Wien's displacement law, Wien's Raleigh Jeans law – Quantum theory of radiation – Planck's law Deduction of Wien's law, Raliegh Jean's Law Wiens Displacement Law from Planck's Law - Measurement of radiation – Types of pyrometers- Disappearing filament optical pyrometer - experimental determination - Angstrom phyeliometer - determination of solar constant

Part – B :OPTICS

UNIT - III

6. MATRIX METHODS IN PARAXIAL OPTICS:

Concept and derivation transition, refraction and system matrices position of the image plans and magnification of the optical system - application of matrix methods to simple optical systems (1) a thick lens (2) thin lenses in contact (3) two thin lenses separated by distance cardinal points of lens system Unit and nodal planes.

7. ABERRATION (MATRIX METHODS)

Chromatic aberration in a lens- achromatic doublet - Achromatic for two lenses in contact and separated by a distance - monochromatic in aberration. The spherical aberration (longitudinal spherical aberration) due to (i) a plane refracting surface and (ii) a spherical surface (expressions without proof) minimization of spherical aberrations - explanation of coma – astigmatism .

8. INTERFERENCE:

The super position principle - coherence – temporal and spatial conditions for interference light. Interference by division of wave front- Fresnel's biprism - determination of wavelength of light change of phase on reflection – determination of thickness of a transparent material using prism. Interference by division of amplitude- oblique incidence of a plane wave on a thin film (the cosine law) colours of thin films - non reflecting thin films - interference by a plane parallel film illuminated by a point surface - interference by film with two non parallel reflecting surfaces (wedge shaped film) - determination of diameter of wire Newton's rings in reflected and transmitted light, Determination of wavelength of monochromatic light - Michelson Interferometer, types of fringes, determination of wavelength of monochromatic light, thickness of a thin plate.

UNIT -IV

9. DIFFRACTION:

Fraunhofer diffraction - diffraction due to a single slit and circular aperture, Limit of resolution - two - slit Fraunhofer diffraction - Fraunhofer diffraction pattern with N – Slits - The Fourier transform and its properties - the shifting theorem and application of the F.T. to Fourier diffraction due to single slit. A double slit and the diffraction grating - The diffraction grating - normal and oblique incidence determination of wavelength of light- Fresnel diffractionfresnel half period Zones - Zone plates diffraction at a straight edge - diffraction of plane waves by a straight edge.

10. POLARIZATION AND DOUBLE REFRACTION:

Polarized light- Brewster's law - Malus Law - phenomenon of double refraction in calcilte- Reftaction of plane wave incident on a negative crystal like calcite – Nichol prism. Analysis of polarized light by quarter wave plate Babinet compensator - Optical activity Laurent's half shade polarimeter experiment.

11.LASERS, FIBER OPTICS AND HOLOGRAPHY:)

Spontaneous, stimulated emission - laser principle - population inversion Einstein coefficients - Types lasers, He-Ne and Ruby lasers and the applications of lasers.

Optical fiber types, rays and modes step and graded index fibers and their structure - fiber materials. Principles of fiber communication (qualitative treatment only)

Basic principles of Holography - Gabor Hologram and its limitations application of Hologram.

REFERENCE BOOKS:

Introduction to modem optics	G.K. Fowles (Holt- Rinehert)
B.Sc. Physics Vol. II	Sastry Shanker Rao and Babu Rao (S.Chand)
B.Sc. Optics	Telugu Academy
B.Sc. Heat	Telugu Academy
Optics	Brijlal and subrahmanyam (S Chand and Co.)
Heat and Thermodynamics	Zemansky (Mc Graw Hill Publ)
Thermodynamics	D.C. Tayal (Himalaya Publ)
Heat and Thermodynamics	Brijlal and subrabmanyam(S Chand and Co.)
Fourier optics	Goodman:
Fiber optic communications	Aggarwal, ped wheeler pub.

PRACTICALS – B.Sc., Physics - paper -II

1. Thermo emf thermometry determination of temperature using a thermocouple.
2. Heating efficiency of electrical kettle with varying voltages
3. Thickness of a wire - wedge method.
4. Determination of wavelength of light - Biprism.
5. Determination of Radius curvature of given convex lens - Newton's rings.
6. Determination of wavelength of light diffraction at thin wire
7. Resolving power of grating
8. Determination of mean diameter of lycopodium powder (Diffraction)
9. Study or optical rotation – polarimeter
10. Dispersive power of prism
11. Determination of wavelength a light using diffraction grating minimum deviation method
12. Pulfrich refractometer - Determination of refractive index of liquid
13. Wavelength of light using diffraction grating - normal incidence method.
14. I -d curves - using spectrometer
15. Resolving power of a telescope.
16. Refractive index of a liquid and glass.
17. Wavelength of a Laser using diffraction grating

ACHARYA NAGARJUNUA UNIVERSITY
B.Sc. DEGREE EXAMINATION (DISTANCE EDUCATION)
(Examination at the end of Second Year)

PAPTE II – Physics Paper II – THERMODYNAMICS AND OPTICS

MODEL QUESTION PAPER

Time: Three hours

Maximum: 100 marks

PART A- (2 x 15 = 30 marks)

Answer any **TWO** questions

1. Basing on Maxwell's distribution law for molecular speeds explain $C_{r.ms.}, \bar{C}$ and C_p and velocities of gas molecules at a given temperature. Deduce the relation between these three velocities.
2. Define Maxwell's thermodynamic potentials. Obtain the relation $(C_p - C_v) = R$ between C_p and C_v from Maxwell's thermodynamical equations.
3. Explain with theory the adiabatic demagnetization process of producing very low temperatures.
4. What are pyrometers? Describe Ferry's total radiation pyrometer.

PART-B (2×15=30 marks)

Answer any **TWO** questions

5. Explain various defects of lens. Describe the methods to minimize each defect.
6. Describe the principle and working of a Michelson interferometer. Describe the method to determine the wavelength of a monochromatic light using this interferometer.
7. What is meant by polarized light? How is polarized light produced? State and explain Malus law and Brewster's law.
8. Write differences between Fraunhofer and Fresnel diffraction. Discuss the Fraunhofer diffraction due to N slits.

PART C—(5x 4 = 20 marks)

Answer any **FIVE** questions.

9. Explain the working of refrigerator.
10. State and explain thermoelectric power with graph.
11. Define mean free path and deduce the expression for it.
12. Define dispersive power and resolving power of a grating.
13. Obtain first T-ds equation.
14. Explain the cardinal points of a lens system.
15. Explain the formation of colours in thin films.
16. Define block body. Describe Farr's black body.

PART-D (4 x 5 =20 marks)

Answer any **FOUR** questions.

17. The efficiency of a Cornot's engine is 60%. Calculate the increase in the temperature of the source so that efficiency becomes 75%.
18. Find the wavelength at which maximum energy is blackbody at a temperature $327^0 C$. Wien constant is $3\times10^{-3} mK$
19. The radius of the 8th ring 12th ring in Newton's ring apparatus is and respectively. Find the diameter of the 20th ring.
20. Calculate the change in entropy when 500gm of water at$0^0 C$ is mixed with 500gm of water at $100^0 C$
21. Calculate the order of the spectrum when grating with width grating having 15,000 lines with resolving power 75,000.
22. A black body at $500^0 C$ has a surface area of $0.5cm^2$ and radiate heat at the rate of. $1.02\times10^4 Js^{-1}$ Calculate the Stefan's constant.
23. Two thin lenses of focal lengths 16cm and 12cm form a combination which is corrected for spherical aberration. Find the distance between the principal points of the combination.
24. Ordinary light is incident on a plane glass plate at angle 58^0.the reflected light is plane polarized. Find the refractive index of glass plate material and also the refracting angle of the light ray.

B.Sc. Physics II Year course
Centre for distance education

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Practical manual

UNIT-1

LESSON-1

KINETIC THEORY OF GASES

Objectives:-

In this lesson the following particulars will be able to understand

1. The velocity of gas molecules range from zero to infinite.
2. There is certain probability for the molecules having the velocity in a particular range.
3. Transfer of momentum of gas molecules produce viscosity of gases.
4. Transfer of kinetic energy (K.E) of gas molecules to produce conduction.
5. Transfer of molecules of gas gives rise to diffusion.

Structure of the lesson:-

1.1 Introduction:-

The molecules are in random motion with all possible velocities and in all directions. The molecules collide with one another and with the walls of containing vessel. All collisions are perfectly elastic. The distance between two successive collisions is called free path of the molecule. The average distance between two successive collisions is called mean free path

$$\lambda = \frac{1}{\sqrt{2}\,\pi d^2 n}$$

Let m be the mass of each molecule, n be the number of molecules per unit volume, V be volume of the container and $\bar{v}$ be the root mean square speed of the molecules . On the basis of the kinetic theory of the gases, the pressure exerted by the gas molecules is given by

$$P = \frac{1}{3}\frac{mn}{V}\bar{v}^2$$

$$\text{Where } \bar{v} = \sqrt{\frac{v_1^2 + v_2^2 + \dots\dots\dots\dots + v_n^2}{n}}$$

$$P = \frac{1}{3}\frac{M}{V}\bar{v}^2 \quad \text{Where M=mn=mass of the gas in the container}$$

$$\Rightarrow P = \frac{1}{3}\rho\bar{v}^2 \quad \text{Where density } \rho = \frac{M}{V}$$

$$= \frac{2}{3}\times\frac{1}{2}\rho\bar{v}^2$$

$$= \frac{2}{3}\times \text{K.E .of the molecule}$$

$$= \frac{2}{3}\times\frac{3}{2}kT \quad \text{Where k is Boltzmann's constant k= } 1.38\times10^{23}\,\text{J/mole}-K$$

$$\therefore P = kT.$$

1.2 Maxwell's speed distribution law:-

The law of distribution of speeds among the molecules of a gas was first established by Maxwell. In order to derive the law the following assumptions are considered.

1. The gas consists of molecules with all possible speeds between 0 and ∞.
2. The average density of a gas enclosed in a vessel in equilibrium is same at all places.
3. Though the speeds of individual molecules are changing. But a definite number of molecules have speeds between definite ranges.
4. The probability that any molecule selected at random has velocities lying between certain limits is a purely function of velocity and the limits considered.
5. The components of velocity of a molecule in three mutually perpendicular directions are considered to be independent of each other.

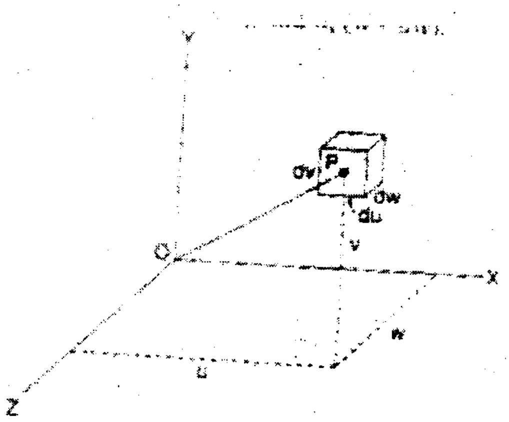


fig1.1

Let us represent the molecules in a velocity diagram. Consider a molecule at point P having a velocity C represented by a vector as shown in fig(1.1). Let the projections of this vector in X, Y, Z. directions be u, v, w respectively. All the molecules having the velocity components lie in the range u and u+du, v and v+du and w and w+dw will be contained in the volume dudvdw. The probability that a given molecule having the velocity between u and u+du is f(u)du. The probability that a given molecule having the velocity between v and v+dv is f(v)dv. The probability that a given molecule having the velocity in between w and w+dw is f(w)dw.

The probability that a molecule having the velocities between

u and u+du, v and v+dv, w and w+dw=f(u)du f(v)dv f(w)dw

=f(u)f(v)f(w)dudvdw----(1.1)

The resultant velocity of the molecules is a single vector C.

The probability for a molecule having the velocity C in the volume dudvdw is f(c) dudvdw. More conveniently it can be written as $\phi\left(C^2\right)dudvdw$.......(1.2)

From equations (1.1) & (1.2) we can write

$$f(u)\, f(v)\, f(w)\, du\, dv\, dw = \phi(C)^2\, du\, dv\, dw$$

$$\Rightarrow f(u)\, f(v)\, f(w) = \phi\left(C^2\right). \rightarrow. (1.3)$$

For a particular value of C chosen, $\phi\left(C^2\right)$ is constant

Diff eqn.(1.3) and equating it to zero we get,

$$d[f(u)\ f(v)\ f(w)] = d[\Phi(C)^2] = 0 \rightarrow (1.4)$$

This equation can be written as

$$f^1(u)\, f(v)\, f(w)\, du + f(u)\, f^1(v)\, f(w)\, dv + f(u)\, f(v)\, f^1(w)\, dw = 0$$

Dividing with f(u)f(v)f(w) we get,

$$\frac{f^1(u)}{f(u)} du + \frac{f^1(v)}{f(v)} dv + \frac{f^1(w)}{f(w)} dw = 0 \rightarrow (1.5)$$

$\because$ C is considered as constant we can write,

$$C^2 = u^2 + v^2 + w^2 = \text{constant}$$

Diff we get,

$$d(C)^2 = d(u^2 + v^2 + w^2) = 0$$

$$\Rightarrow 2u\, du + 2v\, dv + 2w\, dw = 0$$

$$\Rightarrow u\, du + v\, dv + w\, dw = 0$$

$\Rightarrow \lambda u du + \lambda v dv + \lambda w dw = 0 \rightarrow (1.6)$

Where λ is a constant quantity

Adding equations (1.5) & (1.6) we get,

$$\frac{f^1(u)}{f(u)} du + \lambda\, u du + \frac{f^1(v)}{f(v)} dv + \lambda\, v dv + \frac{f^1(w)}{f(w)} dw + \lambda\, w dw = 0$$

$$\Rightarrow \left[\frac{f^1(u)}{f(u)} + \lambda\, u\right] du + \left[\frac{f^1(v)}{f(v)} + \lambda\, v\right] dv + \left[\frac{f^1(w)}{f(w)} + \lambda\, w\right] dw = 0 \rightarrow (1.7)$$

∵ u, v & w are independent upon each other, we can equate each term to zero.

$$\left[\frac{f^1(u)}{f(u)} + \lambda\, u\right] du = 0,$$

$$\left[\frac{f^1(v)}{f(v)} + \lambda\, v\right] dv = 0, \qquad \text{..........(1.8)}$$

$$\left[\frac{f^1(w)}{f(w)} + \lambda\, w\right] dw = 0$$

Consider

$$\left[\frac{f^1(u)}{f(u)} + \lambda\, u\right] du = 0$$

$$\Rightarrow \frac{f^1(u)}{f(u)} du = -\lambda\, u du$$

Integrating on both sides we get,

$$\log_e f(u) = \frac{-\lambda\, u^2}{2} + \log_e a$$

Where $\log_e a$ is a integration constant

$$\log_e f(u) - \log_e a = \frac{-\lambda u^2}{2}$$

$$\Rightarrow \log_e \left(\frac{f(u)}{a}\right) = \frac{-\lambda u^2}{2}$$

$$\Rightarrow f(u) = a\, e^{\frac{-\lambda u^2}{2}}$$

Similarly f(v)= $f(v) = a\, e^{\frac{-\lambda v^2}{2}}, \quad f(w) = a\, e^{\frac{-\lambda w^2}{2}}$

The probability of finding a molecule having the velocity in between u and u+du. v and v+dv, w and w+dw is $f(u)\, f(v)\, f(w)\, du\, dv\, dw = a\, e^{-bu^2}.\ a\, e^{-bv^2}.\ a\, e^{-bw^2}\, du\, dv\, dw$

$$\Rightarrow f(u)\, f(v)\, f(w)\, du\, dv\, dw = a^3\, e^{-b\left(u^2+v^2+w^2\right)}\, du\, dv\, dw \rightarrow (1.9)$$

Let 'n' be the number of molecules per unit volume. The probability of finding the molecules in one unit volume having the velocity in between $-\infty$ and $+\infty$ is n.

$$n \int\int\int_{-\infty}^{+\infty} f(u)\, f(v)\, f(w)\, du\, dv\, dw = n$$

$$\Rightarrow \int\int\int_{-\infty}^{\infty} a^3\, e^{-b\left(u^2+v^2+w^2\right)}\, du\, dv\, dw = 1$$

$$\Rightarrow a^3 \int\int\int_{-\infty}^{\infty} e^{-b\left(u^2+v^2+w^2\right)}\, du\, dv\, dw = 1 \rightarrow (1.10)$$

Solving this integral we can obtain,

$$\int_{-\infty}^{\infty} e^{-bu^2} du = \sqrt{\frac{\pi}{b}}, \quad \int_{-\infty}^{\infty} e^{-bv^2} dv = \sqrt{\frac{\pi}{b}}, \quad \int_{-\infty}^{\infty} e^{-bw^2} dw = \sqrt{\frac{\pi}{b}}.$$

Writing these values in equation (1.10) we get,

$$a^3 \sqrt{\frac{\pi}{b}}\sqrt{\frac{\pi}{b}}\sqrt{\frac{\pi}{b}} = 1$$

$$\Rightarrow a^3\left(\frac{\pi}{b}\right)^{\frac{3}{2}} = 1$$

$$\Rightarrow a\left(\frac{\pi}{b}\right)^{\frac{1}{2}} = 1$$

$$\Rightarrow a = \left(\frac{b}{\pi}\right)^{\frac{1}{2}} \rightarrow (1.11)$$

We can obtain $b = \frac{m}{kT} \rightarrow (1.12)$

Where m is the mass of each molecule

k is the Boltzmann's constant

T is the absolute temperature of gas

Writing equation (1.12) in equation (1.11) we get

$$a = \left(\frac{m}{2\pi kT}\right)^{\frac{1}{2}} \rightarrow (1.13)$$

The probability of finding the molecules in one unit volume having the velocity in between u & u+du, v & v+dv and w & w+dw can be written as

dn = n f(u) f(v)f(w)dudvdw

$$\Rightarrow dn = n a^3 e^{-b\left(u^2+v^2+w^2\right)}\ du\ dv\ dw$$

$$\Rightarrow dn = n\left(\frac{m}{2\pi KT}\right)^{\frac{3}{2}}\ e^{\frac{-m}{2KT}\left(u^2+v^2+w^2\right)} \rightarrow (1.14)$$

It is called Maxwell's speed distribution law.

The probability of finding the molecules having the velocity in between C & C+dC in one unit volume can be obtained by replacing the volume dudvdw with volume between the spheres of radii C & C+dC.

$$dv = \frac{4}{3}\Pi\ (C+dC)^3 - \frac{4}{3}\Pi C^3$$

$$= \frac{4}{3}\Pi\left[(C+dC)^3 - C^3\right]$$

$$= \frac{4}{3}\Pi\left[C^3 + dC^3 + 3C^2dC + 3CdC^2 - C^3\right]$$

$$= \frac{4}{3}\Pi\ 3C^2dC$$

$$\therefore dv = 4\Pi C^2dC$$

We have $u^2 + v^2 + w^2 = C^2$

The no. of molecules having the velocity between c & c+dc in one unit volume can be written as

$$dn(C), = n\left(\frac{m}{2\Pi kT}\right)^{3/2} .\ e^{\frac{-mC^2}{2kT}} .\ 4\Pi C^2dC$$

$$\Rightarrow dn(C) = 4\Pi n\left(\frac{m}{2\Pi kt}\right)^{3/2} .\ e^{\frac{-mc^2}{2kT}} . C^2dC \rightarrow (1.15)$$

It is also called Maxwell's speed distribution law.

Maxwell's speed distribution curves:

If a graph is drawn taking no. of molecules per unit speed interval along Y-axis and velocity of the molecules along X-axis the curves are obtained at different temperatures as shown in fig(1.2).

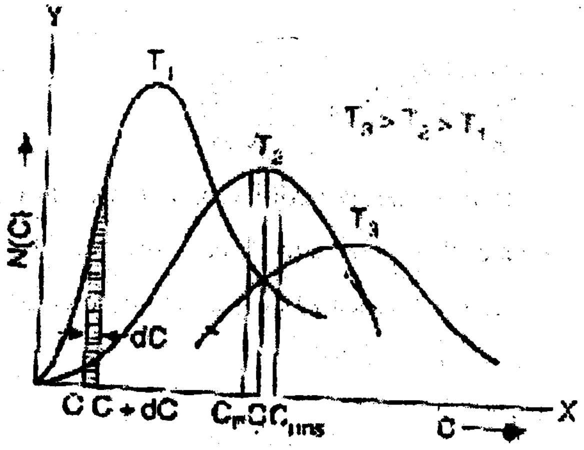


Fig .(1.2)

It is observed that the no. of molecules in a given speed interval increases up to a max. and then decreases towards zero at any temperature .The no. of molecules in a given speed interval is obtained by the area under the curve which is shown as shaded region. The total area under the curve and X-axis gives the total no. of molecules as the temperature is increased. The distribution curve becomes flatten and broaden. The speed at which the no. of molecules is maximum corresponds to peak value shown by curve is called the most probable speed c_p

Most probable speed (c_p): It is the speed possessed by maximum number of molecules in a given sample.

$$c_p = 1.414\sqrt{\frac{kT}{m}}$$

Average speed $(\bar{c})$: It is the speed of all the molecules divided by total number of molecules.

$$\bar{c} = 1.59\sqrt{\frac{KT}{m}}$$

RMS speed(C_{rms}): It is the square root of average of squares of speeds of all the molecules.

$$C_{rms} = \sqrt{\frac{c_1^2 + c_2^2 + \dots\dots\dots + c_n^2}{n}} = 1.732\sqrt{\frac{kT}{m}}$$

Mean free path: The distance between two successive collisions is called free path. The average distance between two successive collision is called mean free path. It is denoted by λ .from Maxwell's speed distribution law. The mean free path is obtained as

$$\lambda = \frac{1}{\sqrt{2}\,\Pi d^2 n}$$

Where d is diameter of each molecule

n is number of molecules per unit volume.

1.3 Transport phenomenon: When the gas is not in equilibrium state. We can obtain the following three cases.

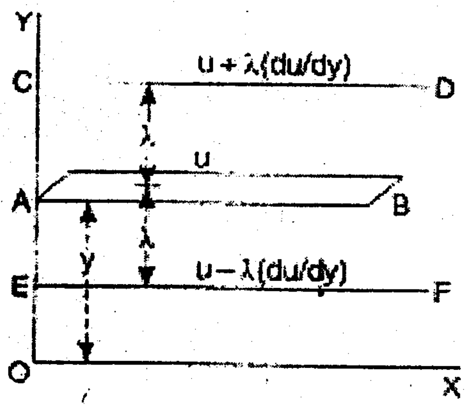
1. The velocity components may not have the same value at all parts of the gas. Now the layers having more momentum transport momentum to the layers having less momentum to get the equilibrium state. This gives rise to the phenomenon of viscosity.

2. The different parts of the gas may be at different temperatures. Now to bring the equilibrium state, the molecule of gas will transport kinetic energy from the regions of higher temperature to the regions of lower temperature. This gives rise to the phenomenon of condition.

3.the different parts of the gas may have different densities i.e., no. of molecules per unit volume .Now to bring the equilibrium state, the molecules of gas transport mass from the regions of higher density to regions of lower density. This gives rise to the phenomenon of diffusion.

The transport of momentum, energy and mass represent conduction, viscosity and diffusion, they are called transport phenomenon.

1.4 Viscosity of gases: Let the molecules in different layers possess different velocities. Let us assume the molecules which are present in the layers which are in touch with X-axis posses zero velocity and velocity of molecules will increase along Y-axis.



Fig(1.3).

The change in velocity per unit distance is called velocity gradient. It is denoted by $\frac{du}{dy}$.

Let u be the velocity of the molecules in layer AB as shown in fig (1.3). Now the velocity of the molecules in the layer CD which is at distance λ is $u + \lambda \frac{du}{dy}$

Consider another layer EF at distance $\frac{du}{dy}$ below the layer AB .the velocity of the molecules in this layer is $u - \lambda \frac{du}{dy}$.

Let the number of molecules pet unit volume be n. now we can write, $\frac{n}{3}$ as number of molecules along any axes.

The number of molecules which are traveling along any axes in a particular direction is $\frac{n}{6}$

Let $\overline{C}$ be the average velocity of the molecule.

The no. of molecules which are traveling through one unit area in one second can be written as

$$= \frac{n\overline{C}}{6} \quad \left[\because \frac{1}{6}n\overline{C} = \frac{1}{6}\frac{N}{V}\times\frac{s}{t} = \frac{1}{6}\times\frac{N}{A\times s}\times\frac{s}{t}\right]$$

Let m be the mass of each molecule.

The mass of the molecule traveling through 1 unit area in 1second

$$= \frac{mn\overline{C}}{6}$$

The momentum of the molecule in the layer CD$= \frac{mn\overline{C}}{6}\left[u + \lambda \frac{du}{dy}\right] \rightarrow (1.16)$

The momentum of the molecule in the layer EF$= \frac{mn\overline{C}}{6}\left[u - \lambda \frac{(du}{dy}\right] \rightarrow (1.17)$

The change in momentum per unit area per second is

$$\frac{F^1}{A} = \frac{mn\bar{C}}{6}\left[u + \lambda\frac{du}{dy}\right] - \frac{mn\bar{C}}{6}\left[u - \lambda\frac{du}{dy}\right]$$

$$= \frac{mn\bar{C}}{6}\left[u + \lambda\frac{du}{dy} - u + \lambda\frac{du}{dy}\right]$$

$$= \frac{mn\bar{C}}{6}\,2\lambda\,\frac{du}{dy}. \quad = \frac{mn\bar{C}}{3}\,\lambda\,\frac{du}{dy}.$$

$$\frac{F^1}{A} = \frac{1}{3}mn\bar{C}\,\lambda\,\frac{du}{dy} \longrightarrow (1.18)$$

The viscous force which is acting between the layers can be written as

$$F^1 = \eta\, A\frac{du}{dy}$$

$$\Rightarrow \frac{F^1}{A} = \eta\,\frac{du}{dy} \rightarrow (1.19)$$

Where η is coefficient of viscosity

From equations (1.18) and (1.19) we can write,

$$\eta\,\frac{du}{dy} = \frac{1}{3}\,mn\bar{C}\,\lambda\,\frac{du}{dy}$$

$$\Rightarrow \eta = \frac{1}{3}\,mn\,\bar{C}\,\lambda \rightarrow (1.20)$$

We know that m n= ρ (density of the gas)

$$\eta = \frac{1}{3}\,\rho\bar{C}\,\lambda$$

We have the expression for the mean free path as

$$\lambda = \frac{1}{\sqrt{2}\pi\ d^2\ n} \rightarrow (1.21)$$

Writing equation (1.21) in equation (1.20) we get.

$$\eta = \frac{1}{3}\,mn\bar{C} \times \frac{1}{\sqrt{2}\pi\ d^2\ n}$$

$$\Rightarrow \eta = \frac{m\bar{C}}{3\sqrt{2}\ \pi\ d^2} \rightarrow (1.22)$$

From this expression we may conclude that

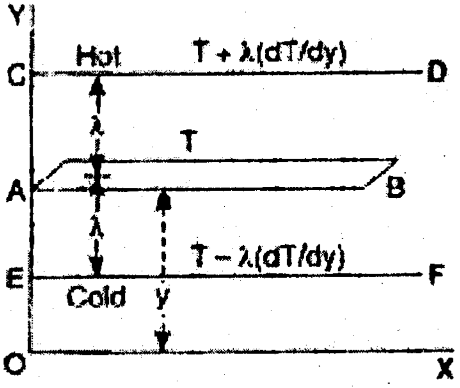
1. The coefficient of viscosity of a gas is independent of pressure or density or number of molecules per unit volume.

2. The coefficient of viscosity of gas is directly proportional to average molecular speed $(\bar{c})$.

3. η is directly proportional to $\sqrt{T}$ since $\bar{c} \propto \sqrt{T}$

4. The coefficient of viscosity η is inversely proportional to square of the diameter of molecule.

1.5 Thermal conductivity: Let us assume the presence of different layers of a gas which are at different temperatures. Let the layer which is in touch with X-axis posses zero temperature and the temperature increases along Y-axis i.e., along OY.



Fig(1 .4)

Let T be the temperature in layer AB. Consider two layers CD&EF at a distance λ on both sides as shown in fig(1.4). The temperature at the upper layer is $T + \lambda \frac{dT}{dy}$,where

$\frac{dT}{dy}$ is temperature gradient i.e, change in temperature per unit distance .The temperature of the lower layer EF is $T - \lambda \frac{dT}{dy}$. Let n be the number of molecules per unit volume.The number of molecules travel along any axis is $\frac{n}{3}$.The number of molecules traveling along any axis in a particular direction is $\frac{n}{6}$.

Let $\bar{c}$ be the average velocity of the molecules and m be the mass of each molecule.

The number of molecules which are passing through an unit area in one second is $\frac{n\bar{c}}{6}$.

The mass of the molecules traveling along any axis in one direction is $\frac{mn\bar{c}}{6}$.

The heat energy transferred through in one unit area in one second from the upper layer CD is

$$Q_1 = mass \times specific\ heat \times temp$$

$$Q_1 = \frac{1}{6}\ mn\bar{c} \times c_v \times \left[T + \lambda \frac{dT}{dy}\right] \rightarrow (1.23)$$

Where c_v is specific heat of gas at constant volume. The heat energy flowing lower layer EF through one unit area in one second in the upward direction is

$$Q_2 = \frac{1}{6}\ mn\bar{c} \times c_v \times \left(T - \lambda \frac{dT}{dy}\right) \rightarrow (1.24)$$

The resultant transfer of heat energy per unit area per second is

$$Q = \frac{1}{6}\ mn\bar{c}\ c_v \left[T + \lambda \frac{dT}{dy}\right] - \frac{1}{6}\ mn\bar{c}\ c_v \left[T - \lambda \frac{dT}{dy}\right]$$

$$= \frac{1}{6}\ mn\bar{c}\ c_v \left[T + \lambda \frac{dT}{dy} - T + \lambda \frac{dT}{dy}\right]$$

$$= \frac{1}{6}\ mn\bar{c}\ c_v\ 2\lambda \frac{dT}{dy}$$

$$Q = \frac{1}{3} mn\bar{c}\, c_v\, \lambda\, \frac{dT}{dy} \rightarrow (1.25)$$

The coefficient of thermal conductivity (k) is defined as the transfer of heat energy per unit area per second per unit temperature gradient.

$$k = \frac{Q}{At\left(\frac{dt}{dy}\right)}$$

Here A=1, t=1 sec

$$k = \frac{Q}{\left(\frac{dT}{dy}\right)} \Rightarrow Q = k\,\frac{dT}{dy} \rightarrow (1.26)$$

From equations(1.25) &(1.26) we can write,

$$k = \frac{dT}{dy} = \frac{1}{3} mn\bar{c}\, c_v\, \lambda\, \frac{dT}{dy}$$

$$\Rightarrow k = \frac{1}{3} mn\bar{c}\, c_v\, \lambda \rightarrow (1.27)$$

We have m n=ρ

$$k = \frac{1}{3} \rho\, \bar{c}\, c_v\, \lambda \rightarrow (1.28)$$

We have the expression for the mean free path as

$$(\lambda) = \frac{1}{\sqrt{2}\,\pi\, d^2 n}$$

Write equation (1.27) in above equation,

$$k = \frac{1}{3} mn\bar{c}\, c_v\, \frac{1}{\sqrt{2}\,\pi\, d^2 n}$$

$$k = \frac{m\bar{c}\, c_v}{3\sqrt{2}\,\pi\, d^2} \rightarrow (1.29)$$

Equation (1.27), (1.28) & (1.29) represents the expressions for coefficients of thermal conductivity.

We have

$$\eta = \frac{m\bar{c}}{3\sqrt{2}\,\pi d^2}$$

$$k = \eta\, c_v \quad \rightarrow (1.30)$$

It is the relation between k and η

The conclusions are as below.

1. For a given temp. k is independent of n or pressure or density.

2. $k \;\alpha\; \bar{c}$, $\bar{c} \;\alpha\; \sqrt{T}$ Hence we can write $k \;\alpha\; \sqrt{T}$

1.6 Diffusion of gases:-

Let us assume the gas in a container having different concentrations in different layers. Let n be the number of molecules in the layer AB i.e. number of molecules per unit volume in that layer.

Let us imagine the two layers CD and EF which are present at a distance on either side. Let the concentration is increasing along positive z-axis and increase in concentration per unit distance is $\frac{dn}{dz}$. It is called concentration gradient. The change in concentration for λ units distance is $\lambda\left(\frac{dn}{dz}\right)$ One third of the concentration of molecules will travel along any axis. $\frac{1}{6}$ of total molecules will travel along any axis in a particular direction. Let $\bar{c}$ be the average velocity. The number of molecules travelling from the layer CD to EF per unit area per sec is $= \frac{1}{6}\left[n + \lambda\frac{dn}{dz}\right]\bar{c}$ ....(1.31)

The no. of molecules traveling from the layer EF to CD per unit area per sec is

$$= \frac{1}{6}\left[n - \lambda\frac{dn}{dz}\right]\bar{c} \quad \rightarrow (1.32)$$

The resultant number of molecules crossing through 1 unit area in 1 sec

$$= \frac{1}{6}\left[n + \lambda \frac{dn}{dz}\right]\bar{c} - \frac{1}{6}\left[n - \lambda \frac{dn}{dz}\right]\bar{c}$$

$$= \frac{1}{6}\left[n + \lambda \frac{dn}{dz} - n + \lambda \frac{dn}{dz}\right]\bar{c}$$

$$= \frac{1}{6} 2\lambda \frac{dn}{dz}\bar{c} \qquad = \frac{1}{3}\lambda \bar{c}\frac{dn}{dz} \qquad \rightarrow (1.33)$$

Diffusion of a gas is defined as the ratio of the number of molecules traveling through one unit area in one sec and concentration gradient.

$$D = \frac{\frac{1}{3}\lambda \bar{c} \frac{dn}{dz}}{\left(\frac{dn}{dz}\right)}$$

$$D = \frac{1}{3}\lambda \bar{c} \qquad \rightarrow (1.34)$$

$$D = \frac{1}{3}\frac{\lambda \rho \bar{c}}{\rho}$$

$$\Rightarrow D = \frac{\eta}{\rho} \rightarrow (1.35) \because \eta = \frac{1}{3}\rho \bar{c}\lambda$$

We have $\lambda = \dfrac{1}{\sqrt{2}\pi d^2 n}$ ....(1.35)

Write above equation (1.35) in eq (1.34) we get,

$$D = \frac{1}{3}\frac{\bar{c}}{\sqrt{2}\pi d^2 n} \qquad \rightarrow (1.36)$$

Equations (1.34), (1.35), (1.36) represent the expression for diffusion of a gas.

(i) $D \,\alpha\, \bar{c} \,\alpha\, \sqrt{T}$

(ii) $D \,\alpha\, \frac{1}{n} \,\alpha\, \frac{kT}{p} \,\alpha\, \frac{T}{P} \left(\therefore p = nkT \Rightarrow n = \frac{p}{kT}\right)$

Combining these two we get,

$$D \,\alpha\, \sqrt{T}\ \alpha \frac{T}{p}$$

$$\Rightarrow D \,\alpha\, T^{\frac{3}{2}} \,.\, p^{-1} \quad \rightarrow (1.37)$$

Diffusion for a gas is directly proportional to $T^{\frac{3}{2}}$ and inversely proportional to pressure p.

1.7. Solved Examples :-

1.Particles of mass $6.2 \times 10^{-17}\ kg$ are suspended in a liquid at $27^{\circ} c$. They are found to have a rms speed of 1.4×10^{-2} m/sec .Calculate Avagadro's number.

Sol:

$m = 6.2 \times 10^{-17}$ kg

$T = 27^{0} c = 27 + 273 = 300k$

$c_{rms} = 1.4 \times 10^{-2}$ m/sec $\quad N = ?$

$$c_{rms} = 1.732 \sqrt{\frac{kT}{m}}, \quad c^2\ rms = (1.732)^2 \left(\frac{kT}{m}\right) = \frac{3kT}{m}$$

$$k = \frac{m}{3T} c^2 rms = \frac{6.2 \times 10^{-17} \times \left(1.4 \times 10^{-2}\right)^2}{3 \times 300} = \frac{6.2 \times 1.96 \times 10^{-21}}{9 \times 10^2}$$

$$= \frac{12.152 \times 10^{-23}}{9} = 1.35 \times 10^{-23}\ {}^{J}\!/_{K}.$$

$$k = 1.35 \times 10^{-23} J/K$$

$$kN = R$$

$$N = \frac{R}{k}$$

$$= \frac{8.314}{1.35 \times 10^{-25}} = 6.1585 \times 10^{23}\ molecule/(mole)$$

2. At what temperature, pressure remaining constant will the rms speed of a gas be half of its value at $0^\circ C$.

Sol:

$$T_1 = 0^0C = 0 + 273 = 273K$$

$$C_{rms\,1} = C_{rms} \quad : \quad C_{rms\,2} = \frac{C_{rms}}{2} \quad T_2 = ?$$

$$C_{rms\,1} = 1.732\sqrt{\frac{KT_1}{m}} \quad C_{rms\,2} = 1.732\sqrt{\frac{KT_2}{m}}$$

$$\frac{C_{rms\,1}}{C_{rms\,2}} = \frac{1.732\sqrt{\frac{KT_1}{m}}}{1.732\sqrt{\frac{KT_2}{m}}} = \sqrt{\frac{T_1}{T_2}}$$

$$\frac{C_{rms}}{\left(\frac{C_{rms}}{2}\right)} = \sqrt{\frac{T_1}{T_2}} \Rightarrow 2 = \sqrt{\frac{T_1}{T_2}} \Rightarrow 4 = \frac{T_1}{T_2}$$

$$T_2 = \frac{T_1}{4} = \frac{273}{4} = 68.25k$$

$$T_2 = 68.25 - 273^0c = -204.75^0C$$

3.Deduce rms value speed of hydrogen molecule at 5000K. Given $K = 1.38 \times 10^{-23} J/K$.

Sol:

$$T = 5000K, K = 1.38 \times 10^{-23} J/K$$

$$m = 2 \times 1.67 \times 10^{-27} kg$$

$$C_{rms} = 1.732\sqrt{\frac{KT}{m}} = 1.732 \times \sqrt{\frac{1.38 \times 10^{-23} \times 5000}{2 \times 1.67 \times 10^{-27}}}$$

$$= 7.87 \times 10^3 m/\sec$$

4. At what temperature will the average speed of hydrogen molecule will same as that of hydrogen molecule at $35^\circ C$.

Sol:

Hydrogen:

$\overline{c_{H_2}} = \bar{c}$

$m_1 = 2\ units$

$T_1 = ?$

Nytrogen:

$\overline{c_{N_2}} = \bar{c}$

$m_2 = 28\ units$

$T_2 = 35^\circ c = 35 + 273 = 308K$

$$\overline{c_{H_2}} = 1.59\sqrt{\frac{KT_1}{m_1}}, \overline{c_{N_2}} = 1.59\sqrt{\frac{KT_2}{m_2}}$$

$$1.59\sqrt{\frac{KT_1}{m_1}} = 1.59\sqrt{\frac{KT_2}{m_2}}$$

$$\frac{KT_1}{m_1} = \frac{KT_2}{m_2}$$

$$T_1 = \frac{m_1 T_2}{m_2} = \frac{2\times 308}{28} = 22k$$

$$T_1 = 22 - 273 = -251^\circ C$$

5. Calculate the rms speed of hydrogen.(i) at NTP (ii)At a pressure of $2\times10^5 N/m^2$

   Given that the density of hydrogen is $8.9\times10^{-2}\ Kg/m^3$ and

   $R = 8.4\times10^2\ J/Kg-mole-K$

Sol: **Consider**

$$\frac{KT}{M} = \frac{R}{N} \times \frac{T}{M} [\because KN = R]$$

$$= \frac{RT}{NM} = \frac{PV}{NM} \begin{bmatrix} \because PV = NRT \\ PV = RT \text{ if } n = 1 \end{bmatrix}$$

$$= \frac{PV}{M} [\because M = NM]$$

$$= \frac{P}{\rho} \left(\because \rho = \frac{M}{V} \right)$$

$$\therefore \frac{KT}{M} = \frac{P}{\rho}$$

$$C_{rms} = 1.732\sqrt{\frac{KT}{M}} = 1.732\sqrt{\frac{P}{\rho}}$$

$$C_{rms} = 1.732\sqrt{\frac{P}{\rho}}$$

$$\rho = 8.9 \times 10^{-2} \, Kg/m^3$$

(i)NTP:

$$T = 0^0 C = 0 + 273 = 273$$

$\rho = 1atm = 76cm$ of Hg=0.76m of Hg

$$= 0.76 \times 13.6 \times 10^3 \times 9.8 \, N/m^2$$

$$= 1.013 \times 10^5 \, N/m^2$$

$$C_{rms} = 1.732\sqrt{\frac{P}{\rho}} = 1.732 \times \sqrt{\frac{1.0138 \times 10^5}{8.9 \times 10^{-2}}}$$

$$= 1.732 \times \sqrt{0.138 \times 10^7}$$

$$= 1.732 \times 1.0667 \times 10^3$$

$$= 1.847 \times 10^3 \, m/s$$

(ii)P$= 2 \times 10^5 \, N/m^2$

$$C_{rms} = 1.732\sqrt{\frac{p}{\rho}} = 1.732 \times \sqrt{\frac{2\times10^5}{8.9\times10^{-2}}}$$

$$= 1.732 \times \sqrt{0.2247\times10^7}$$

$$= 1.732 \times \sqrt{2.247\times10^6}$$

$$= 1.732 \times 1.4989\times10^3$$

$$= 2.596\times10^3 m/s$$

6. Find the values of $\bar{C}, C_{rms}, C_p$ for a molecule of a gas with density of $0.3\,{}^{gm}/_{lit}$ at a pressure of 300mm of Hg.

Sol:

$$\rho = 0.3\,{}^{gm}/_{lit} = \frac{0.3}{10^3} gm/cc = \frac{0.3}{10^3}\times\frac{10^{-3}}{10^{-6}} Kg/m^3 = \frac{0.3}{10^3}\times10^{-3} Kg/m^3$$

$$\therefore \rho = 0.3 Kg/m^3$$

$$p = 300mm \text{ of Hg}$$

$$= 300\times10^{-3} m \text{ of Hg} = 0.3\text{m of Hg}$$

$$= 0.3\times13.6\times10^3\times9.8 N/m^2$$

$$p = 39.984\times10^3 N/m^2$$

we know that $\frac{KT}{M} = \frac{P}{\rho}$

$$C_p = 1.414\times\sqrt{\frac{KT}{M}} = 1.414\sqrt{\frac{P}{\rho}}$$

$$= 1.414\sqrt{\frac{39.984\times10^3}{0.3}}$$

$$= 1.414\sqrt{133.28\times10^3}$$

$$= 1.414\sqrt{13.328\times10^4}$$

$$= 1.414\times3.6507\times10^2$$

$$= 5.162\times10^2 m/s$$

$$\bar{C} = 1.59\sqrt{\frac{KT}{M}} = 1.59\sqrt{\frac{P}{\rho}} = 1.59 \times 3.6507 \times 10^2$$
$$= 5.804 \times 10^2 m/s$$

$$C_{rms} = 1.732\sqrt{\frac{KT}{M}} = 1.732\sqrt{\frac{P}{\rho}} = 1.732 \times 3.6507 \times 10^2$$
$$= 6.323 \times 10^2 m/s$$

7. The rms speed of hydrogen molecule is 1.84 km/s. What will be the rms speed of oxygen molecule at the same temperature. The molecular weights of hydrogen and oxygen are 2 and 32 respectively.

Sol:

$$m_1 = 2\ amu$$
$$m_2 = 32\ amu$$
$$(C_{rms})H_2 = C_1 = 1.84\ {}^{km}\!/_{s} = 1.84\ X\ 10^3\ {}^{m}\!/_{s}$$
$$(C_{rms})o_2 = C_2 = ?$$
$$C_{rms} = 1.732\sqrt{\frac{KT}{m}}. \quad (C_{rms})H_2 = 1.732\sqrt{\frac{KT}{m_1}},$$
$$(C_{rms})o_2 = 1.732\sqrt{\frac{KT}{m_2}}$$
$$\frac{(C_{rms})_{O_2}}{(C_{rms})_{H_2}} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{m_{H_2}}{m_{O_2}}} = \sqrt{\frac{2}{32}} = \sqrt{\frac{1}{16}} = \frac{1}{4}$$
$$\frac{(C_{rms})o_2}{1.84 \times 10^3} = \frac{1}{4}$$
$$\Rightarrow (C_{rms})o_2 = \frac{1.84 \times 10^3}{4} = 0.46 \times 10^3 m/s$$

8. Calculate the diameter of molecule of benzene, if $n = 2.79\times10^{19}\ molecules/cc$ and mean free path for benzene is 2.2×10^{-6} cm

Sol:

$$\lambda = \frac{1}{\sqrt{2}\ \pi\ d^2\ n} \Rightarrow d^2 = \frac{1}{\sqrt{2}\ \pi\ \lambda\ n}$$

$$d^2 = \frac{1}{1.414 \times 3.14 \times 2.2 \times 10^{-6} \times 2.79 \times 10^{19}} = \frac{1}{27.252 \times 10^{13}}$$

$$= \frac{1}{2.7252 \times 10^{14}} = 0.36694 \times 10^{-14}$$

$$d = 0.6057 \times 10^{-7}$$

$$= 6.057 \times 10^{-8}\ cm.$$

9. **The** mean free path of CO_2 molecule fill in a container is $\lambda = 7.9\times10^{-5}m$. If the density of a gas is $1.7Kg/m^3$. Find the diameter of CO_2 molecule.

Sol: $\lambda = 7.9\times10^{-5}m$, $\rho=1.7Kg/m^3$, d=?

$$m = CO_2 = C + O + O = 12+16+16 = 44amu$$

$$= 44x1.67x10^{-27} = 73.48x10^{-27}\,Kg$$

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}; mn = \rho \Rightarrow n = \frac{\rho}{m}$$

$$\lambda = \frac{m}{\sqrt{2}\pi d^2 \rho} \Rightarrow d^2 = \frac{m}{\sqrt{2}\pi\lambda\rho}$$

$$d^2 = \frac{73.48\times10^{-27}}{1.44\times3.14\times7.9\times10^{-5}\times1.7}$$

$$= \frac{73.48\times10^{-22}}{59.628}$$

$$= 1.232\times10^{-22}$$

$$\therefore d = 1.11\times10^{-11}m$$

10. Find the coefficient of thermal conductivity of air from the following data.

$$c_v = 15.5 \times 10^3 \ {}^{J}\!\big/_{kg-mole-k} \qquad k = 1.38 \times 10^{-23} \ {}^{J}\!\big/_{k}$$

$$m = 8 \times 10^{-26} \ kg \qquad d = 3.2 \times 10^{-10} \ m, \ T = 10^0 \ c$$

Sol:

$$T \ 10^0 c = 10 + 273 = 283k, \ k = ?$$

$$k = \frac{m\bar{c}\,c_v}{3\sqrt{2}\ \pi\ d^2} \quad and \quad \bar{c} = 1.59\sqrt{\frac{kT}{m}}$$

$$= 1.59\sqrt{\frac{1.38 \times 10^{-23} \times 283}{8 \times 10^{-26}}} =$$

$$\bar{c} = 1.59 \times \sqrt{\frac{390.54 \times 10^3}{8}}$$

$$\bar{c} = 1.59\sqrt{48.8175 \times 10^3} = 1.59\sqrt{4.88175 \times 10^4}$$

$$\bar{c} = 1.59 \times 2.209 \times 10^{2,} = 3.513 \times 10^2 \ {}^{m}\!\big/_{s}$$

$$k = \frac{m\bar{c}\,c_v}{3\sqrt{2}\,\pi\ d^2} = \frac{8 \times 10^{-26} \times 3.513 \times 10^2 \times 15.5 \times 10^3}{3 \times 1.414 \times 3.14 \times \left(3.2 \times 10^{10}\right)^2} =$$

$$3.193 \times 10^{-1} \ {}^{J}\!\big/_{ms - K}$$

$$k = 0.3193 \ {}^{J}\!\big/_{m.s.K}$$

11. If the coefficient of viscosity of hydrogen gas is under certain conditions. Find the coefficient of thermal conductivity. Given

Sol:

Cv = 1.1 X 10^4 J/m.k.

$$\eta = 8.6 \times 10^{-6} \ N.{}^{s}\!\big/_{m^2} \quad c_v = 1.1 \times 10^4 \ {}^{J}\!\big/_{m-k} \quad k = ?$$

$$k = \frac{1}{3} mn\bar{c}\,c_v\ \lambda \qquad \text{and} \quad \eta = \frac{1}{3} mn\bar{c}\,\lambda .$$

$$\therefore \ k = \eta c_v = 8.6 \times 10^{-6} \times 1.1 \times 10^4$$

$$= 9.46 \times 10^{-2}\ {}^{J}\!/_{m.s.K}$$

1.8.Summary:

Even though the gas molecules are moving randomly in all possible directions with all possible velocities, Maxwell derived the expression for certain probability of the molecules having certain range. Transport phenomena explained the transfer of momentum, kinetic energy and molecules from one layer to another layer to get equilibrium state. In the transport phenomena we obtained viscosity of gases, thermal conduction and diffusion.

1.9. Key words:

Maxwell's speed distribution law, mean free path, viscosity, conduction, diffusion.

1.10. Self Assessment Questions:

1. Derive an expression for Maxwell's laws of distribution of molecular speeds in a gas?
2. What are transport phenomena? Derive the expression for the coefficient of viscosity?
3. Derive the expression for thermal conductivity of a gas and diffusion of a gases on the basis of kinetic theory.

Exercises:

1.Determine the rms velocity of N_2 molecules at S.T.P.(density of) N_2 is 1.25 gm/c.c)

$$\left[\text{Hint:}C_{rms} = \sqrt{\frac{3P}{\rho}} \qquad Ans: 4.93\times10^{4} cm/s\right]$$

2.At what temperature is the ems speed of oxygen molecules will be double of its rms at $27^0 c$.

$$[\text{Hint:}c_{rms} = \sqrt{\frac{3KT}{m}}, \frac{C_{rms1}}{C_{rms2}} = \sqrt{\frac{T_1}{T_2}} \qquad Ans: 927^o c]$$

3.Find the rrms velocity of hydrogen molecule at N.T.P.and at $127^0 c$.

$$\left[\text{Hint:}C_{rms} = \sqrt{\frac{3P}{\rho}}, \frac{C_{rms1}}{C_{rms2}} = \sqrt{\frac{T_1}{T_2}} \qquad \text{Ans:}1.85\times10^{5} cm/s\ and\ 2.3\times10^{5} cm/s\right]$$

1.11. Reference Books:

Unified Physics	Dr.S.L.Gupta&Sanjeev Gupta	Jai Prakash Nath & Co Meerut 2005		
Heat and thermodynamics	Brijlal and Subrahmanyam	S.Chand and Co		
B.Sc.Heat	Telugu Academy			
Thermodynamics	D.C. Thayal	Himalaya Publication		
D.S.MASTHUR'S Heat and Thermodynamics	Revised by Dr.M.N.BAPAT	SULTAN CHAND &SONS	NEW DELHI	1997
B.Sc .Physics Thermodynamics &Heat	Dr.J.P.Agarwal &Amit Agarwal	Pragati Prakashan	Meerut	1992

UNIT-1
LESSON-2

Fundamentals of Thermodynamics

Objectives:

In this lesson we are be able to learn

1. Main fundamentals of thermodynamics.
2. System, surroundings, equilibrium state
3. Carnot's heat engine construction, working, efficiency.
4. First law of thermodynamics and second law of thermodynamics.
5. Carnot's theorem.
6. Absolute scale of temperature.

Structure of the lesson:

2.1 Introduction

2.2 system and surroundings

2.3 Thermodynamic equilibrium

2.4 Concept of temperature

2.5 Internal energy of the system

2.6 Workdone in an isothermal system

2.7 Workdone in an adiabatic system

2.8 First law of thermodynamics

2.9 Application of First law of thermodynamics

2.10 Reversible and irreversible processes

2.11 Carnot's reversible heat engine

2.12 Efficiency of carnot's heat engine

2.13 Carnot's theorem

2.14 Coefficient of performance

2.15 Clausius- Clapeyron's equation

2.1 Introduction

Thermodynamics is the branch of physics, which deals with the transformation of heat energy to the other forms of energy such as mechanical energy, chemical energy, electrical energy etc and vice versa. Here we discuss the transformation of heat energy into mechanical work and vice versa.

2.2 System and surroundings:

If a definite quantity of matter is present between the specified boundaries, it is called a System. Everything around the system is called surroundings. The gas which is present in a cylinder with movable piston can be taken as a system.

The systems are classified into 3 types

1. Open system: When there is transformation of both energy and matter between the system and the surroundings, it is called open system.
2. Closed system: When there is transformation of energy but not matter between the system and the surroundings, it is called closed system.
3. Isolated system: When there is transformation of neither energy nor matter 4.Between the system and the surroundings, it is called isolated system.

2.3 Thermodynamic equilibrium:

The state of the system can be represented by pressure, volume and temperature provided the system is in equilibrium. These parameters are called thermodynamic variables or thermodynamic coordinates. For a system to be in thermodynamical equilibrium, the following conditions must be fulfilled.

1. Mechanical equilibrium: When there is no unbalanced force between the system and the surroundings, then the system is said to be in Mechanical equilibrium.
2. Thermal equilibrium: If the temperature at each and every part of the system is same and it is same as that of the surroundings, then the system is said to be in thermal equilibrium.
3. Chemical equilibrium: If the chemical combination is same at each and every part of the system and it is same as that of the surroundings, then the system is said to be in chemical equilibrium.

Thermal equilibrium: When all the properties of a body remain unchanged, we say that the state of the body is not changed. On the other hand, if any property changes we say that the state of the body is changed.

When there is no heat exchange between the bodies, the bodies are said to be in thermal equilibrium. Ex: Consider two identical bodies A and B. Body A feels hot to the hand while body B be feels cold to the hand. Both are placed in contact for sufficient time. Due to this, heat flows from body A to body B. Now both the bodies are separated. Both A and B give the same temperature sense. It means Aand B are in thermal equilibrium with each other.

2.4 Concept of temperature: -

All the bodies in thermal equilibrium must have a common property, which has the same value for all of them. This property is called temperature.

Temperature is the physical property that tells whether a body is in thermal equilibrium or not with another body .If the two bodies are in thermal equilibrium their temperature is same.

Heat:

The temperature of a body is a measure of its degree of hotness or coldness. Heat is transferred from one body to another body due to the temperature difference. (Heat is a form of energy, which is transferred from one body to another body due to the temperature difference between them).

Work: When a force acts on a system such that it produces a displacement in the system. the work is said to be done.

If the system is displacing the surroundings in the outward direction the work done is external work and it is positive. Here the work is done by the system on the surroundings.

Ex: Consider the gas present in a cylinder, which is expanding. If the piston displaces then the work done is external work and it is +ve .If the work is done by the surroundings on the system again the work done is external work and it is taken as –ve.

If a part of the system does some work on another part of the system, it is called internal work .In thermodynamics external work is more important.

2.5. Internal energy of the system: -

The stored energy which is not apparently shown by the system is called its internal energy or intrinsic energy.

Ex: 1. A mixture of hydrogen and oxygen gas on explosion provides mechanical energy.

2. When Zinc plates are arranged in Copper sulphate solution (Daniel cell) both react and provide electrical energy.

3.A liquid freezes into a solid by giving heat energy.

According to kinetic theory of gases matter consists of small particles called molecules. The molecules move randomly with different velocities in different directions. Due to this each molecule possess some K.E. The sum of the kinetic energies of all the molecules is called internal kinetic energy.

Due to inter molecular force of attraction each molecule possess some potential energy. The sum of potential energies of all the molecules is called internal potential energy.

The sum of internal kinetic energy and internal potential energy of a system is called internal energy of a system .It is denoted by'U'.

Consider the presence of a gas in a cylinder with movable piston. Let U_i be the initial internal energy. Let some work W is done on the system (gas) by moving the piston in the downward direction. Due to this the temperature of a gas increases .As a result of the internal kinetic energy increases and internal potential energy increases it means internal energy of the gas increases .Let U_f be the final internal energy. Now we can write the change in internal energy as $U_f - U_i = -W$

Here W is the work done by the surroundings on the system. It is taken as –ve.

Indicator diagram:

Consider the presence of a gas in a cylinder with movable piston. Let the initial pressure and volumes of the gas be P_1, V_1. If a graph is plotted taking pressure along y-axis and volume along x-axis the initial position is represented by point A as shown in figure(2.1). Due to the expansion of gas let this piston is moving in the outward direction. Due to this the volume of the gas increases and pressure decreases. It is shown by the curve AB on the graph. After expansion P_2 be the pressure and V_2 be volume.

During expansion consider a small region in which the pressure P is almost constant and volume is changed by dV. The work done during this small expansion is dw=PdV=shaded area CDEF. The work done in the entire process is represented by the area between the curve and volume axis.

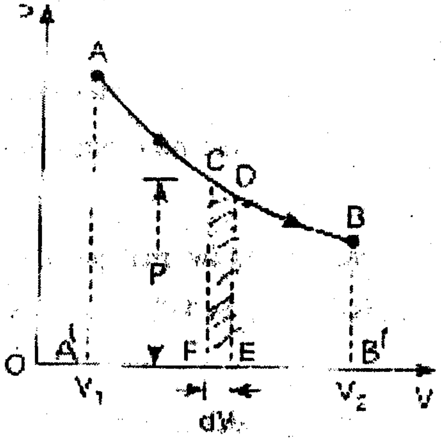


Fig. (2.1)

2.6. Work done in an isothermal process:-.

The changes, which are taking place in pressure and volume by maintaining constant temperature for a thermo dynamical system, are called "isothermal changes". When the volume of the gas increases, pressure and temperature will decrease. But by absorbing heat energy the temperature is maintained constant. When the gas is compressed, volume decreases, pressure and temperature is maintained constant.

Consider 1 gm mole of an ideal gas in a cylinder with movable piston. Let the initial pressure and volume be P_i, V_i. Let T be the temperature of the gas. The initial position is represented by point A on P-V graph (i.e., P along Y-axis & V along X-axis). Let the gas is allowed to expand. Due to this volume increases, pressure and temperature will decrease. But by absorbing heat energy, the temperature T is maintained constant. Let P_f and V_f be the final pressure and volumes. This expansion is represented by the curve AB.

During expansion, consider a small expansion at point C. Here, let the pressure P is constant and the volume is increased by dV. The work done during this small expansion is,

$$dW = PdV \quad \left[PdV = \frac{F}{A} \times A \times S = dW\right]$$

$\because dW =$ shaded area, CDEF.

The work done during the entire expansion is

$$W = \int_{V_i}^{V_f} P.dV$$

$$W = \int_{V_i}^{V_f} PdV = \int_{V_i}^{V_f} \frac{RT}{V} dV (\because PV = RT)$$

$$= RT \int_{V_i}^{V_f} \frac{dV}{V} = RT[\log_e V]_{V_i}^{V_f}$$

$$= RT \log_e \left(\frac{V_f}{V_i}\right)$$

$$= 2.303 \text{ RT} \log_{10} \left(\frac{V_f}{V_i}\right)$$

W = area ABB′A′A

It is the expression for the work done, during isothermal process. If the initial pressure and volumes are P_1, V_1 and the final pressure and volumes are P_2, V_2, then we can write the work done as.

$$W = 2.303 RT \log_{10} \left(\frac{V_2}{V_1}\right)$$

2.7.Work done in an adiabatic process: -

The changes, which are taking place for a thermodynamical system in pressure, volume and temperature without having any exchange of heat energy between the system and the surroundings, are called adiabatic changes.

Consider 1 gm mole of an ideal gas, in a cylinder with movable piston. Here the walls, piston and base are made with perfectly non-conducting material. Let the initial pressure, volume and temperature of the gas be P_i, V_i and T_i. When the gas is allowed to expand, volume increases and pressure and temperature will decrease. After expansion, let the final pressure, volume and temperature be P_f, V_f and T_f .On a graph which is drawn taking pressure along Y-axis and volume along X-axis, we can obtain the curve AB as shown in figure(2.2).

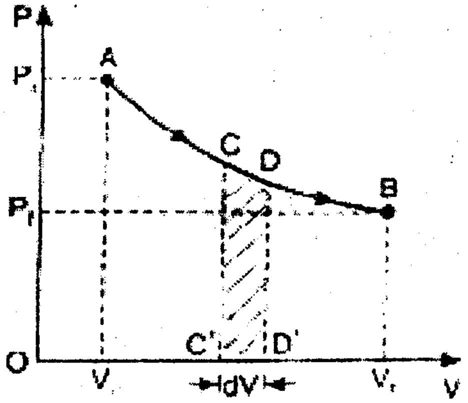


Fig. (2.2).

During the expansion, consider a small expansion at point C. During this small expansion, let the pressure 'P' is constant and the volume is increased by dV. The work done during this small expansion can be written as

$$\because dW = PdV$$

$$= \text{shaded area } pqrs$$

The work done during the entire expansion from A to B is

$$\omega = \int_{V_i}^{V_f} PdV$$

$$= k\int_{V_i}^{V_f} V^{-\gamma} dV$$

$$= k\left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_{V_i}^{V_f}$$

$$= \frac{k}{-\gamma+1}\left[V_f^{-\gamma+1} - V_i^{-\gamma+1}\right]$$

$$=\frac{1}{-\gamma+1}\left[kV_f^{-\gamma+1}-kV_i^{-\gamma+1}\right]\rightarrow(2.1)$$

The points A and B lie on the same adiabatic.

We can write, $P_iV_i^{\gamma}=P_fV_f^{\gamma}=k$

Writing this in eq (1) we get,

$$\therefore w=\frac{1}{1-\gamma}\left[p_f\,v_f^{\gamma}v_f^{-\gamma+1}-p_i\,v_i^{\gamma}v_i^{-\gamma+1}\right]$$

$$\frac{p_f\ v_f-p_iv_i}{1-\gamma}$$

$$=\frac{p_f\ v_f-\ p_i\ v_i}{1-\gamma}\rightarrow(2.2)$$

We have $P_iV_i=RT_i$ and $P_fV_f=RT_f$

$$\therefore w=\frac{1}{1-\gamma}\left[RT_f-RT_i\right]=\frac{R}{1-\lambda}\left[T_f-T_i\right]$$

$$\Rightarrow w=\frac{R}{1-\gamma}\left[T_f-T_i\right]\rightarrow(2.3)$$

Eq. (2.2) is the expression for the adiabatic expansion in terms of pressure and volume.
Eq. (2.3) is the expression for the adiabatic expansion in terms of temperature.

For adiabatic compression, we can obtain the curve from B to A. The work done during the adiabatic compression can be written as,

$$W=\frac{1}{\gamma-1}[P_fV_f-P_iV_i]$$

$$W=\frac{R}{\gamma-1}[T_f-T_i]$$

If the initial pressure, volume and temperature are P_1, V_1 and T_1 and the final pressure, volume and temperatures are, P_2, V_2 and T_2 we can write the work done during adiabatic expansion as

$$W = \frac{1}{1-\gamma}[P_2 V_2 - P_1 V_1]$$

$$\text{or } W = \frac{R}{1-\gamma}[T_2 - T_1]$$

The work done in adiabatic compression can be written as,

$$W = \frac{1}{\gamma-1}[P_1 V_1 - P_2 V_2]$$

$$\text{or } W = \frac{R}{\gamma-1}[T_2 - T_1]$$

2.8 First law of Thermodynamics:-

First law of Thermodynamics is based on the principle law of conservation of energy. It may be stated in two ways.

1. According to the First law of Thermodynamics, if some mechanical work is done, a certain amount of heat will be produced .If some heat energy disappears an equivalent mechanical work will be done.

$W \alpha H$

$W=JH$ where J is called mechanical equivalent of heat

$J = 4.2 J/cal$

$= 4.2 \times 10^{+3} J/K.cal$

2. If some heat energy (dQ) is supplied to a system it is utilized in three ways.

(i) A part of heat energy is utilized to increase the temperature of the gas; As a result the internal kinetic energy of the system increases. It is denoted by dU_k

(ii) A part of the heat energy is utilized against the intermolecular force of attraction .Due to this internal potential energy increases .It is denoted by dU_p

(iii) The remaining part of the heat energy is utilized to do the external work .It is denoted by dW. The total heat energy (dQ) supplied can be written as

$$dQ = dU_k + dU_p + dW$$

$$dQ = dU + dW \rightarrow (2.4)$$

Where dU is increase in internal energy .It is the mathematical or of first law of thermodynamics.

We know that $dW=PdV$

$dQ=dU+PdV$

It is also the mathematical form of first law of thermodynamics.

The first law can be stated, as the heat energy supplied to the system is equal to the sum of increase in internal energy and the external work done.

Significance of first law:-

1. Heat is form of energy in transit.
2. Energy is conserved in thermodynamic system.
3. Every thermodynamic system in equilibrium state posses internal energy, which is a function of the state of the system.

2.9 Applications of first law of thermodynamics: -

1. Isothermal process: In isothermal process, the temperature of the gas is maintained constant .Due to this the internal energy 'U' remains constant.

$\because$. U is constant we can write $dU=0$

From first law of thermodynamics we have,

$$dQ = dU + dW$$

$$dQ = 0 + dW$$

$$\therefore dQ = dW$$

In this process the entire energy which is supplied to the system is utilized to do the external work only.

2. Adiabatic process: In this process the exchange of heat energy between the system and the surroundings is equal to zero.

i.e., $dQ=0$

From first law of thermodynamics we have,

$$dQ = dU + dW$$
$$0 = dU + dW$$
$$\therefore dW = -dU$$

In this process the work done is equal to change in internal energy.

3. Cyclic process: A Cyclic process is the process in which the system gains the initial state after number of changes. Since the system is gaining the initial state, the temperature of the gas is same for the initial and final states .Due to this the internal energy U is same.

The change in internal energy dU=0.

From first law of thermodynamics we have,

$$dQ = dU + dW$$
$$dQ = 0 + dW$$
$$\therefore dQ = dW$$

In this process the heat energy, which is absorbed in the cyclic process, is equal to the work done in the cyclic process.

4. Isochoric process: Isochoric process is the process in which the volume of the gas is constant.

$$V=\text{constant}$$
$$\Rightarrow dV=0$$
$$\Rightarrow PdV=0$$

It means the external work done dW=PdV=0

From first law of thermodynamics we have,

$$dQ = dU + dW$$
$$dQ = dU + 0$$
$$\therefore dQ = dU$$

In this process, the entire heat, which is supplied to the system, is utilized to increase the internal energy only.

5. Free expansion: Free expansion is an adiabatic process is which no work is performed on the system or by the system. Consider two vessels A and B, which are identical and the walls are made with perfectly rigid material. Both the bulbs are connected by means of a narrow tube. Both are separated by using stopcock. Bulb A is completely filled with gas where as bulb B is evacuated. When the stop cock is pulled suddenly the gas A rushes from bulb A to bulb B. since the gas is not doing any work, the expansion is called free expansion.

$$dW = 0$$

Since the walls are perfectly rigid, there is no exchange of heat energy between the system and the surroundings.

From first law of thermodynamics we have,

$$dQ = dU + dW$$
$$0 = dU + 0$$
$$\therefore dU = 0$$
$$U = \text{constant}$$
$$i.e., U_f - U_i = 0$$
$$\therefore U_f = U_i$$

Since the internal energy is constant in this expansion, the temperature of the gas is constant.

2.10 Reversible and irreversible process:-

Reversible process: A reversible process is one, which can be reversed in such a way that all changes occurring in the direct process are exactly repeated in the opposite order, and inverse sense and no changes are left in the system or in the surroundings. If heat is absorbed in the direct process, the same amount of heat would be given in the reverse process. If work is done on the working substance in the direct process and the work will be done by the working substance in the reverse process.

Examples of reversible process:

1. Slow isothermal and adiabatic processes are reversible.
2. If the spring stretched or compressed slowly, it is reversible.
3. If a perfectly elastic ball is allowed to incident on a perfectly elastic surface. it is reversible.

4. In thermo electricity, Peltier effect and Thomson effect are reversible.
5. Slow evaporation or slow condensation is reversible.
6. By absorbing heat energy, ice converts into water. If same amount of heat energy is removed from water, it converts into ice.

Irreversible process: A process which is not reversible is called irreversible process. Almost all processes in nature such as conduction, radiation, radio activity are irreversible.

Examples of irreversible process:

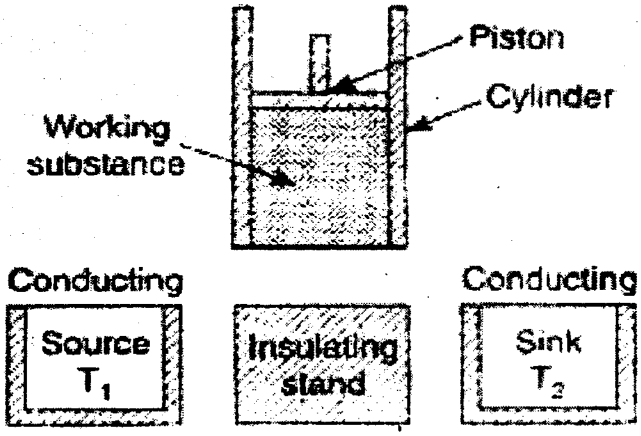
1. Sudden isothermal and adiabatic processes are irreversible.
2. Sudden stretching or compression of a spring is irreversible.
3. If a steel ball is allowed to fall on the lead surface, it is irreversible.
4. Sudden evaporation or condensation is irreversible.
5. The heat produced according to joule's effect (i^2Rt) due to the passes of current through a resistance wire is irreversible.
6. Joule-Kelvin effect is irreversible.

2.11 CARNOT'S REVERSIBLE HEAT ENGINE:-

In 1824, Carnot conceived a theoretical heat engine, which cannot be obtained in actual practice. But it is useful to determine the efficiency of the other heat engines. The Carnot's heat engine mainly consists of 4 parts

1. Working substance
2. Source
3. Sink
4. Insulating stand

1. Working substance: As the working substance an ideal gas is taken in a cylinder with movable piston. The walls and piston are made with perfectly non-conducting material, where as the base is made with perfectly conducting material.



Fig(2.3)

2. Source: Source is maintained at high constant temperature T (K). Even though it gives heat energy continuously its temperature should be maintained constant. It happens, if the thermal capacity is very high.

3. Sink: Sink is maintained at low constant temp T_2 k.. Even though it absorbs heat energy continuously its temperature should be maintained. It happens if the thermal capacity is very high.

4. Insulating stand: In order to isolate the working substance from the surroundings, this insulating stand is used. When the cylinder is placed on the insulating stand, the working substance is isolated.

The working substance undergoes the following 4 operations.

1. Isothermal expansion
2. Adiabatic expansion
3. Isothermal compression
4. Adiabatic compression

1. Isothermal expansion: One-gram mole of an ideal gas is taken in a cylinder with movable piston. Let the initial pressure, volume and temperatures of the gas be P_1,V_1,T_1.On a graph which is drawn taking pressure along Y-axis and volume along X-axis, the initial state is represented by point A as shown in fig (2.4). Now the cylinder is placed on the source. The gas is allowed to expand. When the gas is expanding volume increases, pressure decreases and temperature decreases. But by absorbing heat energy from the source, the temperature of the gas is maintained constant. This isothermal expansion is represented by the curve AB. At the point B, the pressure and volumes are P_2 and V_2.

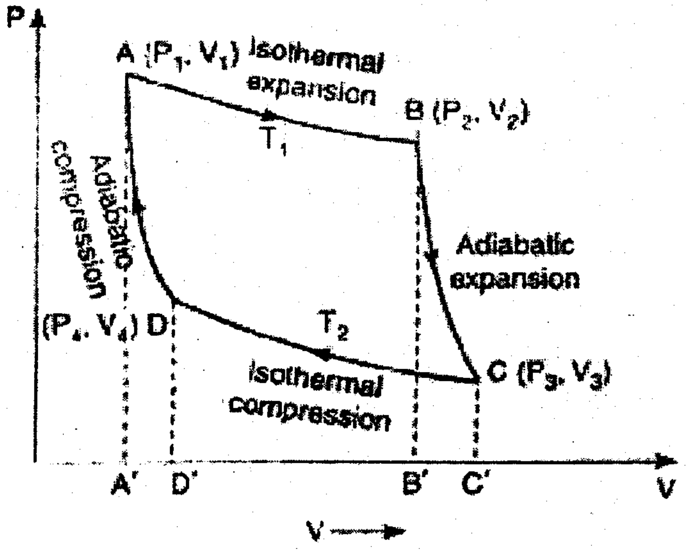


Fig.(2.4)

The work done during this isothermal expansion is

$$W_1 = \int_{V_2}^{V_1} Pdv = \int_{V_2}^{V_1} \frac{RT_1}{V} dV (\because PV = RT)$$

$$= RT_1 \int_{V_2}^{V_1} \frac{dv}{V} = RT_1 [\log_e V]_{V_2}^{V_1}$$

$$W_1 = RT_1 \log_e \left(\frac{v_1}{v_2} \right)$$

$$W_1 = area ABB'A'A$$

If Q_1 is the heat energy absorbed from the source we can write,

$$W_1 = Q_1 = RT_1 \log_e \left(\frac{V_2}{V_1} \right) ....(2.5)$$

2. Adiabatic expansion: The cylinder is separated from the source and placed on the insulating stand. The gas is allowed to expand further. Due to the expansion, volume increases, pressure decreases and temperature decreases. The gas is allowed to expand until the temperature of the gas becomes the temperature of the sink i.e., after expansion let the pressure be P_3 and volume be V_3. This expansion is represented by the curve BC. The work done during this adiabatic expansion is

$$W_2 = \int_{V_2}^{V_3} Pdv = \int_{V_2}^{V_3} \frac{K}{\gamma} dv [\therefore PV^\gamma = k]$$

$$W_2 = K \int_{v_2}^{v_3} v^{-\gamma} dv$$

$$= k \left[\frac{v^{-\gamma+1}}{-\gamma+1} \right]_{V_2}^{V_3}$$

$$= \frac{k}{1-\gamma} \left[V_3^{-\gamma+1} - V_2^{-\gamma+1} \right]$$

$$= \frac{1}{1-\gamma} \left[kV_3^{-\gamma+1} - kV_2^{-\gamma+1} \right]$$

Since B and C lie on the same adiabatic,

$$p_2 V_2^{\gamma} = p_3 V_3^{\gamma} = k = \text{constant}$$

$$W_2 = \frac{1}{1-\gamma}\left[p_3 V_3^{\gamma} V_3^{-\gamma+1} - p_2 V_2^{\gamma} V_2^{-\gamma+1}\right]$$

$$W_2 = \frac{1}{1-\gamma}\left[P_3 V_3' - P_2 V_2'\right]$$

At point B, $P_2V_2=RT_1$

At point C, $P_3V_3=RT_2$

Writing these values in the above equation we get,

$$W_2 = \frac{1}{1-\gamma}\left[RT_2 - RT_1\right]$$

$$W_2 = \frac{R}{1-\gamma}\left[T_2 - T_1\right] \text{..........(2.6)}$$

$$W_2 = area \quad BCC^1 B^1 B$$

3.Isothermal compression: The cylinder is separated from the insulating stand and placed on the sink. The gas is allowed to compress. Due to compression, volume decreases, pressure increases and temperature increases. But by rejecting the excess heat energy to the sink, the temperature of the gas $T_2 K$ is maintained constant. After compression, let the pressure be P_4 and the volume be V_4

This isothermal compression is represented by the curve CD.

The work done during this process is

$$W_3 = \int_{V_3}^{V_4} p\, dV = \int_{V_3}^{V_4} \frac{RT_2}{V} dV \qquad \text{since } pv = RT_2$$

$$= RT_2 \int_{V_3}^{V_4} \frac{dV}{V} = RT_2 \log_e\left(\frac{V_4}{V_3}\right)$$

$$W_3 = -RT_2 \log_e\left(\frac{V_3}{V_4}\right)$$

$$= area\ CD\ D^1\ C^1\ C$$

Q_2 is the heat energy rejected to the sink we can write,

$$W_3 = -Q_2 = -RT_2 \log_e\left(\frac{V_3}{V_4}\right) \quad \ldots\ldots\ldots\ldots\ldots(2.7)$$

4.Adiabatic compression: Now the cylinder is separated from the sink and placed on insulating stand. The gas is allowed to compress further. Due to this, volume decreases, pressure increases and temperature increases. The gas is allowed to compress until it gains the initial state A. i.e., pressure P_1 volume V_1 and temperature T_1. This adiabatic compression is represented by the curve DA.

The work done during the adiabatic compression is

$$W_4 = -\frac{R}{1-\gamma}\left[T_2 - T_1\right] \ldots\ldots\ldots\ldots\ldots(2.8)$$

The total work done in the entire cyclic process is

$$W = RT_1 \log_e\left(\frac{V_2}{V_1}\right) + \frac{R}{1-\gamma}\left[T_2 - T_1\right] - RT_2 \log_e\left(\frac{V_3}{V_4}\right) - \frac{R}{1-\gamma}\left[T_2 - T_1\right]$$

$$W = RT_1 \log_e\left(\frac{V_2}{V_1}\right) - RT_2 \log_e\left(\frac{V_3}{V_4}\right)$$

$\because$ Q_1 is the heat absorbed from the source and Q_2 is the heat rejected to the sink we can write the work done as

$$\therefore W = Q_1 - Q_2$$

$$= RT_1 \log_e \left(\frac{V_2}{V_1}\right) - RT_2 \log_e \left(\frac{V_3}{V_4}\right) \quad \text{............(2.9)}$$

2.12 EFFICIENCY OF CARNOT'S HEAT ENGINE :-

The efficiency of Carnot's heat engine is defined as the ratio of work done in the cyclic process and heat absorbed from the source.

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} \quad \text{............(2.10)}$$

Writing the values of W & Q_1 from equation (2.9) &(2.5) we get,

$$\eta = \frac{W}{Q_1} = \frac{RT_1 \log_e \left(\frac{V_2}{V_1}\right) - RT_2 \log_e \left(\frac{V_3}{V_4}\right)}{RT_1 \log_e \left(\frac{V_2}{V_1}\right)} \quad \text{................(2.11)}$$

In the adiabatic process we have

$$PV^{\gamma} = constant$$

$$\frac{RT}{V} V^{\gamma} = constant$$

$$TV^{\gamma-1} = \frac{constant}{R}$$

$$TV^{\gamma-1} = constant$$

The points B and C lie on the same adiabatic we can write,

$$T_1V_2^{\gamma-1} = T_2V_3^{\gamma-1}$$

$$\frac{T_1}{T_2} = \frac{V_3^{\gamma-1}}{V_2^{\gamma-1}}$$

$$\frac{T_1}{T_2} = \left(\frac{V_3}{V_2}\right)^{\gamma-1} \quad \text{........(2.12)}$$

Similarly,

$\because$ The points D and A lie on the same adiabatic we can write,

$$T_2 V_4^{\gamma-1} = T_2 V_4^{\gamma-1}$$

$$\left(\frac{V_4}{V_1}\right)^{\gamma-1} = \frac{T_1}{T_2}$$

$$\Rightarrow \frac{T_1}{T_2} = \left(\frac{V_4}{V_1}\right)^{\gamma-1} \quad \text{.............(2.13)}$$

From equations (2.12) & (2.13) we can write,

$$\left(\frac{V_3}{V_2}\right)^{\gamma-1} = \left(\frac{V_4}{V_1}\right)^{\gamma-1}$$

$$\frac{V_3}{V_2} = \frac{V_4}{V_1}$$

$$\frac{V_3}{V_4} = \frac{V_2}{V_1} \quad \text{.................(2.14)}$$

Writing equations (2.14) in (2.11) we get,

$$\eta = \frac{RT_1 \log\left(\frac{V_2}{V_1}\right) - RT_2 \log_c \left(\frac{V_2}{V_1}\right)}{RT_1 \log_2 \left(\frac{V_2}{V_1}\right)}$$

$$= \frac{T_1 - T_2}{T_1}$$

$$\eta = 1 - \frac{T_2}{T_1} \text{..................(2.15)}$$

$$\therefore \ \eta = \frac{w}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

It is the expression for the efficiency of the Carnot's heat engine.

IMPRATICABILITY OF CARNOT'S HEAT ENGINE:-

1. Even though the heat engine is extracting heat energy from the hot body (source), its temperature should be maintained constant. Even though the heat engine is giving the heat energy to the body (sink), its temperature should be maintained constant. Both are impossible.

2 The walls and piston are made with perfectly non-conducting material where as the base is made with perfectly conducting material. It is also impossible.

3. The heat engine is slowly working. Whatever may be the efficiency, slow working heat engines are not useful in our day-to-day life.

4. If the engine is operated in the reverse direction, it has to take heat energy from the cold body and has to supply heat energy to the hot body. It is the principle of refrigerator. It is impossible without taking any help from the external agency.

2.13 Carnot's Theorem:-

According to Carnot's theorem no heat engine is more efficient than the reversible heat engine, which are operating between the same two temperatures. The second part of the statement is, all reversible heat engines, which are operating between the same two temperatures, possess some efficiency (irrespective of the nature of the working substance taken)

Proof:

Consider two heat engines one is irreversible (I) and the other is reversible(R) which are operating between the hot body (source) of temperature T_1 and cold body (sink) of temperature T_2 as shown in figure(2.5) .

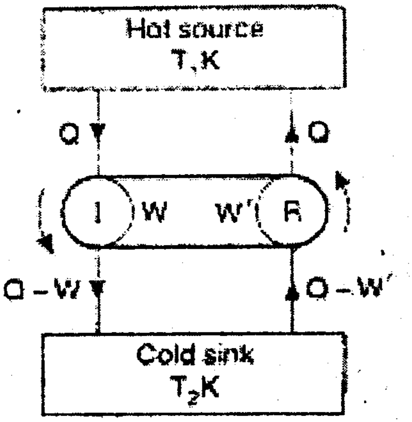


Fig. (2.5)

Let the reversible engine I is absorbing Q units heat energy from the source. Converting a part of heat energy W into work and rejecting the remaining heat energy (Q-W) to the sink. Let the reversible engine 'R' is absorbing "Q'" heat energy from the cold body, some work "W'" is done on the working substance and rejecting the total heat energy $Q' + W'$ to the source. Let the heat energy rejected to the source be 'Q' i.e $Q' + W' = Q$ Due to this the heat content of the source is maintained constant. Let us assume that the efficiency of irreversible heat engine (η_I) is more than the efficiency of reversible heat engine (η_R).

$$\eta_I > \eta_R$$

$$\frac{W}{Q} > \frac{W}{Q'}$$

$$W > W_1^1$$

$$W - W' \quad \textit{is positive}$$

Let both the engines are connected by means of a belt and they are allowed to more combine.

The combined engine is drawing $Q' = Q - W'$ heat energy from the sink and giving $Q - W$ heat energy to the sink.

The resultant heat energy taken by the combined engine from the sink is,

$$\left(Q - W'\right) - \left(Q - W\right)$$

$$\Rightarrow Q - W' - Q + W$$

$$\Rightarrow W - W'$$

But it is obtained as positive.

It indicates that $Q - W'$ is greater than $Q - W$. It means the combined engine is extracting more amount of heat energy from the cold body and giving less amount of heat energy to the hot body. But it is contradiction to second law of thermodynamics. It means our assumption is wrong.

The efficiency of reversible heat engine (η_R) is more than the efficiency of irreversible heat engine (η_I). To prove the second part of the statement considers two reversible heat engines A and B, which are operating between same two temperatures $T_1 K$ and $T_2 K$. Let both the engines A and B are combined by a belt. If A is moving in the forward direction and B is moving in the back ward direction. It means A drives B. In this case $\eta_B > \eta_A$. If engine B moves in the farward direction, A moves in the backward direction. It means B drives A. In this case we can write $\eta_A > \eta_B$.

From the above two cases we can write $\eta_A = \eta_B$. It means the efficiency of both the reversible engines is same.

2.14 Coefficient of performance (K):-

The coefficient of performance K is defined as the ratio of the heat taken from the cold body and the work needed to run the refrigerator.

$$= \frac{Q_2}{w}$$

$$= \frac{Q_2}{Q_1 - Q_2}$$

$$= \frac{1}{\frac{Q_1}{Q_2} - 1}$$

$$= \frac{1}{\left(\frac{T_1}{T_2} - 1\right)}$$

$$k = \frac{1}{\frac{T_1 - T_2}{T_2}}$$

$$= \frac{T_2}{T_1 - T_2}$$

$$\therefore k = \frac{T_2}{T_1 - T_2}$$

It is the expression for coefficient of performance. For a good refrigerator, the coefficient of performance is more.

Relation between η &K:-

We have the expressions for η and K as,

$$\eta = 1 - \frac{T_2}{T_1} \text{ and } k = \frac{T_2}{T_1 - T_2}$$

Consider $k = \frac{T_2}{T_1 - T_2}$

Adding one on both sides we get,

$$k+1 = \frac{T_2}{T_1 - T_2} + 1$$

$$K+1 = \frac{T_2 + T_1 - T_2}{T_1 - T_2}$$

$$K+1 = \frac{T_1}{T_1 - T_2}$$

$$K+1 = \frac{1}{1 - \frac{T_2}{T_1}}$$

$$K+1 = \frac{1}{\eta}$$

$$\therefore \eta = \frac{1}{K+1}$$

It is the relation between η and K.

2.15 Clausius – Clapeyron's equation:-

Consider two isothermals ABCD and $A'B'C'D'$ at temperatures T and T- dT on a graph which is drawn taking pressure along Y-axis and volume along X-axis as shown in figure(2.6). Consider two adiabatics BE and CF to complete the carnot's cycle. At points B and E the substance is in liquid state while at points C and F the substance is in the vapour state. Let V_1 and V_2 be the specific volumes of liquid and vapor. Let L be the latent heat of vaporizations. Let m be mass of the liquid at temperature T and pressure P. At point B, the volume of the liquid is mV_1. At point C, the volume of the liquid is mV_2

Change in volume

$$= mV_2 - mV_1$$

$$= m\left(V_2 - V_1\right)\text{............}(2.16)$$

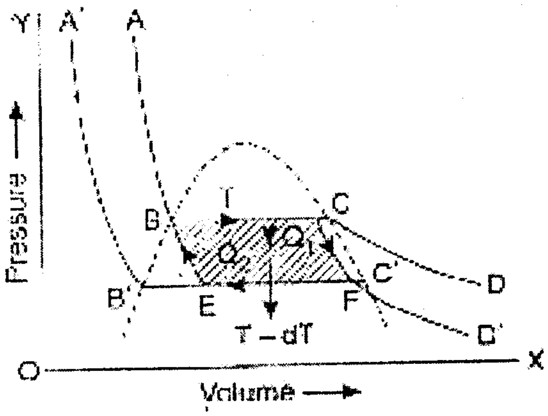


Fig.(2.6)

The working substance undergoes the following cycle.

1. **Isothermal expansion BC:-**During this isothermal expansion ,the substance is changed from liquid to vapour at constant temperature T. the heat energy absorbed for this process is $Q_1 = mL$......(2.17)

2. **Adiabatic expansion CF:-**During this process, the pressure is decreased from P to P-dP and the temperature is decreased from T to T-dT

3. **Isothermal compression FE:-** During this process, by rejecting $T = 27^0C = 300K$ heat energy at temperature T-dT, the substance changes from vapor to liquid state.

4. **Adiabatic compression EB:-** During this process, the pressure is increased from P- dP to P and the temperature is increased from T- dT to T.

the work done in the entire cycle process is ,

W=area BCFEB

=BC × CF

= Change in volume × Change in pressure

$$W = m\left(V_2 - V_1\right)[P - (P - dP)]$$

$$W = m\left(V_2 - V_1\right)dP \rightarrow (2.18)$$

we have the expression for the efficiency of carnot cycle as,

$$\eta = \frac{W}{Q_1} = 1 - \frac{T_2}{T_1}$$

$$\Rightarrow \frac{m\left(V_2 - V_1\right)}{mL} dP = 1 - \frac{T_2}{T_1}$$

2.18 Formulae:-

1. In isothermal process , PV= constant $P_1 V_1 = P_2 V_2$
2. In adiabatic process, PV^{γ} = constant $P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$P_1^{1-\gamma} T_1^{\gamma} = P_2^{1-\gamma} T_2^{\gamma}$$

3. work done in an adiabatic isothermal process $W = 2.303 RT \log_{10}\left(\frac{V_2}{V_1}\right)$
4. work done during adiabatic expansion $W = \frac{1}{\gamma - 1}\left[P_1 V_1 - P_2 V_2\right]$

$$= \frac{1}{\gamma - 1}\left[T_2 - T_1\right]$$

work done in adiabatic compression $W = \frac{1}{\gamma - 1}\left[P_2 V_2 - P_1 V_1\right] = \frac{1}{\gamma - 1}\left[T_2 - T_1\right]$

5. from first law of thermodynamics we have $dQ = dU + dW = dU + PdV$
6. From cornots cycle we have $\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$

7. From thermodynamic scale of temperature $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$

8. coefficient of performance $K = \frac{T_2}{T_1 - T_2} = \frac{Q_2}{W}$

9. Relation between η *and* k *is* $\eta = \frac{1}{k+1}$

10. Clausius-Clapeyron's equation $\frac{dP}{dT} = \frac{L}{T(V_2 - V_1)}$

2.19 Solved examples:-

1. A certain mass of a gas at NTP is expanded to three times its volume under adiabatic conditions. Calculate the resulting temperature and pressure.

Sol:- $T_1 = 0^0c = 273k, P_1 = 1atm, V_1 = V,\ V_2 = 3V$

$T_2 = ?, P_2 = ?,\ \gamma = 1.4$

$$(a)\ P_1 v_1^{\gamma} \Rightarrow P_2 V_2^{\gamma} = P_2 = \frac{P_1 V_1^{\gamma}}{V_2^{\gamma}} = P_1 \left(\frac{V_1}{V_2}\right)^{\gamma}$$

$$= P_1 \left(\frac{V}{3V}\right)^{\gamma} = P_1 \times \frac{1}{3^{\gamma}}$$

$$P_2 = 1 \times \frac{1}{3^{1.4}} = \frac{1}{4.65} = 0.215atm$$

$$(b)\ T_1 V_1^{\gamma-1} = T_2 v_2^{\gamma-1} \Rightarrow T_2 = \frac{T_1 V_1^{\gamma-1}}{V_2^{\gamma-1}} = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$= 273 \left(\frac{V}{3V}\right) 1.4-1$$

$$\Rightarrow T_2 = 273 \left(\frac{1}{3}\right)^{0.4} = \frac{273}{3^{0.4}} = \frac{273}{1.552}$$

$$= 175.9k = 176k$$

$$T_2 = 176 - 273^0c$$

$$= -97^0c$$

2. A certain volume of a gas at $10^5 N/m^2$ pressure expands isothermally until its volume is doubled and then adiabatically until its volume is redoubled. Find the final pressure of the gas. $[\gamma = 1.4]$

Sol:-

$V_1 = V \qquad P_1 = 10^5 N/m^2$

Isothermal:-

$V = 2V \qquad P_2 = ?$

$$P_2 = \frac{P_1V_1}{V_2} = \frac{10^5 \times V}{2V}$$

$$\therefore P_2 = 0.5 \times 10^5 N/m^2$$

Adiabatic:-

$V_3 = 2V_2 \qquad P_3 = ?$

$$P_2V_2^\gamma = P_3V_3^\gamma$$

$$P_3 = \frac{P_2V_2^\gamma}{V_3^\gamma}$$

$$P_3 = 0.5 \times 10^5 \left(\frac{V_2}{V_3}\right)^\gamma$$

$$= 0.5 \times 10^5 \left(\frac{V_2}{2V_2}\right)^{1.4}$$

$$= 0.5 \times 10^5 \times \frac{1}{2^{1.4}}$$

$$= \frac{0.5 \times 10^5}{2.63}$$

$$\therefore P_3 = 0.19 \times 10^5 N/m^2$$

3. A fixed mass of air at 1 atm pressure is compressed adiabatically to 5 atm allowed to expand isothermally to its original volume. Calculate the pressure at the end of isothermal process. $[\gamma_{air} = 1.4]$

Sol:-

$P_1 = 1 atm \qquad V_1 = V$

Adiabatic compression:-

$$P_2 = 5atm \qquad V_2 = ?$$

$$P_1V_1^\gamma = P_2V_2^\gamma$$

$$V_2^\gamma = \frac{P_1V_1^\gamma}{P_2}$$

$$V_2^\gamma = \frac{1\times V^{1.4}}{5} = 0.2\times V^{1.4}$$

$$V_2 = 0.2^{\frac{1}{1.4}}\times V$$

$$V_2 = 0.3169V$$

Isothermal expansion:-

$$V_3 = V \qquad P_3 = ?$$

$$P_2V_2 = P_3V_3$$

$$P_3 = \frac{P_2V_2}{V_3}$$

$$= \frac{5\times 0.3169V}{V}$$

$$P_3 = 1.5845 \approx 1.6\text{atm}$$

4. A mass of a gas occupies a volume of 4 litre at s pressure of 1atm and a temperature of 300 C .It is compressed adiabatically to a volume of 1 litre .Determine a)The final pressure and b)the final temperature .Assuming it to be an ideal gas for which $(\gamma = 1.4)$

$$V_1 = 4 \text{ litre} \quad P_1 = 1\text{atm} \quad T_1 = 300k$$

$$V_2 = 1\text{litre} \qquad P_2 = ?, T_2 = ?$$

$$(a) = P_2V_2^\gamma = P_1V_1^\gamma \Rightarrow P_2 = P_1\left(\frac{V_1}{V_2}\right)^\gamma = 1\times\left(\frac{4}{1}\right)^{1.4}$$

$$= 4^{1.4} = 8 \; atm$$

$$P_2 = 8\text{atm}$$

(b) $p_1^{1-\gamma}\, T_1^{\gamma} = p_2^{1-\gamma} .\, T_2^{\gamma} \Rightarrow \left(\frac{p_1}{p_2}\right)^{1-\gamma} .\ T_1^{\gamma} = T_2^{\gamma}$

$$T_2 = \left(\frac{p_1}{p_2}\right)^{\frac{1-\gamma}{\gamma}} \times T_1 = \left(\frac{1}{8}\right)^{\frac{1-1.5}{1.5}} \times 300$$

$$= \left(\frac{1}{8}\right)^{\frac{-1}{3}} \times 300 = \frac{1}{2^{-1}} \times 300$$

$$= 2 \times 300 = 600K$$

$$T_2 = 600K = 600 - 273 = 327^0\text{C}$$

5. A tyre pumped to a pressure of 3atm ,suddenly burst .Calculate the fall in temperature due to adiabatic expansion .Temperature of the air before expansion is 27^{0}C and γ=1.4.

sol:-

$P_1 = 3atm \qquad P_2 = 1atm$

$T_1 = 27^0C = 300K, \qquad T_2 = ?$

In adiabatic process,

$$P^{1-\gamma}T^{\gamma} = \text{constant}$$

$$P_1^{1-\gamma}T_1^{\gamma} = P_2^{1-\gamma}T_2^{\gamma}$$

$$T_2^{\gamma} = \frac{P_1^{1-\gamma}T_1^{\gamma}}{p_2^{1-\gamma}} = \left(\frac{P_1}{P_2}\right)^{1-\gamma} \times T_1^{\gamma}$$

$$T_2 = \left(\frac{P_1}{P_2}\right)^{1-\gamma} \times T_1^{\gamma}$$

$$T_2 = \left(\frac{3}{1}\right)^{\frac{1-1.4}{1.4}} \times 300 = (3)^{-\frac{0.4}{1.4}} \times 300$$

$$T_2 = (3)^{-\frac{0.4}{1.4}} \times 300 = 3^{-0.2857} \times 300$$

$$T_2 = \frac{1}{3^{0.2857}} \times 300 = \frac{1}{1.3687} \times 300 = 219.18K$$

$$T_2 = 219.18 - 273^0 C$$

$$T_2 = -53.82^0 C$$

6. 1gm of water (volume1cc) becomes 1671CC of steam when boiled at a pressure of 1atm .The latent heat of vapourization it 540 cal/gm .Compute the external work done and increase in internal energy $\left(1\ atm\ =\ 1.013\ \times\ 10^5\ \ {}^{N}\!/_{m^2}\right)$

sol:-

$V_1\ =\ 1cc\ =\ 10^{-6}\ m^3,\ \ m = \ 1\ gm\ =\ 10^{-3}\ kg,\ V_2\ = 1671cc = 1671 \times 10^{-6} m^3$

$$L\ =\ 540\ {}^{cal}\!/_{gm}\ =\ 540\ \times\ 4.2\ {}^{J}\!/_{gm}\ =\ \frac{540 \times\ 4.2}{10^{-3}}\ {}^{J}\!/_{kg}$$

$$=\ 2.268\ \times\ 10^6\ {}^{J}\!/_{kg}$$

$$dW\ =\ PdV\ =\ p\left(V_2\ -\ V_1\right)$$

$$= 1.013\ \times\ 10^5\ \left(1671 \times\ 10^{-6} -\ 1\ \times\ 10^{-6}\ \right)$$

$$=\ 1.013\ \times\ 1670 \times\ 10^5\ \times\ 10^{-6}$$

$$=\ 169.171\ J$$

$$dQ\ =\ mL\ =\ 10^{-3}\ \times\ 2.268\ \times\ 10^6\ =\ 2.268\ \times\ 10^3 J$$

$$dQ\ =\ 2268\ J$$

$$dQ\ =\ dU\ +\ dW\ \ \Rightarrow\ dU\ =\ dQ\ -\ dW$$

$$=\ 2268\ -\ 169.2$$

$$=\ 2098.8J\ =\ \frac{2098}{4.2}$$

$$dU =\ 499.7\ cal$$

7. 1 m^3 of water is converted into 1671 m^3 of steam at atmospheric pressure and 100^0 C temperature .The latent heat of vapourization of water is $2.3\times10^6 J/Kg$. If 2Kg of water be converted into stem at atmospheric pressure 100^0 C, then how will be the increase in it's internal energy.

Sol:-

$$P = 1atm = 1.01\times10^5 N/m^2$$

$$L_{vapour} = 2.3\times10^6 J/Kg$$

$$\rho = 1.0\times10^3 Kg/m^3$$

$$V_1 = \frac{m}{\rho} = \frac{2}{10^3} = 2\times10^3 Kg/m^3$$

$$V_2 = 1671\times2\times10^{-3} m^3$$

$$dW = PdV = 1.01\times10^5\times(1671\times2\times10^{-3} - 2\times10^{-3})$$

$$= 1.01\times10^5\times1671\times2\times10^{-3}$$
$$= 33734\times10^2$$
$$= 3.3734\times10^5 J$$

$$dQ = mL_{vapour}$$

$$= 2\times2.3\times10^6$$
$$= 4.6\times10^6 J$$
$$= 46\times10^5 J$$

$$dQ = dU + dW$$

$$dU = dQ - dW$$

$$= 46\times10^5 - 3.3734\times10^5$$
$$= 42.6266\times10^5$$

$$\therefore dU = 4.263\times10^6 J$$

8. A reversible engine takes heat a source at 527^0 C and gives heat to the sink at 127^0 C. If the power of engine is 750watts .What is the heat it takes per second from the source .

sol:-

$$T_1 = 527^0 c = 527 + 273 = 800\text{K}$$

$$T_2 = 127^0 c = 127 + 273 = 400\text{K}$$

$$p = \frac{W}{T} = 750 \text{ Watt}, Q = ? , T = 1\text{sec}$$

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{400}{800} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\eta = \frac{W}{Q_1} = \frac{1}{2} = \frac{750}{Q_1} \Rightarrow Q_1 = 750 \times 2$$

$$= 1500 J$$

9. The efficiency of a cornot's engine is 25% .On reducing the temperature of the sink by 50^0 C the efficiency becomes 50% what are the source and sink.

Sol:-

$$\eta_1 = 25\% = \frac{1}{4}, \eta_1 = 1 - \frac{T_2}{T_1}$$

$$\eta_1 = 1 - \frac{T_2}{T_1} = \frac{1}{4}$$

$$\Rightarrow 1 - \frac{1}{4} = \frac{T_2}{T_1}$$

$$\Rightarrow \frac{3}{4} = \frac{T_2}{T_1} \rightarrow (1)$$

$$T_2^1 = T_2 - 50$$

$$\eta_2 = 50\% = \frac{1}{2}$$

$$\eta_2 = 1 - \frac{T_2^1}{T_1^1} = \frac{1}{2} \Rightarrow 1 - \frac{T_2 - 50}{T_1} = \frac{1}{2}$$

$$\Rightarrow 1 - \frac{1}{2} = \frac{T_2 - 50}{T_1}$$

$$= \frac{1}{2} = \frac{T_2 - 50}{T_1} = \frac{T_1}{T_2 - 50} = 2 \rightarrow (2)$$

$$(1) \times (2) \quad \frac{T_2}{T_1} \times \frac{T_1}{T_2 - 50} = \frac{3}{4} \times 2 = \frac{3}{2}$$

$$= \frac{T_2}{T_2 - 50} = \frac{3}{2} \Rightarrow 3\,T_2 - 2T_2 = 150$$

$$T_2 = 150\text{K}$$

$$\frac{T_1}{T_2} = \frac{4}{3} = T_1 \ \frac{4T_2}{3} = \frac{4 \times 150}{3}$$

$$= 200\text{K}$$

10. The carnot's engine with the cold body 17^0 C has 30% efficiency .By how much should be hot source alone be raised in temperature if it requires 60% efficiency.

Sol:-

$$T_2 = 17^0 c = 17 + 273 = 290\text{K}$$

$$\eta_1 = 30\% = 0.3$$

$$\eta_1 = 1 - \frac{T_2}{T_1} = 0.3$$

$$\Rightarrow 1 - 0.3 = \frac{T_2}{T_1}$$

$$\Rightarrow \frac{T_2}{T_1} = 0.7 \Rightarrow T_1 = \frac{T_2}{0.7}$$

$$T_1 = \frac{290}{0.7} = 414.28\text{K}$$

$$\eta_1 = 60\% = 0.6$$

$$\eta_1 = 1 - \frac{T_2}{T_1} = 0.6$$

$$\Rightarrow 1-0.6=\frac{T_2}{T_1'}$$

$$\Rightarrow \frac{T_2}{T_1'}=0.4 \Rightarrow T_1'=\frac{T_2}{0.4}$$

$$T_1'=\frac{290}{0.4}=725\mathrm{K}$$

$$T_1'-T_1=725-414.28K=310.72\mathrm{K}$$

11. Carnot's engine has the same efficiency between 1500K and 500K and T K and 1000K(being the temp of the sink in this case).Find the value of T.

Sol:-

1st case:-

$$T_1=1500\mathrm{K}, \quad T_2=500\mathrm{K}$$

$$\eta_1=1-\frac{T_1}{T_2}=1-\frac{500}{150}=1-\frac{1}{3}=\frac{2}{3}$$

2nd case:-

$$T_1=T, \quad T_2=1000K$$

$$\eta_2=1-\frac{T_2}{T_1}=1-\frac{1000}{T_1}$$

Given that $\eta_1 > \eta_2$

$$\Rightarrow \frac{2}{3}=1-\frac{1000}{T}$$

$$\frac{1000}{T}=\frac{1}{3}$$

$$\Rightarrow T\times 1=3\times 1000$$

$$\therefore T=3000K$$

12. A carnot's refrigerator takes heat from water at 0^0 C and discards it to the room at the temperature of 27^0 C. If 100Kg of water at 0^0 C are to be changed to ice what is the required work in Joules. Latent heat of ice $=3.4\times10^5 J/Kg$

Sol:-

$$T_2 = 0^0C = 273K$$

$$T_1 = 27^0C = 27 + 273 = 300K$$

$$m = 100kg,\ Lice = 3.4 \times 10^5\ {}^{J}\!/_{kg},\ W = ?$$

$$Q = m\,Lice = 100 \times 3.4 \times 10^5 = 3.4 \times 10^7 J$$

$$W = Q_1 - Q_2 = Q_2\left[\frac{Q_1}{Q_2} - 1\right] = Q_2\left[\frac{T_1}{T_2} - 1\right]$$

$$= 3.4 \times 10^7 \left(\frac{300}{273} - 1\right)$$

$$W = 3.4 \times 10^7 \times \frac{27}{273} = 0.336 \times 10^7 J$$

$$W = 3.36 \times 10^6 J$$

13. Calculate the work done when a gm mole of a perfect gas expands isothermally at 27^0 C to double its original volume (Given R=8.32 J/Kcal-mole)

sol:-

$$T = 27^0C = 300K$$

$$R = 8.32\ {}^{J}\!/_{kcal.mole},\ W = ?,\ V_1 = V,\ V_2 = 2V$$

$$W = 2.303\,RT\,\log_{10}\left(\frac{V_2}{V_1}\right) = 2.303 \times 8.32 \times 300 \times \log\left(\frac{2V}{V}\right)$$

$$= 2.303 \times 8.32 \times 300 \times 0.3010$$

$$= 1726J$$

2.20 Summary:

This lesson explains the complete basic fundamentals of thermodynamics. Hence the student learns about system and surroundings. Work done in an isothermal process and adiabatic process are explained efficiency of carnot's reversible heat engine is explained. First &second law of thermodynamics are explained relation between Carnot's theorem and second law of thermodynamics is discussed. Absolute scale of temperature is explained with division of scale.

2.21 Key words:

System, surroundings, equilibrium, isothermal, adiabatic, carnot's reversible heat engine, carnot's theorem, absolute scale, first and second law of thermodynamics, reversible and irreversible process.

2.22 Self assessment questions:

1.Distinguish between isothermal and adiabatic processes. Derive the expressions for work done in isothermal and adiabatic process.

2.Describe the working of carnot's engine and derive an expression for efficiency.

3.State and explain carnot's theorem. Explain the absolute scale of temperature.

Short answer questions.

1.explain first law of thermodynamics.

2.explain second law of thernodynamics.

3.derive clausius-clapeyron's equation.

4.Explain irreversible and reversible process.

5.State and prove Carnot's theorem.

6.What are the applications of first law of thermodynamics.

Exercies:-

1.How mech work must be supplied to transfer 1000J of heat from a cold reservour at -73^0C to the hot reservoir at 27^0C by means of refrigerator .

Ans:500J.

$$\left[\begin{array}{l} \text{Hint:}Q_2 = 1000J, T_2 = -73^0C = -73+273 = 273K \\ T_1 = 27^0\text{C}=27+273=300\text{K} \\ \text{W}=Q_1 - Q_2 = Q_2\left[\dfrac{Q_1}{Q_2}-1\right] = Q_2\left[\dfrac{T_1}{T_2}-1\right] = 1000\left[\dfrac{300}{200}-1\right] \\ = 1000\times\dfrac{1}{2} = 500J \end{array}\right]$$

2.Assuming that a domestic refregerator can be regarded as a reversible engine working between 0^0C and 17^0C Calculate the energy that must be supplied to freeze 1Kg of water already at 0^0C

$L=4.3\times10^5 J/Kg$

$$\left[\begin{array}{l} \text{Hint:}T_2 = 0^0C = 0+273 = 273K, m = 1Kg. \\ T_1 = 17^0C = 17+273 = 290K, L_{ice} = 4.3\times10^5 J/Kg \\ Q_2 = mL_{ice} = 1\times4.3\times10^5 J \\ \text{W}=(Q_1 - Q_2) = Q_2\left[\dfrac{Q_1}{Q_2}-1\right] = Q_2\left[\dfrac{T_1}{T_2}-1\right] = 2.677\times10^4 J \end{array}\right]$$

3.Calculate the workdone when a mole of an ideal gasexpande isothermally at $123^0C$to double its original volume,R=8.314 J/degree-mole. Ans:2305.3J

$$\left[\begin{array}{l} \text{Hint:T}=127^0C = 127+273 = 400K, \text{R=8.314J/deg-mole.} \\ \text{W=?}, V_1 = V, V_2 = 2V. \\ \text{W}=2.303\text{RT}\log_{10}\left(\dfrac{V_2}{V_1}\right) \end{array}\right]$$

4.Calculate the workdone when a gm mole of an ideal gas expands isothermally at 27^0C to double its original volume. R=8.3J/Cal-mole. Ans:1726J.

$$\left[\text{Hint:T}=27^0C = 300K, V_1 = V, V_2 = 2V, \text{W}=2.303\text{RT}\log_{10}\left(\dfrac{V_2}{V_1}\right)\right]$$

2.23. Reference books:

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Heat and Thermodynamics	Brijlal and Subrahmanyam	S.Chand & co		
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D.S.MATHUR'S Heat and Thermodynamics	Revised by Dr.M.N.BAPAT	SULTAN CHAND &SONS	NEW DELHI	1997
B.Sc.Physics Thermodynamics &Heat	Dr.J.P.Agarwal &Amit Agarwal	Pragati Prakashan	Meerut	1992

UNIT - I
LESSON-3

ENTROPY

Objectives:

In this lesson student will be able to know the following

1. Concept of entropy
2. Physical significance of entropy
3. Entropy in reversible and irreversible processes
4. Entropy of universe
5. Change in entropy due to change of state.

Structure of lesson:-

3.1 Concept of entropy:-

Entropy is a Greek expression used for transformation concept we know that in an isothermal process, temperature of the system remains constant. In an adiabatic process, Clausius showed that one property remains constant in an adiabatic process and it is called entropy. The entropy is denoted by S.

Consider a graph which is drawn by taking pressure along y-axis and volume along x-axis. Consider three isothermals at temperatures, T_1, T_2 and T_3 as shown in Fig(3.1).

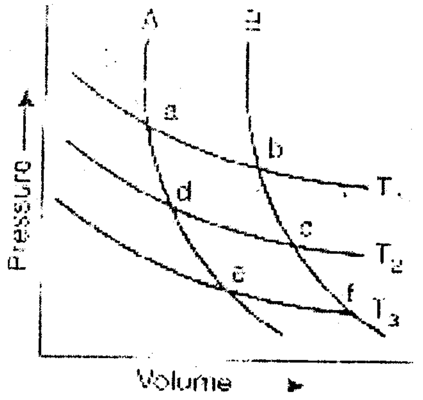


Fig. (3.1).

Consider two adiabatic A and B on these isothermals. Let the intersecting points be a, b, c, d, e, f. Let us assume that the system absorbs Q_1 heat entropy at temperature T_1 from a to b. Let the system rejects Q_2 heat energy at temperature T_2 from c to d. We can consider abcda as carnot's reversible cycle. We have the expression for the efficiency of carnot's engine as,

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} \Rightarrow \frac{Q_2}{Q_1} = \frac{T_2}{T_1} \Rightarrow \frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \frac{Q_3}{T_3} = ... = \frac{Q}{T} = \text{constant}$$

The quantity $\frac{Q}{T}$ is a definite thermal property of the working substance and is termed as entropy.

Physical significance of entropy:-

We have the expression for the change of entropy as,

$$\text{change of entropy} = \frac{\text{heat energy}}{\text{temperature}}$$

$$ds = \frac{dQ}{T}$$

Heat energy =change of entropy × temperature......(3.1)

In mechanics we have the formula

Gravitational potential energy =mgh

Gravitational potential energy= mass × height....(3.2)

If we compare equations (3.1) & (3.2) the height in gravitational potential energy corresponds to temperature in heat energy. The mass in gravitational potential energy corresponds to change of entropy in thermodynamics. Hence entropy may be thought of as a property which bears to heat motion similar to that of mass in linear motion. Hence entropy is also called as thermal inertia.

Entropy is not a physical quantity because there is nothing to represent it physically. Hence it is called as thermal property.

3.2 Measurement of entropy:-

It is impossible to measure the absolute value of energy at any state.We always measure the change in entropy.

Change in entropy =heat energy/temperature

If a small quantity of heat energy dQ is absorbed or rejected at constant temperature T, the change in entropy is, $\Delta S = \frac{dQ}{T}$

If a system changes from state A to state B ,the change in entropy can be written as,

$$S_B - S_A = \int_A^B \frac{dQ}{T}$$

Let us assume that the entropy in the initial state is zero. i.e., S_A=0

$$\therefore S_B - 0 = \int_A^B \frac{dQ}{T} \Rightarrow S_B = \int \frac{dQ}{T}$$

So, by considering the entropy in the initial state as zero, we can determine the entropy in the final state.

Consider a carnot's reversible cycle ABCDA .Let us calculate the change in entropy between the points D&B along the paths DAB and DCB .Change in entropy in an adiabatic process from D to A is equal to zero .In an isothermal process from A to B ,the heat energy absorbed is Q_1 at temperature T_1.Change in entropy from A to B=$\frac{Q_1}{T_1}$

Change in entropy between the points D and B along the paths DAB $= 0 + \frac{Q_1}{T_1} = \frac{Q_1}{T_1}$

In isothermal process from D to C, Q_2 is the heat energy absorbed at temperature T_2.

Change in entropy from D to C $= \frac{Q_2}{T_2}$

In the adiabatic process from C to B the change in entropy =0.Change in entropy between the paths D and B along the path DCB $= \frac{Q_2}{T_2} + 0 = \frac{Q_2}{T_2}$

From carnot's reversible cycle we have, $\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$

It means change in entropy between any two states depends only upon the initial and final states but not upon the path.

3.3 Change in entropy in a reversible cycle:-

To calculate the change in entropy in a general reversible process, first of all let us calculate the change in entropy in carnot's reversible cycle.

Consider a carnot's reversible cycle ABCDA.In isothermal process from A to B, Q_1 heat energy in absorbed at temperature T_1.

Increase in entropy from A to B $= +\frac{Q_1}{T_1}$

In adiabatic expansion from B to C, change in entropy =0.In isothermal expansion from C to D , Q_2 heat energy is rejected at temperature T_2 .

Decrease in entropy from C to D $= -\frac{Q_2}{T_2}$

In adiabatic compression from D to A, change in entropy =0

Total change in entropy in carnot's reversible cycle

$$= \frac{Q_1}{T_1} + 0 - \frac{Q_2}{T_2} + 0 = \frac{Q_1}{T_1} - \frac{Q_2}{T_2} = 0 \left[\because \frac{Q_1}{T_1} = \frac{Q_2}{T_2} \right]$$

It means the change in entropy in a carnot's reversible cycle is zero .

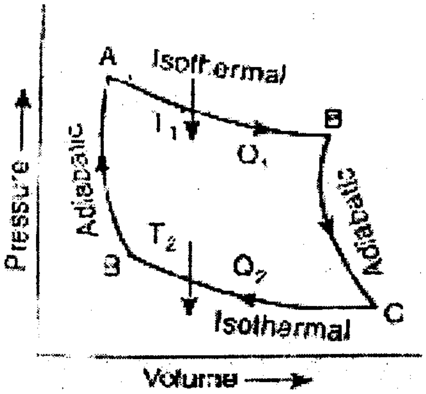


Fig. (3.2)

Now let us apply this to a general reversible process in which the system is changing from state A to state B and then to state A. Now let us consider the number of carnot cycles abcd, efgh, ijkl, mnop,--------on this general reversible process. But the paths bh,fl,jp-----------are traversed for twice in opposite directions. Due to this, the effect of these paths is zero. The resultant path is abefijmn-----opklghcda.

Since the change in entropy in each carnot cycle is zero, the change in entropy in these carnot's cycles can be written as $\Delta S = \Sigma \frac{dQ}{T} = 0$

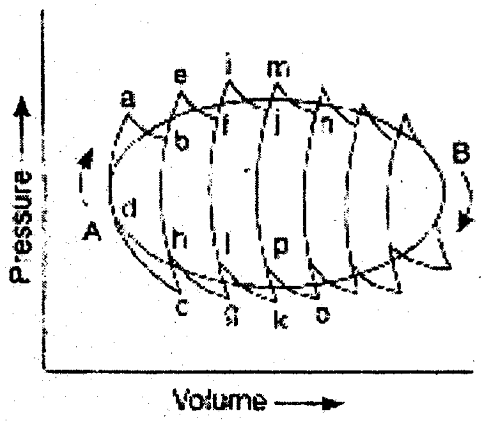


Fig. (3.3)

If we consider a very large number of carnot cycles, the zigzag path converts into a smooth curve from A to B and B to A.

The change in entropy in the reversible process is

$$dS = \int \frac{dQ}{T} = 0$$

It means in all reversible process the change in entropy is zero.

3.4 change in entropy in an irreversible process:-

The examples for irreversible process are conduction of heat, radiation diffusion of gas, electricity through conductors etc.

consider an irreversible heat engine which is absorbing Q_1 heat energy from the hot body of temperature T_1 and rejecting Q_2 heat energy to the cold body of temperature T_2 .its efficiency can be written as,

$$\eta_1 = 1 - \frac{Q_2}{Q_1}$$

Consider a reversible heat engine which is operating between same two temperatures $T_1 K$ and $T_2 K$ .its efficiency can be written as, $\eta_R = 1 - \frac{T_2}{T_1}$.

We know that, $\eta_1 < \eta_R$

$$\Rightarrow 1-\frac{Q_2}{Q_1}<1-\frac{T_2}{T_1}$$

$$\Rightarrow -\frac{Q_2}{Q_1}<-\frac{T_2}{T_1}$$

$$\Rightarrow \frac{Q_2}{Q_1}>\frac{T_2}{T_1}$$

$$\Rightarrow \frac{Q_2}{T_2}>\frac{Q_1}{T_1}$$

$$\Rightarrow \frac{Q_2}{T_2}>\frac{Q_1}{T_1}$$

$$\Rightarrow \frac{Q_2}{T_2}-\frac{Q_1}{T_1}>0$$

$$\Rightarrow dS>0$$

$\Rightarrow dS$ is positive

Since the change in entropy in a reversible cycle is zero, this increase in entropy is taking place in irreversible process. It means in all irreversible processes the value of entropy increases.

3.5 Entropy and second law of thermodynamics:-

The second law of thermodynamics is a law which is connected with the direction in which any chemical or physical process involving energy change takes place.In reversible process the change in entropy is zero and in irreversible process the value of entropy increases.Basing on this the second law of thermodynamics is expressed as below.

A natural process that starts in one equilibrium state and ends in another equilibrium state will go in a direction that causes the entropy of the system plus environment to increase.

Let the system is changed from initial state A to final state B. In this process the change in entropy is,

$$S_B - S_A = \int_A^B \frac{dQ}{T}$$

If the system absorbs a small quantity of heat energy dQ at constant temperature T, the change in entropy is

$$dS = \frac{dQ}{T} [\because dQ = TdS]$$

This equation is called mathematical form of second law of thermodynamics.

The basic principle of entropy explains both the statements of second law of thermodynamics as below.

1. According to Kelvin's statement "it is impossible to draw the heat energy from a body by decreasing it to temperature less than the temperature of the surroundings". Let us assume that it is possible.

If the heat engine draws Q units heat energy from a body of temperature T, the decrease in entropy is $-\frac{Q}{T}$. Since the working substance gains the initial state, its change in entropy is zero.Hence the total change in entropy is $-\frac{Q}{T}$ .The decrease in entropy of the system is against the principle of entropy.

2. According to Clausius statement "it is impossible for any self acting machine to transfer heat energy from cold body to hot body without having any aid from external agency."Let us assume that it is possible, a heat engine (refrigerator) is transferring Q units heat energy from the cold body of temperature T_2 to the hot body of temperature T_1.

Decrease in entropy of the cold body $= -\frac{Q}{T_2}$

Increase in entropy of the hot body $= +\frac{Q}{T_1}$

The entropy of the working substance is considered as remains constant.

Total change in entropy

$$dS = \frac{Q}{T_1} - \frac{Q}{T_2} = Q\left[\frac{1}{T_1} - \frac{1}{T_2}\right] = -ve\left[\because T_1 > T_2\right]$$

This shows that the total energy of the system and surroundings is decreasing. It is against the principle of entropy.

3.6 Entropy and Disorder:-

The solid state is an orderly state as the arrangement of molecules can be specified because they are held by intermolecular forces. Here the orderliness is more and disorder is less. When the solid is converted into liquid, the liquid molecules can move from one place to another place within the liquid. As a result the disorder increases and entropy increases .When the liquid is converted into vapour the disorder further increases, because the gas molecules can move randomly in all possible directions with all possible velocities. Due to this the entropy also increases. The converse of this effect is also true. it means if steam is converted into water disorder and entropy will decrease. If water is converted into ice again disorder and entropy will decrease.

The entropy of a system is the measure of the degree of the disorder prevailing among the molecules.

Examples:-

1. Free expansion: - Consider two bulbs A and B which are connected by means of a narrow tube and separated by using a stop cock. The bulb A is completely filled with gas and bulb B is evacuated. If the stopcock is pulled suddenly the gas rushes from bulb A to bulb B. It is an irreversible process.

Before pulling the stopcock all the gas molecules are present in bulb A. When the stopcock is pulled the gas molecules are present in bulb A and bulb B. Due to this the disorder is increased and entropy also increases.

2.Conduction of heat:- Consider two bodies at temperatures T_1K and T_2K. Let $T_1 > T_2$ .Now the rms velocity of the molecules in the first body is greater than the rms velocity of the molecules in the second body. When both the bodies are made in contact conduction of heat takes place. They acquire a common temperature TK. Now both the bodies are separated. The rms velocity of the molecules in these bodies is same. It means the classification is less when compared to the initial stage. It indicates that the disorder is increased. It is also an irreversible process. Therefore entropy increases due to conduction of heat.

3.7 Entropy of universe:-

In a reversible process the entropy remains constant where as in an irreversible process the entropy is increases. In the universe almost all the process are irreversible and entropy is increasing.

According to the principle of degradation of energy the available energy of the universe is tending towards zero.

Consider a carnot's engine which is operating between the temperatures T_1 and T_2.Let it is absorbing Q_1 heat energy from the hot body of temperature T_1 and rejecting Q_2 heat energy to the cold body of temperature T_2. Available energy for work $= Q_1 - Q_2$

$$= Q_1\left[1 - \frac{Q_2}{Q_1}\right]$$

$$= Q_1\left[1 - \frac{T_2}{T_1}\right]$$

If Q is the heat energy absorbed from the hot body available energy for work

$$= Q\left[1 - \frac{T_2}{T_1}\right]$$

Now let us consider still lower temperature T_0. If the engine is operated between the temperatures T_1 and T_0.

available energy for work $= Q\left[1 - \frac{T_0}{T_1}\right]$

If the engine is operated between temperatures T_2 and T_0,

available energy for work $= Q\left[1 - \frac{T_0}{T_2}\right]$

when the engine is operated between the temperatures T_1 and T_2,

$$\text{Loss of available energy} = Q\left[1-\frac{T_0}{T_1}\right]-Q\left[1-\frac{T_0}{T_2}\right]$$

$$= Q-\frac{QT_0}{T_1}-Q+\frac{QT_0}{T_2}$$

$$= \frac{QT_0}{T_2}-\frac{QT_0}{T_1}$$

$$= T_0\left[\frac{Q}{T_2}-\frac{Q}{T_1}\right]$$

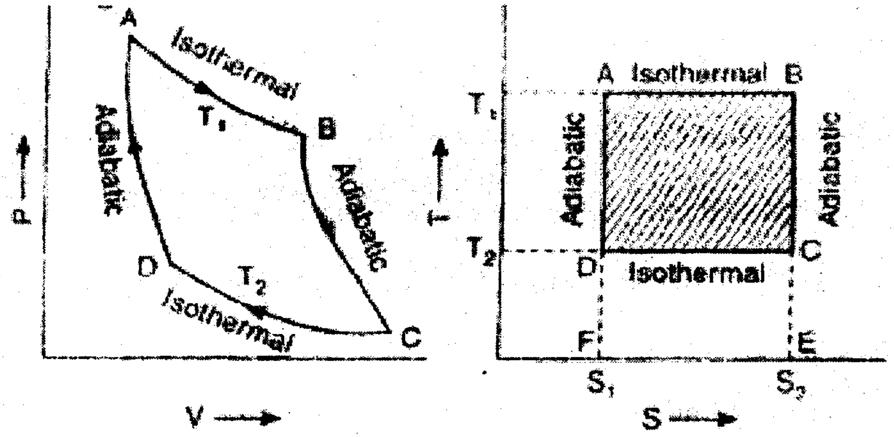
$$= T_0 \times dS$$

where dS is change in entropy.

As irreversible process is continuously going on in nature, the entropy is increasing while the available energy of the universe is continuously decreasing.

3.8 Temperature -entropy diagram of T-dS diagram or T-S diagram:-

In carnot's reversible cycle, the working substance undergoes the following four operations.



(a) (b)

Fig. (3.4)

1. During isothermal expansion from A to B, Q_1 heat energy is absorbed at constant temperature T_1.
2. During adiabatic expansion from B to C, the temperature decreases from T_1 to T_2.
3. During isothermal compression from C to D, Q_2 heat energy is rejected at temperature T_2.
4. During adiabatic compression from D to A, the temperature is increased from T_2 to T_1.

In a graph which is drawn taking pressure and volume along Y and X-axis, the curves are obtained as shown.

In these four operations are drawn in a graph taking temperature along Y-axis and entropy along X-axis we can obtain four lines as shown.

1. During isothermal expansion, temperature T_1 is constant and the entropy is increased from S_1 to S_2 by absorbing Q_1 heat energy.
2. During adiabatic expansion from B to C, the temperature decreases from T_1 to T_2 at constant entropy S_2.
3. During isothermal compression from C to D, Q_2 heat energy is rejected at constant temperature T_2 and the entropy is decreased from S_2 to S_1.
4. During adiabatic compression from D to A, the temperature raises from T_2 to T_1 at constant entropy S_1.

From second law of thermodynamics we have, $ds = \frac{dQ}{T} \Rightarrow dQ = TdS$

The heat energy absorbed Q_1 during isothermal expansion from A to B is

$$Q_1 = T_1(S_2 - S_1) = AF(OE - OF) = AF \times FE$$

$$Q_1 = \text{area AFEB}$$

The heat energy rejected (Q_2) during isothermal compression from C to D is

$$Q_2 = T_2(S_2 - S_1) = DF(OE - OF) = DF \times FE$$

$$Q_2 = \text{area DFEC}$$

The heat energy which is converted into work is $W = Q_1 - Q_2$

=area AFEB-area DFEC

=area ABCD

The efficiency of the heat engine is

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = \frac{AreaABCD}{Area\mathrm{AFEB}} = \frac{AD \times AB}{\mathrm{AF} \times AB} = \frac{\mathrm{AF} - DF}{\mathrm{AF}} = \frac{T_1 - T_2}{T_1}$$

$$\eta = 1 - \frac{T_2}{T_1}$$

Thus the efficiency of carnot's reversible cycle can be obtained using entropy - temperature diagram.

Uses of T-S diagram:-

1. They are used in meteorology.
2. They are of great use in predicting the defects in the performance of the engine.
3. To check the efficiency of heat engine.
4. To obtain the work value of the fuel used.

3.9 Entropy of the perfect gas:-

Consider one mole of perfect gas .Let its pressure be P, volume be V and temperature be T.

From first law of thermodynamics we have, $dQ = dU + dW$ ......(3.3)

Due to absorption of heat, let us assume that the internal energy is increased initially. If C_V is the specific heat at constant volume and dT is the increase in temperature, the increase in internal energy is $dU = C_V dT$

Later on, if the external work is done and the volume is increased by dV, the external work done is $dW = PdV$

Writing these values in equation (1) we get,

$dW = C_V dT + PdV$....(3.4)

$dQ = TdS ...(3.5)$

From equations (3.4) and (3.5) we can write, $TdS = C_V dT + PdV$

$$\Rightarrow \frac{C_V}{T} dT + \frac{RdV}{V} .....(3.6) \begin{bmatrix} \because PV = RT \\ \frac{P}{T} = \frac{R}{V} \end{bmatrix}$$

$$dS = \frac{C_V dT + PdV}{T} \Rightarrow dS = \frac{C_V}{T} dT + \frac{PdV}{T}$$

Let the initial volume, temperature and entropy be V_i, T_i and S_i

Let the final volume, temperature and entropy be V_f, T_f and S_f.

Integrating equation (3.6) and applying these limits we get,

$$\int_{Si}^{Sf} dS = \int_{T_i}^{T_f} \frac{C_V}{T} dT + \int_{V_i}^{V_f} \frac{RdV}{V} \Rightarrow [S]_{S_i}^{S_f} = C_V [\log_e T]_{T_i}^{Tf} + R[\log_e V]_{V_i}^{V_f}$$

$$\Rightarrow S_f - S_i = C_v \log_e \left(\frac{T_f}{T_i}\right) + R \log_e \left(\frac{V_f}{V_i}\right)$$

$$\therefore S_f - S_i = 2.303 \left[C_v \log_{10} \left(\frac{T_f}{T_i}\right) + R \log_{10} \left(\frac{V_f}{V_i}\right) \right] ....(3.7)$$

It is the change in entropy in terms of temperature and volume

To calculate the change in entropy in terms of temperature and pressure we have to eliminate volume.

For one mole of perfect gas we have PV=RT

differentiating we get, PdV + VdP= RdT

$$PdV = RdT - VdP \Rightarrow dV = \frac{RdT - VdP}{P} .....(3.8)$$

writing equations (3.8) in equation (3.6) we get,

$$dS = \frac{C_V}{T} dT + \frac{R}{V} \left[\frac{RdT - VdP}{P} \right]$$

$$=\frac{C_V}{T}dT+\frac{R}{RT}[RdT-VdP][\because PV=RT]$$

$$=\frac{C_V}{T}dT+\frac{RdT}{T}-\frac{VdP}{T}$$

$$=(C_V+R)\frac{dT}{T}-\frac{VdP}{T}$$

$$=C_p\frac{dT}{T}-V\frac{dp}{T}\quad[\because C_p-C_v=R\Rightarrow C_p=C_v+R]$$

$$dS=C_p\frac{dT}{T}-\frac{R}{P}dP\quad[\because PV=RT]\ldots\ldots\ldots\ldots(3.9)$$

Let the initial pressure, temperature and entropy be P_i, T_i and S_i

Let the final pressure, temperature and entropy be P_f, T_f and S_f

Integrating the above equations and applying these limits we get,

$$\int_{S_i}^{S_f} dS=C_p\int_{T_i}^{T_f}\frac{dT}{T}-R\int_{P_i}^{P_f}\frac{dP}{P}$$

$$[S]_{S_i}^{S_f}=C_P[\log_e T]_{T_i}^{T_f}-R[\log_e P]_{P_i}^{P_f}$$

$$\Rightarrow S_f-S_i=C_P\log_e\left(\frac{T_f}{T_i}\right)-R\log_e\left(\frac{P_f}{P_i}\right)$$

$$S_f-S_i=2.303\left[C_P\log_{10}\left(\frac{T_f}{T_i}\right)-R\log_{10}\left(\frac{P_f}{P_i}\right)\right]\ldots\ldots(3.10)$$

It is the change in entropy in terms of temperature and pressure.

To calculate the change in entropy in terms of pressure and volume we have to eliminate the temperature .

For one mole of perfect gas we get, $PV=RT$

Differentiating this we get

$$PdV+VdP=RdT$$

$$dT=\frac{PdV+VdP}{R}\ldots\ldots\ldots\ldots(3.11)$$

writing equation (3.11) in equation (3.6) we g

$$dS = \frac{C_V}{T}\left(\frac{PdV + VdP}{R}\right) + \frac{RdV}{V}$$

$$= C_V\left(\frac{PdV + VdP}{PV}\right) + \frac{RdV}{V}$$

$$= C_V\left(\frac{dV}{V} + \frac{dP}{P}\right) + \frac{RdV}{V}$$

$$= C_V\frac{dV}{V} + C_V\frac{dP}{P} + \frac{RdV}{V}$$

$$= \left(C_V + R\right)\frac{dV}{V} + C_V\frac{dP}{P}$$

$$= C_P\frac{dV}{V} + C_V\frac{dP}{P}\left[\because C_P - C_V = R \Rightarrow C_P = C_V + R\right]$$

$$dS = C_P\frac{dV}{V} + C_V\frac{dP}{P} \text{........(3.12)}$$

Let the initial pressure,volume and entropy be P_i, T_i and S_i

Let the final pressure, volume and entropy be P_f, T_f and S_f.

Integrating this equation and applying these limits we get,

$$\int_{S_i}^{S_f} dS = \int_{V_i}^{V_f} \frac{C_P}{V} dV + C_V \int_{P_i}^{P_f} \frac{dP}{P}$$

$$[S]_{S_i}^{S_f} = C_P[\log_e V]_{V_i}^{V_f} + C_V[\log_e P]_{P_i}^{P_f}$$

$$\Rightarrow S_f - S_i = 2.303 C_P[\log_{10} V]_{V_i}^{V_f} + C_V[\log_{10} P]_{P_i}^{P_f}$$

$$\Rightarrow \mathrm{S_f} - \mathrm{S_i} = 2.303\left[\mathrm{C_P}\log_{10}\left(\frac{\mathrm{V_f}}{\mathrm{V_i}}\right) + \mathrm{C_V}\log_{10}\left(\frac{\mathrm{P_f}}{\mathrm{P_i}}\right)\right] \text{......(3.13)}$$

It is the expression for change in entropy in terms of pressure and volume .

3.10.Change in entropy on heating a substance :-

Consider a substance (solid or liquid) of mass m and specific heat c .By absorbing small amount of heat energy dQ , let the temperature of the substance is increased by dT.

$\therefore dQ = mcdT$

The increase in entropy is

$$\Delta S = \frac{dQ}{T} = \frac{mcdT}{T}$$

By absorbing the heat energy if the temperature of the substance is changed from T_1 to T_2 ,then the change in entropy is

$$dS = \int_{T_1}^{T_2} \frac{dQ}{T} = \int_{T_1}^{T_2} \frac{mcdT}{T} = mc\int_{T_1}^{T_2} \frac{dT}{T} = mc\left[\log_e T\right]_{T_1}^{T_2}$$

$$dS = mc\log_e\left(\frac{T_2}{T_1}\right)$$

$$\therefore dS = 2.303\log_{10}\left(\frac{T_2}{T_1}\right) \ldots\ldots (3.14)$$

It is the expression for the change in entropy of the substance in the same state .

3.11. Change in entropy when ice converts into steam:-

Consider m units mass of ice at T_1K (0^0C).Let L_{ice} be the latent heat of ice and L_{steam} be the latent heat of steam. Let C be the specific heat of water.

1. For the conversion of ice at T_1K to water at T_1K, the heat energy required is $dQ = mL_{ice}$

Change in entropy is $dS_1 = \frac{dQ}{T_1} = \frac{mL_{ice}}{T_1} \ldots\ldots (3.15)$

It is the expression for change in entropy when ice converts into water at T_1K

2.Now we have to obtain the change in entropy when ice converts into water at T_2K. By absorbing small amount of heat energy dQ, let the temperature of water is raised by dT.

$\therefore dQ = mcdT$

Here the change in entropy is $\Delta S_2 = \frac{dQ}{T} = \frac{mcdT}{T}$

The change in entropy when water at T_1 K is changed into water at T_2 K is

$$dS_2 = \int_{T_1}^{T_2} \frac{dQ}{T} = \int_{T_1}^{T_2} \frac{mcdT}{T} = mc\left[\log_e T\right]_{T_1}^{T_2}$$

$$dS_2 = mc\log_e\left(\frac{T_2}{T_1}\right)$$

$$dS_2 = 2.303mc\log_{10}\left(\frac{T_2}{T_1}\right) \text{........}(3.16)$$

It is the expression for change in entropy when water at T_1 K is changed into water at T_2 K.

3. Now we have to calculate the change in entropy when water at T_2 K converts into steam at T_2 K

The amount of heat energy required to convert water into steam is $dQ = mL_{steam}$

The change in entropy when water converts into steam at T_2 K is

$$dS_3 = \frac{dQ}{T_2} = \frac{mL_{steam}}{T_2} \text{....}(3.17)$$

The total change in entropy when ice at T_1 K is changed into steam at T_2 K is obtained adding equation (3.15).(3.16) and (3.17)

$$\therefore dS = dS_1 + dS_2 + dS_3$$

$$dS = \frac{mL_{ice}}{T} + 2.303mc\log_{10}\left(\frac{T_2}{T_1}\right) + \frac{mL_{steam}}{T_2} \text{...}(3.18)$$

3.12.Change in entropy during free expansion:-

Consider μ moles of an ideal gas in vessel A. Both vessels A and B are thermally insulated, connected by a narrow tube and separated by a stop cock .The bulb B is completely evacuated.

When the stop cock is pulled suddenly, the gas rushes from bulb A to B. The expansion is called free expansion. Since the gas is entering into vacuum, the work done by the gas is zero. At the same time, the internal energy U is constant due to constant temperature. The change in internal energy is zero i.e., dU=0

The heat energy supplied is zero i.e., dQ=0

Here we are not in a position to calculate the change in entropy. But actually free expansion is an irreversible process in which entropy increases .So we consider an equivalent reversible process to obtain the change in entropy.

From first law of thermodynamics we have, dQ = dU + dW

For an ideal gas, U is constant in free expansion, it means dU=0

$$dQ = 0 + dW = dW$$

$$dQ = PdV$$

$$dQ = \frac{\mu RT}{V} dV \left[\because PV = \mu RT\right]$$

$$\frac{dQ}{T} = \mu R \frac{dV}{V}$$

The change in entropy is

$$\Delta S = \frac{dQ}{T} = \mu R \frac{dV}{V}$$

The change in entropy when the volume of the gas is changed fromV_1 to V_2 is

$$dS = \int_{V_1}^{V_2} \frac{dQ}{T} = \int_{V_1}^{V_2} \frac{\mu R dV}{V} == \mu R \int_{V_1}^{V_2} \frac{dV}{V} = \mu R \left[\log_e V\right]_{V_1}^{V_2}$$

$$dS = \mu R \log_e \left(\frac{V_2}{V_1}\right)$$

$$dS = 2.303 \mu R \log_{10} \left(\frac{V_2}{V_1}\right)$$

It is the increase in entropy during free expansion. Since $V_2 > V_1$. The increase in entropy of the universe due to free expansion can also be written as

$$dS = 2.303 \mu R \log_{10} \left(\frac{V_2}{V_1}\right)$$

3.13 Solved examples:-

Formulae:

1. Change in entropy when the temperature of the substance is changed is

$$\therefore dS = 2.303mc\log_{10}\left(\frac{T_2}{T_1}\right) ------(3.14)$$

2. Change in entropy when the state of the substance in changed is

$$dS = \frac{dQ}{T} = \frac{mL}{T}$$

3. Change in entropy when the volume of substance is changed is

$$dS = 2.303\mu R\log_{10}\left(\frac{V_2}{V_1}\right)$$

where μ is number of moles.

4. Change in entropy when the same substance of equal masses at different temperature are mixed with each other is $dS = 2\,\mathrm{mc}\log_e\left(\frac{T_1+T_2}{2\sqrt{T_1T_2}}\right)$

1. One gram mole of a perfect gas expands isothermally to four times its initial value. Calculate the change in its entropy in terms of gas constant R?

Solution: $dS = \int_{V_1}^{V_2}\frac{dQ}{T} = \int_{V_1}^{V_2}\frac{dW}{T} = \int_{V_1}^{V_2}\frac{PdV}{T} = \int_{V_1}^{V_2}\frac{R}{V}dV = R\int_{V_1}^{V_2}\frac{dV}{V}$

$$= R\left[\log_e V\right]_{V_1}^{V_2} = R\log_e\left(\frac{V_2}{V_1}\right) = 2.303R\log_{10}\left(\frac{V_2}{V_1}\right)$$

$$V_1 = V, V_2 = 4V,$$

$$dS = 2.303\times R\log_{10}\left(\frac{4V}{V}\right)$$

$$= 2.303\times R\times 0.6020$$

dS=1.386R J/K

2. One Kg of ice at 0^0C is melted and converted to water at 0^0C. Compute the change in entropy . Given that latent heat of $L_{ice} = 3.34$ J/Kg

$$m = 1Kg, T = 0^0C = 273K, L_{ice} = 3.34\times10^5 J/Kg$$

$$dS = \frac{dQ}{T} = \frac{mL_{ice}}{T} = \frac{1\times3.34\times10^5}{273} = 0.01222\times10^5$$

$$dS = 1.222\times10^3 J/Kg$$

3. One Kg of boiling water at 100^{0}C is mixed with one Kg of water at 0^{0}C . Calculate the change in entropy.

Solution:-

$$dS = 2mc\log_e\left\{\frac{T_1+T_2}{2\sqrt{T_1T_2}}\right\}$$

m=1Kg, $C = 4.2\times10^3\ J/K-Kg$

$T_1 = 0^0C = 0+273 = 273K$

$T_2 = 100^0C$=100+273=373K

dS=?

$$dS = 2.303\times2\times1\times4.2\times10^3\times\log_{10}\left\{\frac{273+373}{2\sqrt{273+373}}\right\}$$

$$=19.345\times10^3\times\log_{10}\left\{\frac{646}{2\sqrt{101829}}\right\}$$

$$=19.345\times10^3\times\log_{10}\left\{\frac{323}{319.106}\right\}$$

$$=19.345\times10^3\times\log_{10}1.0122 = 19.345\times10^3\times0.0052$$

$$=0.10059\times10^3$$

$$dS = 100.59J/K$$

4. Calculate the change in entropy when 10 Kg of ice at 0^0C is changed to water at 2^0C $\left(L_{ice} = 80\ KCal/Kg\right)$

Solution:-

$m = 10Kg, T_1 = 0^0C = 273k, T_2 = 2^0C = 2+273 = 275k.$

$L_{ice} = 80kcal/Kg = 80\times10^3\times4.2J/Kg = 336\times10^3 = 3.36\times10^5J/Kg$

specific heat of water

$$C = 1 \, {cal}/{gm-K} = \frac{4.2J}{10^{-3} Kg-K} = 4.2\times10^{3} \, {J}/{Kg-mole}$$

when ice T_1k is changed to water at T_1k we have

$$dS_1 = \frac{mL_{ice}}{T}$$

when ice T_1k is changed to water at T_2k we have

$$dS_2 = 2.303mc\log_{10}\left(\frac{T_2}{T_1}\right)$$

$$dS = dS_1 + ds_2 = \frac{mL_{ice}}{T} + 2.303mc\log_{10}\left(\frac{T_2}{T_1}\right)$$

$$= \frac{10\times3.36\times10^{5}}{273} + 2.303\times10\times4.2\times10^{3}\times\log_{10}\left(\frac{275}{273}\right)$$

$$= 0.1236\times10^{5} + 0.028\times10^{4}$$

$$= 1.236\times10^{4} + 0.028\times10^{4} = (1.236+0.028)\times10^{4}$$

$$dS = 1.264\times10^{4} \, {J}/{K}$$

3.14 Summary:-

This lesson explains, what is entropy.How entropy can be measured. What is its physical significance? How the entropy of universe is changing.

3.15 Keywords:-

Entropy, free expansion, reversible process, irreversible process, universe.

3.16 Self assessment questions:-

Long answer questions

1. Define entropy? what is the physical significance of entropy? Derive change in entropy in reversible and irreversible process.

2. Explain the concept of write about entropy. Explain T-dS diagram. Derive the expression for efficiency using it.What are the uses of T-dS diagram.

Short answer questions

1. Explain change in entropy in free expansion
2. Explain the concept of entropy and second law of thermodynamics
3. Describe the change in entropy of universe.

Exercises:-

1. Calculate the change in entropy when 300 gm of lead melts at 373^0C; Lead has the latent heat of fusion as 5.85 cal/gm.

$$\left[\text{hint:}ds=\frac{dQ}{T}=\frac{mL_{fussion}}{T} \qquad ans:12.28J/k\right]$$

2.Find the increase in entropy when 10 gm of ice at 0^0C is converted into water at same temperature, given that latent heat of fusion of ice is 80 cal/gm.

$$\left[\text{hint:}ds=\frac{dQ}{T}=\frac{mL_{fussion}}{T}=\frac{10\times 80}{273}=2.93\text{cal/k}\right]$$

3.Find the change in entropy when 0.1 kg of ice at -10^0C is completely converted into steam at 100^0c.Given specific heat of ice =2100 J/kg-k; Latent heat of fusion of ice=80 cal/gm.

$$L_{steam}=540cal/gm$$

$$\left[\begin{array}{l} dS=dS_1+dS_2+dS_3+dS_4=2.303\times mc\log_{10}\left(\frac{T_2}{T_1}\right)+\frac{mL_{ice}}{T_2}+2.303mc\log_{10}\left(\frac{T_3}{T_2}\right)+\frac{mL_{steam}}{T_3} \\ \text{Ans}=867.9J/k \end{array}\right.$$

3.17 Reference books:-

Unified physics	Dr.S.L.Gupta R Sanjeev Gupta	Jai prakashnath Co.	Meerut2005	
Heat and Thermodynamics	Brijilal and Subramanyam	S. Chand & co		
B.Sc Heat Thermodynamics	Telugu Academy D.C Tayal	Himalaya publications		
D.S.MATHUR'S Heat and Thermodynamics	Revised by Dr.M.N.BAPAT	SULTAN CHAND &SONS	NEW DELHI	1997
B.Sc.Physics Thermodynamics &Heat	Dr.J.P.Agarwal &Amit Agarwal	Pragati Prakashan	Meerut	1992

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UNIT - I

LESSON-4

MAXWELL'S EQUATIONS AND THERMODYNAMIC POTENTIALS

Objectives:

We will be able to learn in this following lesson:

1. The energy functions of thermodynamics which are called as thermodynamic potentials.
2. Using thermodynamic potentials, Maxwell equations are derived.
3. Alternative derivation of maxwell equations is discussed.
4. Difference of specific heats and ratio of specific heats are obtained.
5. Joule-Kelvin effect and its expression using Maxwell's equations.

Structure of the lesson :

4.1 Introduction

4.2 Thermodynamic potentials

- a) Internal energy (U)
- b) Helmholtz function (F)
- c) Enthalpy or total heat energy function (H)
- d) Gibb's function (G)

4.3 Maxwell's equations (alternative treatment)

4.4 Ratio of specific heats

4.5 Difference of specific heats

4.6 Joule Kelvin effect and Joule Kelvin coefficient

4.7 First and second T-dS equations

4.8 Solved problems

4.9 Summary

4.10 keywords

4.11Self assessment questions

4.12Reference books

4.1 Introduction:

Thermodynamical state of a system may be represented by thermodynamic variables like pressure P, volume V, temperature T and entropy S.

We have first and second law of thermodynamics as

$dQ=dU+PdV$ and $dQ=TdS$

Using four thermodynamic variables and these equations we can formulate number of relations. In these only four relations are important and they are called as Maxwell's equations. At each time two variables are independent and other two are dependent.

4.2 Thermodynamic Potentials:

Thermodynamic functions or potentials are energy functions which are formed by combining the basic thermodynamic variables. They are internal energy U, Helmholtz function F, enthalpy or total heat energy functions H and Gibb's function G.

a) Internal Energy (U):-

The internal energy (U) of the system is the energy which it possess due to its molecular constitution and motion. This is the sum of kinetic energy and potential energy of the molecule.

From first and second law of thermodynamics we have,

$$dQ = dU + PdV........(4.1)$$
$$dQ = TdS.......(4.2)$$

From these two equations we can write,

$$dU + PdV = TdS \Rightarrow dU = TdS - PdV...(4.3)$$

Taking the partial derivative of U with respect to S at constant volume V we get,

$$\left[\frac{\partial U}{\partial S}\right]_V = T\times 1 - P\times 0$$
$$\left[\frac{\partial U}{\partial S}\right]_V = T........(4.4)$$

Taking the partial derivative of U with respect to V at constant entropy S we get,

$$\left[\frac{\partial U}{\partial V}\right]_S = T\times 0 - P\times 1$$

$$\left[\frac{\partial U}{\partial V}\right]_S = -P \text{......}(4.5)$$

Since U is perfect differentiable we can write,

$$\left[\frac{\partial}{\partial S}\left(\frac{\partial U}{\partial V}\right)_S\right]_V = \left[\frac{\partial}{\partial V}\left(\frac{\partial U}{\partial S}\right)_V\right]_S$$

$$\left[\frac{\partial}{\partial S}(-P)\right]_V = \left[\frac{\partial}{\partial V}(T)\right]_S$$

$$\left(\frac{\partial P}{\partial S}\right)_V = -\left(\frac{\partial T}{\partial V}\right)_S \text{..........} I$$

It is called first Maxwell's thermodynamic relation.

b) Helmholtz function (F) :-

From equation (4.3) we have,

$$dU = TdS - PdV$$

$$dU - TdS = -PdV \text{........}(4.6)$$

If the process is isothermal the temperature T is constant.

Now we can write TdS=d(TS)

Writing this in equation (4.6) we get,

$$dU - d(TS) = -PdV$$

$$d[U - TS] = -PdV$$

Let U-TS=F, where F is called as Helmholtz free energy function or work function.

$$\therefore F = U - TS \text{...........}(4.7)$$

Taking small change in F we get,

$$dF = dU - TdS - SdT$$

$$dF = TdS - PdV - TdS - SdT$$

$$dF = -PdV - SdT$$

Taking the partial derivative of F with respect to V at constant temperature we get,

$$\left(\frac{\partial F}{\partial V}\right)_T = -P \times 1 - 0$$

$$\left(\frac{\partial F}{\partial V}\right)_T = -P \text{..........}(4.8)$$

Taking the partial derivative of F with respect to T at constant volume V we get

$$\left(\frac{\partial F}{\partial T}\right)_V = 0 - S \times 1$$

$$\left(\frac{\partial F}{\partial T}\right)_V = -S \text{.........}(4.9)$$

Since F is perfect differentiable we can write,

$$\left[\frac{\partial}{\partial V}\left(\frac{\partial F}{\partial T}\right)_V\right]_T = \left[\frac{\partial}{\partial T}\left(\frac{\partial F}{\partial V}\right)_T\right]_V$$

$$\left[\frac{\partial}{\partial V}(-S)\right]_T = \left[\frac{\partial}{\partial T}(-P)\right]_V$$

$$-\left(\frac{\partial S}{\partial V}\right)_T = -\left(\frac{\partial P}{\partial T}\right)_V$$

$$\because \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V \text{............}(II)$$

It is called the second Maxwell's thermodynamic relation.

c) Enthalpy of Total energy function(H) :-

The Enthalpy or Total energy function is defined as

$$H = U + PV \text{...........}(4.10)$$

Considering a small change in H we get,

$$dH = dU + PdV + VdP$$

$$= TdS - PdV + PdV + VdP$$

$$dH = TdS + VdP \text{.........}(4.11)$$

Taking the partial derivative of H with respect to S at constant pressure P we get,

$$\left(\frac{\partial H}{\partial S}\right)_P = T \times 1 + 0$$

$$\left(\frac{\partial H}{\partial S}\right)_P = T$$

Taking the partial derivative of H with respect to P at constant S we get,

$$\left(\frac{\partial H}{\partial P}\right)_S = 0 + V \times 1$$

$$\left(\frac{\partial H}{\partial P}\right)_S = V$$

Since H is perfect differentiable we can write,

$$\left[\frac{\partial}{\partial P}\left(\frac{\partial H}{\partial S}\right)_P\right]_S = \left[\frac{\partial}{\partial S}\left(\frac{\partial H}{\partial P}\right)_S\right]_P$$

$$\left[\frac{\partial}{\partial P}(T)\right]_S = \left[\frac{\partial}{\partial S}(V)\right]_P$$

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P \text{...........}(III)$$

This equation is called third Maxwell's thermodynamic relation.

d) Gibb's function (G) :-

Enthalpy is an extensive thermal property and it is mathematically defined as $H = U + PV$..........(4.10)

Considering a small change in H we get,

$$dH = dU + PdV + VdP$$

$$dH = TdS - PdV + PdV + VdP \text{ (using equation 4.3)}$$

$$dH = TdS + VdP \text{..........(4.11)}$$

If the process is isothermal ; the temperature T is constant

$$\therefore TdS = d(TS)$$

If the process is isobaric, the pressure P is constant

$$\therefore dP = 0$$

If the process is isothermal as well as isobaric equation (4.11) becomes

$$dH = d(TS) + 0$$
$$dH - d(TS) = 0$$
$$d(H - TS) = 0$$
$$dG = 0$$
$$Where G = H - TS ....(4.12)$$

and it is called as Gibb's function.

The Gibb's function G is constant and minimum, if the process is isothermal and isobaric.

Considering a small change in G we get,

$$dG = dH - TdS - SdT$$

Using equation (4.11) in this equation we get,

$$dG = TdS + VdP - TdS - SdT$$
$$dG = VdP - SdT ....(4.13)$$

Taking the partial derivative of G with respect to P at constant T we get,

$$\left(\frac{\partial G}{\partial P}\right)_T = V \times 1 - 0$$
$$\left(\frac{\partial G}{\partial P}\right)_T = V$$

Taking the partial derivative of G with respect to T at constant pressure P we get,

$$\left(\frac{\partial G}{\partial T}\right)_P = 0 - S \times 1$$
$$\left(\frac{\partial G}{\partial T}\right)_P = -S$$

Since G is perfect differentiable we can write,

$$\left[\frac{\partial}{\partial P}\left(\frac{\partial G}{\partial T}\right)_P\right]_T = \left[\frac{\partial}{\partial T}\left(\frac{\partial G}{\partial P}\right)_T\right]_P$$

$$\left[\frac{\partial}{\partial P}(-S)\right]_T = \left[\frac{\partial}{\partial T}(V)\right]_P$$

$$-\left(\frac{\partial S}{\partial P}\right)_T = \left(\frac{\partial V}{\partial T}\right)_P$$

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P \text{..........}(IV)$$

It is called the fourth Maxwell's thermodynamic relation.

Importance :-

1. The change in internal energy when a system changes from one state to another is independent of the path followed between the two states.

2. In an isothermal process, the decrease in Helmholtz free energy (F) is a measure of the maximum work obtainable from change in state(-dF=dW)

3. Enthalpy (H) is an extensive thermal property of the system. The change in enthalpy is equal to the quantity of heat given to the system for the change of state at constant pressure.

4. Gibb's free energy function (G) is a characteristic property of a system. In an isothermal and isobaric process dG=0 . It means Gibb's function G is constant and minimum.

4.3 Maxwell's thermodynamics relations(Alternative derivation) :

The internal Energy of a system is completely described by volumeV and temperature T. Due to this we can write the independent variables as pressure P, volumeV, temperature T, entropy S and internal energy U.

From 1st and 2nd law of thermodynamics we have,

$$dQ = dU + PdV \text{..........}(4.1)$$

$$dQ = TdS \text{.........}(4.2)$$

Taking two independent variables at each time and using first and second law of thermodynamics we can obtain number of relations .In these relations only four relations are important. They are called Maxwell's thermodynamic relations.

From equations (4.1) and (4.2) we can write,

$$dU + PdV = TdS \Rightarrow dU = TdS - PdV \text{ .... } (4.3)$$

We have to take two independent variables at each time .Let us assume the independent variables as x and y. Now we can write dU, dS and dV in terms of partial derivations as below.

$$dU = \left(\frac{\partial U}{\partial x}\right)_y dx + \left(\frac{\partial U}{\partial y}\right)_x dy$$

$$dS = \left(\frac{\partial S}{\partial x}\right)_y dx + \left(\frac{\partial S}{\partial y}\right)_x dy \quad \text{----------------------------}(4.14)$$

$$dV = \left(\frac{\partial V}{\partial x}\right)_y dx + \left(\frac{\partial V}{\partial y}\right)_x dy$$

Writing equation (4.14) in equation(4.3) we get,

$$\left(\frac{\partial U}{\partial x}\right)_y dx + \left(\frac{\partial U}{\partial y}\right)_x dy = T\left[\left(\frac{\partial S}{\partial x}\right)_y dx + \left(\frac{\partial S}{\partial y}\right)_x dy\right] - P\left[\left(\frac{\partial V}{\partial x}\right)_y dx + \left(\frac{\partial V}{\partial y}\right)_x dy\right]$$

Comparing the coefficients of dx and dy we get ,

$$\left(\frac{\partial U}{\partial x}\right)_y = T\left(\frac{\partial S}{\partial x}\right)_y - P\left(\frac{\partial V}{\partial x}\right)_y \quad \text{- - - - - - - - - - - - - - -}(4.15)$$

$$\left(\frac{\partial U}{\partial y}\right)_x = T\left(\frac{\partial S}{\partial y}\right)_x - P\left(\frac{\partial V}{\partial y}\right)_x \quad \text{- - - - - - - - - - - - - - -}(4.16)$$

Differentiating equation (4.15) with respect to y we get

$$\frac{\partial^2 U}{\partial x \partial y} = \left(\frac{\partial T}{\partial y}\right)_x \left(\frac{\partial S}{\partial x}\right)_y + T\frac{\partial^2 S}{\partial x \partial y} - \left(\frac{\partial P}{\partial y}\right)_x \left(\frac{\partial V}{\partial x}\right)_y - P\frac{\partial^2 V}{\partial x \partial y} \text{- - - - - - - - - - - - - - -}(4.17)$$

Differentiating equation (4.16) with respect to y we get

$$\frac{\partial^2 U}{\partial x \partial y} = \left(\frac{\partial T}{\partial y}\right)_x \left(\frac{\partial S}{\partial x}\right)_y + T\frac{\partial^2 S}{\partial x \partial y} - \left(\frac{\partial P}{\partial y}\right)_x \left(\frac{\partial V}{\partial x}\right)_y - P\frac{\partial^2 V}{\partial x \partial y} \text{- - - - - - - - - - - - - - -}(4.18)$$

Since u, s and v are perfect differentiable we can write,

$$\frac{\partial^2 U}{\partial y \partial x} = \frac{\partial^2 U}{\partial x \partial y}, \frac{\partial^2 S}{\partial y \partial x} = \frac{\partial^2 S}{\partial x \partial y}, \frac{\partial^2 V}{\partial y \partial x} = \frac{\partial^2 V}{\partial x \partial y} \text{-------------------}(4.19)$$

Writing equations(4.19) in equations (4.17) and (4.18) we get,

$$\left(\frac{\partial T}{\partial y}\right)_x\left(\frac{\partial S}{\partial x}\right)_y-\left(\frac{\partial P}{\partial y}\right)_x\left(\frac{\partial V}{\partial x}\right)_y=\left(\frac{\partial T}{\partial x}\right)_y\left(\frac{\partial S}{\partial y}\right)_x-\left(\frac{\partial P}{\partial x}\right)_y\left(\frac{\partial V}{\partial y}\right)_x \text{ ---------------(4.20)}$$

It is the general expression using which we can obtain the required thermodynamic relations.

(i) Let the independent variables be S and V

$$x = S, y = V$$

$$\frac{\partial S}{\partial x}=1, \frac{\partial V}{\partial y}=1, \frac{\partial S}{\partial y}=0, \frac{\partial V}{\partial x}=0$$

Writing these values in equation (4.20) we get,

$$\left(\frac{\partial T}{\partial V}\right)_S \times 1-0=0-\left(\frac{\partial P}{\partial S}\right)_V \times 1$$

$$\Rightarrow\left(\frac{\partial T}{\partial V}\right)_S=-\left(\frac{\partial P}{\partial S}\right)_V \text{ -------------(I)}$$

It is called first Maxwell's thermodynamic relation.

(ii) Now let us choose the independent variables as temperature 'T' and volume 'V'

$$x = T, y = V$$

$$\frac{\partial T}{\partial x}=1, \frac{\partial V}{\partial y}=1, \frac{\partial T}{\partial y}=0, \frac{\partial V}{\partial x}=0$$

Writing these values in equation(4.20) we get,

$$0-0=1\times\left(\frac{\partial S}{\partial V}\right)_T-\left(\frac{\partial P}{\partial T}\right)_V \times 1$$

$$\Rightarrow\left(\frac{\partial S}{\partial V}\right)_T=\left(\frac{\partial P}{\partial T}\right)_V \text{ -------------(II)}$$

It is called second Maxwell's thermodynamic relation.

(iii) Let us choose the independent variables as entropy S and pressure P.

i.e., $x = S, y = P$

$$\frac{\partial S}{\partial x}=1, \frac{\partial P}{\partial y}=1, \frac{\partial S}{\partial y}=0, \frac{\partial P}{\partial x}=0$$

Writing these values in equation (4.20) we get,

$$\left(\frac{\partial T}{\partial P}\right)_S \times 1 - 1 \times \left(\frac{\partial V}{\partial S}\right)_P = 0$$

$$\Rightarrow \left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P \text{ --------------(III)}$$

It is called third Maxwell's thermodynamic relation.

(iv)Let us choose the independent variables as temperature T and pressure P.

$$\text{i.e., } x = T, y = P$$

$$\frac{\partial T}{\partial x} = 1, \frac{\partial P}{\partial y} = 1, \frac{\partial T}{\partial y} = 0, \frac{\partial P}{\partial x} = 0$$

Writing these values in equation (4.20) we get,

$$0 - 1 \times \left(\frac{\partial V}{\partial T}\right)_P = 1 \times \left(\frac{\partial S}{\partial P}\right)_T \times -0$$

$$\Rightarrow \left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T \text{ --------------(IV)}$$

It is called fourth Maxwell's thermodynamic relation

4.4 Ratio of specific heats:

In the case of solids and liquids ,the change in volume is very small with the increase in temperature .But In case of gases , volume and pressure will changes with change in temperature to the greater extent by the absorbing the heat energy.

Specific heat at constant pressure(C_P):

Specific heat at constant pressure is defined as the amount of heat energy required to raise the temperature of unit mass of the substance by 1 ^{o}C or 1K at constant pressure.

$$C_P = \left(\frac{\partial Q}{\partial T}\right)_P = T\left(\frac{\partial S}{\partial T}\right)_P \text{ ....(4.21)}$$

Specific heat at constant volume(C_V):

Specific heat at constant volume is defined as the amount of heat energy required to raise the temperature of unit mass of the substance by 1C or 1K at constant volume.

$$\therefore C_V = \left(\frac{\partial Q}{\partial T}\right)_V = T\left(\frac{\partial S}{\partial T}\right)_V \text{ ------------(4.22)}$$

The Bulk modulus of elasticity can be written as,

$$\text{Elasticity} = \frac{\text{Stress}}{\text{Strain}} = \frac{\partial P}{-\left(\frac{\partial V}{V}\right)} = -V\left(\frac{\partial P}{\partial V}\right).$$

In an adiabatic process entropy S is constant .So we can write,

$$\text{Adiabatic elasticity} = E_S = -V\left(\frac{\partial P}{\partial V}\right)_S \text{ ....(4.23)}$$

In an isothermal process temperature T is constant .so we can write

$$\text{Isothermal elasticity} = E_T = -V\left(\frac{\partial P}{\partial V}\right)_T \text{ ....(4.24)}$$

The ratio of adiabatic and isothermal elasticities can be written as

$$\frac{E_S}{E_T} = \frac{-V\left(\frac{\partial P}{\partial V}\right)_S}{-V\left(\frac{\partial P}{\partial V}\right)_T} = \frac{\left(\frac{\partial P}{\partial V}\right)_S}{\left(\frac{\partial P}{\partial V}\right)_T}$$

$$\frac{E_S}{E_T} = \frac{\left(\frac{\partial P}{\partial T}\cdot\frac{\partial T}{\partial V}\right)_S}{\left(\frac{\partial P}{\partial S}\cdot\frac{\partial S}{\partial V}\right)_T} = \frac{\left(\frac{\partial P}{\partial T}\right)_S\left(\frac{\partial T}{\partial V}\right)_S}{\left(\frac{\partial P}{\partial S}\right)_T\left(\frac{\partial S}{\partial V}\right)_T}$$

Using four Maxwell equations, in this equation we get,

$$\frac{E_S}{E_T}=\frac{\left(\frac{\partial S}{\partial V}\right)_P}{\left(\frac{\partial T}{\partial V}\right)_P}\cdot\frac{\left(\frac{\partial P}{\partial S}\right)_V}{\left(\frac{\partial P}{\partial T}\right)_V}=\left(\frac{\frac{\partial S}{\partial V}}{\frac{\partial T}{\partial V}}\right)_P\cdot\left(\frac{\frac{\partial P}{\partial S}}{\frac{\partial P}{\partial T}}\right)_V=\left(\frac{\partial S}{\partial T}\right)_P\left(\frac{\partial T}{\partial S}\right)_V$$

$$\therefore\frac{E_S}{E_T}=\left(\frac{\partial S}{\partial T}\right)_P\left(\frac{\partial T}{\partial S}\right)_V$$

This equation can be written as

$$\frac{E_S}{E_T}=\frac{\left(\frac{\partial S}{\partial T}\right)_P}{\left(\frac{\partial S}{\partial T}\right)_V}=\frac{T\left(\frac{\partial S}{\partial T}\right)_P}{T\left(\frac{\partial S}{\partial T}\right)_V}=\frac{C_P}{C_V}$$

$$\therefore\frac{E_S}{E_T}=\frac{C_P}{C_V}=\gamma$$

It means the ratio of adiabatic and isothermal elasticity is equal to the ratio of the two specific heats.

4.5 Difference of two specific heats:

In the case of solids and liquids, the change in volume is very small with the increase in temperature. But in case of gases, the volume and pressure changes with temperature to the greater extent by the absorbing the heat energy. Due to this reason, two specific heats are considered for gases.

Specific heat at constant pressure(C_P):

Specific heat at constant pressure is defined as the amount of heat energy required to raise the temperature of unit mass of the substance by $1^\circ C$ or 1K at constant pressure.

$$C_P=\left(\frac{\partial Q}{\partial T}\right)_P=T\left(\frac{\partial S}{\partial T}\right)_P \text{ --------(4.2 1)}$$

Specific heat at constant volume(C_V):

Specific heat at constant volume is defined as the amount of heat energy required to raise the temperature of unit mass of the substance by $1^\circ C$ or 1K at constant volume.

$$C_V = \left(\frac{\partial Q}{\partial T}\right)_V = T\left(\frac{\partial S}{\partial T}\right)_V \text{ ---------(4.22)}$$

C_P is always greater than C_V ($C_P > C_V$).

The reason is, in the case of C_P, the heat energy which is supplied to the gas is utilised to increase the temperature of the gas and to do the external work. In the case of C_V, the entire heat energy is utilized to increase the temperature of the gas only. Hence C_P is always greater than C_V

$$C_P - C_V = \left(\frac{\partial Q}{\partial T}\right)_P - \left(\frac{\partial Q}{\partial T}\right)_V$$

$$= T\left(\frac{\partial S}{\partial T}\right)_P - T\left(\frac{\partial S}{\partial T}\right)_V$$

$$\therefore C_P - C_V = T\left[\left(\frac{\partial S}{\partial T}\right)_P - \left(\frac{\partial S}{\partial T}\right)_V\right] \text{ ------(4.25)}$$

Let us assume the entropy 'S' as a function of volume V and temperature T.

$$\therefore S = f(V,T)$$

Writing S in terms of partial derivatives we can write,

$$dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV$$

Dividing with dT and taking pressure P as constant we get,

$$\left(\frac{\partial S}{\partial T}\right)_P = \left(\frac{\partial S}{\partial T}\right)_V \times 1 + \left(\frac{\partial S}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P$$

$$\left(\frac{\partial S}{\partial T}\right)_P - \left(\frac{\partial S}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P \text{ ------(4.26)}$$

Writing equation (4.26) in equation(4.25) we get,

$$C_P - C_V = T\left(\frac{\partial S}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P \text{ ----------(4.27)}$$

From second Maxwell's thermodynamic relation we have,

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

Substituting this equation in (4.27) we get

$$C_P - C_V = T\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_P \text{------}(4.28)$$

It is the expression for the difference of specific heats.

Case (1) perfect gas:-

For 1mole of a perfect gas we have, PV=RT-----------------(4.29)

Differentiating equation (4.29) with respect to temperature T at constant volume V we get,

$$\left(\frac{\partial P}{\partial T}\right)_V \times V = R \times 1 \Rightarrow \left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{V} \text{------}(4.30)$$

Differentiating equation (4.29) with respect to temperature T at constant pressure P

We get $$P \times \left(\frac{\partial V}{\partial T}\right)_T = R \times 1 \Rightarrow \left(\frac{\partial V}{\partial T}\right)_P = \frac{R}{P} \text{------}(4.31)$$

Writing equation(4.30) and (4.31) in equation (4.28) we get,

$$C_P - C_V = T \cdot \frac{R}{V} \cdot \frac{R}{P} = \frac{T \cdot R \cdot R}{R \cdot T} = R$$

$$\therefore C_P - C_V = R \text{---------------}(4.32)$$

(ii) Vanderwaal's equation:

For ordinary gases we have the Vanderwaal's equation as

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

$$\left(P + \frac{a}{V^2}\right) = \frac{RT}{(V - b)}$$

$$P + aV^{-2} = RT(V - b)^{-1} \text{------}(4.33)$$

Differentiating equation (4.33) with respect to 'T 'at constant volume 'V' we get,

$$\left(\frac{\partial P}{\partial T}\right)_V + 0 = R \times 1 \times (V - b)^{-1}$$

$$\Rightarrow \left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{(V - b)} \text{---------}(4.34)$$

Differentiating equation (4.33) with respect to temperature T at constant pressure P we get,

$$0+a(-2)V^{-3}\left(\frac{\partial V}{\partial T}\right)_P = RT(-1)(V-b)^{-2}\left(\frac{\partial V}{\partial T}\right)_P + R\times 1\times (V-b)^{-1}$$

$$\Rightarrow -\frac{2a}{V^3}\left(\frac{\partial V}{\partial T}\right)_P = -\frac{RT}{(V-b)^{-2}}\left(\frac{\partial V}{\partial T}\right)_P + \frac{R}{(V-b)}$$

$$\Rightarrow \frac{RT}{(V-b)^2}\left(\frac{\partial V}{\partial T}\right)_P - \frac{2a}{V^3}\left(\frac{\partial V}{\partial T}\right)_P = \frac{R}{(V-b)}$$

$$\Rightarrow \left(\frac{\partial V}{\partial T}\right)_P\left[\frac{RT}{(V-b)^2}-\frac{2a}{V^3}\right] = \frac{R}{(V-b)}$$

$$\Rightarrow \left(\frac{\partial V}{\partial T}\right)_P = \frac{R/V-b}{\left[\frac{RT}{(V-b)^2}-\frac{2a}{V^3}\right]} ------(4.35)$$

Writing equations (4.35) and (4.34) in equation(4.28) we get,

$$C_P - C_V = T\times\frac{R}{(V-b)}\times\frac{R/V-b}{\left[\frac{RT}{(V-b)^2}-\frac{2a}{V^3}\right]}$$

$$C_P - C_V = \frac{TR^2}{(V-b)^2\left[\frac{RT}{(V-b)^2}-\frac{2a}{V^3}\right]}$$

$$C_P - C_V = \frac{TR^2}{RT-\frac{2a(V-b)^2}{V^3}}$$

$$\therefore V >> b, (V-b)^2 \approx V^2$$

$$C_p - C_v = \frac{TR^2}{RT-\frac{2aV^2}{V^3}} = \frac{TR^2}{RT\left[1-\frac{2a}{VRT}\right]}$$

$$= R\left[1-\frac{2a}{VRT}\right]^{-1}$$

$$\therefore C_p - C_v = R\left[1+\frac{2a}{VRT}\right] ------(4.36)$$

It is the expression for difference of specific heats for ordinary gases.

Another form for difference of specific heats ($C_P - C_V$):-

The difference of specific heats in terms of bulk modulus (E) and coefficient of volume expansion (α) can be obtained as below.

We have the expression for the difference of specific heats as,

$$C_p - C_v = T\left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_P \text{ ------(4.37)}$$

Let the pressure P be the function of temperature T and volume V.

$$P = f(T, V)$$

In terms of partial derivatives we can write,

$$dP = \left(\frac{\partial p}{\partial T}\right)_V dT + \left(\frac{\partial p}{\partial V}\right)_T dV$$

If the resultant pressure P is constant then $dP = 0$

$$\therefore 0 = \left(\frac{\partial p}{\partial T}\right)_V dT + \left(\frac{\partial p}{\partial V}\right)_T dV$$

Dividing with dT we get,

$$0 = \left(\frac{\partial p}{\partial T}\right)_V + \left(\frac{\partial p}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)$$

$$0 = \left(\frac{\partial p}{\partial T}\right)_V + \left(\frac{\partial p}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p$$

$$\left(\frac{\partial p}{\partial T}\right)_V = -\left(\frac{\partial p}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p \text{ --------(4.38)}$$

Writing equation (4.38) in equation (4.37) we get,

$$C_p - C_v = T(-)\left(\frac{\partial p}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial V}{\partial T}\right)_P$$

$$= -T\left(\frac{\partial p}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P^2$$

$$= \frac{T}{V}\left[\frac{\partial P}{-\left(\frac{\partial V}{V}\right)}\right]_T \times V^2 \left[\frac{\left(\frac{\partial V}{V}\right)}{\partial T}\right]_P^2$$

$$C_p - C_v = \frac{T}{V} \times E \times V^2 \times \alpha^2$$

$$Since E = \frac{stress}{stain} = \frac{\partial P}{-\left(\frac{\partial V}{V}\right)}$$

$$\alpha = \frac{\text{change in volume per unit volume}}{\text{Temperature diffrence}} = \frac{\left(\frac{\partial V}{V}\right)}{\partial T}$$

$$\therefore C_P - C_V = TEV\alpha^2$$

It is another form of difference of specific heats.

From this equation, we draw the following conclusions.

(1) If T=0, then $C_P - C_V = 0 \Rightarrow C_P = C_V$.

It means at absolute zero, the two specific heats are equal.

(2) As T, E, V and α are always positive, $C_P - C_V$ can never be negative. It means Cp is always greater than C_V

4.6. Joule-Kelvin effect or Joule-Kelvin coefficient :-

In this lesson we derive the expression for Joule-Kelvin effect using Maxwell's equations.

Joule-Kelvin effect :-

When a gas is passed from high constant pressure place to low constant pressure place through porous plug (like cotton), there is a change in the temperature of the gas. This is called Joule-Kelvin effect or Joule-Thomson effect . The experimental results of Joule-Kelvin effect are as below.

1. All the gases (expect H_2 and He) suffer cooling effect when they are passed from high constant pressure place to low constant pressure place.
2. The change in temperature is directly proportional to the change in pressure.
3. If the initial temperature of the gas is increased, the cooling effect decreases
4. At one particular temperature called inversion temperature(T_i)the gas is suffering neither heating effect nor cooling effect.
5. If the temperature of the gas is more than the inversion temperature ($T>T_i$), the gas suffers

heating effect. For H_2 and He, the room temperature is more than their inversion Temperature. Due to this reason only H_2 and He are exhibiting heating effect at room temperature.

6.If temperature of the gas is less than inversion temperature (T>T$_i$), the gas suffers cooling effect.

Expression :-

When a gas suffers Joule-Kelvin effect the enthalpy of a gas remains constant

$$\therefore H = U + PV = \text{Constant}$$

Considering a small change in H we get,

$$dH = dU + PdV + VdP = 0$$

From first and second law of thermodynamics we can write

$$dU + PdV = TdS$$

Writing this in the above equation we get,

$$dH = TdS + VdP = 0 ------(4.39)$$

Let us assume the entropy S as a function of pressure P and temperature T,

$$S = f(P,T)$$

$$dS = \left(\frac{\partial S}{\partial P}\right)_T dP + \left(\frac{\partial S}{\partial T}\right)_P dT ---(4.40)$$

Writing equation (4.40) in equation (4.39) we get,

$$T\left[\left(\frac{\partial S}{\partial P}\right)_T dP + \left(\frac{\partial S}{\partial T}\right)_P dT\right] + VdP = 0$$

$$T\left(\frac{\partial S}{\partial P}\right)_T dP + T\left(\frac{\partial S}{\partial T}\right)_P dT + VdP = 0 -----(4.41)$$

From fourth Maxwell's equation we have,

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P --------(4.42)$$

we can write,

$$T\left(\frac{\partial S}{\partial T}\right)_P = \left(\frac{T\partial S}{\partial T}\right)_P = \left(\frac{\partial Q}{\partial T}\right)_P = C_P ....(4.43)$$

writing equations (4.42) and (4.43) in equation (4.41) we get,

$$-T\left(\frac{\partial V}{\partial T}\right)_P dP + C_P dT + VdP = 0$$

$$C_P dT = T\left(\frac{\partial V}{\partial T}\right)_P dP - VdP$$

Dividing with dP we get,

$$C_P \frac{\partial T}{\partial P} = T\left(\frac{\partial V}{\partial T}\right)_P - V$$

$$\left(\frac{\partial T}{\partial P}\right)_H = \frac{1}{C_P}\left[T\left(\frac{\partial V}{\partial T}\right)_P - V\right] \text{---------} (4.44)$$

It is the expression for Joule-Kelvin effect. or Joule-Thomson effect.

(i) If $\left(\frac{\partial T}{\partial P}\right)_H$ is positive, the gas suffers cooling effect.

(ii) If $\left(\frac{\partial T}{\partial P}\right)_H$ is negative, the gas suffers heating effect.

(iii) If $\left(\frac{\partial T}{\partial P}\right)_H$ is zero, the gas suffers neither heating effect nor cooling effect.

a) Joule Kelvin effect for perfect gas :-

For one mole of perfect gas we have, PV=RT

Differentiating with respect to T at constant P we get,

$$P\left(\frac{\partial V}{\partial T}\right) = R \times 1 \Rightarrow \left(\frac{\partial V}{\partial T}\right) = \frac{R}{P} \text{............} (4.45)$$

Substituting equation (4.45) in equation (4.44) we get,

$$\left(\frac{\partial T}{\partial P}\right)_H = \frac{1}{C_P}\left[T \times \frac{R}{P} - V\right]$$

$$\left(\frac{\partial T}{\partial P}\right)_H = \frac{1}{C_P}\left[\frac{RT}{P} - V\right] = \frac{1}{C_P}\left[\frac{PV}{P} - V\right]$$

$$\left(\frac{\partial T}{\partial P}\right)_H = \frac{1}{C_P}[V - V] = 0 [\therefore PV = RT]$$

$$\therefore \left(\frac{\partial T}{\partial P}\right)_H = 0$$

It means perfect gas suffers neither heating effect nor cooling effect.

b) Joule-Kelvin effect for vanderwaal's equation :-

For ordinary gases we have the vanderwaal's equation as,

$$\left(P+\frac{a}{V^2}\right)(V-b)=RT------(4.45)$$

Differentiating equation (4.45) with respect to T at constant P we get,

$$\left(P+\frac{a}{V^2}\right)\left(\frac{\partial V}{\partial T}\right)_P+(V-b)\left[0+a(-2)V^{-3}\left(\frac{\partial V}{\partial T}\right)_P\right]=R$$

$$\frac{RT}{(V-b)}\left(\frac{\partial V}{\partial T}\right)_P+\left[\frac{-2a}{V^3}\left(\frac{\partial V}{\partial T}\right)_P\right](V-b)=R$$

Multiplying with (v-b) we get,

$$RT\left(\frac{\partial V}{\partial T}\right)_P-\frac{2a}{V^3}\left(\frac{\partial V}{\partial T}\right)_P(V-b)^2=R(V-b)$$

$$\left(\frac{\partial V}{\partial T}\right)_P\left[RT-\frac{2a}{V^3}(V-b)^2\right]=R(V-b)$$

$$\left(\frac{\partial V}{\partial T}\right)_P=\frac{R(V-b)}{RT-\frac{2a}{V^3}(V-b)^2}$$

$$\because V>>b\Rightarrow(V-b)^2\approx V^2$$

$$\therefore\left(\frac{\partial V}{\partial T}\right)_P=\frac{R(V-b)}{RT-\frac{2a\times V^2}{V^3}}=\frac{R(V-b)}{RT-\frac{2a}{V}}=\frac{R(V-b)}{RT\left[1-\frac{2a}{VRT}\right]}$$

$$\left(\frac{\partial V}{\partial T}\right)_P=\frac{V-b}{T\left[1-\frac{2a}{VRT}\right]}$$

$$T\left(\frac{\partial V}{\partial T}\right)_P=\frac{V-b}{\left[1-\frac{2a}{VRT}\right]}=(V-b)\left[1-\frac{2a}{VRT}\right]^{-1}$$

$$=(V-b)\left[1+\frac{2a}{VRT}\right]$$

$$T\left(\frac{\partial V}{\partial T}\right)_P = V - b + \frac{2aV}{VRT} - \frac{2ab}{VRT}$$

The term $\frac{2ab}{VRT} \approx 0$

$$\therefore T\left(\frac{\partial V}{\partial T}\right)_P - V = \frac{2a}{RT} - b \text{-------}(4.46)$$

Substituting equation (4.46) in equation (4.44) we get,

$$\left(\frac{\partial T}{\partial P}\right)_H = \left[\frac{2a}{RT} - b\right]\frac{1}{C_P} \text{-------}(4.47)$$

It is the expression for Joule-Kelvin effect in the case of vanderwaal's equation.

(i) If $\frac{2a}{RT} = b$, then $\left(\frac{\partial T}{\partial P}\right)_H = 0$

Now the gas suffers neither heating effect nor cooling effect. The corresponding temperature is called Inversion temperature (T_i).

$$\therefore \frac{2a}{RT_i} = b \Rightarrow T_i = \frac{2a}{Rb} \text{---------}(4.48)$$

It is the expression for inversion temperature.

(ii) If $T > T_i$, then $\left(\frac{\partial T}{\partial P}\right)_H$ is negative. Now the gas suffers heating effect.

(iii) If $T < T_i$, then $\left(\frac{\partial T}{\partial P}\right)_H$ is positive. Now the gas suffers cooling effect.

4.7 First and Second T-dS equations :-

First T-dS equation: -

Let us assume the entropy S be the function of temperature T and volume V.

S= f(T,V)

In terms of partial derivatives we have,

$$dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV$$

$$TdS = T\left(\frac{\partial S}{\partial T}\right)_V dT + T\left(\frac{\partial S}{\partial V}\right)_T dV \text{-------}(4.49)$$

we can write, $T\left(\frac{\partial S}{\partial T}\right)_V = \left(\frac{T\partial S}{\partial T}\right)_V = \left(\frac{\partial Q}{\partial T}\right)_V = C_V$ ------(4.50)

From second Maxwell's thermodynamic relation we have,

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V \text{------(4.51)}$$

Substituting equations (4.50) and (4.51) in equation (4.49) we get,

$$TdS = C_V dT + T\left(\frac{\partial P}{\partial T}\right)_V dV \text{----(4.52)}$$

It is called first T-dS equation.

Second T-dS equation:-

Let us assume the entropy S be the function of temperature T and pressure P.

$$\therefore S = f(T,P)$$

In terms of partial derivatives we have,

$$dS = \left(\frac{\partial S}{\partial T}\right)_P dT + \left(\frac{\partial S}{\partial P}\right)_T dP$$

$$TdS = T\left(\frac{\partial S}{\partial T}\right)_P dT + T\left(\frac{\partial S}{\partial P}\right)_T dP \text{-----(4.53)}$$

we can write, $T\left(\frac{\partial S}{\partial T}\right)_P = \left(\frac{T\partial S}{\partial T}\right)_P = \left(\frac{\partial Q}{\partial T}\right)_P = C_P$ ------(4.54)

From fourth Maxwell's thermodynamic relation we have,

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P \text{-----(4.55)}$$

substituting the equations (4.54)and(4.55) in equation (4.53) we get,

$$\mathrm{Tds} = \mathrm{C_p dT} + \mathrm{T}\left(-\frac{\partial \mathrm{v}}{\partial \mathrm{T}}\right)_\mathrm{p} \mathrm{dp}$$

$$\therefore \mathrm{Tds} = \mathrm{C_p dT} - \mathrm{T}\left(\frac{\partial \mathrm{v}}{\partial \mathrm{T}}\right)_\mathrm{p} \mathrm{dp} \text{-----(4.56)}$$

It is called second T-dS equation.

4.8. Solved problems: -

1.Considering mercury at 0°C and 1 atmospheric pressure calculate γ. Given $C_p = 28 J/mole-k$, specific volume $C_v = 1.47\times10^{-5} m^3/mole$, volume expansively $\alpha = 1.81\times10^{-6} k^{-1}$ and compressibility is $3.94\times10^{-6} atm^{-1} or 3.89\times10^{-11} pa^{-1}$.

<u>Solution: -</u>

Given $C_p = 28J/mole-k, V = 1.47\times10^{-5} m^3/mole, \alpha = 1.81\times10^{-6} k^{-1}$

$$E' = 3.94\times10^{-6} atm^{-1} or 3.89\times10^{-11} pa^{-1}, T = 0^\circ c = 273K$$

$P = 1$ atm $= 1.01\times10^5 N/m^2 or pa, \gamma = ?$

$$C_p - C_v = TE\alpha^2V = \frac{T\alpha^2V}{E'} = \frac{273\times(1.81\times10^{-6})^2\times1.47\times10^{-5}}{3.89\times10^{-11}}$$

$$= \frac{273\times3.2761\times1.47\times10^{-12}\times10^{-5}\times10^{11}}{3.89}$$

$$= 337.977\times10^{-6}$$

$$= 0.0003379 J/mole-K$$

$$C_v = C_p - 0.0003379 = 28 - 0.0003379 = 27.9996 J/mole-K$$

$$\gamma = \frac{C_p}{C_v} = \frac{28}{27.9996} \approx 1$$

$$\therefore \gamma = \frac{C_p}{C_v} = 1$$

2. Calculate the molar heat capacity at constant volume of mercury at $0^\circ c$ and 1 atmospheric pressure. From expansion we have, $C_p = 28J/mole-degree$, V=0.0147litre/mole $\alpha = 1.81\times10^{-6} deg^{-1}$ and compressibility is $3.94\times10^{-6}/atm$.

Solution:-

$$C_p = 28J/\text{mole}-\text{degree}, T = 0^\circ c = 0+273 = 273K$$

$$\alpha = 1.81\times10^{-6}\,\deg^{-1}, E' = 3.94\times10^{-6}/\text{atm} = \frac{3.94\times10^{-6}}{1.01\times10^{5}}\,\text{pa}^{-1}$$

$$= 3.89\times10^{-11}\,\text{pa}^{-1}$$

$$V = 0.0147\times10^{-3}\,m^3/\text{mole}, C_v = ?$$

$$C_p - C_v = TEV\alpha^2 = \frac{TV\alpha^2}{E^1}$$

$$= \frac{273\times0.0147\times10^{-3}\times\left(1.81\times10^{-6}\right)^2}{3.89\times10^{-11}}$$

$$= \frac{13.1473}{3.89}\times10^{-4} = 3.379\times10^{-4}$$

$$C_p - C_v = 3.379\times10^{-4} = 28 - 0.0003379$$

$$C_v = C_p - 0.0003379 = 28 - 0.0003379$$

$$\therefore C_v = 27.9996J/\text{mole}-K$$

4.9. Summary :-

The four important energy functions are discussed using thermodynamic variables pressure, volume, temperature and entropy. Using these thermodynamic potentials Maxwell's equations are derived.The ratio of specific heats and difference of specific heats are explained. Joule-Kelvin effect is explained in detail and it is obtained using Maxwell's equations.

4.10. Keywords :-

Thermodynamic potentials, Maxwells equations, specific heats, Joule-Kelvin effect.

4.11. Self assessment questions:-

Long answer questions

1. Define four thermodynamic potentials. Obtain Maxwell's thermodynamic equations using these potentials.
2. What is Joule-Kelvin effect? Derive the Joule-Kelvin effect from Maxwell's thermodynamic relations.

3.Explain specific heats. Derive the expression for difference of specific heats.

Short answer questions:

4. Explain specific heats. Derive the expression for the ratio of specific heats.

5. Derive first and second T-dS equations.

Exercise:

1.For a metallic copper disc at 300K, the following values are known

$C_p = 24.5\mathrm{J/mole-k}, \alpha = 50.4\times10^{-6}\mathrm{k}^{-1}, V = 7.06\mathrm{cm}^3/\mathrm{mole}$

$\text{Isothermalcompressibility} = 7.78\times10^{-12}\mathrm{N/m}^2$

[Hint $C_p - C_v = TEV\alpha^2 = \dfrac{TV\alpha^2}{E^1}$ Ans:23.8085 J/mole-k]

4.12 Reference books:

Unified Physics	Dr.S.L.Gupta & Sanjeev Gupta	Jaiprakash Nath & Co., Meerut -2005	
Heat and Themodynamics	Brijlal and Subrahmanyam	S. Chand & co	
B.Sc heat	Telugu Academy		
Themodynamics	D.C.Tayal	Himalaya Publications	
D.S.Mathure's Heat and Thermodynamics	Revised by Dr.M.N.BAPAT	SULTAN CHAND &SONS	NEW DELHI-1997
B.Sc.Physics Thermodynamics &Heat	Dr.J.P.Agarwal &Amit Agarwal	Pragati Prakasam	Meerut-1992

UNIT II
LESSON 5

LOW TEMPERATURE PHYSICS

Objectives:-

In this lesson the following topics will be discussed

1. Joule Kelvin effect process in detail
2. Difference between Joules expansion, adiabatic expansion and Joule-Kelvin effect
3. How the gases can be liquefied
4. The process of adiabatic demagnetization to obtain very low temperatures

Structures of the lesson:-

5.1 Introduction

5.2 Joule-Kelvin effect or porous plug experiment

5.3 Expression for Joule-Thomson expansion

5.4 Distinction between Joule expansion, adiabatic expansion and Joule-Kelvin effect

5.5 Liquefaction of gases

- a) Principle of regenerating cooling
- b) Liquefaction of air
- c) Liquefaction of hydrogen
- d) Liquefaction of helium

5.6 Adiabatic demagnetization

5.7 Refrigerator

5.8 Solved examples

5.9 Summary

5.10 Keywords

5.11 Self assessment questions

5.12 Reference books

5.1 INTRODUCTION:-

Phase change of first order

The thermodynamics state of the system is explained by the variables pressure P, volume V and temperature T. In thermodynamics we use the word the phase for transition of matter. For examples ice, water and steam are the phases of a single distance. A first order phase change is one which is accompanied by emission or absorption with out any change in temperature.

For each substance, there is a set of temperature and pressure at which any of the two states of the three (solid, liquid, gas) may exist in equilibrium .We can consider the following curves.

1. Liquid vapour curve on curve of vaporisation:-

Consider an enclosure filled with liquid and its saturated vapour. The vapour pressure increases with the increase in temperature. Both liquid and vapour are in equilibrium at some temperature and pressure. It is shown by the curve OA in fig 5.1 and it is called curve of vaporisation. Above the curve, the substance is liquid and below the curve the substance is vapour.

2. Fusion curve:-

The solid may coexist in equilibrium with liquid .The temperature and pressure at which the solid and liquid may exist in equilibrium, is shown in curve OB in fig5.1.The curve is known as fusion curve. On the right side of the curve the phase of the substance is liquid and on the left side of the curve it is solid.

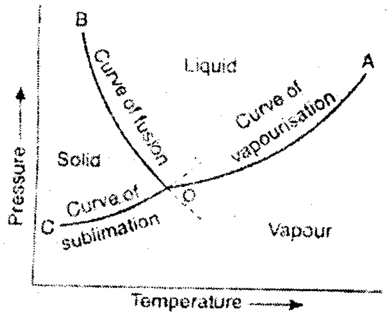


Fig. (5 · 1)

Fig. 5.1

3. **Sublimation curve:-**

The solid may co-exist with vapour in equilibrium .The temperature and pressure at which the solid and vapour may exist in equilibrium is shown by curve OC in fig 5.1. The curve is known as sublimation curve .Above OC the substance is in solid phase, and below the curve the substance is in vapour phase.

When all the three curves are plotted on the same graph with the same scale, they intersect at point O. The point 'O' is called the "triple point". The triple point may be defined as the point at which the temperature and pressure are such that the solid, liquid and vapour phase co-exist in equilibrium .The corresponding temperature is called "triple point temperature (T_t)"and the corresponding pressure is called "triple point pressure (P_t).

5.2 Joule -Kelvin effect (or) porous plug experiment:-

According to Joule -Kelvin effect when a gas is passed from high constant pressure place to low constant pressure place the gas suffers change in temperature.

The results of Joule –Kelvin effect are as shown.

1. Almost all gasses (except hydrogen and helium) suffer cooling effect, when they are passed from high constant pressure place to low constant pressure place at room temperature.
2. The variation in temperature is directly proportional to variation in pressure.
3. If the initial temperature of the gas is increased ,the cooling effect decreases.
4. At one particular temperature called inversion temperature (T_i) the gas suffers neither heating effect nor cooling effect.
5. If the temperature of the gas is less than the "inversion temperature" ($T<T_i$) the gas suffers cooling effect.

6. If the temperature of the gas is more than the "inversion temperature" ($T> T_i$) the gas suffers heating effect.

For hydrogen and helium the inversion temperatures are $-80^0 C$ and $-240^0 C$ respectively due to this reason only hydrogen and helium are exhibiting heating effect at room temperature.

5.3 Expressions for Joule -Thomson cooling:-

Consider a thermally insulated cylinder PQ which is divided into two compartments by a porous plug as shown in fig(5.2). It is fitted with two non-conducting pistons A and B .Suppose the piston A forces one gram mole of a gas through the porous plug at a constant pressure (P_1).The gas coming out from the porous plug pushes the piston B in out word direction to a small constant pressure (P_2). Let V_1, V_2 be the volumes of the gas before and after passing through the porous plug.

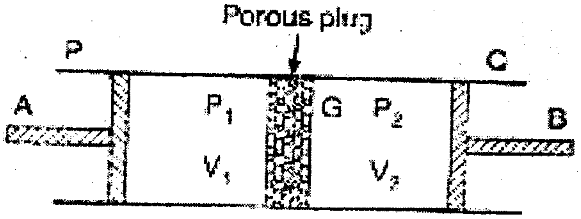


Fig :(5.2)

Work done by the piston A on the gas in the left compartment = P_1V_1

Work done on the piston B by the gas in the right compartment = P_2V_2

Net work done by the gas = $P_2V_2 - P_1V_1$ ------5.1

Let, initial internal energy= U_1

Final internal energy= U_2

Decrease in internal energy= $U_1 - U_2$------5.2

As the system is thermally isolated, this work must have been done at the expense of the internal energy of the gas. Hence we can write,

$$p_2V_2 - p_1V_1 = U_1 - U_2$$
$$p_2V_2 + U_2 = U_1 + p_1V_1$$
$$p_1V_1 + U_1 = p_2V_2 + U_2$$
$$i.e H = U + PV$$

=constant----------------5.3

Hence enthalpy H is constant in Joule-Kelvin effect. From equation (1) we have,

Net work done by the gas $(W_1) = P_2V_2 - P_1V_1$

In addition to this some internal work is done against the inter molecular force of attraction .If we assume the gas to obey the Vanderwaal's equation, the attraction between the molecules is equal to internal pressure $\frac{a}{V^2}$, where a is constant and V is volume.

The work done against the inter molecular force of attraction

$$W_2 = \int_{V_1}^{V_2} \frac{a}{V^2} dv = a \int_{V_1}^{V_2} \frac{1}{V^2} dv = a\left(\frac{-1}{V}\right)_{V_1}^{V_2}$$

$$W_2 = a\left(\frac{1}{V_1} - \frac{1}{V_2}\right) \text{................}(5.4)$$

Total work done by the gas is

$$W = W_1 + W_2$$

$$= p_2V_2 - p_1V_1 + a\left(\frac{1}{V_1} - \frac{1}{V_2}\right) \text{......}(5.5)$$

From Vander waal's equation we have,

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

$$\Rightarrow PV - Pb + \frac{a}{V^2}.V - \frac{ab}{V^2} = RT$$

$$\Rightarrow PV - Pb + \frac{a}{V} = RT \qquad \left[\because \frac{ab}{V^2} \approx 0\right]$$

$$\Rightarrow PV = RT + Pb - \frac{a}{V} \text{--------}(5.6)$$

For the values in the left compartment we can write,

$$P_1V_1 = RT + P_1b - \frac{a}{V_1} \text{--------}(5.7)$$

For the values in the right compartment we can write,

$$P_2V_2 = RT + P_2b - \frac{a}{V_2} \text{--------}(5.8)$$

$$5.8-5.7 \Rightarrow$$

$$P_2V_2 - P_1V_1 = RT + P_2b - a/V_2 - RT - P_1b + a/V_1$$

$$P_2V_2 - P_1V_1 = RT + P_2b - \frac{a}{V_2} - RT - P_1b + \frac{a}{V_1}$$

$$P_2V_2 - P_1V_1 = (P_2 - P_1)b + a\left(\frac{1}{V_1} - \frac{1}{V_2}\right) \text{........(5.9)}$$

Substituting equation (5.9) in equation (5.5)

$$W = (P_2 - P_1)b + \left(\frac{a}{V_1} - \frac{a}{V_2}\right) + a\left(\frac{1}{V_1} - \frac{1}{V_2}\right)$$

$$W = (P_2 - P_1)b + 2a\left(\frac{1}{V_1} - \frac{1}{V_2}\right) \text{..........(5.10)}$$

Since values $\frac{1}{V_1}$ and $\frac{1}{V_2}$ are small we can use the approximate relation

$$PV = RT \Rightarrow \frac{1}{V} = \frac{P}{RT}$$

for the values of left and right compartments we can write,

$$\frac{1}{V_1} = \frac{P_1}{RT} \, and \, \frac{1}{V_2} = \frac{P_2}{RT}$$

Substituting these values in equation (5.10) we get

$$W = (P_2 - P_1)b + \frac{2a}{RT}(P_1 - P_2)$$

$$W = (P_1 - P_2)\left(\frac{2a}{RT} - b\right) \text{..........(5.11}$$

As the system is isothermally insulated this work is drawn from the internal energy of the gas .If C_p is the specific heat at constant pressure and dT is the change in temperature ,the heat energy drawn by the gas is $C_p dT$ calories $= C_p dTJ$ joules ------------(5.12)

From equations (5.11) and (5.12) we can write,

$$C_p dTJ = (P_1 - P_2)\left[\frac{2a}{RT} - b\right]$$

$$dT = \frac{(P_1 - P_2)}{C_p J}\left[\frac{2a}{RT} - b\right] \text{--------(5.13)}$$

It is the required expression for Joule-Kelvin effect

(i)If $\frac{2a}{RT}$ =b ,then dT=0, now the gas suffers neither heating effect nor cooling effect .The corresponding temperature is called inversion temperature .It is denoted by T_i

$$\frac{2a}{RT_i} = b \Rightarrow T_i = \frac{2a}{Rb} ......6.14)$$

(ii)If $T > T_i$, then dT is negative, now the gas suffers heating effect.

(iii)If $T < T_i$, then dT is positive now the gas suffers cooling effect.

5.4 Distinction between joules expansion, adiabatic expansion and Joule-Kelvin expansion (or) Distinction between different types of expansions:-

(i)Joule's expansion:-It is a free expansion in which the gas expands into vacuum and hence does no external work.In this expansion ,some internal work is done by the against the intermolicular force of atraction .Due to large heat capacityof apparatus , the temperature of the gas is maintained constant.

(ii)Adiabatic expansion:- In this expansion ,the compressed gas suddenly released into atmosphere .The gas performs External work .The process is so fast ~~and external~~ work is drawn from the internal energy of the gas .Thus the gas cools.

(iii)Joule-Kelvin expansion:- In this expansion ,the gas at high pressure is forced to pass through porous plug through a region of lower pressure .Here external work is done on the gas as well as by the gas ,in additional to the internal work .Due to this .there may be a heating effect or cooling effect depending upon the initial temperature of the gas .At room temperature, almost all gases (except Hydrogen and Helium)exhibit the cooling effect.

5.5 Liquefaction of gases:-

The principles involved in the liquefaction of gases are Joule-Thomson effect and regenerating cooling.

a) Principle of regenerating cooling :-

According to Joule-Kelvin effect when a compressed gas, whose initial temperature is less than the inversion temperature, is forced through a fine nozzle its temperature falls. The fall in temperature is more, when the initial temperature of the gas is less. The Joule-Kelvin effect is not only sufficient to liquefy the gases .This draw back is removed by the principle of regenerating cooling.

In regenerative cooling, the gas which has been suffered Joule-Thomson expansion is made to flow back over the tube containing the incoming gas .Thus the incoming gas is cooled .Again it suffers Joule-Kelvin effect .Due to this, the gas is still more cooled .In this way, by using principles of regenerative cooling and Joule-Kelvin effect the desired low temperature can be obtained at which the gas is liquefied.

b) Liquefaction of air:-

(i)Hampson air liquefier: - Hampson air liquefier is as shown fig(5.3).The air which is free from dust, carbon dioxide and moisture is compressed to 150 atmospheres by using a compressor P .The compressed air is passed through a tube on which the cold water is circulating .Due to this, the heat produced due to compression is removed .Now the air flows into the spiral tube of heat exchanger, where it suffers Joule Thomson effect at nozzle 'N'. The pressure out side the nozzle is one atmosphere Due to this the air is cooled.

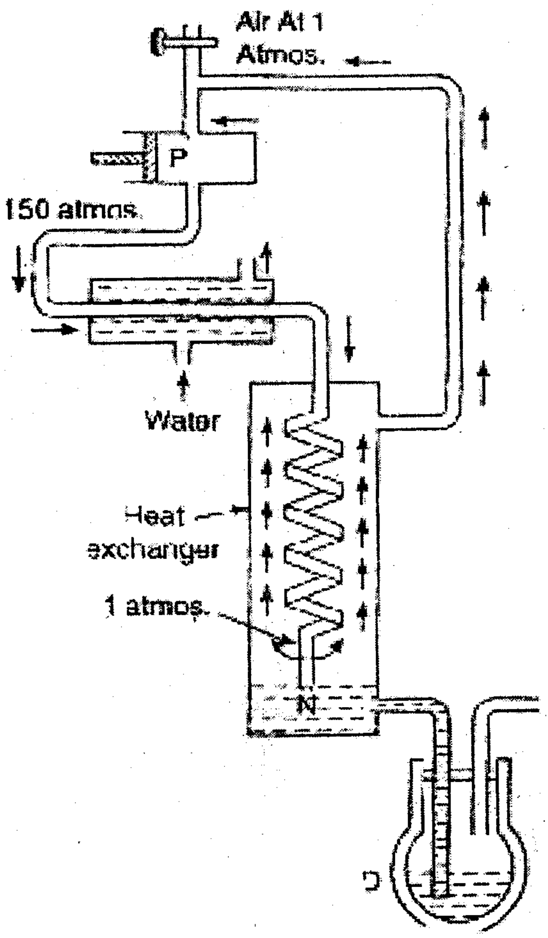


Fig:(5.3)

The cold air flows back over the spiral tube .in this process ,it cools the incoming gas and returns back to the compressor P .By continuing the process ,the temperature of air coming out from the nozzle N, becomes at about –188 °C. At this temperature the air is liquefied .The liquid air is collected into Dewar flask.

<u>(ii) Linde's air liquefier:-</u> The Lindes air liquefier is as shown in Fig. 5.4. It consists of two compressors P_1 and P_2 .In compressor P_1 the air is compressed to 20 atmospheres. This compressed air is passed through a tube on which cold water circulation is taking place .Due to this ,the heat of compression will be removed .Now the air is passed through the solution of caustic potash (KOH), calcium chloride ($CaCl_2$) and Phosphorous pentoxide($P_2 O_5$).Due to this, the carbon dioxide and water present in air are removed .If CO_2 and water vapour are not removed choke will be formed in the entire system.

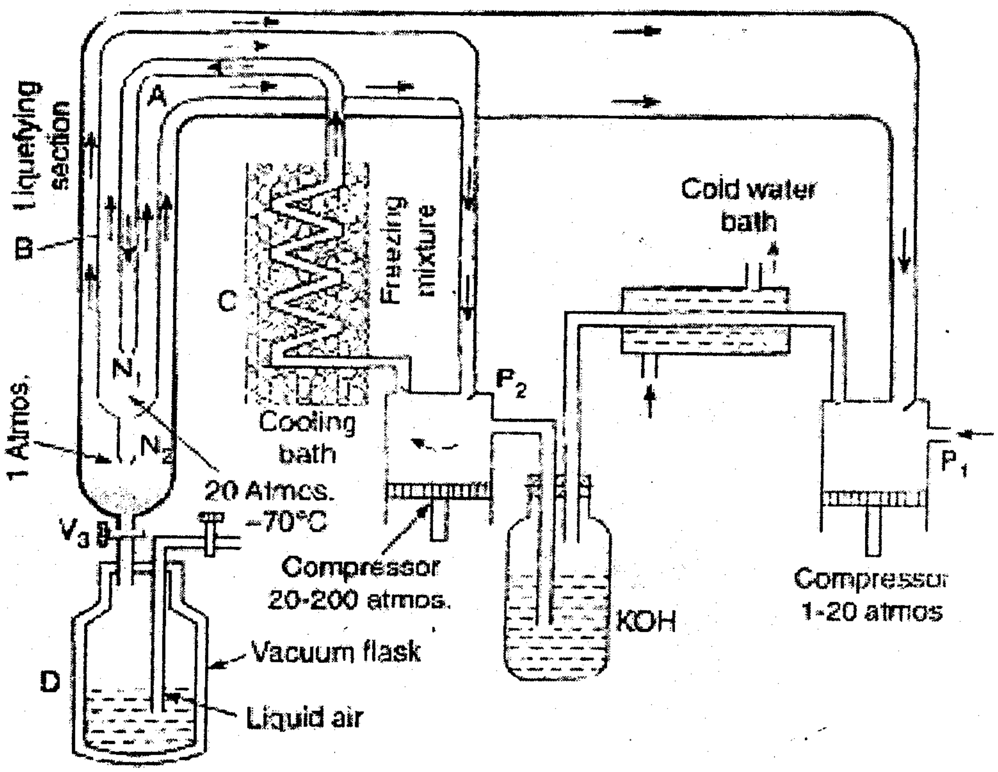


Fig:(5.4)

The air which is free from CO_2 and water vapour is compressed to 200 atmospheres using a compressor P_2.This compressed air is passed through a spiral tube immersed in a freezing mixture (ice + NaCl). Due to this, the temperature of the air falls to at about -20°C. Now the air is passed through the tube A. When the air is coming out through the nozzle N_1 of tube A, the pressure decreases to 20 atmospheres. Due to this, temperature decreases to at about -70 °C. This air is passed through the tube B. When it is passed through tube B, it cools the incoming air in tube A. Then this air enters into the compressor P_2.

With in a few cycles, the temperature of the air coming out from the nozzle N_1 becomes at about $-183^0 C$. Now the nozzle N_2 is open. Here the pressure is changing from 20 atmospheres to 1 atmospheres .The temperature of air coming out from nozzle N_2 becomes at about $-183^0 C$.At this temperature the air is liquefied .The liquid air is collected into Dewar flask.

The air which is not liquefied is allowed to enter into the compressor P_1 after passing through tube C. when the air is passing through tube, it cools the air present in the tubes A and B .The cyclic process is continued to obtain more liquid air.

c) Liquefication of hydrogen:-

The hydrogen gas is free from dust , CO_2 and water vapour is compressed to 200 atm by means of compressor P .This compressed hydrogen gas is passed through a spiral tube placed in a chamber containing solid CO_2 and alcohol as shown in fig.(5. 5). Then the hydrogen gas is passed through coil C_1 in chamber 'A' which is cooled by the out going hydrogen gas.

Then the hydrogen gas is passed through coil C_2 in chamber B .The chamber B is filled with liquid air. Due to this the temperature of hydrogen gas in coil C_2 becomes at about -170^0C .Then the hydrogen gas is passed through the coil C_3in chamber C. In this chamber liquid air is evaporated at low pressure. Due to this, the temperature further decreases. The temperature of hydrogen becomes at about -200^0C ..

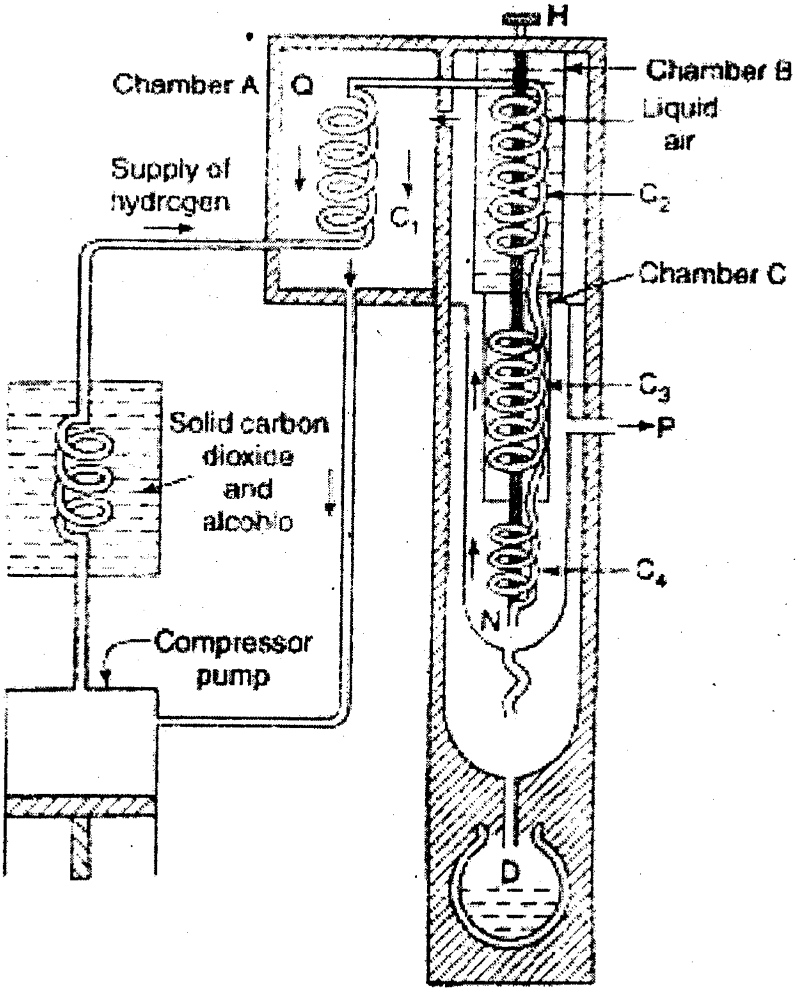


Fig:(5.5)

Now the hydrogen gas is passed through another coil C_4 which is cooled by the outgoing hydrogen gas. When the hydrogen gas is coming out from nozzle N of coil C_4, the pressure decreases from 200 atm to 1 atm. Due to Joule-Kelvin effect the temperature further decreases. This cooled hydrogen gas is allowed to enter into compressor P after cooling the incoming hydrogen gas in the coils C_4,C_3, C_2

and C_1. Within a few cycles, the temperature of hydrogen gas near the nozzle becomes at about $-250^0 C$. When hydrogen gas is coming out from the nozzle, the temperature becomes less than $-253^0 C$. At this temperature hydrogen gas is liquefied. The liquid hydrogen is collected into Dewar flask.

d) Liquefaction of Helium:-

Helium is the last gas to be liquefied. All attempts made before 1908 to liquefy helium are failed. The reason is inversion temperature (T_i) of helium $-240^0 C$ and critical temperature of helium is $-268^0 C$. In order to obtain cooling effect due to Joule Kelvin effect, the initial temperature of the gas should be less than $-240^0 C$. After attaining $-268^0 C$, the helium gas is liquefied.

(i) KAMMERLINGH ONNES HELIUM LIQUEFIER:-

Kammerlingh Onnes helium liquefier is as shown in fig.5.6.. Pure, dry and pre-cooled helium is compressed to about 40 atmospheres in a compressor P.

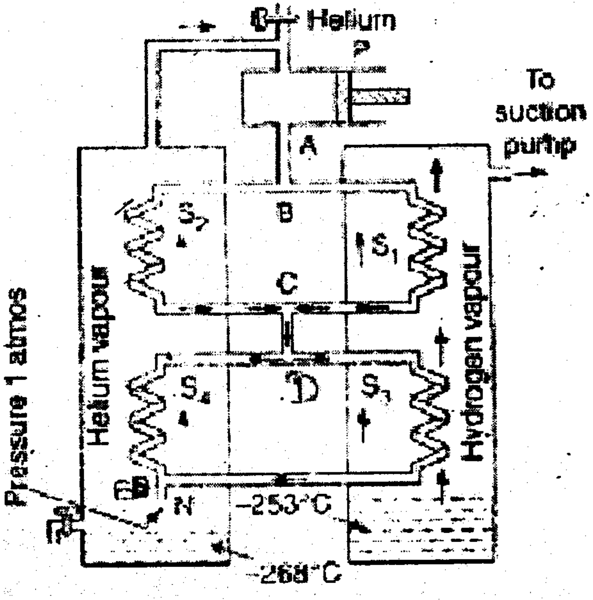


Fig:(5.6)

The compressed helium is passed through tube AB. At B, the helium gas is divided into two parts. A part is passed through spiral tube S_1 and a part is passed through spiral tube S_2. Both parts are allowed to combine at C and this helium gas is allowed to travel from C to D. At D, the helium gas is again divided into two parts. A part is passed through spiral tube S_3 and a part is passed through spiral tube S_4. Both are allowed to combine at E. The spiral tubes S_2 and S_4 are placed in a chamber which contains the outgoing helium gas. The spiral tubes S_1 and S_3 are placed in another chamber. In this chamber, liquid hydrogen evaporated at low pressure using suction pump. The temperature of helium gas near point E becomes less than $-240^0 C$. When it is coming out from nozzle N, the temperature further decreases due to Joule-Thomson effect. This cooled helium gas is allowed to enter into compressor P after cooling the incoming helium gas in the spiral tubes S_1 and S_3. Within few cycles, the temperature of the helium gas coming out from the nozzle becomes at about $-268^0 C$. At this temperature helium gas is liquefied. The liquid helium is collected into Dewar flask.

(ii) KAPITZA'S HELIUM GAS LIQUIFIER:-

The most important modification by Kapitza's was made in the expansion engine where no lubricant is used. All lubricants will become solidified at such

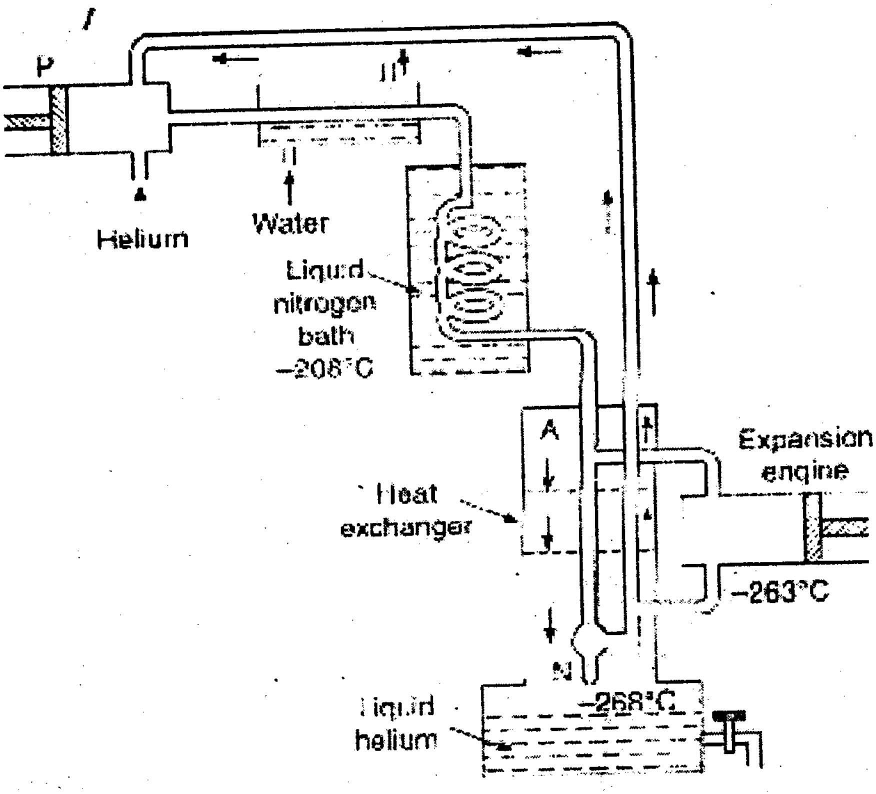


Fig:(5.7)

low temperatures. He used the compressor where the piston is loosely fitted with the cylinder with a very small clearance as shown in fig. 5.7. When adiabatic expansion takes place a little gas escapes through the clearance which is negligible.

5.6) Adiabatic Demagnetization:-

In order to obtain very low temperatures, the adiabatic demagnetization process is used. A paramagnetic substance contains molecules which are lying in an orderly manner when the substance is magnetized. The molecules are set in the direction of magnetic field. Due to this the temperature of the substance rises. This magnetized substance is kept at very low temperature. Now if the magnetic field is removed, the temperature of the substance decreases.

The paramagnetic substance 'P' i.e., (gadolinium sulphate) is suspended in vessel A as shown in the fig.5.8.

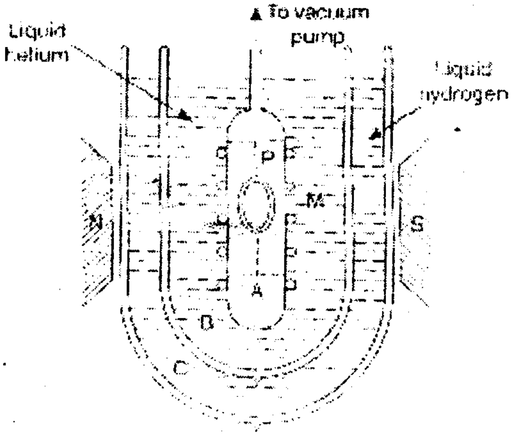


Fig. 5.8

vessel A is filled with liquid helium evaporated at low pressure. The temperature of helium vapour becomes at about 1K. The vessel A is placed in another vessel B which is filled with liquid He at 1K. The vessel B is placed in vessel C which is filled with liquid H_2 at 20 K. A strong magnetic field is applied to magnetize the paramagnetic substance. Coils M are placed to measure the temperatures by susceptibility measurements. Due to magnetization, the temperature of paramagnetic substances slightly increases. But this heat is absorbed by He gas, from it and this heat is transferred to liquid He. Again the temperature of paramagnetic substance becomes 1K. Using vacuum pump helium gas is removed from vessel A. Due to this the paramagnetic substance is thermally isolated. Now the applied magnetic field is switched off. Due to this adiabatic demagnetization the temperature of the paramagnetic substance decreases.

Expression:-

If B is the strength of the magnetic field applied dI is the increase in intensity of magnetization, work done on the substance=B dI

When the substance is demagnetized work done by the substance= -BdI

Total external work done is $dW=PdV-BdI$-------------(5.15)

From first law of thermodynamics we have $dQ=dU+dW$------------(5.16)

From these two equations (5.15 & 5.16) we can write,

$$dQ=dU+PdV-BdI$$

Since Volume V is constant, $dV=0$ $\Rightarrow PdV=0$

$$dQ=dU-BdI \text{----------}(5.17)$$

From second law of thermodynamics we have,

$$dQ=TdS \text{--------------}(5.18)$$

From equations 5.17 and 5.18 we can write,

$$dU-BdI=TdS \text{----------}(5.19)$$

From Maxwell's third thermodynamic relation we have,

$$\left(\frac{\partial T}{\partial P}\right)_S=\left(\frac{\partial T}{\partial S}\right)_P \ldots\ldots\ldots\ldots..(5.20)$$

In order to obtain the required relation let us replace pressure P with -B and volume V with intensity of magnetization I.

$$-\left(\frac{\partial T}{\partial B}\right)_s=\left(\frac{\partial I}{\partial s}\right)_B$$

$$\left(\frac{\partial T}{\partial B}\right)_s=-\left(\frac{T\left(\frac{\partial I}{\partial T}\right)_B}{T\left(\frac{\partial s}{\partial T}\right)_B}\right)=\frac{-\left(\frac{\partial I}{\partial T}\right)_B.T}{\left(\frac{\partial Q}{\partial T}\right)_B}\ldots.(5.21)$$

If m is mass of the substance and C_B is the specific heat at constant magnetic field B, we can write

$$\left(\frac{\partial Q}{\partial T}\right)_B=mC_B\ldots\ldots\ldots(5.22)$$

Writing equation (5.22) in equation (5,21) we get,

$$\left(\frac{\partial T}{\partial B}\right)_s=\frac{-T}{mC_B}\left(\frac{\partial I}{\partial T}\right)_B\ldots\ldots\ldots(5.23)$$

The susceptibility of a material (χ) is defined as the ratio of intensity of magnetization (I) and strength of the magnetic field applied (B).

$$\text{Susceptability}(\chi)=\frac{\text{intensity of the magnetic field(I)}}{\text{strength of the magnetic field}(B)}$$

$$\chi=\frac{I}{B}\ ..(5.24)$$

From Curie's law the susceptibility of a material is inversely proportional to its absolute temperature.

$$\chi \alpha \frac{1}{T} \Rightarrow \chi = \frac{C}{T} ......5.25$$

Where C is Curie's constant

From equations (5.24) and (5.25) we can write

$$\frac{I}{B} = \frac{C}{T} \Rightarrow$$

$$I = \frac{BC}{T} ....5.26$$

Substituting equations (5.26) in equation (5.23) we get,

$$\left(\frac{\partial T}{\partial B}\right)_S = \frac{-T}{mc_B}\left[\frac{\partial\left(\frac{Bc}{T}\right)}{\partial T}\right] = \frac{-TBC}{mc_B}\left(\frac{-1}{T^2}\right)$$

$$= \left(\frac{\partial T}{\partial B}\right)_s = \frac{BC}{mc_B T}$$

$$\left(\frac{\partial T}{\partial B}\right)_s = \frac{KB}{c_B T}$$

where $K = \frac{c}{m}$

'K' is curies constant per unit mass.

$$\partial T = \frac{KB\partial B}{c_B T} ----5.27$$

When the magnetic field is changed from B_i to B_f, let the temperature of paramagnetic substance is changed from T_i to T_f. Integrating equation (5.27) and applying these limits we get,

$$\int_{T_i}^{T_f} \partial T = \int_{B_i}^{B_f} \frac{KB\partial B}{C_B T} \Rightarrow [T]_{T_i}^{T_F} = \frac{K}{C_B T}\left[\frac{B^2}{2}\right]_{B_i}^{B_f}$$

$$\Rightarrow T_f - T_i = \frac{K}{2C_B T}\left[B_f^{\,2} - B_i^{\,2}\right] ------(5.28)$$

Let the strength of initial magnetic field be B i.e., $B_i=B$. When the substance is demagnetized, the final magnetic field is zero i.e., $B_f=0$.

Writing these values in equation (5.28) we get,

$$T_f - T_i = \frac{k}{2C_B T}\left[o^2 - B^2\right]$$

$$T_f - T_i = \frac{-kB^2}{2C_B T} \text{--------} (5.29)$$

Here negative sign indicates that the temperature of the substance decreases due to demagnetization.

Refrigerator:-

A machine used for producing low temperature below surroundings and maintaining an enclosure at that temperature is called as refrigerator. Actually a refrigerator is a Carnot's engine working in backward direction. The liquid which on evaporation produces cooling is called refrigerant. Commonly used refrigerators are Ammonia (NH_3) for large plants and sulphur dioxide (SO_2), Methyl Chloride (CH_3Cl), Freon (CCl_2F_2) and so on is used in small plants.

Refrigerating machines are of two types

1. Vapour Compression machine
2. Vapour Absorption machine

Here we discuss vapour compression machine only.

Vapour Compression machine (Frigidaire):-

The Vapour compression machine is as shown in fig.5.9. The refrigerant is compressed by a compressor 'P'. When the piston P moves in the downward direction valve V_1 opens and valve V_2 is closed. If the piston is moved upward direction valve V_2 opens and V_1 is closed.

When the piston P is pushed in the downward direction valve V_1 opens and valve V_2 is closed. Now the refrigerant under high pressure enters in to the condenser coil (C) on which cold water is circulating. Due to this, refrigerant converts into liquid.

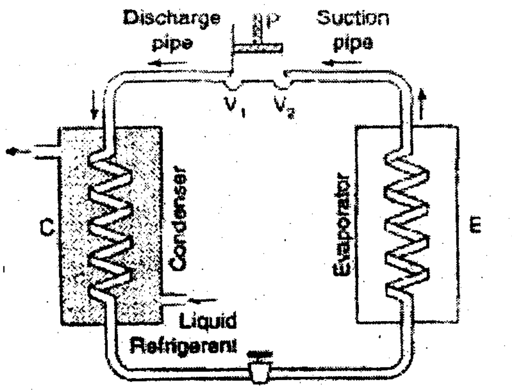


Fig. 5.9

The liquid refrigerant passed through regulator valve V and enters into the evaporator coil. Due to low pressure, the liquid refrigerant evaporates in the evaporator E. As more and more liquid evaporates in an evaporator coil (E), more cooling is produced, because the required latent heat of evaporization is extracted from the surroundings. After a certain time the temperature reaches the desired value. The cycle is repeated again and again.

5.8. Solved examples:-

1. Calculate the change in temperature when the gas suffers Joule -Thomson effect expansion at -173^0 C The pressure difference on the two sides of the plug being 20 atmospheres. Does the gas show a heating effect (or) a cooling effect on the expansion.

Given a= $a = 0.0341 lit^2 - atm/mole^2$, $b = 0.02737 lit/mole$ and $C_p = 5\frac{cal}{mole-Kelvin}$

$a = 0.0341 lit^2 - atm/mole^2$

$$=0.0341\times\left(10^{-3}m^{3}\right)^{2}\times 1.01\times 10^{5}\,{}^{N}\!/_{m^{2}-mole^{2}}$$

$$=3.444\times 10^{-3}\,\frac{N-m^{4}}{mole^{2}}$$

$$b=0.0237\,{}^{lit}\!/_{mole}=0.0237\times 10^{-3}\,{}^{m^{3}}\!/_{mole}$$

$$T=-173^{0}c=-173+273=100k$$

$$dp=20atm=20\times 1.01\times 10^{5}\,{}^{N}\!/_{m^{2}}=20.2\times 10^{5}\,{}^{N}\!/_{m^{2}}$$

$$R=8.31\ {}^{J}\!/_{mole-kelvin}$$

$$Cp=5\,{}^{cal}\!/_{mole-kelvin}=5\times 4.2\,{}^{J}\!/_{mole-kelvin}$$

$$dT=?$$

$$dT=\frac{-p_{1}-p_{2}}{c_{p}}\left[\frac{2a}{RT}-b\right]$$

$$=\frac{-20.2\times 10^{5}}{21}\left[\frac{2\times 3.444\times 10^{-3}}{8.31\times 100}-0.0237\times 10^{-3}\right]$$

$$=-0.9619\times 10^{2}[0.0083897-0.0237]$$

$$=-96.19\times(-0.01531)$$

$$=1.4726k$$

$dT \doteq 1.5k$ (Heating effect.)

5.9 Summary :

Change of phase of a substance was discussed. Joule-Kelvin effect process was explained. Principle of regenerative cooling and Joule-Kelvin effect are commonly used to liquefy gases such as air, hydrogen and helium. Adiabatic demagnetization process was discussed to obtain low temperatures. The process of refrigeration was discussed.

5.10 Key words:-

Phase, Joule-Kelvin effect, Liquefaction, Regenerative cooling, Adiabatic demagnetization, Refrigeration.

5.11 self assessment questions:-

Long answer questions:-

1. Describe Joule -Kelvin effect porous plug experiment. Derive the concerned expression.
2. Explain what is meant by adiabatic demagnetization. Derive the concerned expression.

3. What is the principle of regenerative cooling? Explain Kapitza's method of liquefaction of helium.

Short answer questions:-

1) What is Joule-Kelvin effect? Explain.

2) Compare different types of expansions.

3) What is the principle of regenerative cooling?

4) Explain adiabatic demagnetization process.

5) Explain the principle of refrigeration.

Exercises:-

1. Calculate the temperature of oxygen after expanding through a 150 atmosphere having c_p =7 cal/mole-K and vanderwals constants $a = 1.32liktre^2 - atm/mole$ and $b = 3.12\times10^{-2}\,{}^{litre}/_{mole}$ and the initial temperature 27 and R= $8.31\times10^{-2}\,litre - atm/mole$

$$\left[\text{Hint}: dp = \frac{-(p_1-p_2)}{C_p}\left[\frac{2a}{RT}-b\right]\right] \quad \left[ans: -11.7^{o}C\right]$$

2. The vanderwaal's constants a and b for one mole of a gas are $0.245litre^2 - atm/mole$ and $b = 2.67\times10^{-2}\,{}^{litre}/_{mole}$ respectively. Calculate the temperature of inversion.

$$\left[\begin{array}{l}\text{Hint}: T_i = \dfrac{2a}{Rb} = \dfrac{2\times0.245}{8.3\times10^{-2}\times2.67\times10^{-2}} = \dfrac{0.49\times10^{4}}{22.161} \\ Ans: 221K \quad \text{(or)} \quad -52^{0}C\end{array}\right]$$

5.12 Reference books:-

Unified Physics	Dr.S.L.Gupta and Sanjeev Gupta	Prakash Nath & Co Meerut 2005.		
Heat and Thermodynamics	Brijlal and Subrahmanyam	S.Chand & Co		
B.Sc. Heat	Telugu Academy.			
Thermodynamics	D.C.Tayal	Himalaya publications.		
D.S.MATHUR'S Heat and Thermodynamics	Revised by Dr.M.N.BAPAT	SULTAN CHAND &SONS	NEW DELHI	1997
B.Sc.Physics Thermodynamics &Heat	Dr.J.P.Agarwal &Amit Agarwal	Pragati Prakashan	Meerut	1992

UNIT II

Lesson 6

QUANTUM THEORTY OF RADIATION

Objectives :

This lesson covers the following and also will be able to learn.

1. Black body and its radiation and energy distribution.
2. Important laws of Thermal radiation.
3. Pyrometers to measure high temperatures.
4. Solar constant and determination of the temperature of the sun.

Structure of the lesson :

6.1 Introduction

6.2 Black body and its radiation.

  (a) Fery's black body

  (b) Wien's black body

6.3 Energy distribution in black body radiation.

6.4 Laws of Thermal radiation.

6.5 Wien's displacement law.

6.6 Rayleigh – Jeans law.

6.7 Planck's radiation law.

6.8 Deduction of laws from planck's radiation law

6.9 Pyrometers

6.10 Fery's total radiation Pyrometer

6.11 Modification for the measurement of very high temperatures.

6.12 Disappearing filament optical Pyrometer,

6.13 Solar constant and its determination.

6.1 Introduction

Transfer of heat takes place in three processes called conduction, convection and radiation (i) If the transfer of heat is taking place without the actual involvement of the movement of the molecules along with heat flow, is called conduction. (ii) The transfer of heat due to the actual movement of the molecules from one place to another place is called convection. (iii) The transfer of heat from one place to another place irrespective of the presence of material medium is called radiation.

The radiation emitted by a body by virtue of its temperature is called Thermal Radiation. Thermal Radiation is a particular range of electromagnetic radiation. Thermal radiation wavelength band extends from 1000 A^{O} to 10,00,000 A^{O}. The radiation of wavelength between 1000 A^{O} to 3800 A^{O} is called Ultra violet radiation. The radiation of wavelength between 3800 A^{O} to 7500 A^{O} is called visible region. The radiation ranging from 7500A^{O} to 100,000 A^{O} belong to infrared region.

According to prevost's theory of heat exchanges, every body emits and absorbs radiant energy continuously as long as its temperature is more than OK.

6.2 Black body and its radiation :

A perfect black body is one which completely absorbs all the radiations of all wave lengths incident on it. Since it may neither reflect nor transmits any radiation. It appears black whatever be the colour of radiation. A perfectly black body is a good absorber as well as a good radiator. As the radiations emitted by a black body possess maximum possible wavelengths, the radiation is called full radiation or total radiation. The radiation emitted by a black body depends only on its temperature, but not on material of the body.

There is no perfect black body in character. The nearest approach is lamp black or platinum black. Here we shall discuss Ferry's black body and Wien's black body.

a) Fery's Black Body:

The apparatus consists (fig. 6.1) of a double walled conducting sphere. The inter space between the walls is evacuated to prevent the loss of heat by conduction and convection. It has a small hole'O' and a conical projection P just opposite to the hole 'O'. The inside surface is coated with lamp black and outer surface is polished with Nickel.

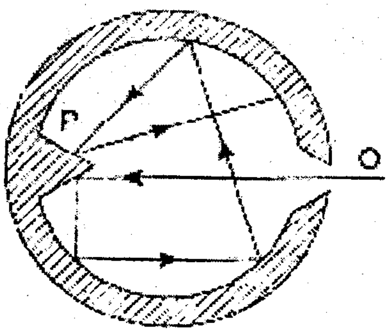


Fig:(6.1)

When the radiation is incident on the hole, it passes inside the enclosure. The radiation suffers multiple reflections and is completely absorbed. The radiation which is incidenting on the opposite surface may be reflected back. To eliminate this projection P is arranged. Since all the radiations which are entering are absorbed, it behaves like a black body. If the enclosure is heated to a definite temperature, it is filled with radiation of all possible wave lengths.

b) Wien's Black Body :

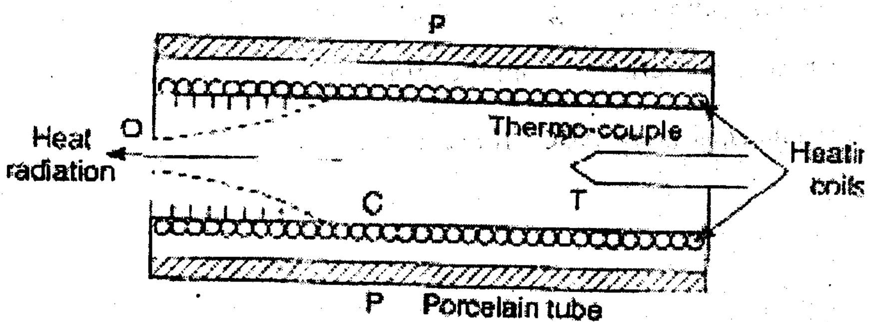


Fig:(6.2)

The wien's black body consists of a long metallic tube C blackened inside(Fig. 6.2). The tube is heated by electric current passing in the coil. These coils are called heating coils. Then the tube is surrounded by P, P (porcelain tubes). The temperature of the Central part of the tube is measured by using a thermocouple.

When current is passed through the coils heat radiations emerge out from the hole O. The radiation from the inner chamber can be limited with the help of diaphragm shown by dotted lines. The hole 'O' acts as a black body radiator.

6.3 Energy distribution in black body radiation:

First of all Stefan gave the fourth power law. According to Stefan's law, the total amount of radiant energy by a black body per unit area per second due to all wave lengths is directly proportional to fourth power of absolute temperature.

$E \alpha T^4 \Rightarrow E = \sigma T^4$ where σ is Stefan's constant

$$\sigma = 5.67\times10^{-8}\, {}^{J}\!/_{s-m^2-K^4} \quad \text{(or)} \quad {}^{W}\!/_{m^2-K}$$

This law gives the total energy radiated by a black body. But it is not giving any information about the energy distribution.

The distribution of energy in black body radiation for different wave lengths and its different temperatures was experimentally determined by Lummer and Pringsheim. The radiation from a black body

'O' was focussed on a slit S by a concave mirror M_1 (fig. 6.3). The slit 'S' is placed at the focus of another concave mirror M_2. Due to this the radiation reflected from the mirror M_2 becomes parallel. The parallel beam of radiation is passed through Fluorspar prism ABC. If ordinary glass prism is used it absorbs certain radiations, hence it is not used.

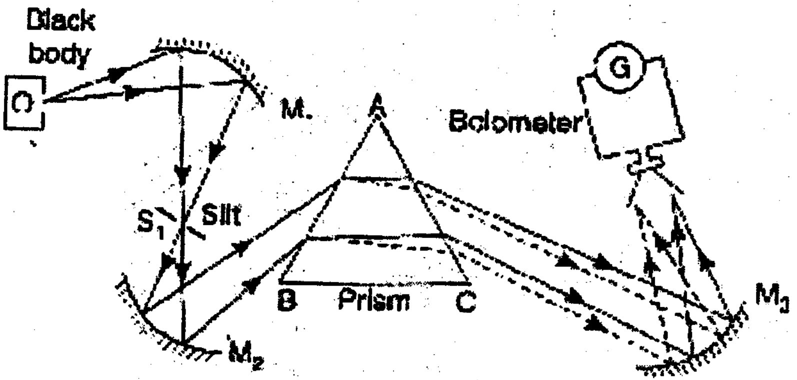


Fig:(6.3)

The Prism produces the spectrum of radiation. The dispersed beam coming out from the prism is allowed to incident on concave mirror M_3. This mirror M_3 focuses different wave lengths at different angles. The intensity of the radiation E_λ is measured by using Bolometer. The deflection produced in the galvanometer G is directly proportional to intensity of radiation E_λ. Different wave length radiations are allowed to enter into bolometer by moving the bolometer from one place to another place or by rotating the mirror M_3.

The intensity of radiation E_λ is defined as $E_\lambda d\lambda$ is energy for wave lengths in between λ and $\lambda + d\lambda$ emitted per second per unit surface area of a black body. The experiment was repeated at different temperatures of black body. A graph was then drawn taking E_λ along y axis and λ along X-axis. The curves are obtained as shown in fig. 6.4.

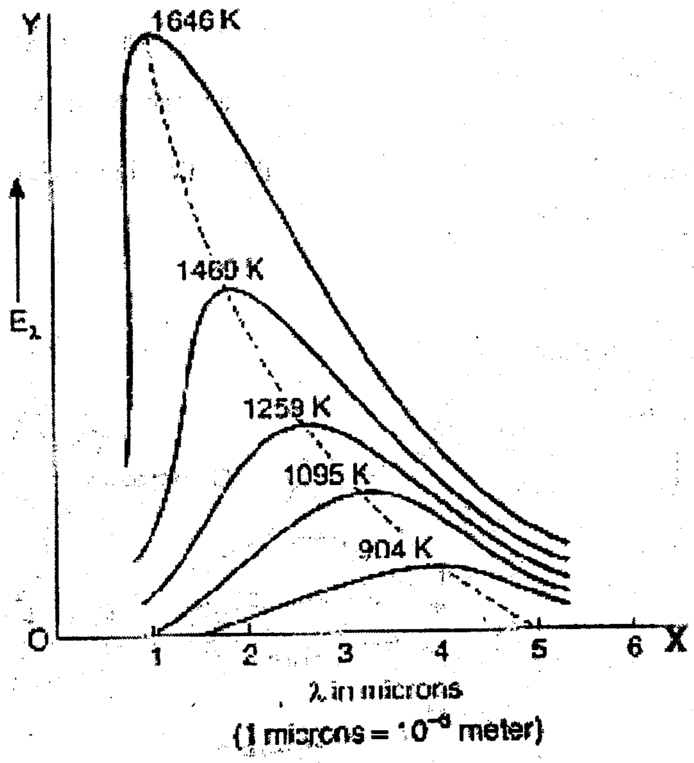


Fig:(6.4)

The experimental results are as below:

1. The emission from a black body at any temperature is composed of radiation from all wave lengths.
2. At a given temperature the energy is not uniformly distributed. As the temperature of the black body increases, the intensity of radiation for each wave length increases. After certain wave length the intensity of radiation is decreasing.
3. The total energy of radiation at any temperature is given by the area between the curve corresponding to that temperature and the horizontal axis.
4. The amount of radiant energy emitted is small at very short and at very long wave lengths. At a particular temperature the intensity of radiation E_λ is maximum for a particular wave length λ_m. Most of the energy is present near λ_m.
5. The wave length corresponding to maximum energy and absolute temperature of the black body is always constant.

$$\lambda_m T = \text{constant} \Rightarrow \lambda_m T = \lambda_m{}^1 T^1$$

6.4 Laws of Thermal Radiation:

1. *Stefan's law or Stefan – Boltz mann's law* :

According to Stefan's law the total amount of radiant energy emitted by a black body per second per unit area is directly proportional to fourth power of its absolute temperature.

$$E \alpha T^4 \Rightarrow E = \sigma T^4$$

Let the temperature of the hot body be T_1 and it is surrounded by another black body of absolute temperature T_2 . Then the energy radiated from the black body is

$$E = \sigma(T_1^4 - T_2^4)$$

2. Wien's displacement law :

According to wien's displacement law the product of wave length corresponding to maximum energy and absolute temperature is constant.

$$\lambda_m T = cons\tan t$$

3. Wien's Law :

According to wien's law, the energy distribution in the thermal spectrum is given by

$$E_\lambda = A\lambda^{-5} e^{-B/\lambda T}$$

This formula agrees with the experimental curves for shorter wave lengths.

4. Rayleigh – Jean's law:

According to Rayleigh Jean's law, the energy distribution in the thermal spectrum is given by

$$E_\lambda = \frac{8\pi kT}{\lambda^4}$$

where k is Boltzmann's constant.

This formula agrees with the experimental results for longer wave lengths.

5. Planck's law :

On the basis of quantum theory, Planck derived the formula for energy distribution in the therma spectrum as

$$E_\lambda = \frac{8\pi kT}{\lambda^5\left(e^{\frac{hc}{\lambda kT-1}}-1\right)}$$

where h is planck's constant, C is velocity of light and K is Boltzmann's constant.

Planck's formula covers the entire range of thermal radiation.

6.5 Wien's displacement law:

According to wien's displacement law, the product of wavelength corresponding to maximum energy (λ_m) and the absolute temperature T is constant.

$$\lambda_m T = constant = b$$

The value of constant (b) is $0.2896\times10^{-2} m-k$

Expression :- Consider a spherical enclosure with perfectly reflecting walls and expanding like foot ball bladder. Let it be filled with black body radiation of energy density ' u' at a temperature T. If V be the volume of the enclosure, then total internal energy U of radiation is given by

$$U=uV \quad ....(6.1)$$

By electromagnetic theory, the pressure exerted by the radiation on the wall of enclosure is 1/3rd of internal energy density

i.e $P = \frac{u}{3}$ ....(6.2)

From first law of thermodynamics we have,

$$dU + PdV = dQ$$

Since there is supply of energy from outside dQ = 0

$$dU + PdV = 0$$

writing equations (6.1) and (6.2) in this equation we get,

$$d(u.V) + \frac{u}{3}dV = 0$$

$$V.du + udV + \frac{4}{3}dV = 0$$

$$Vdu + \frac{4}{3}udV = 0$$

dividing with uv we get,

$$\frac{du}{4}+\frac{4}{3}.\frac{dv}{V}=0$$

Integrating on both sides we get,

$$\log_e^u+\frac{4}{3}\log_e^V=\text{constant}$$

$$\log_a^u+\log_e^{V^{4/3}}=\text{constant}$$

$$\log u.V^{\frac{4}{3}}=\text{constant}$$

$$\Rightarrow \quad u.V^{\frac{4}{3}}=\text{constant}$$

From Stefan's law, we have, $u=\sigma.T^4$

$$\sigma.T^4.V^{\frac{4}{3}}=\text{constant}$$

$$T^4.V^{\frac{4}{3}}=\text{constant}$$

Taking fourth root on both sides,

$$T.V^{\frac{1}{3}}=\text{constant}$$

If r is the radius of the spherical enclosure, $V=\frac{4}{3}\pi r^3$

$$\Rightarrow T\left(\frac{4}{3}\pi r^3\right)=\text{constant}$$

$$\Rightarrow T.r \quad =\text{constant-}\_\_\_\_\_\_\_\_\_\_\_\_(6.3)$$

According to Doppler effect, when the spherical enclosure is expanding the wavelength of radiation increases. Let the radiation is incidenting at point 'A' on the wall at position S_1. Let θ be the angle of incidence. After reflection the radiation travels along AC. Let λ be the wavelength. During the time of time period T, let the wall is displaced to another position S2. If v is the velocity of the enclosure, the displacement of wall is T. Now the next radiation which is traveling in the same direction travels along path ABD. Let ABD = λ^1. AN is the perpendicular drawn from point A onto the line BD.

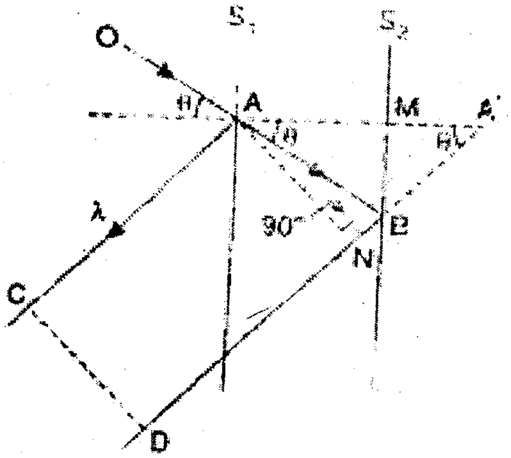


Fig:(6.5)

The change in wavelength can be written as

$$\begin{aligned}\Delta\lambda = \lambda^1 - \lambda &= AB + BD - AC \\ &= AB + BN + ND - AC \\ &= AB + BN + ND - ND \\ &= AB + BN \text{....................}(6.4)\end{aligned}$$

The line BD is extended. The normal drawn at point A to the walls is also extended and it is AMA^1.

From figure 6.5, Δ^{le} ABM and A^1BM are identical.

$\Delta\lambda = A^1B + BN = A^1N$

Writing AB= A^1B in equation (6.4) we get

$$\therefore \Delta\lambda = A'B + BN = A'N ------(6.5)$$

From A A^1N we can write,

$$Cos\theta = \frac{A'N}{AA'} = \frac{A'N}{AM + MA'} = \frac{A'N}{2AM} \qquad (\because AM = MA)$$

We have $AM = VT$

$$\therefore Cos\theta = \frac{A'N}{2vT} \Rightarrow A'N = 2vT\cos\theta$$

$$\Delta\lambda = 2VT\cos\theta \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}(6.6)$$

It is the expression for change in wave length between two successive collisions. Consider a spherical enclosure of radius 'r'. The radiation will reflect on the walls as shown in fig.6.6.

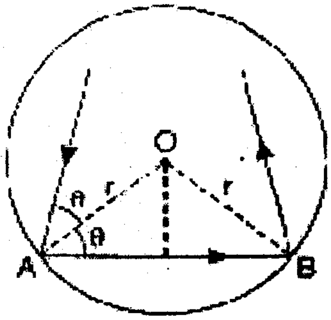


Fig:(6.6)

The distance between two successive collisions is
$r\cos\theta + r\cos\theta = 2r\cos\theta$

The velocity of radiation is c. It is the distance traveled by the radiation in one second.

Number of collisions in one second $= \dfrac{c}{2r\cos\theta}$

Number of collisions in dt seconds $= \dfrac{c.dt}{2r\cos\theta}$ ______________(6.7)

Change in wave length in dt seconds $= d\lambda = 2vT.\cos\theta \times \dfrac{c.dt}{2r.\cos\theta}$

$$= \frac{vTcdt}{r}$$

$$= \frac{v\lambda}{cr}cdt \quad (\because T = \frac{\lambda}{C})$$

$$d\lambda = \frac{\lambda.v.dt}{r} = \frac{\lambda.dr}{r}$$

$$\Rightarrow \frac{d\lambda}{\lambda} = \frac{dr}{r}$$

Integrating on both sides we get,

$$\log_e^{\lambda} = \log_e^{r} + \text{constant}$$

$$\Rightarrow \log_e\left(\frac{\lambda}{r}\right) = \text{constant} \quad \Rightarrow \frac{\lambda}{r} = \text{constant} \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (6.8)$$

From equation (6.3) we have, r x T = constant

From equations (6.3) and (6.8) we can write,

$$r \times T \times \frac{\lambda}{r} = \text{constant} \quad \Rightarrow \lambda.T = \text{constant}$$

$$\therefore \lambda T = \lambda'.T' \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (6.9)$$

This equation shows that the wave length is inversely proportional to the temperature of black body.

From first law of thermodynamics we have,

$$dU + pdV = 0 \; (\therefore dQ = 0)$$

$$d(uV) + \frac{u}{3}dV = 0 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (6.10)$$

If $u_\lambda d\lambda$ is the energy density between the wave lengths λ and $\lambda + d\lambda$ per unit volume, then we can replace u with $u_\lambda d\lambda$

$$d(u_\lambda dV) + \frac{1}{3} \times u_\lambda d\lambda dV = 0$$

$$d(u_\lambda d\lambda)V + u_\lambda d\lambda dV + \frac{1}{3} \times u_\lambda d\lambda dV = 0$$

$$d(u_\lambda d\lambda)V + \frac{4}{3} \times u_\lambda d\lambda dV = 0$$

dividing with $u_\lambda d\lambda$ we get,

$$\frac{d(u_\lambda d\lambda)}{u_\lambda d\lambda} + \frac{4}{3}\frac{dV}{V=0} = 0 - - - - - - (6.11)$$

Volume of the enclosure is

$$V = \frac{4}{3}\pi r^3, dV = \frac{4}{3}\pi \times 3r^2 dr \quad = 4\pi r^2 dr$$

$$\frac{dV}{V} = \frac{4\pi r^2 dr}{\frac{4}{3}\pi r^3} = \frac{3dr}{r}$$

Writing this in equation (6.11) we get

$$\frac{d(u_\lambda d\lambda)}{u_\lambda d\lambda}+\frac{4}{3}.\frac{3dr}{r}=0$$

$$\frac{d(u_\lambda)}{u_\lambda d\lambda}+4\frac{dr}{r}=0$$

Integrating on both sides we get,

$\log_e(u_\lambda .d\lambda)+4\log_e r = \text{Constant}$

$\log_e(u_\lambda .d\lambda)+\log_e r^4 = \text{Constant}$

$\log_e(u_\lambda .d\lambda .r^4) = \text{Constant}$

$u_\lambda .d\lambda .r^4 = \text{Constant}$

we know that, $\lambda \propto r, d\lambda \propto dr$

$u_\lambda .dr.r^4 = Cons\tan t$

$u_\lambda .r^4 .dr = Cons\tan t$

Integrating on both sides,

$$\frac{u_\lambda .r^5}{5} = cons\tan t$$

$u \times r^5 =$ Constant……………… (6.12)

From equation (6.3) we have r x T = constant.

From equations (6.3) and (6.12) we can write

$u_\lambda .T^{-5} =$ constant.

Since the spectral radiancy $E_\lambda \alpha \cdot u_\lambda$ We can write

$E_\lambda .T^{-5} = E_\lambda^{'} \times T^{'-5}$

$$\frac{E_\lambda}{T^5}=\frac{E_\lambda^{'}}{T^{'5}} \qquad \text{…………………….(6.13)}$$

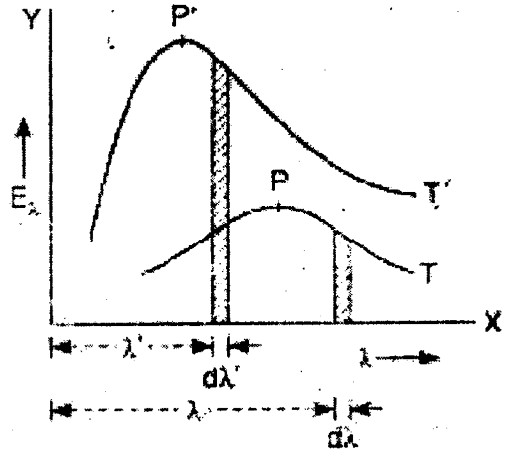


Fig:(6.7)

Experimentally one curve is drawn on a graph taking wave length λ along X – axis and spectral radiancy E_λ' along Y – axis (fig.6.7). Taking the values the graph and using equation (6.13) we can draw the curve at any unknown temperature T'.

From the curves, it was found that the product of wavelength corresponding to maximum energy and absolute temperature is constant.

$$\lambda_m T = \text{constant}$$

It is wien's displacement law.

6.6 Rayliegh – Jeans law:-

Rayleigh and Jean developed a theory for the spectral distribution of black body radiation by the application of electrodynamics and statistical mechanics.

Consider a hollow cubic enclosure of side 'l' with perfectly reflecting walls. Let us place a black particle inside the enclosure. The radiations emitted by the particle will be reflected by the walls. The waves travel in all directions with all wavelengths. In a course of time, the enclosure will be filled with stationary waves of all wavelengths. In 'l' unit length, if n loops are present

$$\frac{n\lambda}{2} = l \Rightarrow \lambda = \frac{2l}{n}$$

$$\Rightarrow \nu = \frac{c}{\lambda} = \frac{cn}{2l}$$

Every allowed frequency is called a mode of vibration.

Let n_x, n_y, n_z be the number of loops present along x, y and z axis. As the radiation is diffuse (scatter), the waves can be inclined at any angle to the rectangular axis. For waves making the angles α, β, γ with three axes, We can write,

$$l\cos\alpha = \frac{n_x\lambda}{2}$$

$$l\cos\beta = \frac{n_y\lambda}{2}$$

$$l\cos\gamma = \frac{n_z\lambda}{2}$$

these equation can be written as

$$\Rightarrow \cos\alpha = \frac{n_x\lambda}{2l}; \cos\beta = \frac{n_y\lambda}{2l}; \cos\gamma = \frac{n_z\lambda}{2l}$$

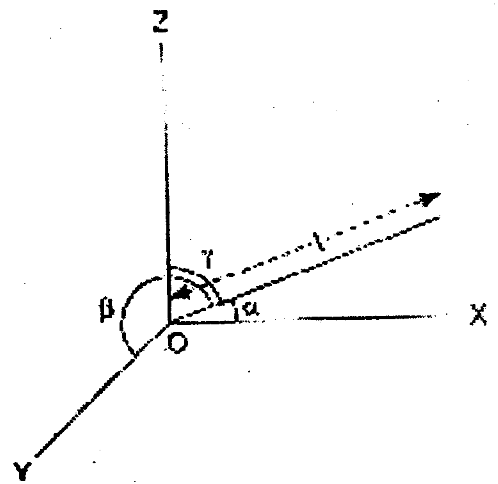


Fig:(6.8)

Here $\cos\alpha$ $\cos\beta$ and $\cos\gamma$ are direction cosines(fig.6.8). they obey the relation,

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$$

$$\left(\frac{n_x\lambda}{2l}\right)^2+\left(\frac{n_y\lambda}{2l}\right)^2+\left(\frac{n_z\lambda}{2l}\right)^2=1$$

$$\Rightarrow n_x^2+n_y^2+n_z^2=\left(\frac{2l}{\lambda}\right)^2 \quad \text{.................(6.12)}$$

This equation represents an ellipsoid or sphere of radius $\frac{2l}{\lambda}$. Each set of values of n_x, n_y, n_z satisfying the above equation corresponds to one mode of vibration.

The total number of modes of vibrations is the total number of possible set of $(n_x, n_y\ n_z)$

First of all, let us consider the number of modes of vibrations in two dimensional plane. For two dimensional plane equation (6.14) becomes,

$$n_x^2+n_y^2=\left(\frac{2l}{\lambda}\right)^2$$

This equation represents the circle when we plot n_x along X-axis and n_y along Y-axis. Each point of intersection represents one mode of vibration. Since only positive values of n_x, n_y are allowed, we have to consider in positive quadrant only (fig.6.9). The area of each square is unity.

Hence the number of Squares is equal to 1/4 th of the area of the circle and it is equal to number of modes of vibrations.

Number of modes of vibrations.

$$=\frac{1}{4}\pi\left(\frac{2l}{\lambda}\right)^2=\frac{\pi l^2}{\lambda^2}$$

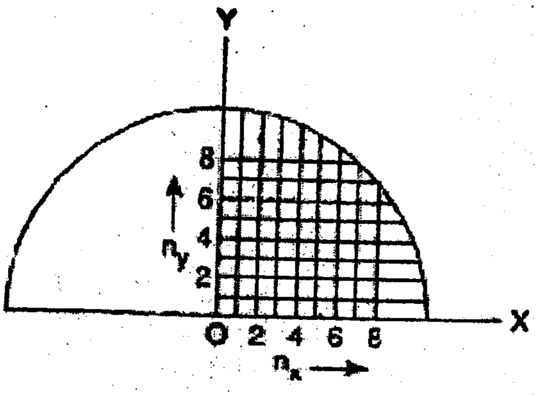


Fig:(6.9)

If we extend the above idea to3 – dimensional space, the circle will be a sphere and each unit square will be a unit cube. Now the number of modes of vibrations upto wave length λ is equal to 1/8th of **the volume of the sphere.**

Number of modes of vibrations $= \frac{1}{8}\times\frac{4\pi}{3}\times\left(\frac{2l}{\lambda}\right)^3$

$$=\frac{1}{8}\times\frac{4\pi}{3}\times\frac{8l^3}{\lambda^3}=\frac{4\pi l^3}{3\lambda^3}\text{.....................}(6.15)$$

Volume of the cube = l^3

Number of modes of vibrations per unit volume $=\frac{4\pi}{3\lambda^3}$

Number of modes of vibrations having the wave length in between λ and $\lambda+d\lambda$ per unit volume

$$=d\left(\frac{4\pi}{3\lambda^3}\right)=\frac{4\pi}{3}(-3).\lambda^{-4}.d\lambda=\frac{-4\pi}{\lambda^4}.d\lambda$$

$$=\frac{4\pi}{\lambda^4}.d\lambda$$

As the transverse electromagnetic waves have two polarizations for each mode, the total number of modes of vibrations is given by

$$f=\frac{8\pi.d\lambda}{\lambda^4}\text{...................}(6.16)$$

According to law of equipartition of energy, the average energy for each value of vibration is KT, where K is Boltzmann's constant and T is absolute temperature.

The energy between the wavelengths λ and $\lambda+d\lambda$

$$E_\lambda.d\lambda=\frac{8\pi d\lambda}{\lambda^4}.kT$$

$$\Rightarrow E_\lambda.d\lambda=\frac{8\pi d\lambda}{\lambda^4}.kT\text{.............}(6.17)$$

We have $v=\frac{c}{\lambda}\Rightarrow\lambda=\frac{c}{v}\Rightarrow|d\lambda|=\frac{-c}{v^2}dv$

$$\Rightarrow d\lambda=\frac{c}{v^2}dv$$

Writing these values in equation (6.16) we can obtain number of modes of vibration having the frequency in between v and $v+dv$ as,

$$f=\frac{8\pi}{\left(\frac{c}{v}\right)^4}\times\frac{c}{v^2}dv=\frac{8\pi cv^4dv}{c^4v^2}$$

$$f=\frac{8\pi v^2dv}{c^3}\text{.........}6.18$$

Taking the average energy as KT from the law of equipartition of energy, the energy between the frequencies v and $v+dv$

can be written as

$$E_\lambda d\lambda=\frac{8\pi kTv^2dv}{c^3}\text{...........}6.19$$

6.7 Planck's radiation law :

In 1900, Max. Planck introduced the revolutionary concept of radiation known as quantum theory of radiation. He made the following assumptions.

1. A black body radiator contains simple harmonic oscillators of possible frequencies.
2. The oscillators cannot emit or absorb energy continuously which is contradiction to electro magnetic theory.
3. Emission or absorption of energy takes place in discrete amounts. For an oscillator of frequency ν, the energies are O, h ν , 2hν , ... It means, it is the integral multiple of small unit of energy hν which is called as quantum or photon. For an oscillator of frequency ν the energy is given by

$E = nh\upsilon \rightarrow (6.20)$ Where n=0,1,2,3,...............

And h is Planck's constant.

Average energy of an oscillator:

If N is the total number of oscillators in the black body and E is the total energy of these oscillators, the average energy can be written as,

$$\bar{\in} = \frac{E}{N} ------6.21$$

Let N₀,N₁,N₂.........Nr,..... be the number of oscillators having the energy $0, \in, 2\in, ...r\in ....$

The total energy of all the oscillators is

$$E = (N_O \times O) + (N_1 \times \bar{\in}) + (N_2 \times 2\bar{\in}) + (N_3 \times 3\bar{\in}) + ----- + (Nr \times r\bar{\in}) + ---- \quad ....(6.22)$$

total number of oscillators in the black body is

$$N = N_0 + N_1 + N_2 + N_3 + ...N_r + ...(6.23)$$

According to Maxwell's distribution formula, the number of oscillators having energy $r\in$ is given by

$$N_r = N_0.e^{-e\frac{r\bar{\in}}{kT}}$$

The number of oscillators having the energy o is $N_0 = N_0.e^0 = N_0$

The number of oscillators having the energy 1 is $N_1 = N_0.e^{-\frac{\in}{kT}}$

The number of oscillators having the energy 2 is $N_2 = N_0.e^{-\frac{2\epsilon}{kT}}$

writing these values in equation (6.23)we get,

$$N = N_0 + N_0.e^{-\frac{\epsilon}{kT}} + N_0.e^{-\frac{2\epsilon}{kT}} + ... + N_0.e^{-\frac{r\epsilon}{kT}} + ...$$

$$= N_0\left(1 + e^{-\frac{\epsilon}{kT}} + e^{-\frac{2\epsilon}{kT}} + ... + e^{-\frac{r\epsilon}{kT}}\right)$$

$$= N_0\left(1 - e^{-\frac{\epsilon}{kT}}\right)$$

$$\therefore 1 + x + x^2 + ... = (1-x)^{-1}$$

$$N = \frac{N_0}{1 - e^{-\frac{\epsilon}{kT}}} ....(6.24)$$

writing these values of $N_0, N_2, N_3,..........$ in equation (6.22)we get the total energy of all the oscillators as below.

$$E = \in N_O.e^{\frac{-\in}{kT}} + 2\in N_O e^{\frac{-2\in}{kT}} + 3\in .N_O\, e^{\frac{-3\in}{kT}} + ------+-----+ r\in N_o e^{\frac{-r\in}{kT}} ----$$

$$= \in N_O.e^{\frac{-e}{kT}}\left[1 + 2e^{\frac{-E}{kT}} + 1 + 3e^{\frac{-2\in}{kT}} + ----+ r.e^{\frac{-(r-1)\in}{kT}} + ----\right] \qquad -----6.25$$

$$E = \in N_O.e^{\frac{-e}{kT}}\left[1 - 2e^{\frac{-\in}{kT}}\right]^{-2} \qquad \left[\because 1 + 2x + 3x^2 + ---(1-x)^{-2}\right]$$

writing equations (6.24) and (6.25) and equation (6.21) we get,

$$\bar{\in} = \frac{\dfrac{\in N_0.e^{-\frac{v}{kT}}}{\left(1 - e^{\frac{\epsilon}{kT}}\right)^2}}{\left(\dfrac{N_0}{1 - e^{\frac{\epsilon}{kT}}}\right)} = \frac{\varepsilon.e^{\frac{-\in}{kT}}}{1 - e^{\frac{-\in}{kT}}} = \frac{\varepsilon.}{\dfrac{1}{e^{\frac{-\in}{kT}}} - 1} = \frac{\varepsilon.}{e^{\frac{\epsilon}{kT}} - 1}$$

we know that $\varepsilon = h\nu$

$$\bar{\in} = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1} ....(6.26),$$ It is the expression for the average energy of oscillator.

Planck's formula :-

We know that the number of oscillators per unit volume in the frequencies range ν and $\nu + d\nu$ is given by

$$f = \frac{8\pi \upsilon^2 d\upsilon}{C^3} \rightarrow (6.27)$$

The energy per unit volume between the frequencies ν and $\nu + d\nu$ can be obtained by multiplying equations (6.26) & (6.27) is

$$E_\nu d\nu = \frac{8\pi \upsilon^2 d\upsilon}{C^3} \times \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$$

$$E_\nu d\nu = \frac{8\pi h\upsilon^3 d\upsilon}{C^3\left(e^{\frac{h\nu}{kT}} - 1\right)} \rightarrow (6.28)$$

This is known as Planck's radiation law.

$$\upsilon = \frac{c}{\lambda} \Rightarrow d\upsilon = \frac{-c}{\lambda^2} d\lambda$$

$$|d\nu| = \frac{c}{\lambda^2} d\lambda$$

writing these values in equation (6.28) we get, the Planck's radiation law in terms of wavelength,

$$E_\lambda d\lambda = \frac{8\pi h}{c^3} \times \frac{c^3}{\lambda^3} \times \frac{c}{\lambda^2} d\lambda \times \frac{1}{\left(e^{\frac{hc}{\lambda kT}} - 1\right)}$$

$$E_\lambda d\lambda = \frac{8\pi hc d\lambda}{\lambda^5 \left[e^{\frac{hc}{\lambda KT}-1}\right]} \text{..............} (6.29)$$

This is known as Planck's radiation law.

It represents energy per unit volume between the wavelength λ and $\lambda + d\lambda$. This formula agrees well with the experimental curves throughout the whole range of wavelengths. This formula converts into,

(i) Wein's formula for shorter wavelength.

(ii) Rayleigh-Jean's law for longer wavelength.

(iii) Wein's displacement law for maximum energy radiation.
(iv) Stefan – Boltzman's law for total radiation.

6.8 Deduction of laws from Planck's radiation law :-

(i) Wein's formula :-

We have the expression for the energy between the wavelengths λ and $\lambda + d\lambda$ as

$$E_\lambda d_\lambda = \frac{8\pi hc d\lambda}{\lambda^5\left(e^{\frac{hc}{\lambda kT}} - 1\right)} \text{.............}(6.29)$$

If "λ" is small, then $\frac{hc}{\lambda kT}$ is more

$$\frac{hc}{\lambda kT} >> 1$$

Due to this one can be neglected when compared to $e^{\frac{hc}{\lambda kt}}$

$$E_\lambda d_\lambda = \frac{8\pi hc.d\upsilon}{\lambda^5 \times e^{\frac{hc}{\lambda kT}} - 1}$$

Let $8\pi hc = A$ and $\frac{hc}{K} = B$

$$E_\lambda d\lambda = A.\lambda^{-5}.e^{-\frac{B}{\lambda T}} d\lambda$$

It is wein's formula.

(ii)Rayleigh – Jeans law :-

If "λ" is more, $\frac{hc}{\lambda kT}$ is small

$$e^{\frac{hc}{\lambda kT}} = 1 + \frac{hc}{\lambda kT} + \left(\frac{hc}{\lambda kT}\right)^2 + ----$$

$$e^{\frac{hc}{\lambda kT}} = 1 + \frac{hc}{\lambda kT}$$

neglecting higher order terms.

writing this in equation (6.29) we get,

$$E_\lambda d_\lambda = \frac{8\pi hc d\lambda}{\lambda^5\left(1 + \frac{hc}{\lambda kT} - 1\right)} = \frac{8\pi hc}{\lambda^5} \cdot \frac{\lambda kT}{hc}$$

$$E_\lambda d_\lambda = \frac{8\pi kT d\lambda}{\lambda^4}$$, It is Rayleigh-Jeans law.

(iii) Wein's displacement law :-

For the wavelength corresponding to maximum energy we can use the condition $\frac{dE_\lambda}{d_\lambda}=0$

We have $E_\lambda = \frac{8\pi hc}{\lambda^5\left(e^{\frac{hc}{\lambda kT}}-1\right)} = 8\pi hc\lambda^{-5}.\left[e^{\frac{hc}{\lambda kT}}-1\right]^{-1}$

$$\frac{dE_\lambda}{d\lambda} = A\left[\lambda^{-5}.(-1)\left(e^{\frac{hc}{\lambda kT}}-1\right)^{-2} e^{\frac{hc}{\lambda kT}} \times \frac{(-1)hc}{\lambda^2 kT} + (-5)\lambda^{-6}\left(e^{\frac{hc}{\lambda kT}}-1\right)^{-1}\right] = 0$$

$\because A \# 0,$

$$\frac{1}{\lambda^5}\times\frac{hc}{\lambda^2 kT}\times\frac{e^{\frac{hc}{\lambda kT}}}{\left(e^{\frac{hc}{\lambda kT}}-1\right)^2} - \frac{5}{\lambda^6}\times\frac{1}{\left(e^{\frac{hc}{\lambda kT}}-1\right)} = 0$$

multiplaing with $\lambda^6 \times \left(e^{\frac{hc}{\lambda kT}}-1\right)$ we get

$$\frac{hc}{\lambda\ kT}.\frac{e^{\frac{hc}{\lambda kT}}}{\left(e^{\frac{hc}{\lambda kT}}-1\right)} - 5 = 0$$

$$\frac{hc}{\lambda\ kT} = x$$

$$x.\frac{e^x}{e^x-1} - 5 = 0$$

The solution to this equation are x=0 and x=4.965

But $x = \frac{hc}{\lambda kT} = 0$ is not allowed

$$x = \frac{hc}{\lambda kT} = 4.965 \Rightarrow \frac{hc}{4.965} = \lambda T$$

$$\lambda T = \frac{hc}{4.965k} = 0.2896\times10^{-2}\,m.k$$

λT = constant

Since it is the wavelength corresponding to maximum energy we can write,

$\lambda_m.T$ = constant

$$\Rightarrow \lambda_m . T = \lambda_m^1 . T^1$$

(iv)Stefan's Law :-

According to Planck's formula in terms of frequency we have,

$$E_v d_v = \frac{8\pi h v^3 d\upsilon}{c^3\left(e^{\frac{hv}{kT}} - 1\right)}$$

The spectral radiance E_v (spectral) and the energy density E_v are related by the relation

$$(E_v)spectral = \frac{c}{4}.E\upsilon$$

$$(E_v)spectral.d\upsilon = \frac{c}{4}.E\upsilon.d\upsilon$$

$$= \frac{c}{4}\frac{8\pi h\upsilon^3 d\upsilon}{c^3\left[e^{\frac{hv}{kT}} - 1\right]}$$

The total radiant energy due to all frequencies is given by

$$E = \int_0^{\alpha} (E_v)spectral.d\upsilon$$

$$E = \int_0^{\alpha} \frac{c}{4}.\frac{8\pi h\upsilon^3 d\upsilon}{c^2\left[e^{\frac{hv}{kT}} - 1\right]}$$

Let $\frac{hv}{kT} = x$, $v = \frac{kTx}{h}$

writing these values in the above equation we get,

$$E = \int_0^{\alpha} \frac{2\pi h}{c^2(e^x - 1)} \times (\frac{kTx}{h})^3 \times \frac{kTdx}{h} = \frac{2\pi h}{c^2} \times (\frac{k^3T^3}{h^3}) \times \frac{kT}{h} \times \int_0^{\infty} \frac{x^3 dx}{e^x - 1}$$

$$= \frac{2\pi}{c^2} \times \frac{k^4}{h^3} \times T^4 \times \frac{\pi^4}{15} \left[\because \int_0^{\alpha} \frac{x^3 \, dx}{e^x - 1} = \frac{\pi^4}{15} \right]$$

$$E = \frac{2\pi^5 k^4}{15 c^2 h^3} \times T^4 \Rightarrow E = \sigma T^4;$$

$$when \ \ \sigma = \frac{2\pi^5 k^4}{15 c^2 h^3} = 5,67 \times 10^{-8} \, walts / m^2 = k^4$$

It is stefan's law.

6.9 Pyrometers :-

Pyrometers are the instruments which are used to measure the high temperatures. There are two types of pyrometers.

1. *Total radiation pyrometer* :

The instruments that measure total radiation emitted by the body under test are called as total radiation pyrometers. Here the radiations of all wavelengths are considered. Here the temperature is measured making use of Stefan's law.

2. *Optical or Spectral Pyrometer* :-

The Optical pyrometers compare the intensity of radiation of a certain wavelength emitted by the body with that the radiation of same wavelength emitted by a standard body at a known temperature. The temperature of the body is obtained by applying wein's plancks displacement law or planks law.

6.10. Ferry's total radiation pyrometer :-

The apparatus consists of a large concave mirror C of copper and nickered on the front surface. The mirror is provided with a small hole at its centre and an eye is fitted in it. The mirror can be displaced from its one position to another position with the help of rack – pinion screw. Before the mirror C, two plane mirrors M_1, and M_2 are arranged. They are inclined at an angle of 5^0 and separated by a distance of 1.5mm. In the gap, a blackened strip S is arranged. The mirrors

M_1 and M_2 and the strip S are placed in an enclosure in order to prevent the direct incidence of radiation from the hot body on these substances. The blackened strip S is connected to one junction of a thermocouple. Depending upon the deflection in milli voltmeter, the temperature of the hot body can be estimated.

The radiation from the hot body for which the temperature is to be determined ate allowed to incident on concave mirror C. By moving the concave mirror C to a suitable position the images of the two mirrors overlap exactly to form a complete circle as shown in fig.(6.10).

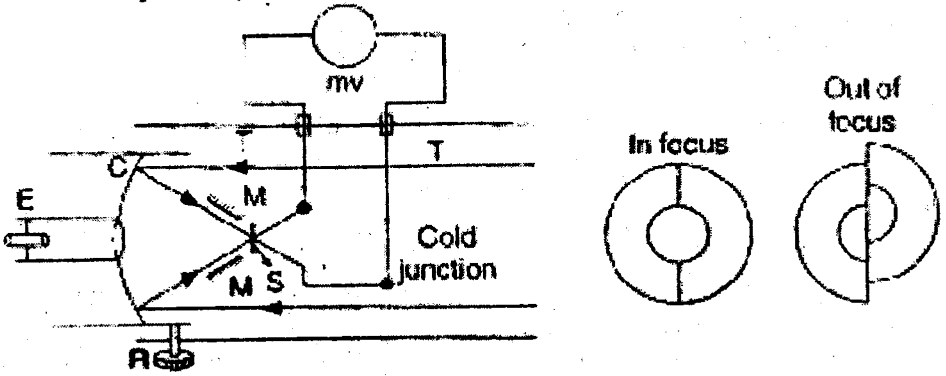


Fig:(6.10)

If T and To are absolute temperatures of the hot body and that of the pyrometer case and θ is the deflection in milli voltmeter, we can write, $\theta \propto T^k$

But the power lies in between 3.8 and 4.2

Hence we can write

$$\theta \propto T^k$$
$$i.e., \theta \propto T^k$$
$$Log\theta \propto LogT^k$$
$$Log\theta \propto kLogT$$
$$Log\theta = ckLogT;$$

where c is constant

Initially, using bodies whose temperatures are accurately known, the deflections in the milli voltmeter are noted. A graph is drawn between $\log\theta$ and log T. The graph comes out to be a straight line and it is called calibration graph.

An unknown temperature can be obtained with the help of this graph, taking the corresponding deflection in milli voltmeter experimentally.

6.11 Modification for the measurements of very high temperatures (Rotating sector method):-

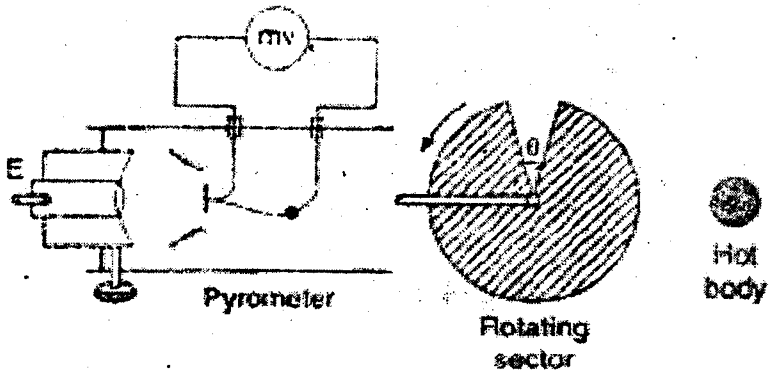


Fig:(6.11)

In order to measure the very high temperatures a rotating sector is used. This is an opaque circular disc in which apart is cut off in the shape of a sector. This is placed in between the hot body and pyrometer (fig.6.11) and made to rotate rapidly on the axis of the eye piece. The sector allows only a fraction of total rotation is equal to $\frac{\theta}{360}$ to enter the pyrometer, where θ is the angle of the sector.

Due to the radiation coming from the sector we can measure the temperature less than the actual temperature of the body. If the pyrometer measures the temperature T_1 and the actual temperature of the hot body is T_2, they can be related by

$$\frac{T_1^4}{T_2^4}=\frac{\theta}{360}\Rightarrow\frac{T_2^4}{T_1^4}=\frac{360}{\theta}\Rightarrow\left(\frac{T_2}{T_1}\right)=\left(\frac{360}{\theta}\right)^{\frac{1}{4}}$$

$$\Rightarrow T_2=T_1\left(\frac{360}{\theta}\right)^{\frac{1}{4}}$$

6.12Disappearing filament optical pyrometer:-

The apparatus(fig.6.12) consists of a telescope filled with an objective 'O' at one and an eyepiece 'E' at other end. The distance between objective and eye –piece can be adjusted using rack and pinion arrangement. The point at which the light rays using coming from the objective are incidenting the cross wires are to be arranged.

The cross wires are replaced by an electric lamp S. The filament F of the lamp is connected to battery of emf B, key K, rheostat Rh and ammeter A in series. The filament is heated by passing current through it. Diaphragms D_1 and D_2 are arranged to limit the core of radiation. The red glass filter G is placed before the eye piece E in order to observe the radiation of particular wave length.

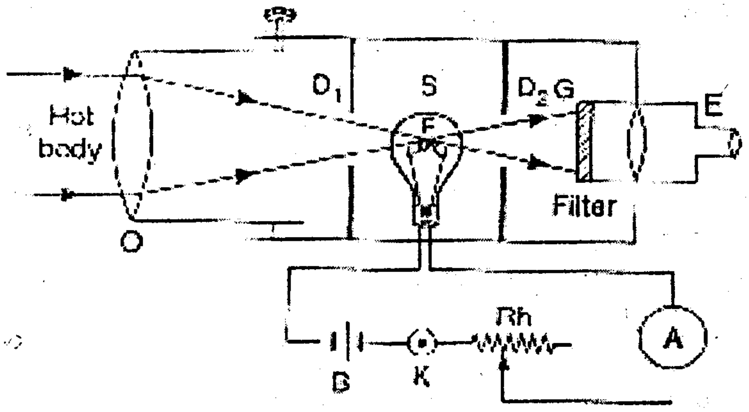


Fig:(6.12)

The pyrometer is turned towards the hot body. The radiations coming from the hot body are made to focus on the filament by adjusting the position of objective with the help of rack and pinion arrangement. The image is viewed with the help of eyepiece E. The current passing through the filament is increased gradually. Due to this its temperature increases. The current through the filament is adjusted until the filament just disappears.When the filament just disappears. The current in the filament are recorded with the help of ammeter. At this stage both filament and image are equally bright.

If T is the temperature of the hot body of filament and I is the current flowing through the filament, they are related by

$$I=a+bT+cT^2$$

Where a , b and c are constants.

The constants a ,b and c are calculated as below.

Three hot bodies whose temperatures are accurately known are taken. The pyrometer is turned towards each hot body and current in the ammeter is noted when the filament disappears, at each time. Substituting these values in the above equation we can obtain three equations. on solving these three equations we can obtain the values of a ,band c.

Now the pyrometer is turned towards the hot body of unknown temperature. The current I in the ammeter is noted when the filament just disappears. Writing the values of a, b, c and I in the equation $I = a + bT + cT^2$;

The temperature of the hot body can be determined.

6.13Solar constant and its determination:-

Sun is radiating energy in all directions. The earth receives only a fraction of this energy. A considerable portion of the incoming radiation is lost by reflection and scattering by the atmosphere. The radiation is also heavily absorbed by the earth's atmosphere. The radiation absorbed by the earth atmosphere depends upon the time of the day and the season of the year. Thus we need a more constant quantity regarding the solar radiation received by the earth. It is termed as solar constant.

Solar constant is defined as the rate at which the solar energy is received by a black surface per unit area placed normal to the sun rays at the average distance of the earth from the Sun in the absence of the atmosphere. It's value is $1340 w/m^2$ (or) $1.92 cal/cm - \min$

Angstrom pyrohelimeter for the determination of solar constant

The instruments which is used to determine the solar constant is called pyroheliometer

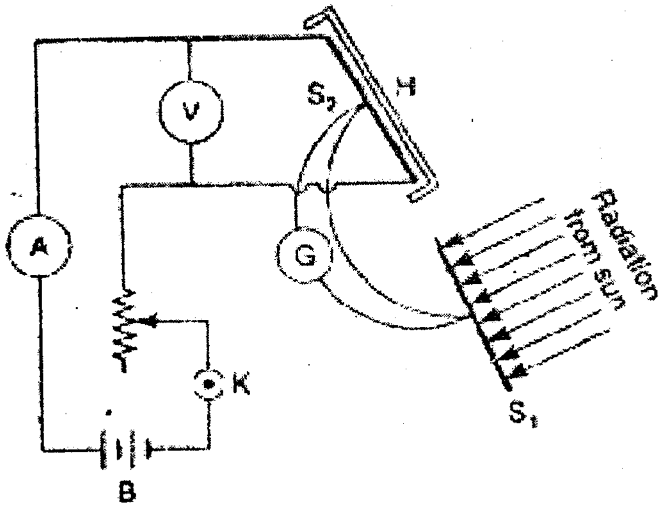


Fig:(6.13)

Angstrom pyrohelimeter is as shown fig (6.13)

The apparatus consists of two thin exactly similar blackened strips S_1 and S_2 of platinum or constantan. The two strips are arranged such that one is open to receive radiation from the sun normally while the other is protected by a double walled shield H. The two strips S_1 and S_2 are connected to the two junctions of a thermo couple by means of a galvanometer G. The metals in the thermo couple are made of copper and constantan. The strip S_2 is heated by passing electric current through it. A voltmeter V is connected in parallel and an ammeter I is connected in series to measure the voltage and current. When both S_1 and S_2 are at the same temperature, the galvanometer shows no deflection. Now S_1 is exposed for sun radiation, the galvanometer shows deflection. Now current is passed through and the strength of current is adjusted using rheostat R h such that the galvanometer shows no deflection. At this stage the heat received by both S_1 and S_2 are equal. Now the solar constant can be calculated using the formula

$$S = \frac{VI \times 60}{Aa \times 4.2}\ {cal}\big/{cm^2 - \min ute}$$

Where 'V' is voltmeter reading, 'I' is ammeter reading, 'A' is area of cross section of strip and 'a' is its absorption coefficient.

The experiment was repeated several times on the same day under constant sky conditions with different elevations of the sun. The average value of solar constant was calculated. The same experiment was repeated throughout the year. If 'S' is the solar constant obtained S_0 is the true value of solar constant and Z is the zenith distance i.e., angular altitude and τ is the transition coefficient, the relation between them is

$$S = S_0 \tau^{\sec z}$$
$$\log S = \log S_0 \tau^{\sec z}$$
$$\log S = \log S_0 + \log \tau^{\sec z}$$
$$\log S = \log S_0 + \sec z \log \tau$$

It is in the form of $y = mx + c$

From the experimental data, the graph is drawn in between logs and Secz. A straight line is obtained as shown fig . (6.14) . The Y intercept value gives log So.

From this, the value of so can be calculated. It is obtained as 1.92 cal/cm^2-m (or) 1340w/m^2

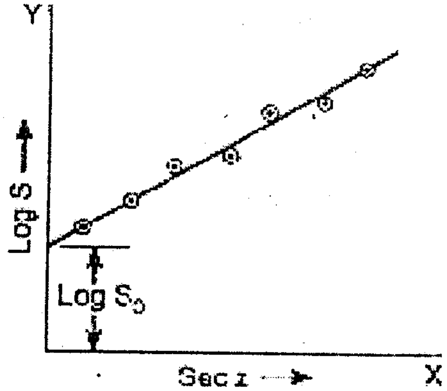


Fig:(6.14)

6.14 Temperature of the Sun:-

Different temperature regions of the sun is shown in fig. (fig.6.15) The temperature of the central core of the sun is at about 20 million degrees. The boundary of the central core is a thick shell called the photosphere. Temperature of the sun usually means the temperature of photosphere which is around 6000 K . The next 300 to 400 Km thick layer is called as reversing layer which contains most of the elements in gaseous state. The extension of reversing layer is chromosphere. This is only seen during total solar eclipse. The outer gaseous layer which extends to a more than half a million Kilometers is called corona.

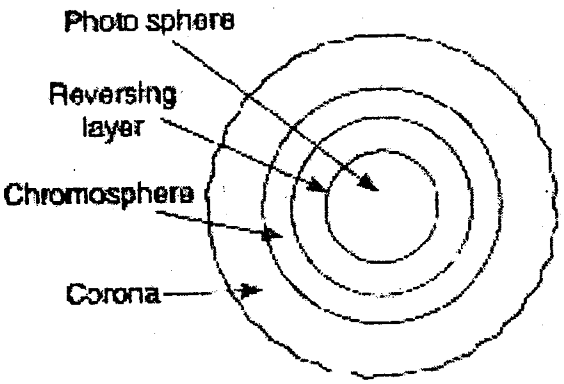


Fig:(6.15)

Let R be the radius of photosphere of the sun. Then its surface area is $4\pi R^2$.

Let T be the absolute temperature of the sun. Then according to stefan's law, the amount of energy emitted by the sun per minute is given by

$E = 4\pi R^2 \sigma T^4 \times 60,$ *where* $'\sigma'$ is stefan's constant_____________(6.30)

This energy is spread in all directions. Let r be the average distance between the sun and the earth. Let us consider a sphere of radius r with sun as the centre. The radiant energy coming from the sun will spread over the surface area $4\pi r^2$. If S is solar constant, the energy received by the surface area $4\pi r^2$ per minute is $4\pi r^2 s$

From equations (6.30) and (6.31) we can write,

$$4\pi R^2 \sigma T^4 \times 60 = 4\pi r^2 s$$

$$T^4 = \frac{r^2 s}{R^2 \sigma 60}$$

$$T = \left[\left(\frac{r}{R}^2\right) \times \frac{1}{60} \times \frac{s}{\sigma}\right]^{\frac{1}{4}}$$

In S .I . Units $T = \left[\left(\frac{r}{R}^2\right) \times \frac{1}{60} \times \frac{s}{\sigma}\right]^{\frac{1}{4}}$

Substituting the values of r ,R ,S and σ ,the temperature of the sun can be calculated.

6.15solved problems:-

1. Determine the temperature of the sun with the help of wien's law.

Given $(b = 2.92\times10^{-3} m-k; \lambda_{\max} = 4900A^0)$

$$\lambda_m = 4900A^0 \Rightarrow 4900\times10^{-10} m = 4.9\times10^{-7} m$$

$$b = 2.92; \qquad T = ?$$

Solution:- $b = \lambda m T$

$$\Rightarrow T = \frac{b}{\lambda m} = \frac{2.92\times10^{-3}}{4.9\times10^{-3}} = 5959K$$

$$\therefore T = 5959K$$

2. A body at 1500K emits maximum energy at a wavelength 20,000 A^o . If the sun emits maximum energy at a wave length 5500 A^o then what will be the temperature of the sun.

$$T_1 = 1500K; \qquad \lambda m = 20,000A^0$$

$$T^1 = ?; \qquad \lambda m^1 = 5500A^0$$

Solution:

$$\lambda m T = \lambda m^1 T^1 \Rightarrow T^1 = \frac{\lambda m.T}{\lambda m^1} = \frac{20,000}{5500}\times1500K$$

$$T^1 = 5454K$$

3.A black body at 1500K radiates maximum wavelength of 2 microns, if the wavelength of maximum energy of moon is 14 microns, what is the temperature of the moon.

Solution:

$T_1 = 1127^0 c \Rightarrow 1127+273 = 1400K$

$\lambda m = 2microns = 2\times10^{-6} m$

$\lambda m^1 = 14microns \Rightarrow 14\times10^{-6} m$

$T_1^1 = ?$

$$\lambda m T_1 = \lambda m^1 T_1^1 \Rightarrow T_1^1 = \frac{\lambda m.T_1}{\lambda m^1} = \frac{2\times10^{-6}\times1400}{14\times10^{-6}} = 200K$$

$$T^1 = 200K$$

4. Calculate the surface area of tungsten filament of 100 watts electric bulb. The operating temperature of tungsten is 2450K and emissivity is 0.3

Solution:

$E = 100W; T = 2450K; e = 0.3$

$E = e.\sigma.T^4 \qquad \sigma = 5.67\times10^{-8}$

m^2

$E = A.e.\sigma.T^4$

$A = \frac{E}{e.\sigma.T^4} = \frac{100}{0.3\times5.67\times(2450)^4\times10^{-8}}$. where A is area

$A = 1.6316\times10^{-4}\,m^2$

5. Calculate the maximum amount of heat lost per second by radiation by a sphere 10 Cm in diameter at a temperature of 227^oC when placed in an enclosure at $27^oC(\sigma = 5.67\times10^{-8}W/m^2 - k^4)$

Solution:

$T = 227^0c = 227 + 273 = 500K$

$T_0 = 27^0c = 27 + 273 = 300K$

$2r = 10cm = r = 5cm = 5\times10^{-2}\,m$

$\sigma = 5.67\times10^{-8}W/m^2 - K^4$

$Q = ?$

According to stefan's law $E = \sigma(T^4 - T_0^4)$

The net rate of loss of heat (or) by the total surface area of the sphere is given by

$Q = E\times4\pi r^2 = \sigma\left(T^4 - T_0^4\right)\times4\pi r^2$

$= 5.67\times10^{-8}\left[500^4 - 300^4\right]\times4\times3.14\times(5\times10^{-2})^2$

$Q = 0.0969\,{}^{J}\!/_{S} = 0.0969\,watt$

6. The luminosity of a star is 20000 times that of the sun. temperature T of sun is 6000K. Find the surface temperature of sun star.

Solution:

$$E_1 = 20000.E_2 ; T_2 = 6000K ; T_1 = ?$$

$$E_1 = \sigma T_1^4$$

$$E_2 = \sigma T_2^4$$

$$\frac{E_1}{E_2} = \frac{T_1^4}{T_2^4} = \left(\frac{T_1}{T_2}\right)^4 \Rightarrow \frac{T_1}{T_2} = \left(\frac{E_1}{E_2}\right)^{\frac{1}{4}}$$

$$T_1 = T_2\left(\frac{E_1}{E_2}\right)^{\frac{1}{4}}$$

$$\frac{T_1}{T_2} = [20000]^{\frac{1}{4}} = 11.89, T_1 = 11.89 \times 6000 = 71352.4K$$

$$T_1 = 71352.4\ K$$

7.Calculate the temperature of sun from the following data adius of the sun $(R) = 7.92 \times 10^5\ km$, distance of sun from earth $r = 1.5 \times 10^8\ Km$ and stefan's constant $\sigma = 5.7 \times 10^{-8} W/m^2 - K^4 \quad S = 1.34 KW/m^2$,

Solution:

$$T = \left[\frac{r^2}{R^2}.\frac{S}{\sigma}\right]^{\frac{1}{4}} = \left[\left(\frac{r}{R}\right)^2.\frac{S}{\sigma}\right]^{\frac{1}{4}}$$

$$= \left[\left(\frac{1.5 \times 10^8}{7.92 \times 10^5}\right)^2 \times \left(\frac{1.34 \times 10^3}{5.7 \times 10^{-8}}\right)\right]^{\frac{1}{4}} = \left(834.56 \times 10^{12}\right)^{\frac{1}{4}} = 5374.8k$$

8. Calculate the average energy of an oscillator of frequency $5.6 \times 10^{12}\ \sec^{-1}$ at 330k .Treating it as (i) classical oscillator (ii) planks oscillation

Solution: $\nu = 5.6 \times 10^{12}\ \sec^{-1} = 5.6 \times 10^{12}\ Hz$

$$T = 330K$$

(1) Classical oscillator

$$\bar{\epsilon} = KT = (1.38 \times 10^{-23} \times 330)$$

$$= 44.54 \times 10^{-23} = 4.454 \times 10^{-21} J$$

(2)Planck's oscillator

$$\bar{\epsilon} = \frac{h\upsilon}{e^{\frac{h\upsilon}{KT}}-1}$$

$$h\upsilon = 6.62\times10^{-34}\times5.6\times10^{12} = 37.07\times10^{-34}$$

$$\frac{h\upsilon}{KT} = \frac{37.07\times10^{-34}}{4.454\times15^{-21}} = 8.223\times10^{-13}$$

$$\bar{\epsilon} = \frac{37.07\times10^{-34}}{e^{8.323\times10^{-3}}-1} = 2.945\times10^{-21}J$$

9.Calculate the number of modes of vibrations in wavelength range 5000 A^0 to 5002 A^0 for radiation in a chamber of volume 100 cm^3

Solution:

$$V = 100cm^3 = 100\times10^{-6}m^3 = 10^{-4}m^3$$

$$\lambda_1 = 5000A^0, \lambda_2 = 5002A^0$$

$$d\lambda = \lambda_2 - \lambda_1 = 2A^0 = 2\times10^{-10}m$$

$$\lambda = \frac{\lambda_1+\lambda_2}{2} = 5001A^0 = 5001\times10^{-10}m$$

$$f = \frac{8\pi V d\lambda}{\lambda^4} = \frac{8\times3.14\times10^{-4}\times2\times10^{-10}}{\left(5001\times10^{-10}\right)^4}$$

$$f = \frac{50.24\times10^{-14}}{\left(5.001\times10^{-7}\right)^4} = 0.0803\times10^{14} = 8.03\times10^{12}$$

$$f = 8.03\times10^{12}\,Hz$$

6.16Summary:-

In this lesson the types of black bodies are discussed. The energy distribution for black body radiation was explained by various laws like wien's displacement law, Rayleigh –Jean's law and Planck's quantum radiation law. Pyrometers are discussed to measure high temperatures. Solar constant is determined and the expression for the temperature of the sun was obtained.

6.17Keywords:

Black body, Absorption,. radiation, energy distribution, quantum theory, pyrometer, solar constant.

6.18.Self Assessment Questions

Long answer questions:-

1. What is a black body ? Explain Lummer and pringsheim experiment for energy distribution of black body radiation.
2. Derive wien's displacement law.
3. Derive Rayleigh –Jeans law.
4. Derive plancks radiation law. Obtaine various laws from it.
5. Explain types of pyrometers. Explain filament disappearing optical pyrometer.

Short answer questions:-

1. Write a short notes on back body.
2. What is solar constant ? How it can be determined.
3. Derive the expression for temperature of the sun.

Exercises:

1. In an atomic explosion, the maximum temperature produced was of the order of $10^7 k$ Calculate the wavelength of maximum energy. Wien's constant 0.292 $cm-k$

Hint: $\lambda_m T = b$

Ans:2.92A^0

2. An energy of 8.25cal/sec is radiated from an area of 6.1cm^2 in furnace. Assuming the furnace to be a back body find its temperature.

$\left[\text{Hint:}E = eA\sigma T^4 \qquad Ans = 1000.4k\right]$

3.A 8cm diameter copper sphere is smeared with black spot. This is heated to 627c. What amount of heat energy is radiated per second.

$\left[\text{Hint:}E = eA\sigma T^4 \qquad Ans = 752.34 watt\right]$

4.A sphere of 20cm diameter at a temperature of $327^\circ C$ placed in an evacuated vessel at a temperature of $27^\circ C$. Calculate the heat lost by the sphere per second by radiation. Assume the sphere to be a black body.

$\sigma = 5.7 \times 10^{-12}\ {}^{J}\!/_{cm^2 - s - des^4}$

$\left[\text{Hint:} E = eA\sigma\left(T^4 - T_0^4\right) \qquad Ans: 869.81J\right]$

5. Calculate the surface temperature of the sun from the following data

$$S = 13.4\ {}^{W}\!/_{m^2}$$
$$R = 6.92 \times 10^{8}\ m$$
$$r = 1.5 \times 10^{11}\ m$$
$$\sigma = 5.67 \times 10^{-8}\ {}^{W}\!/_{m^2 - k^4}$$

$$\text{Hint: } T = \left[\left(\frac{r}{R}\right)^2 \times \frac{s}{\sigma}\right]^{\frac{1}{4}} \qquad Ans: 5773K$$

The activity of a black body radiator is in the shape of cube. Find the number of modes of vibrations per unit volume in the wave length range between 4995 A^0 and 5005 A^0

$\text{Hint:} f = \dfrac{8\pi d\lambda}{\lambda^4} \qquad Ans: 4.019 \times 10^{17}\Big]$

6.19 Reference books:

Unified physics	Dr.S.L.Gupta& Sanjeev Gupta	JaiprakashNath & Co Meerut 2005	
Heat and thermodynamics	Brijlal and Subrahmanyam	S. Chand & Co	
B.Sc. Heat	Telugu Academy		
Thermodynamics	D.C.Tayal	Himalaya Publications	
D.S.Mathure's Heat and Thermodynamics	Revised by Dr.M.N.BAPAT	SULTAN CHAND &SONS	NEW DELHI-1997
B.Sc.Physics Thermodynamics & Heat	Dr.J.P.Agarwal &Amit Agarwal	Pragati Prakasam	Meerut-1992

UNIT II

Lesson-7

STATISTICAL THERMODYNAMICS

Objectives :

This lesson gives the information and deals about the following

1. What is necessity of statistical mechanics?
2. Difference between classical and quantum statistics.
3. Various distribution laws comparison.

Structure of the lesson:-

7.1. Introduction
7.2 Methods of statistical mechanics
7.3 Statistical equilibrium
7.4 Ensemble
7.5 Concept of probability
7.6 Maxwell -Boltzmann distribution law
7.7 Application of Maxwell –Boltzmann's law to an ideal gas
7.8 Quantum statistics
7.9 Base-Einstein distribution
7.10 Bose-Einstein distribution law.
7.11 Photon gas or Blackbody radiation energy distribution
7.12 Fermi- Dirac distribution law
7.13 Fermi energy of electron gas
7.14 Comparison between Maxwell-Boltzmann, Base-Einstein and Fermi-Dirac statistics.
7.15 Solved problems.
7.16 Summary
7.17 Keywords
7.18 Self Assignment questions
7.19 Reference books.

7.1 Introduction:-

Consider the case of gas consisting of 10^{23} molecules in order to obtain the complete information concerning the motion of the system. We have to solve 10^{23} equations of motion or more. But it is not possible. It means, it is impossible to apply the ordinary laws of mechanics to such a physical system. Such problems have been successfully solved by the method of statistical mechanics.

Statistical mechanics is a branch of science, which establishes the relation between macroscopic behaviour (Bulk properties) of the system in terms of its microscopic behaviour (individual particles). So, statistical mechanics investigate the most probable behaviour of the assembly of particles.

7.2 Methods of statistical mechanics:-

The study of statistical mechanics can be classified into two Categories.

1. Classical statistics or Maxwell –Boltzmann's statistics:-

The classical statistics utilizes the results of Maxwell's law of distribution of molecular velocities. The classical statistics successfully explained the phenomenon of temperature, pressure, energy etc., But it is failed to explain black body radiation, photoelectric effect, specific heat at low temperature.

2. Quantum statistics:-

To explain the phenomenon like black body radiation, photoelectric effect, specific heat at low temperature the quantum statistics was developed by Bose, Fermi and Dirac. The quantum statistics is sub divided into two categories.

a)Bose-Einstein statistics

The particles obeying Base-Einstein statistics are known as bosons. Ex: Photons, neutral Helium atoms in ground state, α- particles etc., These particles possess the spin value as integral.

b)Fermi-Dirac statistics

The particles obeying Fermi-Dirac statistics are known as fermions.

Ex: - electrons, protons, neutrons etc., these particles possess **half spin.**

7.3 Statistical equilibrium (Thermodynamical isolated system) :-

Consider a thermodynamically isolated system consisting of N particles. Let the energy states available of these particles be E_1, E_2, ---------- E_n. Suppose that n_1 particles are in state of energy E_1, n_2 particles are in the state of E_2, ------ The total number of particles can be written as,

$$N = n_1 + n_2 + \ldots = \sum_i n_i$$

The total energy of the system can be written as

$$U = n_1 E_1 + n_2 E_2 + n_3 E_3 \ldots\ldots\ldots\ldots\ldots\ldots = \sum_i n_i E_i$$

If the particles are non interacting the total energy U is constant. But due to collisions the transition of molecules from one energy state to another energy state takes place. As a result n_1, n_2,n_3,---------- are continuously changed. Hence a particular most favoured distribution is concerned. When this distribution is reached, the system attains statistical equilibrium.

Statistical Equilibrium (General) :-

If we have a collection of particles, then each single particle is refracted as system. The collection of particles as a whole is called as an assembly. The collection of large number of assemblies is known as ensemble.

Gibb's suggested that any instantaneous state of a particle may be represented by Imagining a 6 dimensional space in which the 6 co-ordinates (X,Y,Z,P_X,P_Y,P_Z)are making along 6 mutually perpendicular axes in space.This is purely a mathematical concept. The 6 dimensional space is known as phase space or μ space. A point in the phase space representing the instantaneous state of particle is called phase point. The number of phase points per unit volume is called the phase density of these points.The density function 'ρ' is called as density of distribution or probability density or distribution function. The density function is a function of position co-ordinates (q_1.q_2.........q_i) and function of momentum co-ordinates (p_1, p_2,....... p_i) corresponding to 2_i combined position – momentum axes in phase space. The density function μ is also a function of time. $\rho = \rho(q_1, q_2, \ldots\ldots\ldots q_i, p_1, p_2, \ldots\ldots\ldots p_i, t)$

Simply $\rho = \rho(q, p, t)$

When the ensemble is a statistical equilibrium, it obeys the following two conditions.

1. The probabilities of finding phase points in various regions of phase should be independent of time.

2. The average values for the properties of the system in the extent should also be independent of time.

Mathematically the conditions of an ensemble in statistical equilibrium is given by

$$\left(\frac{\partial \rho}{\partial t}\right)_{q,p} = 0 \quad \text{For values of q and p}$$

Suppose 'ρ' is taken as a function of energy (α) which is inturn can be expressed as a function of q's and p's. Let the property is expressed by α

We can write,

$$\frac{\partial \rho}{\partial q_i} = \frac{\partial \rho}{\partial \alpha} \cdot \frac{\partial \alpha}{\partial q_i} \rightarrow (7.2)$$

And

$$\frac{\partial \rho}{\partial p_i} = \frac{\partial \rho}{\partial \alpha} \cdot \frac{\partial \alpha}{\partial p_i} \rightarrow (7.3)$$

According to Liouville's theorem we can write,

$$\left(\frac{\partial\rho}{\partial t}\right)_{q,p}+\sum_i\left[\left(\frac{\partial\rho}{\partial q_i}\frac{\partial q_i}{\partial t}\right)+\left(\frac{\partial\rho}{\partial p_i}\frac{\partial p_i}{\partial t}\right)\right]=0$$

$$\Rightarrow\left(\frac{\partial\rho}{\partial t}\right)_{q,p}+\sum_i\frac{\partial\rho}{\partial q_i}\dot{q}_i+\frac{\partial\rho}{\partial p_i}\dot{p}_i=0$$

$$\Rightarrow\left(\frac{\partial\rho}{\partial t}\right)_{q,p}=-\sum_i\left(\frac{\partial\rho}{\partial q_i}\dot{q}_i+\frac{\partial\rho}{\partial p_i}\dot{p}_i\right)$$

$$=-\sum_i\left(\frac{\partial\rho}{\partial\alpha},\frac{\partial\alpha}{\partial q_i}\dot{q}_i+\frac{\partial\rho}{\partial\alpha}\frac{\partial\alpha}{\partial p_i}\dot{p}_i\right)$$

$$=-\frac{\partial\rho}{\partial\alpha}\sum_i\left(\frac{\partial\alpha}{\partial q_i}\dot{q}_i+\frac{\partial\alpha}{\partial p_i}\dot{p}_i\right)$$

$$=-\frac{\partial\rho}{\partial\alpha}\sum_i\left(\frac{\partial\alpha}{\partial q_i}\frac{\partial q_i}{\partial t}+\frac{\partial\alpha}{\partial p_i}\frac{\partial p_i}{\partial t}\right)$$

$$=-\frac{\partial\rho}{\partial t}\sum_i\left(\frac{\partial\alpha}{\partial t}+\frac{\partial\alpha}{\partial t}\right)$$

$$=-\frac{\partial\rho}{\partial\alpha}\sum_i(0+0)$$

$$=0$$

$$\therefore\left(\frac{\partial\rho}{\partial t}\right)_{q,p}=0$$

The ensemble will be in statistical equilibrium if ρ is independent of time.

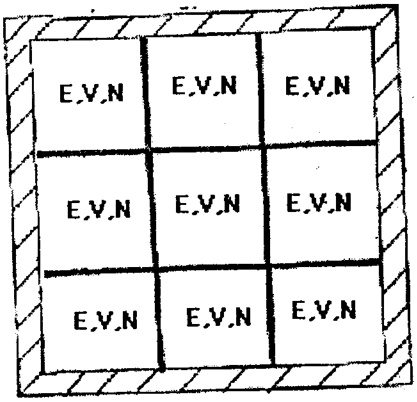
7.4 Ensemble :-

The collection of particle is known as assembly. Each assembly is characterized by the same values of set of macroscopic parameters which uniquely determine the equilibrium state of the assembly. The collection of large number of assemblies is called as an ensemble.

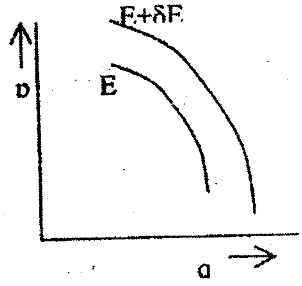
The ensembles are classified into three types.

1. Micro canonical ensemble: -

It is a collection of essentially independent system having the same energy E, volume V and number of particles N. All the assemblies are of the same type. The assemblies Are separated by impermeable and well-insulated walls separate the assemblies such that there is no exchange of E, V & N. We cannot specify the macroscopic energy of ensemble energy.



Fig(7.1)



Fig(7.2)

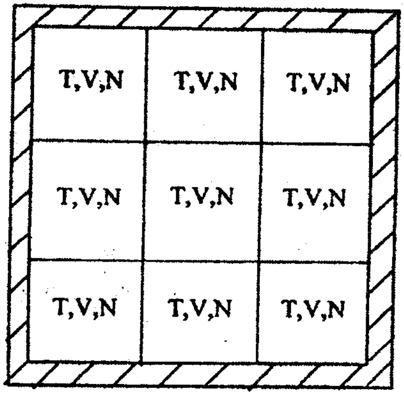
Consider a closed system for which the total energy E is constant

$E= E(q_1, q_2, q_3 \text{---------} q_f, p_1, p_2, p_3 \text{-------} p_f) = \text{constant}$

The surfaces having the same energies are called energy surface or ergodic surface. Consider a graph which is drawn taking momentum p along Y-axis and position q along X-axis. We can obtain two energy surfaces as shown. In between these two surfaces the density is constant Below the 1st curve and above the second curve the density is zero. It means the area between both the curves represents micro canonical ensemble.

2. Canonical ensemble :-

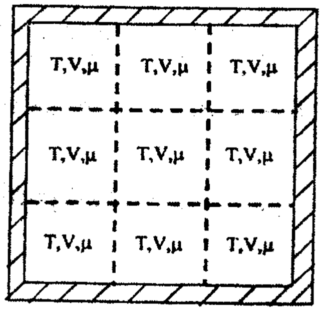
It is a collection of essentially independent assemblies having the same values of temperature T, Volume V and number of identical particles N. As all the assemblies are having the same temperature, we can say that the assemblies are in thermal contact with each other. The individual assemblies are separated by rigid, impermeable but diathermic walls. Since energy can be exchanged between the assemblies they will reach common temperature. Thus in canonical ensemble, system can exchange energy but not particles.



Fig(7.3)

3. Grand Canonical ensemble :

From micro canonical ensemble to canonical ensemble the condition of constant energy is released. This has simplified the calculations in thermodynamics. Now we have to remove the condition of fixing the total number of particles ie. N. As in all the chemical and physical processes the number N changes. So, such an ensemble where exchange of energy and particles takes place with the heat reservoir is known as Grand – Canonical ensemble.



Fig(7.4)

Grand – Canonical ensemble is a collection of independent assemblies having the same temperature T, Volume V and chemical potential.

7.5 Concept of probability

Consider the tossing of a coin. The toss chance that the head or tail falls upward is ½ i.e. 50%. If the coin is tossed for a large number of times, the chances of getting the head or tail upwards are equal i.e. 50%. This fact can be expressed in terms of probability.

The probability of an event may be defined as the ratio of number of cases in which event occurs to the total number of cases, provided the total number of cases is very large.

$$\textit{The probability of an event} = \frac{\textit{Number of cases in which the event occurs}}{\textit{Total number of cases}}$$

Consider that an event occurs in 'a' ways and fail occurs in 'b' ways.

Probability of happening the event = a/a+b

Probability of failing the event = b/a+b

[illegible]bility of composite event = The product of the probabilities of the individual and independent events.

Probability theorems for various distributions laws.

[illegible] of ways in which N distinct particles can be arranged in an order = N!.

Ex. - Let there are 4 particles in a system. The particles can be arranged in 24 ways i.e. 4! Ways.

The number of different ways in which n particles can be selected from N distinguishable particles irrespective of the order of selection = $\frac{N!}{(N-n)!n!}$

Ex :- Suppose we want to select 3 particles out of 4 particles irrespective of the order of selection, we can write n=3 and N=4

$$\text{Number of ways} = \frac{4!}{(4-3)!3!} = \frac{4!}{1!3!} = 4$$

3. The number of different ways in which n distinguishable particles can be arranged in g distinguishable states with not more that one particle in each $= \frac{g!}{n!(g-n)!}$

Ex :- Consider 4 distinguishable states (g=4) and 3 distinguishable particles (n=3)

$$\text{Number of ways} = \frac{4!}{3!(4-3)!} = \frac{4!}{3!1!} = 4$$

7.6 Maxwell Boltzmann distribution law :-

Maxwell Boltzmann distribution law is one of the important distributions in classical statistics. Consider a system of 'n' similar but distinguishable molecules of a gas contained in a vessel .The gas molecules are moving freely with in the vessel in all possible directions . Different gas molecules possess different energies .Maxwell considered that how a fixed total amount of energy is distributed among the various members of an assembly .Basing on this idea Maxwell-Boltzmann's law was derived Out of this 'N' molecules, let n_1 be the number molecules having energy E_1. n_2 molecules have energy E_2, n_3 molecules have energy E_3 and so on .The total number of gas molecules N and also total energy of gas molecules E is constant .we can write,

$$N = n_1 + n_2 + \ldots = \sum_i n_i$$

The total energy can be written as

$$E = n_1\varepsilon_1 + n_2\varepsilon_2 + n_3\varepsilon_3 + \ldots\ldots\ldots\ldots\ldots = \sum_i \varepsilon_i n_i$$

Considering that priory, probability for a molecule have energy ε_i as g_i. Maxwell and Boltzmann derived the following relation.

$$n_i = g_i e^{-\alpha} e^{-\beta\varepsilon_i}$$

Where α and β are constants.

The value of β *is* $\frac{1}{KT}$

$$\therefore n_i = g_i e^{-\alpha} e^{-\frac{\varepsilon_i}{kT}}$$

$$= \frac{g_i}{e^{\alpha} e^{\frac{\varepsilon_i}{kT}}}$$

$$\Rightarrow n_i = \frac{g_i}{e^{\left(\alpha + \frac{\varepsilon_i}{kT}\right)}}$$

This result explains most probable distribution of molecules among the various energy states and known as Maxwell-Boltzman distribution law.

If a graph is drawn by taking number of molecules along the y-axis and energy along x-axis, a curve is obtained as shown . The number of molecules being maximum in zero energy cell and decreases exponentially with energy.

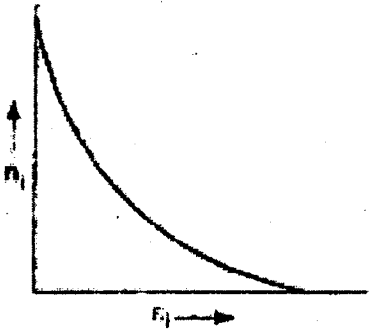


Fig:(7.5)

7.7Application of Maxwell Boltzmann's law to an ideal gas :

Let us consider an assembly of 'n' molecules of an ideal gas contained in a volume V at absolute temperature T. The number of molecules having the energy ε_i can be written as

$$\Rightarrow n_i = g_i e^{-\alpha} e^{-\beta\varepsilon_i} \rightarrow (7.1)$$

Where $e^{-\alpha}$ and β are constants

Instead of considering discrete energies $\varepsilon_1, \varepsilon_2, \varepsilon_3$ ------, here the continuous distribution of energies are considered. If we consider the number of molecules having the energy in between ε and ε+dε equation(7.1) becomes $n(\epsilon)d\epsilon = g(\epsilon)e^{-\alpha}e^{-\beta\epsilon d\epsilon} \rightarrow (7.2)$,

We know that $\epsilon = \frac{p^2}{2m}$ where p is the molecular momentum

The number of molecules having the momentum in between p and p +dp is

$$n(p)dp = g(p)e^{-\alpha}e^{-\beta p dp}$$

$$n(p)dp = g(p)e^{-\alpha}e^{\frac{-\beta p^2}{2m}}dp \rightarrow (7.3)$$

The values in the above equation are obtained as

$$g(p) = \frac{4\pi Vp^2}{h^3} \rightarrow (7.4)$$

$$e^{-\alpha} = \frac{Nh^3}{V}\left(\frac{\beta}{2\pi m}\right)^{\frac{3}{2}} \rightarrow (7.5) \text{ and}$$

$$\beta = \frac{1}{kT} \rightarrow (7.6)$$

Writing equations (7.4), (7.5) AND (7.6) in equation (7.3) we get,

$$n(p)dp = \frac{4\pi Vp^2}{h^3}\frac{Nh^3}{V}\left(\frac{\beta}{2\pi m}\right)^{\frac{3}{2}} e^{\frac{1}{kT}\frac{p^2}{2m}}dp$$

$$n(p)dp = 4\pi Np^2\left(\frac{1}{2\pi mkT}\right)^{\frac{3}{2}} e^{\frac{-p^2}{2mkT}}dp \rightarrow (7.7)$$

We have $\frac{p^2}{2m} = \epsilon \quad \Rightarrow p^2 = 2m\epsilon$

diff.we get

$$2pdp = 2m\, d\epsilon$$

$$\Rightarrow dp = \frac{m\, d\epsilon}{p} = \frac{m\, d\epsilon}{\sqrt{2m\epsilon}}.$$

Writing these values in equation (7.7) we get,

$$n(\in)\ d\in = 4\pi N.2m\in\left(\frac{1}{2\pi mkT}\right)^{\frac{3}{2}} e^{\frac{-\in}{kT}}\ \frac{m\ d\in}{\sqrt{2m\in}}$$

$$= 8\pi Nm\in\frac{1}{2^{\frac{3}{2}}m^{\frac{3}{2}}}\left(\frac{1}{\pi kT}\right)^{\frac{3}{2}} e^{\frac{-\in}{kT}}\ \frac{md\in}{2^{\frac{1}{2}}m^{\frac{1}{2}}\in^{\frac{1}{2}}}$$

$$= \frac{8\pi Nm\in}{2^{\frac{3}{2}+\frac{1}{2}}m^{\frac{3}{2}+\frac{1}{2}}}\left(\frac{1}{\pi kT}\right)^{\frac{3}{2}} e^{\frac{-\in}{kT}}\ \frac{md\in}{\in^{\frac{1}{2}}}$$

$$= \frac{8\pi Nm^2}{2^2 m^2}\left(\frac{1}{\pi kT}\right)^{\frac{3}{2}} e^{\frac{-\in}{kT}}.\in^{\frac{1}{2}}\ d\in$$

$$n(\in)\ d\in = 2\pi N\in^{\frac{1}{2}}\left(\frac{1}{\pi kT}\right)^{\frac{3}{2}} e^{\frac{-\in}{kT}}d\in \rightarrow (7.8)$$

It is the expression for the number of molecules having energy between ε and ε+dε in an ideal gas containing N molecules at absolute temperature.

Equation (7.8) is called Maxwell – Boltzmann distribution of energy.

We have $\in = \frac{1}{2}mV^2, d\in = \frac{1}{2}.m.2VdV = mVdV.$

The number of molecules having the velocity in between V and $V+dV$ is

$$n(V)dV = 2\pi N\left(\frac{1}{2}mV^2\right)^{\frac{1}{2}}\left(\frac{1}{\pi kT}\right)^{\frac{3}{2}}.e^{\frac{-mV^2}{2kT}}mVdV.$$

$$= 2\pi N\frac{m^{\frac{1}{2}}V^2}{2^{\frac{1}{2}}}\left(\frac{1}{\pi kT}\right)^{\frac{3}{2}}.e^{\frac{-mV^2}{2kT}}mVdV.$$

$$= \frac{4\pi Nm^{\frac{3}{2}}V^2}{2.2^{\frac{1}{2}}}\left(\frac{1}{\pi kT}\right)^{\frac{3}{2}}.e^{\frac{-mV^2}{2kT}}dv.$$

$$n(V)dV = 4\pi NV^2\left(\frac{m}{\pi kT}\right)^{\frac{3}{2}}e^{\frac{-mV^2}{2kT}}dv.\rightarrow (7.9)$$

It is the expression for the number of molecules having the velocity in between V and $V+dV$. If a graph is plotted taking number of molecules per unit range n(v) along y-axis and velocity (V) along x-axis, the curve first raises parabolically, reaches maximum and falls exponentially. The fraction of the molecules having the velocities in between v and $v+dv$ is given by

$$\frac{n(V)dV}{N} = 4\pi V^2\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}}.e^{\frac{-mV^2}{2kT}}.dV$$

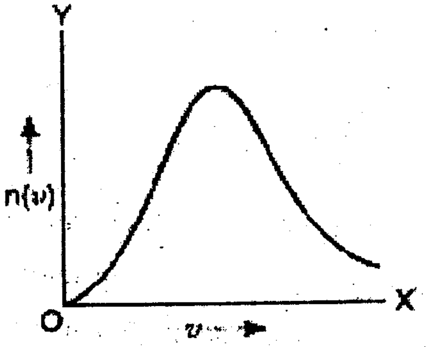


Fig:(7.6)

The fraction $\frac{n(V)dV}{N}$ represents the probability that a molecule will have speed in between v and $v+dv$. It can be written as p(v)dv

$$p(V)dV = 4\pi V^2\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}}.e^{\frac{-mV^2}{2kT}}.dV$$

$$p(V) = 4\pi V^2\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}}.e^{\frac{-mV^2}{2kT}} \rightarrow (7.10)$$

The probability function is defined as the probability that a molecule will possess the speed v.

7.8 Quantum Statistics: -

Classical statistics can explain the phenomena such as temperature, pressure, energy etc. But it is failed to explain the experimental observed phenomena like black body radiation, photoelectric effect, specific heat at low temperature etc. To explain these phenomena, Quantum statistics was introduced. In quantum statistics the particles are considered as in distinguishable.

In 1924, S.N. Bose developed a new statistics and Einstein applied it to active axis. The statistics is known as Bose – Einstein statistics. The particles obeying this statistics are known as Bosons. The particles do not obey Pauli's exclusion principle. The particles are identical, indistinguishable and possess integral spin.

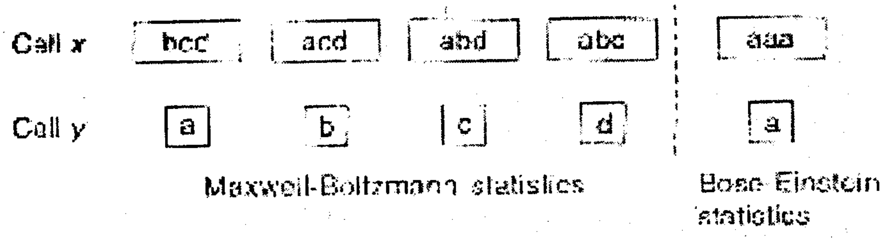
Ex: Photons, neutral Helium atoms, α particles etc,

In 1926, Fermi and Dirac independently modified Bose – Einstein statistics in certain gases. This statistics is called Fermi – Dirac statistics. The particles obeying Fermi – Dirac statistics are known as Fermions. The particles obey Pauli's exclusive principles. The particles are identical, indistinguishable and possess half integral spins (1/2 , 3/2, 5/2 7/2, ...)

Ex :- Electrons, Protons, neutrons etc.,

7.9 Bose – Einstein Distribution :-

In Bose – Einstein distribution the particles are indistinguishable. Consider the distribution of four particles (a,a,a,a) among the two cells x and y such that there are three particles in cell x and one particle in celly. The distribution according to both Maxwell – Boltzmann and Bose – Einstein are as shown fig(7.7)]



Maxwell-Boltzmann statistics

Bose Einstein statistics

Fig:(7.8)

Now let us assume that each cell (x,y) is further divided into four compartments. The distribution is as shown in fig (7.8).

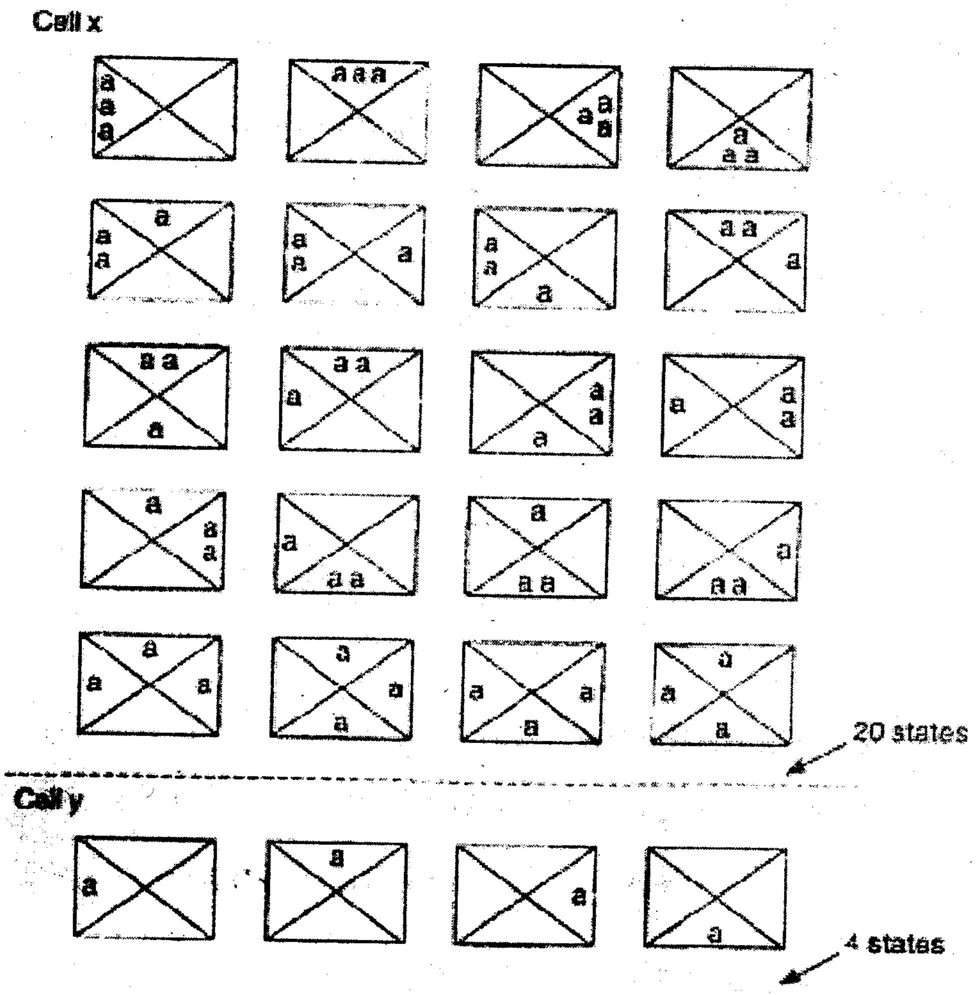


Fig:(7.8)

In general the number of ways in which n particles are distributed along g_i sublevels of its quantum states is

$$\frac{(n_i+g_i-1)!}{n_i!(g_i-1)!}$$

If $n_i=3$, $g_i=4$

Number of states

$$=\frac{(3+4-1)!}{3!(4-1)!}=\frac{6!}{3!3!}=\frac{6\times5\times4\times3}{3\times6}=20$$

If ni=1 and gi=4

Number of states =

$$\frac{(1+4-1)!}{1!(4-1)!}=\frac{4!}{3!}=4$$

Hence the total possible distributions are 20x4=80

Let there are n_1 particles in g_1 states with energy E_1, n_2 particles with energy E_2,------ n_i particles in the g_i state with energy E_i

The total number of arrangements will be

$$G=\left[\frac{(n_1+g_1-1)!}{n_1!(g_1-1)!}\right]\left[\frac{(n_2+g_2-1)!}{n_2!(g_2-1)!}\right]\cdots\left[\frac{(n_i+g_i-1)!}{n_i!(g_i-1)!}\right]$$

$$=\pi_i\left[\frac{(n_i+g_i-1)!}{n_i!(g_i-1)!}\right]$$

As ni and gi are very large we can write $g_i-1=g_i$

$$\therefore G=\pi_i\frac{(n_i+g_i)!}{n_i!g_i!}$$

7.10 Bose – Einstein distribution law :-

In Bose – Einstein statistics, the particles are indistinguishable. The number of different ways in which indistinguishable particles can be distributed among g_i cells is $\frac{(n_i+g_i)!}{n_i!g_i!}$

The total number of distinguishable eigen states is given by

$$\therefore G=\pi_i\frac{(n_i+g_i)!}{n_i!g_i!}$$

For equilibrium state $\delta G=0$

The two subsidiary conditions are

1. The total number of particles in the system is constant.

$(i.e)n=\sum_i n_i$ is constant (or) $\delta n=\sum_i \delta n_i=0$

2. The total energy of the system is constant.

$E=\sum_i E_i=\sum_i n_iE_i$ =constant (or) $\delta E=\delta\sum_i n_iE_i=0$

Basing on these conditions the number of particles having the energy e_i can be written as

$$n_i = \frac{g_i}{e^{\alpha+\frac{E_i}{kT}} - 1}.$$

This equation is known as Bose – Einstein distribution law.

It is the most probable distribution of the particles among various energy states obeying Bose – Einstein statistics.

7.11 Photon gas or Black body radiation energy distribution :-

Consider a black body that is a cavity in same opaque material. Let V be the volume of the cavity which contains a large number of indistinguishable photons of various frequencies. It is called photon gas. According to Bose – Einstein distribution law number of particles having the energy ε_i is

$$n_i = \frac{g_i}{e^{\alpha+\frac{\varepsilon_i}{kT}} - 1}$$

In case of cavity , the number of photons is not fixed due to their absorption and reemission by the walls of the cavity.

In this case we write,

$$\sum_i \delta n_i \neq 0$$

Under this condition $\alpha = 0$,

Writing this equation (7.11) we get,

$$n_i = \frac{g_i}{e^{\frac{\varepsilon_i}{kT}} - 1} ----(7.12)$$

In case of photon gas, the energy difference between successive allowed values is very small. Hence we can replace g_i with g(E)dE The number of particles having the energy in between E and E+dE is

$$dn = \frac{g(E)dE}{e^{\frac{E_i}{kT}} - 1} \Rightarrow \frac{g(E)dE}{e^{\frac{h\nu}{kT}} - 1} \rightarrow (7.13)$$

Replacing g (E) dE with $g(\gamma)d\gamma$ we get

$$dn = \frac{g(\nu)d\nu}{e^{\frac{h\nu}{kT}}-1} \rightarrow (7.14)$$

From Rayleigh – Jeans law, we have the number of states in black body radiations in the frequency range γ and γ+dγ as

$$g(\gamma)d\gamma = \frac{8\pi V\upsilon^2 d\nu}{c^3} \qquad \rightarrow (7.15)$$

Writing equation (7.15) in equation (7.14) we get,

$$dn = \frac{1}{e^{\frac{h\nu}{kT}}-1}.\frac{8\pi V\upsilon^2 d\nu}{c^3}$$

$$\frac{dn}{d\nu} = \frac{8\pi V\upsilon^2}{c^3\left(e^{\frac{h\nu}{kT}}-1\right)} \rightarrow (7.16)$$

Consider 'dn' photons in frequency range υ and υ +dυ .The energy of these photons is $h\upsilon dn$.

Energy per unit volume in the frequency range υ and υ +d υ $= \frac{h\upsilon dn}{v}$.

It can be written as E(υ) d υ of photon gas.

$$\therefore E(\upsilon)d\upsilon = \frac{h\upsilon dn}{V}$$

$$\Rightarrow \frac{E(\upsilon)V}{h\upsilon} = \frac{dn}{d\upsilon} \rightarrow (7.17)$$

From equations (7.16) and(7.17) we can write,

$$\frac{E(\upsilon)V}{h\upsilon} = \frac{8\pi V\upsilon^2}{c^3\left(e^{\frac{h\nu}{kT}}-1\right)}$$

$$E(\upsilon) = \frac{8\pi h\upsilon^3}{c^3\left(e^{\frac{h\nu}{kT}}-1\right)} \rightarrow (7.18)$$

This equation gives energy distribution of a photon gas.

7.12 Fermi – Dirac distribution law:-

Fermi – Dirac statistics is applied to indistinguishable particles which are governed by pauli's –exclusion principle. These particles are known as Fermions.

Consider a system having n indistinguishable particles. Let n_1 be the number of particles of energy ε_1, n_2 be the number of particles of energy ε_2, n_i be the number of particles having energy ε_i .Let g_i denote degenenagy or number of sublevels of ith level.

The total number of distinguishable eigen states for the whole system is given by

$$G=\frac{g_1!}{(g_1-n_1)!n_1!}1\times\frac{g_2!}{(g_2-n_2)!n_2!}\times\cdots\cdots\times\frac{g_i!}{(g_i-n_i)!n_i!}$$

$$G=\pi_i\frac{g_i!}{(g_i-n_i)!n_i!}\rightarrow(7.19)$$

The two subsidiary conditions are

1. The total number of particles of the system is constant

$$n=\sum n_i=\text{constant}\qquad \delta_n=\delta\sum_i n\,i=0$$

2. The total energy of the system is constant.

$$E=\pi_i\sum n_i\varepsilon_i=\text{constant}\qquad \delta E=\delta\sum_i n_i\varepsilon_i=0$$

3. From the above considerations Fermi–Dirac distribution law was represented as ,

$$n_i=\frac{g_i}{e^{\frac{\alpha+E_i}{kT}}+1}\rightarrow(7.20)$$

This equation represents most probable distribution of particles among the various energy states for a system obeying Fermi –Dirac statistics.

7.13 Fermi energy of electron gas:-

For the particles obeying Fermi –Dirac statistics, in thermal equilibrium and at absolute zero, the energy levels from o to a certain values of ε_f are occupied by the particles. Other energy levels are above ε_f are left vacant. The energy ε_f is known as Fermi energy.

From Fermi –Dirac distribution law, we have the expression for the number of particles having the energy ε_i as

$$n_i = \frac{g_i}{e^{\alpha+\frac{\varepsilon_i}{kT}}+1} \rightarrow (7.21)$$

To obtain the number of particles having their energy lie in between E and E + dE, we have to replace with g (E) dE and with dn(E). Now equation (7.21) becomes,

$$dn(E) = \frac{g(E)dE}{e^{\alpha+\frac{E_i}{kT}}+1} \rightarrow (7.22)$$

For a system of free electrons enclosed in a volume V , we have the relation,

$$g(E)dE = \frac{2\times 4\pi p^2 dpV}{h^3}$$

The factor 2 is introduced following the pauli exclusion principle, such that two electrons up spin +1/2 and down spin –1/2 can be accommodated in energy state. Represents the translational energy states per unit volume in phase space. Writing the value of g(E) dE in equation (7.22) we get,

$$dn(E) = \frac{8\pi dpVp^2}{h^3} \times \frac{1}{e^{\alpha+\frac{Ei}{kT}}+1}$$

$$dn(E) = \frac{8\pi p^2 dpV}{h^3} \times f(E) \rightarrow (7.23)$$

where $f(E) = \dfrac{1}{e^{\alpha+\frac{Ei}{kT}}+1}$

and it is called Fermi Dirac Distribution function.

The total number of electrons in the system is obtained by integrating equation (7.23)

$$n = \int dn(E) = \int_{\epsilon=0}^{\epsilon=\epsilon_f} \frac{8\pi p^2 dpV}{h^3} f(E) \rightarrow (7.24)$$

At absolute zero we can write,

$$\begin{aligned} &if \in > \in_f, f(\in) = 0 \\ &if \in > \in_f, f(\in) = 1 \end{aligned}$$

Writing this value in equation (7.24) we get,

$$n = \frac{8\pi V}{h^3} \int_{\in=0}^{\in=\in_f} p^2 dp \times 1 = \frac{8\pi V}{h^3} \left[\frac{p^3}{3} \right]_{\in=0}^{\in=\in_f} \rightarrow (7.25)$$

We have the relation between the energy and momentum

As $\in = \dfrac{p^2}{2m}$

$$\Rightarrow 2m \in = p^2 \qquad \Rightarrow (2m\in)^{\frac{1}{2}} = p$$

$$\Rightarrow (2m\in)^{\frac{3}{2}} = p^3$$

Writing this in equation (7.25) we get,

$$n = \frac{8\pi V}{h^3} \left[\frac{(2m\in)^{\frac{3}{2}}}{3} \right]_{\in=0}^{\in=\in_f}$$

$$n = \frac{8\pi V}{3h^3} \left((2m\in)^{\frac{3}{2}} - 0 \right)$$

$$\Rightarrow \frac{3nh^3}{8\pi V} = (2m\in)^{\frac{3}{2}}$$

$$\left(\frac{3nh^3}{8\pi V} \right)^{\frac{2}{3}} = 2m \in_f$$

$$\in_f = \frac{1}{2m} \left(\frac{3nh^3}{8\pi v} \right)^{\frac{2}{3}} \rightarrow (7.26)$$

It is the expression for Fermi energy of an electron gas.

7.14 Comparison between Maxwell-Boltzmann's statistics, Bose-Einestein statistics and Fermi-Dirac statistics

s.no	M-B statistics	B-E statics	F-D statistics
1	Applies to systems of identical,distinguishable particles such as gases	Applies to systems of identical,distinguishable particles not obeying pauli's exclusion principle such as photons, liquid He etc..	Applies to systems of identical,distinguishable particles not obeying pauli's exclusion principle such as electrons.
2	Particles are called as classical particles	Particles are called Bosons.	Particles are called Fermions.
3	There is no restriction in the number of particles in a given state.	There is no restruction on the number of particles in a given state	Only one particle in a given quantum state is allowed.
4	The distribution law is $n_i = \frac{g_i}{e^{\alpha+\frac{\epsilon_i}{kT}}}$	The distribution law is $n_i = \frac{g_i}{e^{\alpha+\frac{\epsilon_i}{kT}}-1}$	The distribution law is $n_i = \frac{g_i}{e^{\alpha+\frac{\epsilon_i}{kT}}+1}$
5	The distribution Function is $f(\epsilon) = \frac{1}{e^{\alpha+\frac{\epsilon_i}{kT}}}$	The distribution function is $f(\epsilon) = \frac{1}{e^{\alpha+\frac{\epsilon_i}{kT}}-1}$	The distribution function is $f(\epsilon) = \frac{1}{e^{\alpha+\frac{\epsilon_i}{kT}}+1}$
6	The energy of the system such as an ideal atomic gas may be zero at absolute zero	The energy of the system may be zero at absolute zero.	The energy of the system at absolute zero can not be zero.

7.15 SOLVED PROBLEMS:-

1.Caluculate the probability that the speed of an oxygen molecule lies between 100 and 101m/s at a temperature of 200K.The mass of an oxygen molecule is $5.3\mathrm{X}10^{-26}$ kg.The value of K=1.38x10-23J/K

Solution :-

The probability that a molecule has speed between v and v+dv is given by

$$p(v)dv = 4\pi\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2.e^{\frac{-mv^2}{2kT}}.dv$$

$$\frac{m}{2\pi kT} = \frac{5.3\times10^{-26}}{2\times3.14\times1.38\times10^{-23}\times200} = 3\times10^{-6}$$

$$\frac{mv^2}{2kT} = \frac{5.3\times10^{-26}\times100^2}{2\times1.38\times10^{-23}\times200} = 0.1$$

Putting these values we get,

$$p(v)dv = 4\times3.14\times\left(3\times10^{6}\right)^{\frac{3}{2}}\times(100)^2 e^{-0.1}\times1$$

$$= 4\times3.14\times5.2\times10^{-9}\times10^4\times0.9$$

$$= 6\times10^{-4}$$

2. Show that maximum kinetic energy of free electrons in silver at 0k is 5.6ev.

For silver n=5.86x10 28electrons/m^3

Solution:-

$$\in_f = \frac{h^2}{2m}\left(\frac{3n}{8\pi}\right)^{\frac{2}{3}} = \frac{h^2}{8m}\left(\frac{3n}{\pi}\right)^{\frac{2}{3}}$$

$$= \frac{\left(6.624\times10^{-34}\right)^2}{8\times9\times10^{-31}}\times\frac{\left(3\times5.86\times10^{28}\right)^{\frac{2}{3}}}{(3.14)^{\frac{2}{3}}}$$

$$= 9\times10^{-19}\,J$$

$$= \frac{9\times10^{-19}}{1.6\times10^{-19}}\,eV = 5.625\,eV$$

$$\therefore \in_f = 5.625\,eV$$

7.16 SUMMARY

Statistical mechanics deals with macroscopic behavior and microscopic behavior of the system.In Classical statistics or Maxwell-Boltzmann's statistics the particles are distinguishable.In Quantum statistics the particles are indistinguishable.

7.17 KEYWORDS

Statistical equilibrium, Ensemble, Maxwell-Boltzmann statistics, Bose-Einstein statistics, Fermi-Dirac statistics.

7.18 SELF ASSESSMENT QUESTIONS:

LONG ANSWER QUESTIONS:

1.Discuss classical Maxwell-Boltzmann distribution law regarding the distribution of particles into various energy states .

2. Express Bose-Einstein distribution law mathematically. Apply it to a photon gas to obtain the energy distribution.

3. Give the expression for Fermi-Dirac distribution law. Derive an expression for Fermi-Dirac energy of a system of free electrons.

SHORT ANSWER QUESTIONS:

4. What is the difference between classical and Quantum statistics.

5.Compare M-B,B-Eand F-D statistics

EXERCISES:

1. The density of sodium is 970kg/m^2 and its atomic weight is 23.Assuming that one electron per atom contributes to electrical conduction. Compute the fermi -energy at 0° K of this metal.

$$\left[HINT : \eta = \frac{\int N_0}{A} = and \in_F = \frac{h^2}{8m}\left(\frac{3n}{\pi}\right)^{\frac{3}{2}} \right] \qquad (Ans : 3.145ev)$$

7.19 REFERENCE BOOKS

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Thermodynamics	D.C. Tayal	Himalaya Publications

UNIT-III

Lesson 8

MATRIX METHOD IN PARAXIAL OPTICS

Objectives: 1. Convey hte importance of matrix method in defining the final images plane for complex system of lenses.

2. Define the optical cosine and formation of co-ordinates for different rays.

3. Derive the translatin matrix, refraction matrix and hence the system matrix.

4. Derive the eqation for image plane and magnification.

5. Derive the system matrix for thick lens and extended it to thin system.

6. Finally derive the system matrix for two lenses separeted by a distance .

7. Define cordinal points of lens system enumerate their importance.

8. Define uint planes and nodal planes

Structure of the lesson

8.1 Introduction :-

Optics is branch of physics which deals with nature of light and effects of light. It is again subdivided into two branches namely 1 .Physical optics-Deals with nature of light.
2.Geometrical optics -Deals with affects of light when it passes through lenses and mirrors.

When a ray of light from the object is incident on an optical system it either reflects or refracts forming the image of the object.It is quite easy to locate the position,size and orientation of the image with a single lens or mirror.But an optical system like telescopes or microscopes which form the images of the objects contains series of lenses or mirrors.So it is difficult to locate the image plane for such systems geometrically.Matrix method is the solution for such problems.

We can easily define the image plane and magnifications of optical system using matrices

8.2 Concepts of Translation ,Refraction and System matrix :-

When a light ray travels between two points in an optical system,the path of the light ray between two points or between two lenses is represented by translation matrix.

8.2.1 Translation matrix :-

When a ray of light propagates through the homogeneous medium,the effect of translation takes place. Before constructing a translation matrix,let us define the points on the path of light using diretional cosines.

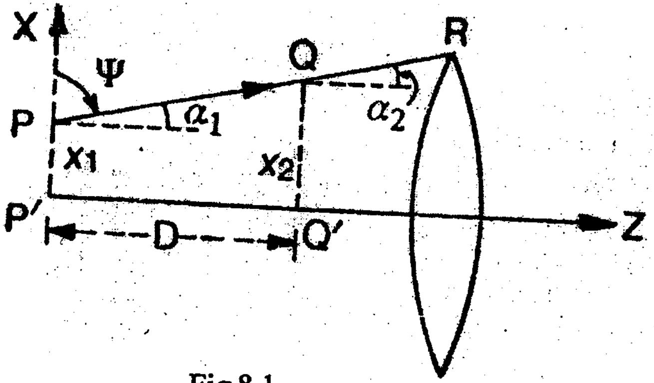

Fig 8.1

Any point on the light ray is specified by two parameters

1. Distance of the point from principal axes 'x'.
2. The angle made by the ray at the given point with principal axis[Z-axis].

In fig8.1 point P is given by (x_1, α_1)

point Q is given by (x_2, α_2).

Optical direction cosine λ :-

It is the product of refractive index of the medium with the cosine of the angle which light ray makes with x-axis.

where $\lambda = \mu \cos \psi = \mu \cos(90 - \alpha)$

hence point P is given by (x_1, λ_1)

Q is given by (x_2, λ_2)

where $\lambda_1 = \mu \cos(90 - \alpha_1)$, $\lambda_2 = \mu \cos(90 - \alpha_2)$

Construction of translation matrix :-

From figure 8.2, the two points Pand Q are defined by (x_1, λ_1) and (x_2, λ_2).The translation between these two points is obtained by finding relations between x_1, x_2 and λ_1 and λ_2

From fig 8.2 $\alpha_1 = \alpha_2$ ____(8.1)

$x_2 = x_1 + D \tan \alpha_1$ ________(8.2)

when angles are small, $x_2 \approx x_1 + D\alpha_1$ ______________(8.3)

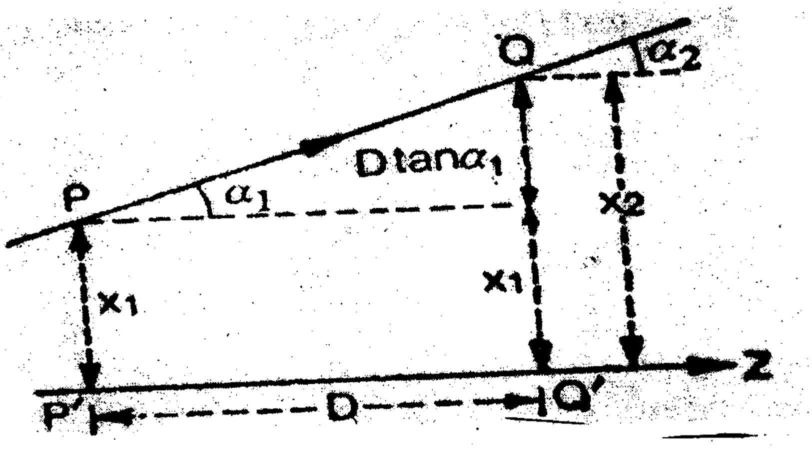

Fig 8.2

$$\lambda_1 = \mu_1 \cos(90-\alpha_1) = \mu_1 \sin\alpha_1 \approx \mu_1\alpha_1 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \quad (8.4)$$

$$\lambda_2 = \mu_1 \cos(90-\alpha_2) = \mu_1 \sin\alpha_2 \approx \mu_1\alpha_2 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \quad (8.5)$$

where μ_1 is the refractive index of homogeneous medium through which the light ray is translating

from eq (8.1), $\alpha_1 = \alpha_2 \Rightarrow \lambda_2 = \lambda_1 \Rightarrow \lambda_2 = 0.x_1 + 1.\lambda_1$ ____________________(8.6)

from eq (8.4), $\alpha_1 = \dfrac{\lambda_1}{\mu_1}$

Substituting in eq.(8.3), $x_2 = x_1 + D\dfrac{\lambda_1}{\mu_1}$

$$x_2 = x_1 + \left(\frac{D}{\mu_1}\right).\lambda_1 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \quad (8.7)$$

Writing the two equations (8.6) and (8.7) in matrix forms leads to

$$\lambda_2 = 1.\lambda_1 + 0.x_1 \quad \text{\_\_\_\_\_\_} \quad (8.8)$$

$$x_2 = \frac{D}{\mu_1}.\lambda_1 + 1.x_1 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \quad (8.9)$$

$$\begin{bmatrix} \lambda_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \dfrac{D}{\mu_1} & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ x_1 \end{bmatrix} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \quad (8.10)$$

Hence we can translate from point (x_1, λ_1) to (x_2, λ_2) using the matrix $\begin{bmatrix} 1 & 0 \\ \dfrac{D}{\mu_1} & 1 \end{bmatrix}$.

Hence it is called translation matrix 'T'.

$$T=\begin{bmatrix}1 & 0\\ \frac{D}{\mu_1} & 1\end{bmatrix}\text{\_\_\_\_\_\_\_\_(8.11)}\quad ,\ \text{Determinant } |T|=1$$

8.2.2 Refraction matrix : -

When a light ray is incident on a lens under go refraction.The path of the light ray after refraction can be obtained from points before refraction N, using refraction matrix.

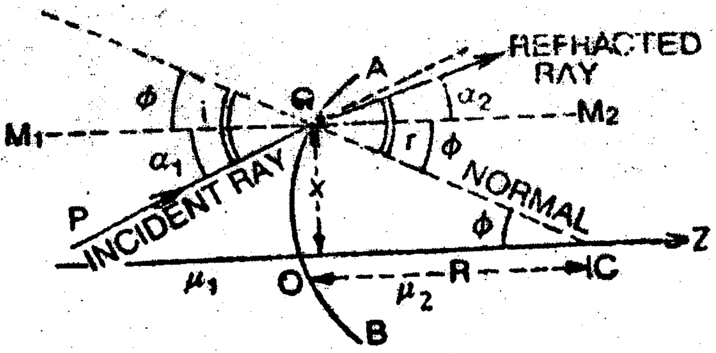

Fig 8.3

rom Fig.8.3,PQ-incident ray ,QS-refracted ray, NC=normal, OC=R= radius of the curvature of the lens .

$OC=QC=R,\ \angle PQN=i, \angle PQM_1=\alpha, \angle M_1QN=\phi,\ \angle SQC=\eta,\ \angle M_2QC=\phi,$

$\angle SQM_2=\alpha_2, \angle OCQ=\phi$

$\therefore i=\phi+\alpha_1,\ \gamma=\phi+\alpha_2$

From Snell's law $\frac{\mu_2}{\mu_1}=\frac{\sin i}{\sin r}\approx\frac{i}{r}\Rightarrow \mu_2 r=\mu_1 i$ where μ_1, μ_2 are refractive indices of air and refracting medium.

Eq --(11) $\Rightarrow \mu_2(\phi+\alpha_2)=\mu_1(\phi+\alpha_1)$

$\Rightarrow \mu_2\alpha_2=\mu_1\alpha_1+\mu_1\phi-\mu_2\phi$

$\Rightarrow \mu_2\alpha_2=\mu_1\alpha_1-\phi(\mu_2-\mu_1)$ ----------------------(8.12)

But from $\Delta^{le}OCQ\left(\mathrm{Tan}\phi = \frac{x}{R}\right) \Rightarrow \frac{x}{R} \approx \phi$,

From Eq.(8.12) $\Rightarrow \mu_2\alpha_2 = \mu_1\alpha_1 - \frac{x}{R}(\mu_2 - \mu_1)$

Optical direction cosines before and after refraction are

$$\lambda_1 = \mu_1\alpha_1, \quad \lambda_2 = \mu_2\alpha_2 \text{\_\_\_\_\_\_\_\_}(8.13)$$

$$\lambda_2 = \mu_2\alpha_2 = \mu_1\alpha_1 - \frac{x}{R}(\mu_2 - \mu_1)$$

$$\Rightarrow \lambda_2 = \lambda_1 - (\mu_2 - \mu_1)\frac{x}{R}$$

Let $\frac{\mu_2 - \mu_1}{R} = P$

$$\therefore \lambda_2 = \lambda_1 - P.x \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_}(8.14)$$

and $\quad x = x_2 = x_1 \Rightarrow x_2 = 0.\lambda_1 + 1.x_1$ _________(8.15)

The point after refraction (x_2, λ_2) can be obtained from point before refraction (x_1, λ_1) by writing (8.15) and (8.14) equations in matrix form.

$$\begin{bmatrix} \lambda_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -P \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ x_1 \end{bmatrix} \text{---------}(8.16)$$

$\therefore R = \begin{bmatrix} 1 & -P \\ 0 & 1 \end{bmatrix}$ is called Refraction matrix.

8.2.3 <u>System matrix</u> 'S':-

The effect of translation between lenses and refraction at the lenses is combindly given by System matrix. The coordinates of light ray when it enters the system are and when it leaves the system the coordinates are (x_2, λ_2). Considering the translation and refraction in same order we can write,

$$\begin{bmatrix} \lambda_2 \\ x_2 \end{bmatrix} = R \times T \begin{bmatrix} \lambda_1 \\ x_1 \end{bmatrix}$$

$$R \times T = S(\text{system matrix}) = \begin{bmatrix} b & -a \\ -d & c \end{bmatrix}$$

$$R \times T = \begin{bmatrix} 1 & -P \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{D}{\mu} & 1 \end{bmatrix} = \begin{bmatrix} 1-\frac{P.D}{\mu} & -P \\ \frac{D}{\mu} & 1 \end{bmatrix} = \begin{bmatrix} b & -a \\ -d & c \end{bmatrix}$$

Determinant of $S = 1 \Rightarrow bc - ad = 1$------------(8.17)
where a, b,c,d are known as Guassian constants representing the elements in final system matrix. Negative signs are taken arbitrarily so that the determinant of S , bc-ad =1

8.3 IMAGE PLANE AND MAGNIFICATION OF OPTICAL SYSTEM :

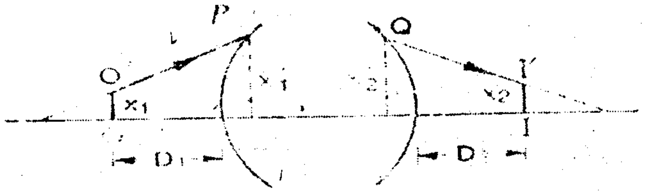

Fig 8.4

From Fig. 8.4, OO' -object plane , II' -image plane
PQ-optical system[series of lenses]
co ordinates of $O'[\lambda_1, x_1], P[\lambda'_1, x'_1], Q[\lambda'_2, x'_2], I'[\lambda_2, x_2]$

D_1 -Distsnce of image plane from lence

D_2 -Distance of image plane from lensesysatem.

All distsnces towards left of refracting system are as negative and right side are taken as positive.

The light ray from object O' translates along $O'P$ .undergoes refraction and translation between Pand Q and translates from Q to I' i.e upto image plane.

Hence the translation matrix for distsnce $O'P$ is

$$T = \begin{bmatrix} 1 & 0 \\ -D_1 & 1 \end{bmatrix}$$

Note:The medium considered is air. Hence

$$\begin{bmatrix} \lambda'_1 \\ x'_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -D_1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ x_1 \end{bmatrix} \text{---------(18) (8.18)}$$

The system matrix for translation and refraction between P and Q is given by

$$S = \begin{bmatrix} b & -a \\ -d & c \end{bmatrix}$$

$$\begin{bmatrix} \lambda_2' \\ x_2' \end{bmatrix} = \begin{bmatrix} b & -a \\ -d & c \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -D_1 & 1 \end{bmatrix} \text{--------(19) (8.19)}$$

Translation matrix for translation between Q and I^1 is given by

$$T = \begin{bmatrix} 1 & 0 \\ D_2 & 1 \end{bmatrix}$$

$$\text{Hence } \begin{bmatrix} \lambda_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ D_2 & 1 \end{bmatrix} \begin{bmatrix} b & -a \\ -d & c \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -D_1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ x \end{bmatrix} \text{------------(8.20)}$$

The product of the matrices [T S T] is given by

$$= \begin{bmatrix} b + aD_1 & -a \\ bD_2 + aD_1D_2 - d - cD_1 & c - aD_2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} \lambda_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} b + aD_1 & -a \\ bD_2 + aD_1D_2 - d - cD_1 & c - aD_2 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ x_1 \end{bmatrix} \text{------------(8.21)}$$

$$\Rightarrow \quad x_2 = (bD_2 + aD_1D_2 - d - cD_1)\lambda_1 + (c - aD_2)x_1$$

When object lies on x-axis,image also lies on x-axis. Hence $x_1 = x_2 = 0$ .Substituting in above equation.

$$0 = (bD_2 + aD_1D_2 - d - cD_1)\lambda_1 + (c - aD_2).0$$

$$bD_2 + aD_1D_2 - d -_1 cD = 0 \text{------------(8.22)} \quad (\text{Since } \lambda_1 \neq 0,)$$

This equation is called equation of image plane.

$$\therefore \begin{bmatrix} \lambda_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} b + aD_1 & -a \\ 0 & c - aD_2 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ x_1 \end{bmatrix} \text{------------(8.23)}$$

From eq-(8.21)

When $x_2 \neq 0$,

$$x_2 = 0.\lambda_1 + (c - aD_2)x_1 = (c - aD_2)x_1 \text{------------(8.24)}$$

Magnification of the system :

$$\text{Magnification } m = \frac{x_2}{x_1} = c - aD_2 \quad [\text{from eq } 8.24]$$

But the determinant of system matrix is 1.

$$\text{Therefore} \begin{vmatrix} b + aD_1 & -a \\ 0 & c - aD_2 \end{vmatrix} = 1$$

$$\Rightarrow (b + aD_1)(c - aD_2) = 1 \Rightarrow b + aD_1 = \frac{1}{c - aD_2} = \frac{1}{m} \text{------------(8.25)}$$

Hence we can write coordinates of image plane as

$$\begin{bmatrix}\lambda_2\\ x_2\end{bmatrix}=\begin{bmatrix}\frac{1}{m} & -a\\ 0 & m\end{bmatrix}\begin{bmatrix}\lambda_1\\ x_1\end{bmatrix}$$

8.4 System matrix for a thick lens:-

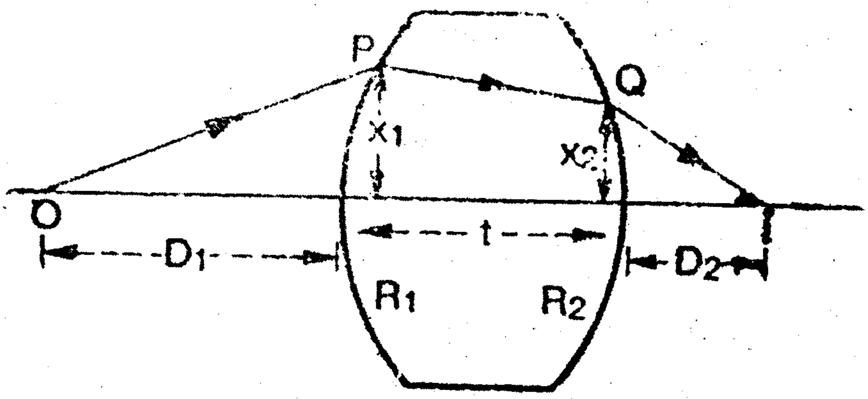

Fig 8.5

Thickness of the lens = t,
Refractive index of the lens material = μ ,
Radius of curvature of two surfaces of lences = R_1, R_2 ,
Distance of object point 'O' from lens = D_1
Distance of image point I from lens = D_2

From fig8.5, Light ray from object point 'O' travels along OP, refracts into lens at P, translates between P and Q, refracts at P,Q and finally travels along QI to form image at I. The coordinates of Pand Q are $(\lambda_1, x_1), (\lambda_2, x_2)$ respectively.

$$\begin{bmatrix}\lambda_2\\ x_2\end{bmatrix}=\begin{bmatrix}1 & -P_2\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ \frac{t}{\mu} & 1\end{bmatrix}\begin{bmatrix}1 & -P_2\\ 0 & 1\end{bmatrix}\begin{bmatrix}\lambda_1\\ x_1\end{bmatrix} \qquad (8.26)$$

where $P_1 = \frac{\mu-1}{R_1}, P_2 = \frac{1-\mu}{R_2} = \frac{-(\mu-1)}{R_2}$ are refracting powers of two surfaces. Hence system matrix for a thick lens is given by

$$S = \begin{bmatrix} 1 & -P_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{t}{\mu} & 1 \end{bmatrix} \begin{bmatrix} 1 & -P_2 \\ 0 & 1 \end{bmatrix} \qquad (8.27)$$

$$S = \begin{bmatrix} 1 - P_2\frac{t}{\mu} & -P_1 - P_2\left(1 - \frac{P_1 t}{\mu}\right) \\ \frac{t}{\mu} & 1 - \frac{P_1 t}{\mu} \end{bmatrix} \qquad (8.28)$$

$$= \begin{bmatrix} b & -a \\ -d & c \end{bmatrix}$$ Comparing the values of a,b,c,d

$$a = P_1 + P_2(1 - P_1\frac{t}{\mu}) = P_1 + P_2 - P_1P_2\frac{t}{\mu},$$

$$b = 1 - P_2\frac{t}{\mu}, c = 1 - P_1\frac{t}{\mu}, \qquad (8.29)$$

$$d = -\frac{t}{\mu}$$

Substituting the values of a,b,c,d in equation of image plane

$bD_2 + aD_1D_2 - d - cD_1 = 0$, We get

$$\left(1 - \frac{P_2 t}{\mu}\right)D_2 + \left(P_1 + P_2 - \frac{P_1P_2t}{\mu}\right)D_1D_2 + \frac{t}{\mu} - \left(1 - \frac{P_1 t}{\mu}\right)D_1 = 0$$

$$\Rightarrow P_1 + P_2 - P_1P_2\frac{t}{\mu} = \frac{1}{D_2} - \frac{1}{D_1} + \frac{P_2 t}{\mu D_1} - \frac{P_1 t}{\mu D_2} - \frac{t}{\mu D_1 D_2} \qquad (8.30)$$

But $D_1, D_2 >> t$. Hence neglecting last three terms in above equations

$$P_1 + P_2 - \frac{P_1P_2t}{\mu} = \frac{1}{D_2} - \frac{1}{D_1}$$, But $$\frac{1}{D_2} - \frac{1}{D_1} = \frac{1}{f}$$

Where D_1 and D_2 are image and object distances, f is the focal length of the lens

Also

$$P_1 = \frac{\mu_1 - 1}{R_1}, P_2 = -\frac{(\mu-1)}{R_2}$$

Substituting in the above equation, $$\frac{1}{f} = (\mu-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(\mu-1)t}{\mu R_1 R_2}\right] \text{--(31)} \qquad (8.31)$$

8.5 System matrix for thin lens :-

System matrix for this lens in general is

$$S=\begin{bmatrix} b & -a \\ -d & c \end{bmatrix}=\begin{bmatrix} 1-\frac{P_2t}{\mu} & -P_1-P_2\left(1-\frac{P_1t}{\mu}\right) \\ \frac{t}{\mu} & 1-\frac{P_1t}{\mu} \end{bmatrix}$$

For this lens is t=0

$$\therefore S=\begin{bmatrix} b & -a \\ -d & c \end{bmatrix}=\begin{bmatrix} 1 & -P_1-P_2 \\ 0 & 1 \end{bmatrix}\_\_\_(8.32)$$

Comparing a,b,c and d values $a=P_1+P_2, b=1, c=1, d=0$

Substituting in equation of image plane

$$bD_2+aD_1D_2-d-CD_1=0$$

$$\Rightarrow D_2+(P_1+P_2)D_1D_2-D_1=0$$

$$\Rightarrow P_1+P_2=\frac{1}{D_2}-\frac{1}{D_1}=\frac{1}{f}$$

$$\Rightarrow \frac{\mu-1}{R_1}-\frac{\mu-1}{R_2}=\frac{1}{D_2}-\frac{1}{D_1}=\frac{1}{f}$$

$$\Rightarrow \frac{1}{f}=(\mu-1)(\frac{1}{R_1}-\frac{1}{R_2})\_\_\_\_\_\_(8.33)$$ (Lens makar's formula)

System matrix of lens $L_1, S_1=\begin{bmatrix} 1 & -f \\ 0 & 1 \end{bmatrix}$

8.6 System matrix for combination of lenses :-

8.6.1 System matrix for two thin lenses seperated by a distance :-

Let us consider two thin lenses L_1 and L_2 of focii f_1 and f_2 respectively (fig. 8.6). Let they be seperated by a distance t. System matrix for these two lenses can be obtained by multiplying the system matrix of lens L_1, Translation matrix between lenses and System

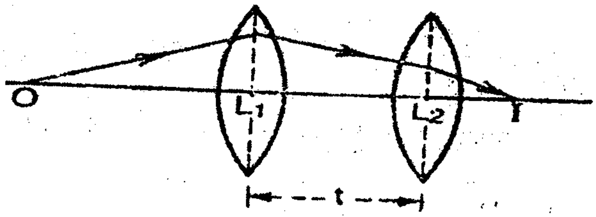

Fig.8.6

System matrix of lens $L_1, S_1=\begin{bmatrix} 1 & -\frac{1}{f_1} \\ 0 & 1 \end{bmatrix}$ ....(8.34)

System matrix of lens $L_2, S_2 = \begin{bmatrix} 1 & -\frac{1}{f_2} \\ 0 & 1 \end{bmatrix}$ ....(.8.35)

Translation matrix between two lenses $T=\begin{bmatrix} 1 & 0 \\ t & 1 \end{bmatrix}$

<u>Note</u> :

$\mu = 1$ for air.

Hence $\frac{t}{\mu} = \frac{t}{1} = t$

Hence system matrix for two thin lenses separated by a distance 't' is given by

$$S = S_1 T S_2 = \begin{bmatrix} 1 & -\frac{1}{f_1} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ t & o \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{f_2} \\ 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} b & -a \\ -d & c \end{bmatrix} = \begin{bmatrix} 1-\frac{t}{f_2} & -\frac{1}{f_1}-\frac{1}{f_2}+\frac{t}{f_2 f_1} \\ t & 1-\frac{t}{f_1} \end{bmatrix}$$

Comparing 'a' value

$$a=\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{t}{f_1 f_2}$$

Hence the combined focal length F of two lenses seperated by a distance is given by

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{t}{f_1 f_2} \text{ ....(36)}$$

8.6.2 <u>Thin lenses incontact</u> :-

When lenses are in contact t=0.
The combined focal length F is given by

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$$

(See Fig. 8.7)

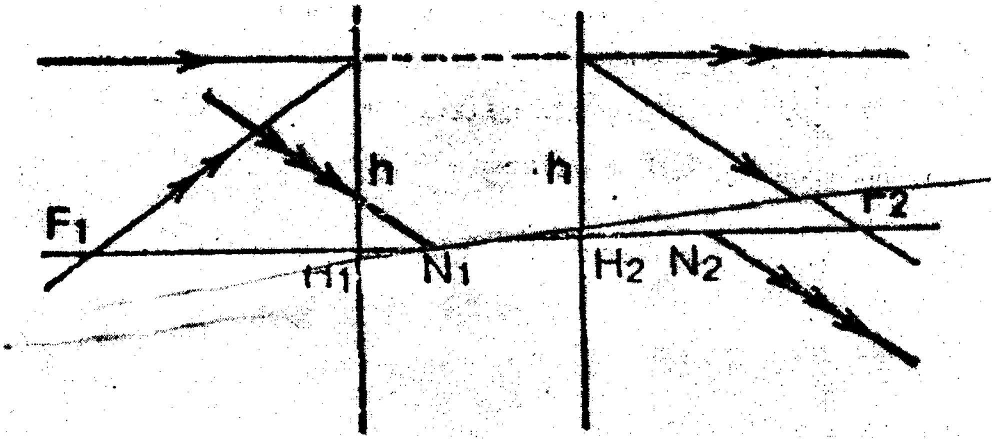

Fig 8.7

8.7 Cardinal points of lens system :-

When we consider co-axial system of lenses [number of lenses having a common principal axis] ,it becomes difficult to locate image and object location because refraction takes place at each lens.So "cardinal points are the points which help in locating the object and image spaces with a co-axial system of lenses."

There are three sets of cardinal points

1. Two Principal or Unit points.
2. Two Focal points.
3. Two nodal points.

8.7.1 Principal or Unit points (H_1, H_2) :-

"The principal points H_1 and H_2 are a pair of conjugate points on principal axis with unit linear transverse magnification."The planes passing through these points and perpendicular to principal axis are called unit planes. if an object is placed at one principal point ,image of same size is formed at another principal point.

8.7.2 Focal points (F_1, F_2) :-

The point on the principal axis where parallel rays meet forming the image of distant object is called focal point. There aretwo such focal points on the principal axis on two sides of a lens system (F_1, F_2) (Fig. 8.7). The planes pasing through these points and perpendicular to principal axis are called "focal planes".The image of the object at the first focal point F_1 is at infinity and the object of the image at F_2 is at infinity.

8.7.3 Nodal points :-

Two conjugate points on the principal axis N_1 and N_2 with unit angular magnification are called Nodal points. The planes passing through the nodal points and perpendicular to principal axis are called nodal planes. If a ray strikes the first nodal point N_1 at an angle, it comes out of the second nodal point N_2 parallel to the object ray.

Note :-If the medium on the both sides of co-axial system of lenses is same, the nodal points and unit points coincide with each other.

8.7.4 Location of unit planes :-

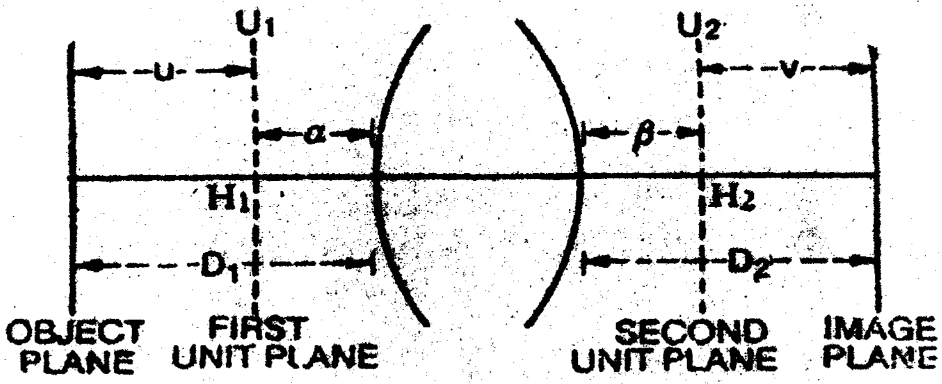

Fig 8.8

Generally object and image distances are measured from unit planes in a co-axial system of lenses. The magnification of optical system m=1 for unit planes (Fig. 8.8)

$$b + aD_1 = \frac{1}{c - aD_2} = \frac{1}{m}$$

$$D_1 = \alpha, D_2 = \beta \Rightarrow b + a\alpha = \frac{1}{c - a\beta} = 1 \qquad \text{(8.37)}$$

$$\Rightarrow \alpha = \frac{1-b}{a}, \beta = \frac{c-1}{a} \qquad \text{(8.38)}$$

For object plane $D_1 = u + \alpha$

For image plane $D_2 = v + \beta$ (8.39)

Image plane equation is $bD_2 + aD_1D_2 - d - cD_1 = 0$

$$\Rightarrow D_2(b + aD_1) = d + cD_1$$

$$D_2 = \frac{d + cD_1}{b + aD_1} \qquad \text{(8.40)}$$

Substituting D_1 and D_2 values in above equation

$$v+\beta=\frac{d+c(u+\alpha)}{b+a(u+\alpha)}$$

$$\Rightarrow v+\frac{(c-1)}{a}=\frac{d+c(u+\frac{1-b}{a})}{b+a(u+\frac{1-b}{a})}$$

Simplifying and re-arranging the the equations $v=\frac{ad-bc+au+1}{a(1+au)}$

but det $|S|=1 \Rightarrow bc-ad=1$

$$\therefore v=\frac{-1+au+1}{a(1+au)}=\frac{u}{1+au} \qquad (8.41)$$

$$\Rightarrow \frac{1}{v}=\frac{1+au}{u}=\frac{1}{u}+a \Rightarrow \frac{1}{v}-\frac{1}{u}=a \_\_\_\_\_\_(42) \quad (8.42)$$

Hence 'a' denotes the reciprocal of the focal length f

$$\therefore a=\frac{1}{f}$$

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \_\_\_\_\_\_(43) \quad (8.43)$$

Hence "when the distances are measured from unit planes in co-axial system of lenses ,the value in position 'a' of the system matrix of co-axial system of lenses gives the combined focal length of the system". α, β values give the positions of unit planes.

8.7.5 Location of Nodal planes :-

γ, δ =distances of nodal planes from the system of lenses.

According to the definition of nodal points N_1 and N_2, See Fig. 8.9

$\lambda_1=\mu\cos\theta=\lambda_2=\mu\cos\theta$,when the medium on two sides is same.

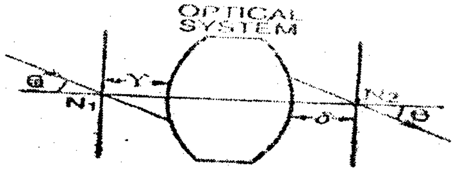

Fig. 8.9

so the image plane is given by

$$\begin{bmatrix}\lambda_2\\ x_2\end{bmatrix}=\begin{bmatrix}b+aD_1 & -a\\ 0 & c-aD_2\end{bmatrix}\begin{bmatrix}\lambda_1\\ x_1\end{bmatrix}$$

For nodal planes $D_1=\gamma$, $D_2=\delta$

$$\Rightarrow\begin{bmatrix}\lambda_2\\ x_2\end{bmatrix}=\begin{bmatrix}b+a\gamma & -a\\ 0 & c-a\delta\end{bmatrix}\begin{bmatrix}\lambda_1\\ x_1\end{bmatrix} \tag{8.44}$$

From above figure, $x_1=0, x_2=0$ [distances of object and image points from principal axis].

$$\therefore\begin{bmatrix}\lambda_2\\ 0\end{bmatrix}=\begin{bmatrix}b+a\gamma & -a\\ 0 & c-a\delta\end{bmatrix}\begin{bmatrix}\lambda_1\\ 0\end{bmatrix}$$

$$\Rightarrow \lambda_2=(b+a\gamma)\lambda_1 \Rightarrow b+a\gamma=1$$

$$\text{or } \gamma=\frac{1-b}{a}=\alpha \tag{8.45}$$

When medium is same on both sides, Nodal points and principal points coincide.

$\therefore$ Linear magnification m=1

$$\Rightarrow b+aD_1=\frac{1}{c-aD_2}=1\Rightarrow c-a\delta=1$$

$$\Rightarrow \delta=\frac{c-1}{a}=\beta \tag{8.46}$$

8.8 SOLVED EXAMPLES

1.The radius of curvature of the surfaces of a double convex lens are 25 cms and -25 cms. The refractive index of the lens is 1.5. Find the power of the lens ? [Ans4 diopters]

Solution :-

R_1=25 cm , R_2=-25cm, μ =1.5

Power of the lens in diopters =100/f

where 'f' is the focal length of double convex lens

'f' is given by the formula

$$\frac{1}{f}=(\mu-1)(\frac{1}{R_1}-\frac{1}{R_2})=(1.5-1)(\frac{1}{25}+\frac{1}{25})$$

$$=0.5\times\frac{2}{25}=\frac{1}{25}$$

$$\frac{1}{f}=\frac{1}{25}$$

$\Rightarrow$ f=25cm

$\therefore$ power of the lens P=100/f

=100/25

=4diopters.

2.Find the focal length, unit planes of Ramsden's eyepiece ?

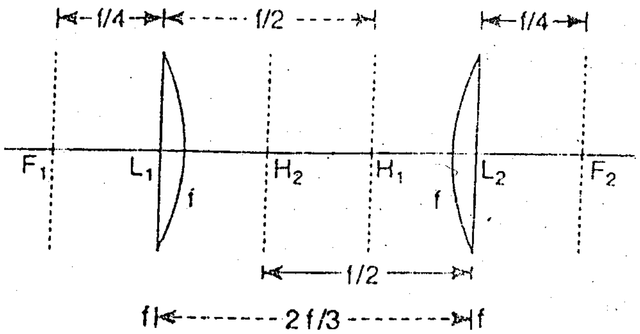

Fig 8.10

Focal length of L_1 =f, Focal length of $L_2 = f$

Distance between lenses =2f/3

System matrix of L_1, $S_1 = \begin{bmatrix} 1 & \frac{-1}{f} \\ 0 & 1 \end{bmatrix}$

Translation matrix from L_1, to L_2, $T = \begin{bmatrix} 1 & 0 \\ \frac{2f}{3} & 1 \end{bmatrix}$

System matrix for lens L_2, $S_2 = \begin{bmatrix} 1 & \frac{-1}{f} \\ 0 & 1 \end{bmatrix}$

System matrix of Ramsden's eyepiece

$$S = \begin{bmatrix} 1 & \frac{-1}{f} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{2f}{3} & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{-1}{f} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{-4}{3f} \\ \frac{2f}{3} & \frac{1}{3} \end{bmatrix}$$

Gaussian constants $a = \frac{4}{3f},\ b = \frac{1}{3},\ c = \frac{1}{3},\ d = \frac{-2f}{3}$

Combined focal length of the eyepiece

$$F = \frac{1}{a} = \frac{3f}{4}$$

The distances of unit planes from respective lenses

$$du_1 = \frac{1-b}{a} = \frac{f}{2},\ du_2 = \frac{c-1}{a} = \frac{-f}{2}$$

Here nodal planes coincide with unit planes

3.Find the focal length unit planes of Huygen's eyepiece ?

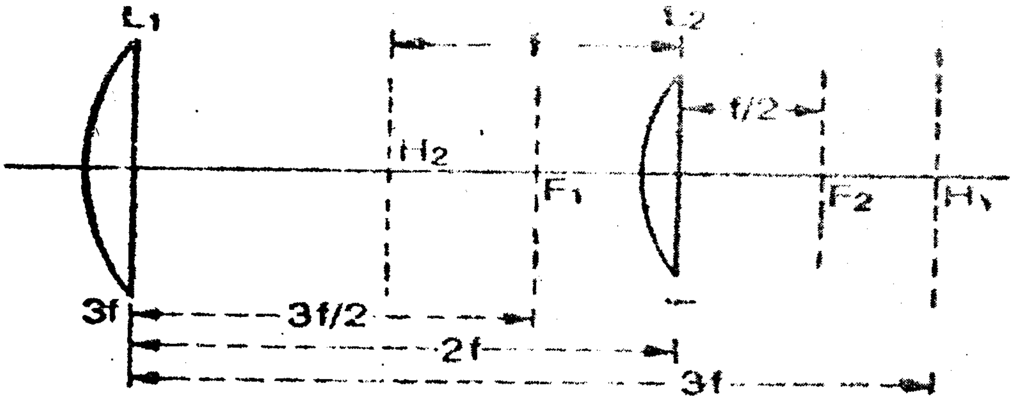

Fig 8.11

The focal length of L_1 =3f,

Focal length of L_2 =f

Seperation distance =2f

System matrix of lens L_1 , $S_1 = \begin{bmatrix} 1 & \frac{-1}{3f} \\ o & 1 \end{bmatrix}$

Translation matrix between L_1 and L_2 , $T = \begin{bmatrix} 1 & 0 \\ 2f & 1 \end{bmatrix}$

System matrix of lens L_2, $S_2 = \begin{bmatrix} 1 & \frac{-1}{f} \\ 0 & 1 \end{bmatrix}$

System matrix of Huygens eyepiece

$$S=\begin{bmatrix} 1 & \frac{-1}{f} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2f & 1 \end{bmatrix}\begin{bmatrix} 1 & \frac{-1}{3f} \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} 1 & \frac{-2}{3f} \\ 2f & \frac{1}{3} \end{bmatrix}$$

Gaussian constants are $a = \frac{2}{3f}$, $b= -1$, $c= 1/3$, $d= -2f$

Combined focal length of Huygens eyepiece

$$F=\frac{1}{a}=\frac{3f}{2}$$

The distance of unit planes from respective lenses

$$du_1 = \frac{1-b}{a} = 3f \quad , \quad du_2 = \frac{c-1}{a} = f$$

4. In Huygen's eyepiece the two plano-convex lenses have focal lengths 12 cm and 4cm respectively .They are seperated by a distance of 8cm. Find the position of unit planes ?

Solution : -

For Huygen's eyepiece the focal length of first lens is three times the focal length of second lens.

$f = 4$cm for L_2 and

$3f = 3 \times 4 = 12$ for L_1

System matrix for Huygen's eyepiece

$$S = \begin{bmatrix} -1 & \frac{-2}{3f} \\ 2f & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} b & -a \\ -d & c \end{bmatrix},$$

$$a = \frac{2}{3f}, b = -1, c = \frac{1}{3}, d = -2f$$

where a,bc,d of system matrix are called Guassian constant.

Combined focal length of the system $F = \frac{1}{a} = \frac{3f}{2}$

The distances of unit planes from respective lenses are given by

$$du_1 = \frac{1-b}{a} = \alpha, \quad du_2 = \frac{c-1}{a} = \beta$$

Substituting $f=4cm$ in above equation we have

$$a=\frac{2}{3f}=\frac{2}{3\times4}=\frac{2}{12}=\frac{1}{6}$$

$b=-1,$

$c=1/3,$

$d=-2f=-2\times4=-8$,

$$du_1=\frac{1-b}{a}=\frac{1+1}{(1/6)}=2\times6=12$$

$$du_2=\frac{c-1}{a}=\frac{1/3-1}{(1/6)}=\frac{-2}{3}\times6=-4$$

Hence distance of unit planes from respective lenses are 12cm and 4cm.

5. Write the system matrix of a convex lens (placed in air) made of glass of refractive index 1.5 and radii of curvature 50 cm .

Solution :-

System matrix of the thin lens $=S=\begin{bmatrix}1 & -P_1-P_2\\0 & 1\end{bmatrix}$

where $P_1=\frac{\mu-1}{R_1}$, $P_2=\frac{1-\mu}{R_2}=-\frac{(\mu-1)}{R_2}$

$\mu=1.5,\quad R_1=50\text{ cm},\quad R_2=-50\text{ cm}$

$P_1=0.5/50=1/100$, $P_2=-(0.5)/-50=1/100$

$-P_1-P_2=-2/100=-1/50$

$$\therefore\ S=\begin{bmatrix}1 & -\frac{1}{50}\\0 & 1\end{bmatrix},$$

$f=1/a=50cm$

6. A convergent thick lens has radii of curvature of +6 cm and -6 cm and thickness 2 cm .The refractive index of its material is 1.6 and is placed in air .Obtain its system matrix . Find the focal length of the lens .

Solution :-

System matrix of a thick lens is given by $S=\begin{bmatrix}1-P_2\frac{t}{\mu} & -P_1-P_2(1-P_1\frac{t}{\mu})\\\frac{t}{\mu} & 1-P_1\frac{t}{\mu}\end{bmatrix}$

where $P_1 = \frac{\mu - 1}{R_1}$,

$$P_2 = \frac{1-\mu}{R_2} = -\frac{(\mu - 1)}{R_2}$$

In the given problem, R1=6 cm, R2= -6 cm, t= 2 cm, μ=1.6
Substituting these values in above forms,

P_1=0.6/6=1/10, P_2=-0.6/-6=1/10, $P_1 t/\mu$ =1/10×2/1.6=1/8

$P_2 t/\mu$ =1/8, t/μ =2/1.6=5/4

$1-P_1 t/\mu$ =1-1/8=7/8,

$1-P_2 t/\mu$ =7/8

$$S=\begin{bmatrix} \frac{7}{8} & (-\frac{1}{10}-\frac{1}{10}(\frac{7}{8})) \\ \frac{5}{4} & \frac{7}{8} \end{bmatrix},$$

$$=\begin{bmatrix} \frac{7}{8} & \frac{-3}{16} \\ \frac{5}{4} & \frac{7}{8} \end{bmatrix} = \begin{bmatrix} b & -a \\ -d & c \end{bmatrix}$$

Gaussian constants are b = 7/8, a = 3/16, d = -5/4, c = 7/8
Combined focal length F = 1/a
= 16/3
= 5.33 cm.

7. The focal lengths of two thin convex lenses are 20 cm and 10 cm. They are at a distance of 10 cm on the axis of symmetry. Find the combined focal length of the whole system?

Solution :-

Combined focal length of two lenses seperated by a distance is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{t}{f_1 f_2}$$

f_1=20 cm, f_2=10 cm, t= 10 cm

$$\frac{1}{F} = \frac{1}{20} + \frac{1}{10} - \frac{10}{20\times10}$$

$$= \frac{1}{10}$$

F=10 cm

8. Two thin converging lenses of powers 5 diopters and 4 diopters are placed co-axially 10 cm apart .Find the focal length of the combination?

Solution :-

Power of the lens P=1/f (in metres)

$f_1 = 1/P_1 = 1/4 = 0.25$ m $= 25$ cm ,

$f_2 = 1/P_2 = 1/5 = 0.20$ m$=20$ cm

The focal length F of the combination of two lenses seperated at a distance t is given by.

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{t}{f_1 f_2}$$

Substituting these values

$$\frac{1}{F} = \frac{1}{25} + \frac{1}{20} - \frac{10}{25 \times 20}$$

$=0.04+0.05+0.02$

$=0.11$

$$F = \frac{1}{0.11}$$

$$= \frac{100}{11}$$

$=9.9$ cm

8.9 SUMMARY :-

1. Translation matrix between two refracting surfaces is given by $T = \begin{bmatrix} 1 & 0 \\ \frac{D}{\mu} & 1 \end{bmatrix}$

2. Refraction matrix at the refracting surface is given by $R = \begin{bmatrix} 1 & -P \\ 0 & 1 \end{bmatrix}$

3. System matrix due to a lens system is given by $S = \begin{bmatrix} b & -a \\ -d & c \end{bmatrix}$

4. Image plane equation us $bD_2 + aD_1D_2 - cD_1 - d = 0$.

5. Magnification of the system $m = \dfrac{1}{b + aD_1} = c - aD_2$.

6. System matrix for a thick lens $S = \begin{bmatrix} 1 - P_2\frac{t}{\mu} & -P_1 - P_2(1 - \frac{t}{\mu}P_1) \\ \frac{t}{\mu} & 1 - \frac{t}{\mu} \end{bmatrix}$

7. Thin lens formula $\frac{1}{f} = (\mu - 1)(\frac{1}{R_1} - \frac{1}{R_2})$

8. Thick lens formula $\frac{1}{f} = (\mu - 1)(\frac{1}{R_1} - \frac{1}{R_2} + \frac{(\mu - 1)}{R_1 R_2}\frac{t}{\mu})$

9. The resultant focal length of two lenses seperated by a distance is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{t}{f_1 f_2}$$

10. The focal length of two lenses in contact(t=0) $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

11. The three cardinal points are principal points, focal points and nodal points.

12. Unit planes : The two conjugate planes for which the lateral magnification is unity.
Nodal planes: The two conjugate planes with unit angular magnification are called Nodal planes.

13. Location of unit planes $\alpha = \frac{1-b}{a}, \beta = \frac{c-1}{a}$

Location of nodal planes $\gamma = \alpha, \delta = \beta$

14. Concept of construction of Translation, refraction and system matrices are explained. Image plane equation is derived and magnification value defined. Thick lens formula and thin lens formula's are derived and their system matrices constructed. Cardinal points are defined and their location derived.

8.10 KEYWORDS: Coaxial system of lenses, object plane, image plane, translation, refraction and system matrices, optical directional cosine lenses, refractive index, magnification, radius of curvature, cardinal points, unit planes, nodal planes, focal planes, gaussian constants.

8.11. SELF ASSESSMENT QUESTIONS:

8.11.1. Long Answer questions :-

1. Explain the translation,refraction and system matrices. Obtain the system matrix for a thick lens and hence obtain the formula for a thin lens ?

2. What is system matrix ? Obtain it in the case of a system of two thin lenses.seperated by a distance and hence find the formula for its focal length ?

3. Show that the focal length of combination of two thin lenses of focal lengths f_1 and f_2 seperated by a distance 't' is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{t}{f_1 f_2}$$

8.11.2 Short Answer questions :-

1. Write translation and refraction matrices?
2. What is a System matrix and its determinant ?
3. Deduce the formula for the focal length of a thin lens using matrix method ?

4. For an object on the principal axis show that co-ordinates corresponding to the image plane can be written as

$$\begin{bmatrix} \lambda_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{m} & -a \\ o & m \end{bmatrix} \begin{bmatrix} \lambda_1 \\ x_1 \end{bmatrix}$$

5. What are the various cardinal points of a lens system ?Explain.

8.11.3 EXERCISE :-

1. A lens of thickness 2 cm and refractive index 1.5 placed in air has radii of curvature 8 cm and 8 cm. Find the system matrix and focal length of this thick lens

[ans; $S = \begin{bmatrix} 0.9167 & -0.1198 \\ 1.333 & 0.9167 \end{bmatrix}$, f=8.347]

2. The focal lengths of two thin convex lenses are 20 cm and 10 cm. They are at a distance 10 cm on the axis of symmetry .If a point object of length 1 cm is placed at a distance of 20 cm from the first lens, find the position of the image and magnification?

[ans:Image plane v=10 cm,magnification m=-1/2]

3. Obtain the system matrix and determine the focal length and position of unit planes of a convex lens of thickness 1 cm and radii +4 cm and -4 cm respectively when placed in air ?

[ans: $\begin{bmatrix} 0.9167 & -0.24 \\ 0.6667 & 0.9167 \end{bmatrix}$,4.2 cm,0.35 cm,-0.35 cm]

4. Two convex lenses of focal lengths 6 cm and 2 cm are co-axially placed at a distance of seperation 4 cm between them. Calculate the power of combination ?

[ans:33.3 diopters]

8.12 REFERENCE BOOKS :-

1. Unified physics vol-2	Dr. S.L.Gupta Sanjeev Gupta Jai Prakash Nath & Co Meerut
2. B.Sc Optics	Telugu Academy
3. Introduction to modern Optics	G.K.Fowles(Holt-Rinchert)
4. Optics	Ghatak
5. B.Sc Second year Physics	Dr.K. Ramakrishna D.V.Brahmaji A.Srinivasa Rao S.L.V. Mallikharjun

Unit III

ABERRATIONS

LESSON 9

OBJECTIVES:-

1.The deviation from the actual size,shape and position of the image formed by an optical lens from ideal image measurements is called "Aberrations".So objective of the present chapter is to define different types of aberrations produced in an image by the lens,and derive the value of aberration mathematically and give methods to minimise the aberration.

2.Derive the equations for deviation produced by a lens and the dispersion produced by an optical instrument.

3.Explain about two types of aberrations namely Chromatic and Monochromatic.

4.Derive an expression for longitudinal chromatic aberration.

5.Explain the principle and working of an achromatic doublet and also achromatism with two lenses seperated by a distance.

6.Define and explain spherical aberration and mention the expressions for spherical aberration in case of plane refracting surgface sperical.

7.Give methods to minimise the spherical aberration.

8.Give an overview of coma and astigmatism.

STRUCTURE

9.1 INTRODUCTION:-

In real optical systems,image formation is due to paraxial rays(Rays coming from object and nearer to principal axis) as well as marginal rays(Rays from object travelling far off from Principal axis).Although we use defectless lenses in optical systems,we obtain a coloured ,spread out image of point object.The reason is the focal points of marginal rays and paraxial rays are different and their deviations at lenses are different.Hence image of single point object may not be a single image.We may obtain images of different colours of at different points and also of different sizes for a single object.When object has longitudinal and lateral dimensions,these defects become more prominent producing distorted image.These defects produced in the image formation are called as "Aberrations". There are mainly two types of aberrations, namely

1.Chromatic aberration
2.Monochromatic aberration

Before going into details of these topics first let us derive the mathematical formulas for deviation produced by lens in a light ray and dispersion produced by the prism in white light ray.These values will be helpful in deriving mathematical formulas for aberrations and in methods to overcome these aberrations.

9.1.1 Deviation produced by a lens :-

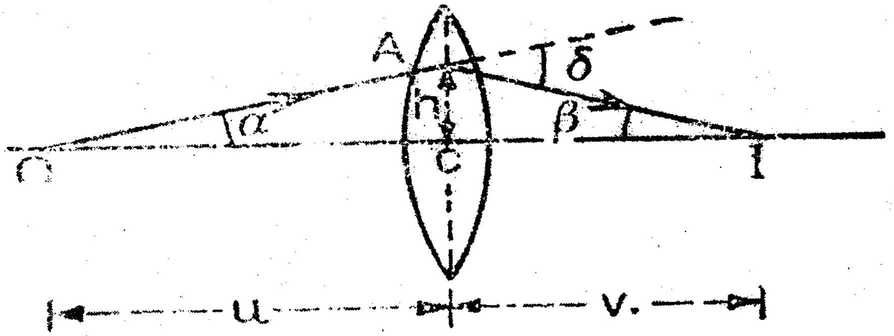

Fig 9.1

OA-incident ray , AI-emergent ray, $\angle\delta$ =Angle of deviation , AC=h- height at which the light ray from object strikes the lens (see Fig. 9.1).

$\angle$POC= α , $\angle$AIC= β ,u=object distance, v=image distance

From Δ AIO,sum of internal angles= external angle.

$\angle \quad \delta$ -------------------(9.1)

when angles are small, $\alpha \approx \tan\alpha, \beta \approx \tan\beta$

from fig 9.1 $\text{Tan}\,\alpha = \frac{h}{-u}, \text{Tan}\,\beta = \frac{h}{+v}$

substituting these values in above equations,

$$\frac{h}{-u} + \frac{h}{+v} = \delta \Rightarrow h\left(\frac{1}{v} - \frac{1}{u}\right) = \delta$$

but $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{h}{f} = \delta \text{ -------------------------- } (9.2)$$

so deviation produced by a lens to a ray is equal to the ratio of the height at which the ray strikes the lens and focal length of the lens.

"Hence although 'f' is same ,rays coming from the object and touching the lens at different heights have different deviations".

9.1.2 Dispersion through a prism:-

The refractive index of a prism 'μ' is given by

$$\mu = \frac{Sin A+\delta/2}{Sin A/2} \text{ ---9.3)}$$

where A-angle of prism and δ is the deviation produced by the prism of a light ray.Refractive index of the prism 'μ' varies with colour, hence produces different deviations to different colours.

$$\mu blue > \mu red$$

From equation 3,when angles are small,we can write

$$\mu = \frac{\frac{A+\delta}{2}}{\frac{A}{2}} = \frac{A+\delta}{A}$$

$$\Rightarrow \mu A = A+\delta \quad \Rightarrow \quad \mu A - A = \delta \quad \Rightarrow \delta = (\mu - 1)A \text{ ------------------ } (9.4)$$

$\because \mu$'s are different for different colours

we have $\delta_V = (\mu_V - 1)A$; $\quad \delta_R = (\mu_R - 1)A$ -------------------------(9.5)

DISPERSIVE POWER:-

It is the ratio of angular dispersion between two colours to the deviation produced for mean ray by the prism.It is denoted by ω.

$$\text{Dispersive power, } \omega=\frac{\delta_V-\delta_R}{\delta} \text{ ------------(9.6),}$$

$$\delta=\frac{\delta_V+\delta_R}{2}\ , \quad \delta=(\mu-1)A \text{------------(9.7)}$$

substituting eq(5),(7) in eq(6)

$$\omega=\frac{(\mu_V-1)A-(\mu_R-1)A}{(\mu-1)A}=\frac{\mu_V-\mu_R}{\mu-1}\ ,$$

$$\omega=\frac{d\mu}{(\mu-1)} \text{ -------------(9.8)}$$

$$\text{where } d\mu=\mu_V-\mu_R \text{ and } \mu=\frac{\mu_V+\mu_R}{2}$$

where μ_V, μ_R are refractive indices of prism or lens material for violet and red colours respectively.

9.2 ABERRATIONS:-

"The deviations from the actual size,shape and positions of an image as calculated by simple equations are called aberrations produced by a lens".

These aberrations or defects are classified mainly into two types.

1.Chromatic aberration : When the light is not monochromatic i.e white light , then the image becomes multicoloured and the defect is known as chromatic aberration.

2.Monochromatic aberration : When light is monochromatic, the defects produced in the shape of image are called monochromatic aberrations,They are subdivided into four categories namely

1.Spherical aberration
2.Coma
3.Astigmatism
4.Curvature

9.3 CHROMATIC ABERRATION:

Chromatic aberration is the result of the fact that the refractive index, μ of an optical system (lens) changes with colour .

According to lens makers formula,

$$\frac{1}{f}=(\mu-1)(\frac{1}{R_1}-\frac{1}{R_2})$$

$$\because \mu_v > \mu_{R,} f_v < f_R \Rightarrow \delta_v > \delta_R$$

So, the white light disperses into seven colours when it is incident on the lens and violet ray deviates more than red ray.So violet coloured rays come to focus before than the red ray.So, each colour forms image at a different focal point.

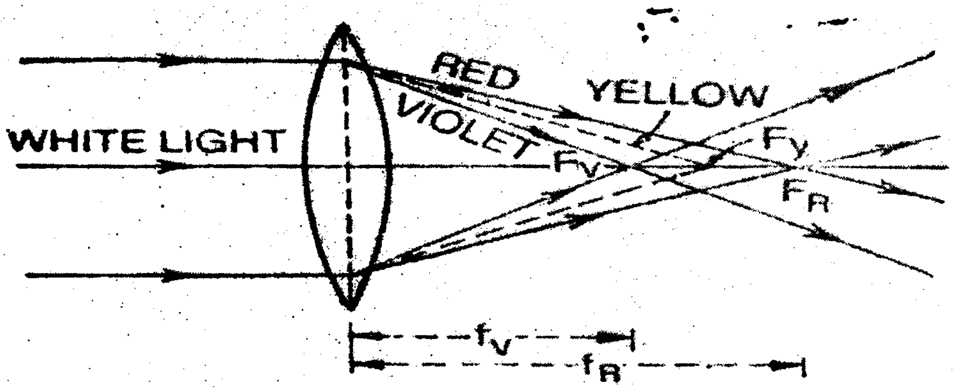

Fig 9.2

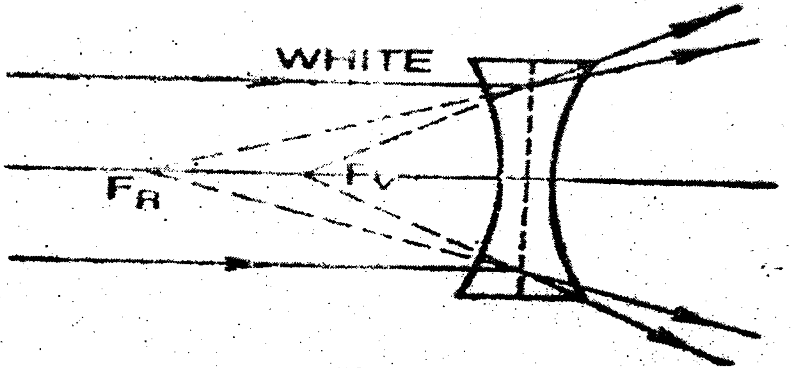

Fig 9.3

"Thus the image of a white object formed by a lens is coloured and blurred. This defect of image is known as Chromatic Aberration".

9.3.1 TYPES OF CHROMATIC ABERRATIONS :-

Chromatic aberration is of two types

1. Longitudinal Chromatic Aberration :-

"The formation of images of different colours in different positions along the axis is known as longitudinal or axial chromatic aberration .

"Thus the image of a white object formed by a lens is coloured and blurred. This defect of image is known as Chromatic Aberration".

9.3.1 . TYPES OF CHROMATIC ABERRATIONS :-

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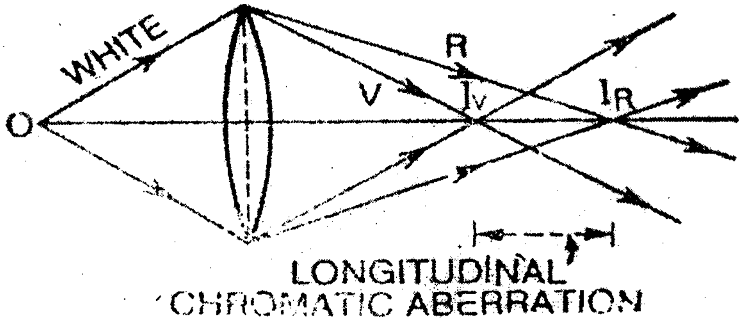

Fig 9.4

Longitudinal chromatic aberation = $I_R - I_v$ ----- (9.9)

When object is at infinity,

Longitudinal Chromatic Aberration = $f_R - f_v$

Longitudinal Chromatic Aberration of convex lens is positive and that of concave lens is negative.

2. Lateral Chromatic Aberration :-

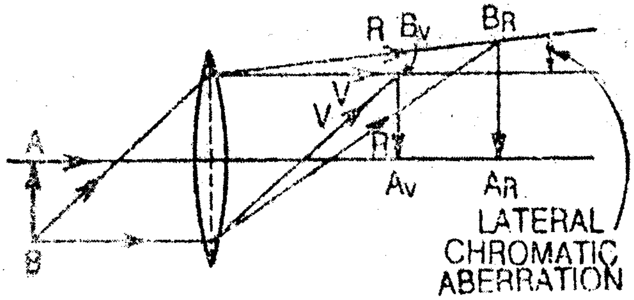

Fig. 9.5

"The images of different colours are formed of different sizes and this defect is called Lateral Chromatic Aberration".From Fig 9.3, it is evident that the height of red coloured image > the height of violet coloured image.

$$\text{Lateral Chromatic Aberration} = B_RA_R - B_VA_V \text{-------------------(9.10)}$$

9.3.2 Calculation of Longitudinal Chromatic aberration:-

Case-1:

When the object is situated at infinity then Lens Makers Formula is

$$\frac{1}{f} = (\mu - 1)(\frac{1}{R_1} - \frac{1}{R_2})$$

since is different for different colours ,we can write expressions for f_r, f_v, f_y where f_r -focal length for red colour f_v -focal length for violet colour f_y -focal length for yellow colour respectively.Hence

$$\frac{1}{f_R} = (\mu_R - 1)(\frac{1}{R_1} - \frac{1}{R_2}) \text{------(9.(11)}$$

$$\frac{1}{f_v} = (\mu_v - 1)(\frac{1}{R_1} - \frac{1}{R_2}), \text{------(9.-(12)}$$

$$\frac{1}{fy} = (\mu_y - 1)(\frac{1}{R_1} - \frac{1}{R_2}) \text{------(9.-(13)}$$

Substracting eq.9.12 fromeq.9.11,

$$\frac{1}{f_v} - \frac{1}{f_R} = \left[(\mu_v - 1) - (\mu_R - 1)\right]\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Multiplying and dividing the above equation with ($\mu_y - 1$),we obtain

$$\frac{1}{f_v} - \frac{1}{f_R} = \frac{(\mu_v - \mu_R)(\mu_y - 1)}{(\mu_y - 1)}(\frac{1}{R_1} - \frac{1}{R_2})$$

Substituting eq .9.13 in above equation and simplifying\

$$\frac{f_R - f_v}{f_Rf_v} = \frac{(\mu_v - \mu_R)}{(\mu_y - 1)}\frac{1}{f_y} \text{----------------(9.14)}$$

$$f_Rf_v \approx f_y^2 \Rightarrow \frac{f_R - f_v}{f_Rf_v} = \frac{(\mu_v - \mu_R)}{(\mu_y - 1)}\frac{1}{f_y}$$

But dispersive power $\omega = \frac{\mu_v - \mu_R}{\mu_y - 1} \Rightarrow f_r - f_v = \omega f_y$ -------------------(9.15)

But for object at infinity $f_R - f_v$ is the measure of longitudinal chromatic aberration.

Longitudinal Chromatic Aberration = ωf_y

Case-2 :

When the object is at finite distance the expression for focal length in terms of object and image distances is given by $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$ ------------------(9.16)

Although the object is placed at a fixed distance u, the image distance v and hence focal length 'f' vary with colour.Hence differentiating eq.16, we obtain

$$\frac{-df}{f^2}=\frac{-dv}{v^2}\Rightarrow\frac{dv}{v^2}=\frac{df}{f^2}\text{ ----------------------------(9.17)}$$

But taking two colours,

$dv = v_R - v_v, df = f_R - f_v$

$\text{Fromeq.15}, df = f_R - f_v = \omega f_y$

Substituting these values in above equation we get

$$\frac{v_R - v_V}{v_y^2}=\frac{\omega f_y}{f_y^2}=\frac{\omega}{f_y}\text{ -------------------(9.18)}$$

vy,fy are for yellow colour (Middile or mean ray)

$$v_R - v_v = \frac{\omega . v_y^2}{f}\text{ ------(9.19)}$$

Here $v_R - v_v$ =Longitudinal chromatic aberration for objects at finite distance.

9.3.3 ACHROMATIC DOUBLET :-

Achromatisation :

The minimisation or removal of chromatic aberration is known as "achromatisation"

Achromatism :

The process in which two lenses are placed such that the image formed is free from chromatic aberration is called as achromatism.

Achromatic doublet :

A pair of lenses selected in such a way that the image is free from chromatic aberration is called Achromatic doublet.

Ex: 1.A convex lens made of crown glass with high power and a concave lens made of flint glass with low power in contact.

2. Two convex lenses seperated by a distance.

9.3.4 Achromatism for two lenses in contact :

Achromatic doublet is selected in such a way that all colours focus at one point i.e. all colours have same focal length.

Lens makers formula is $\frac{1}{f} = (\mu - 1)(\frac{1}{R_1} - \frac{1}{R_2})$ ------------------(9.20)

μ - refractive index of the lens changes with colour.
So differentiating above equation, we get

$$d\left(\frac{1}{f}\right) = d\left[(\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\right] \text{-----------------(9.21)}$$

Dividing equation (9.21) by equation (9.20) we have

$$f.\left(\frac{1}{f}\right) = \frac{d\mu}{\mu - 1}$$

$$d\left(\frac{1}{f}\right) = \frac{d\mu}{\mu - 1}.\frac{1}{f} = \frac{\omega}{f} \text{-----------------------(9.22)}$$

where $\omega = \frac{d\mu}{\mu - 1}$, dispersive power of the lens.

Let us consider two lenses of focal length f_1 and f_2 and dispersive powers ω_1, ω_2, in contact. Hence the resultant focal length of the combination 'F' is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \text{---------------------(9.23)}$$

Differentiating eq 9.23,

$$d\left(\frac{1}{F}\right) = d\left(\frac{1}{f_1}\right) + d\left(\frac{1}{f_2}\right)$$

Substituting eq.22 for $d\left(\frac{1}{f}\right)$ in above equation

$$d\left(\frac{1}{F}\right) = \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2}$$

For an achromatic doublet there should be no change in focal length with colour

$$\therefore d\left(\frac{1}{F}\right) = 0 \Rightarrow \frac{\omega_1}{f_1} = \frac{-\omega_2}{f_2} \Rightarrow \frac{\omega_1}{\omega_2} = \frac{-f_1}{f_2}$$

$$\frac{t_1}{f_2} = \frac{-\omega_1}{\omega_2} \text{----------------------(9.24)}$$

Hence the ratio of focal lengths of two lenses should be equal to the ratio of their dispersive powers and -ve sign indicates one of the lens should be a concave lens in an achromatic doublet.

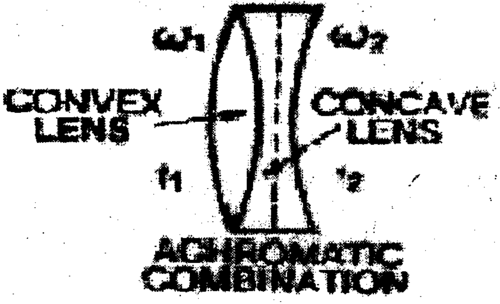

Fig 9.6

9.3.5 Achromatism for two lenses seperated by a distance :-

Let us consider two convex lenses of focal lengths f_1, f_2 and of same dispersive power ω and seperated by a distance x. The combined focal length of two lenses seperated by a distance is given by

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{x}{f_1 f_2} \text{ -----------------------(9.25)}$$

If the combination is to be achromatic, change in focal length 'F' with change in refractive index should be equal to zero.i.e $d\left(\frac{1}{F}\right)=0$

Differentiating equation (25)

$$d\left(\frac{1}{F}\right)=d\left(\frac{1}{f_1}\right)+d\left(\frac{1}{f_2}\right)-\frac{x}{f_2}d\left(\frac{1}{f_1}\right)-\frac{x}{f_1}d\left(\frac{1}{f_2}\right) \text{ --------------------(9.26)}$$

But from the previous section, we can write

$$d\left(\frac{1}{f_1}\right)=\frac{\omega_1}{f_1}, d\left(\frac{1}{f_2}\right)=\frac{\omega_2}{f_2}$$

Substituting these values in above equation ,we get

$$d\left(\frac{1}{F}\right) = \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} - \frac{x}{f_2}.\frac{\omega_1}{f_1} - \frac{x}{f_1}.\frac{\omega_2}{f_2}$$

$$\text{But} d\left(\frac{1}{F}\right) = 0$$

$$\Rightarrow \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} - \frac{x}{f_2}.\frac{\omega_1}{f_1} - \frac{x}{f_1}.\frac{\omega_2}{f_2} = 0$$

$$\Rightarrow \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = \frac{x}{f_2}.\frac{\omega_1}{f_1} + \frac{x}{f_1}.\frac{\omega_2}{f_2}$$

$$\frac{\omega_1 f_2 + \omega_2 f_1}{f_1 f_2} = \frac{x(\omega_1 + \omega_2)}{f_1 f_2}$$

$$\Rightarrow x = \frac{\omega_1 f_2 + \omega_2 f_1}{\omega_1 + \omega_2} \text{-------------(27)} \quad (9.27)$$

Since the dispersive powers of the lenses are equal, $\omega_1 = \omega_2 = \omega$

$$\Rightarrow x = \frac{\omega f_2 + \omega f_1}{2\omega} = \frac{f_1 + f_2}{2} \text{---------------------(9.28)}$$

Hence the two lenses seperated by a distance 'x' will be achromatic if the lenses seperation is equal to average of their respective focal lengths.

9.4 Spherical aberration :-

As mentioned 9.3, spherical aberration is one of the monochromatic aberration.

Definition :-

The inability of the lens to form a point image of a point object is called spherical aberration.

Explanation:

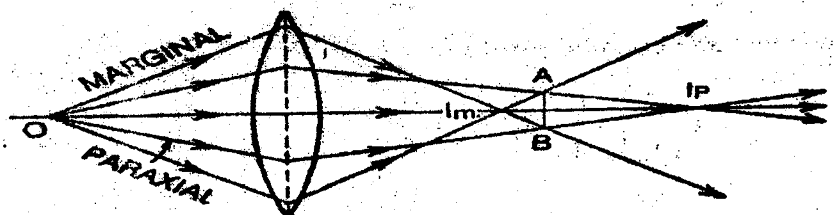

Fig. 9.7

The marginal rays OM,OM from point object 'O' touch the lens at greater height than the paraxial rays OP,OP. According to deviation formula $\delta = h/f$, therefore the marginal rays deviate more than the paraxial rays and come to focus at I_m and paraxial rays farther away at I_p. So we can say the focal lengths of marginal rays < focal length of paraxial rays. Thus there are two images I_m and I_p for same object O. Similarly for remaining rays, images form inbetween I_m and I_p. Hence instead of a point image of a point object 'O' we obtain a spread or spherical image from I_m to I_p. This is termed as spherical aberration. Hence image is a circular disk at each point.

The distance between I_m and I_p is called "longitudinal chromatic aberration".

As we move from I_m to I_p, images are circles of different radii. But at one point between AB, a circular image of minimum radius is formed. This minimum circular image is considered as the image of the object. This circle is called "Circle of least confusion". The radius of the circle is a measure of 'Lateral spherical aberration'.

9.4.1 Spherical aberration by plane refracting surface:-

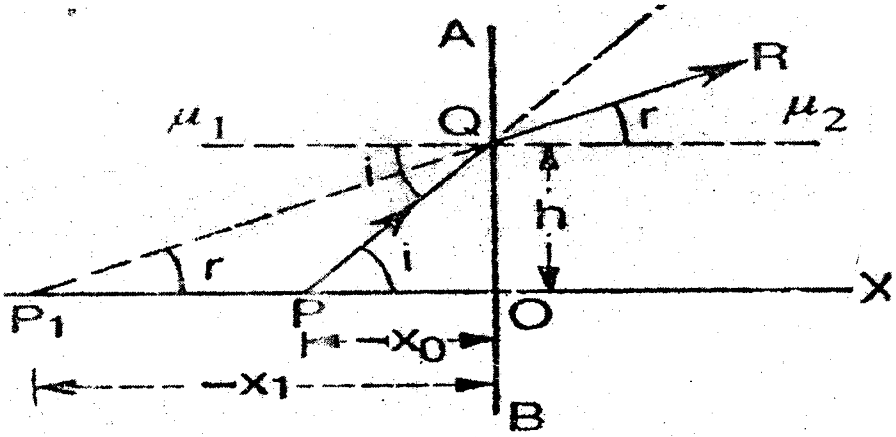

Fig9.8

In above figure AB-Plane refracting surface, P-object point, i-angle of incidence, P_1 -Image point, r-angle of refraction, h- height at which the incident ray touches the plane refracting surface

μ_1, μ_2 -refractive indices on the two sides of refracting surfaces.

As mentioned in previous section,the image point changes from marginal rays to paraxial rays .So the image distance x_1 changes with height 'h'. Hence if we express x_1 in terms of 'h' ,we can find the distance x_1 for marginal rays and paraxial rays seperately. The difference in these two distances is a measure of longitudinal spherical aberration. Hence x_1 in terms of 'h' is given by

$$x_1 = -\mu_0 x_0 - \frac{h^2\mu}{2x_0}\left(1-\frac{1}{\mu^2}\right) \text{---------------------(9.29)}$$

For Paraxial rays, $h \approx 0$

$$x_1 = -\mu x_0$$

For marginal rays,

$$x_1 = -\mu x_0 - \frac{h^2\mu}{2x_0}\left(1-\frac{1}{\mu^2}\right)$$

'h' is maximum ,hence

Longitudinal Chromatic Aberration$= x_1^1 - x_1$

$$= \frac{-h^2\mu}{2x_0}\left(1-\frac{1}{\mu^2}\right) \text{-----------------(9.30)}$$

9.4.2 Spherical Aberration due to Spherical Refracting Surface :-

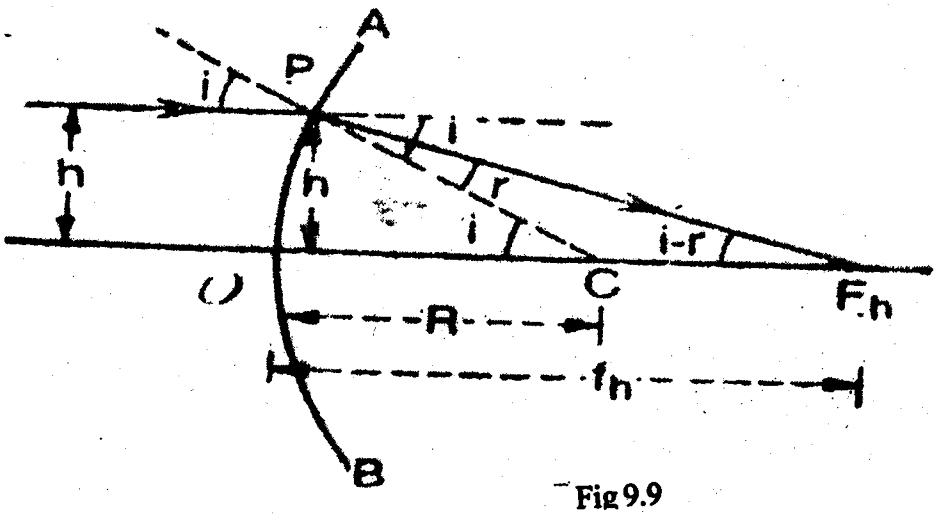

Fig 9.9

In above figure let SP be the parallel incident ray which meets spherical surface AB at a height 'h'. Radius of curvature of the surface OC=R=CP. CN is the normal to the spherical surface at P. $\angle i$ = angle of incidence, $\angle r$ =angle of refraction. The ray refracts and forms the image at F_h.$OF_h = f_h \rightarrow$ focal length of the surface for rays incident at a height 'h'. So f_h varies with 'h'.

f_h is given by

$$f_h = R\left[1 + \frac{1}{\mu \text{Cos} r - \text{Cos} i}\right] \text{------------------------}$$

For paraxial rays , $h \rightarrow 0$ and $\cos i, \cos r \rightarrow 1$

$$\Rightarrow f_p \approx \frac{\mu R}{\mu - 1} \text{----------------------}(9.31)$$

Longitudinal Spherical Aberration is given by

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$$\Delta f_h = f_p - f_h = R\left[\frac{1}{\mu - 1} - \frac{1}{\mu \cos r - \cos i}\right]$$

In terms of 'h' $\Delta f_h = \dfrac{h^2}{2(\mu-1)^2 f_p}$ ----(9.32) This equation is a measure of longitudinal spherical abberation in case of spherical surface. where $f_p = \dfrac{\mu R}{\mu - 1}$ ----(9.33)

9.4.3 Methods of minimisation of spherical aberration :-

1. Since spherical aberration arises due to different focal lengths for different height zones , it can be minimised by closing or stopping some of the zones in the lenses.Hence stops which cover different zones can be used to minimise spherical aberration .But this method reduces the intensity of the image.

2. Spherical aberration can be minimised by using plane convex lens such that the deviation is equally divided at the two surfaces.

Explanation:-

Spherical aberration $\propto \delta^2$

when there are two surfaces $\delta = \delta_1 + \delta_2$

So, spherical aberration $\propto (\delta_1 + \delta_2)^2$

Spherical aberration $\propto (\delta_1 - \delta_2)^2 + 4\delta_1\delta_2$

Spherical aberration will be minimum when

$$\delta_1 - \delta_2 = 0 \Rightarrow \delta_1 = \delta_2 \text{------------------------}(9.34)$$

To attain this condition in microscope objectives or telescope objectives, the curved surface of plano-convex lens should face the side with parallel rays, either incident or emergent.

3. By using suitable combination of convex and concave lenses, spherical aberration can be minimised. The positive spherical aberration of convex lens is nulled by the -ve spherical aberration of concave lens.

4. By using crossed lens :-

If R1 and R2 are the radii of curvatures of two surfaces of the lens, and μ is its refractive index, then ratio of radii of curvature is given by

$$\frac{R_1}{R_2} = \frac{2\mu^2 - \mu - 4}{\mu(2\mu + 1)} \text{ -------------------(9.36)}$$

A lens with refractive index $\mu = 1.686$ satisfies the condition $\frac{R_1}{R_2} = 0$ and such lens is called crossed lens.

5. By using two plano- covex lenses seperated by a distance :-

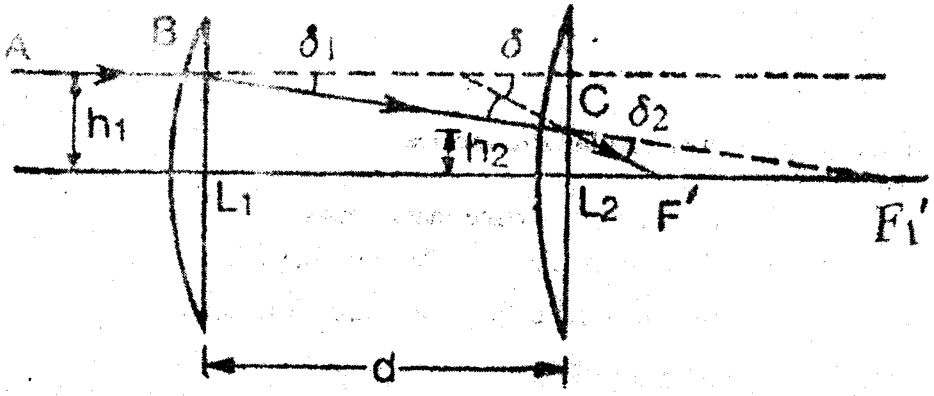

Fig 9.10

The spherical aberration can be minimised by using two plano-convex lenses made of same material and seperated by a distance equal to the difference of their focal lengths.

Proof :-

L_1, L_2 - Two plano convex lenses of focal lengths f_1 and f_2

The deviation produced by lens L_1 to ray AB touching at height h1, $\delta_1 = \frac{h_1}{f_1}$

Similarly the deviation produced in ray by second lens L_2 is $\delta_2 = \frac{h_2}{f_2}$

The emergent ray from two lenses focuses at F_1^1

For minimum spherical aberration

$$\delta_1 = \delta_2$$

$$\Rightarrow \frac{h_1}{f_1} = \frac{h_2}{f_2}$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{f_1}{f_2} \quad \text{-----------------------------(9.37)}$$

From similar triangles $BL_1F_1^1$ and $CL_2F_1^1$ in the above figure

$$\frac{h_1}{h_2} = \frac{L_1F_1^1}{L_2F_1^1} = \frac{f_1}{L_1F_1^1 - L_1L_2} = \frac{f_1}{f_1 - d} \quad \text{----------------------(9.38)}$$

But from eq 9. 37, $\frac{h_1}{h_2} = \frac{f_1}{f_2}$

Substituting these in above equation,

$$\frac{f_1}{f_2} = \frac{f_1}{f_1 - d} \Rightarrow d = f_1 - f_2$$

6. Minimisation by using Aplanatic surface :-

Aplanatic Surface :-

It is a spherical surface characterised by the property of forming a point image of a point object placed on the principal axis. The two focii of the surface are μR and R/μ. When object is placed at a distance of R/μ from the centre of the surface, image "free from spherical aberration " is formed at a distance of μR.

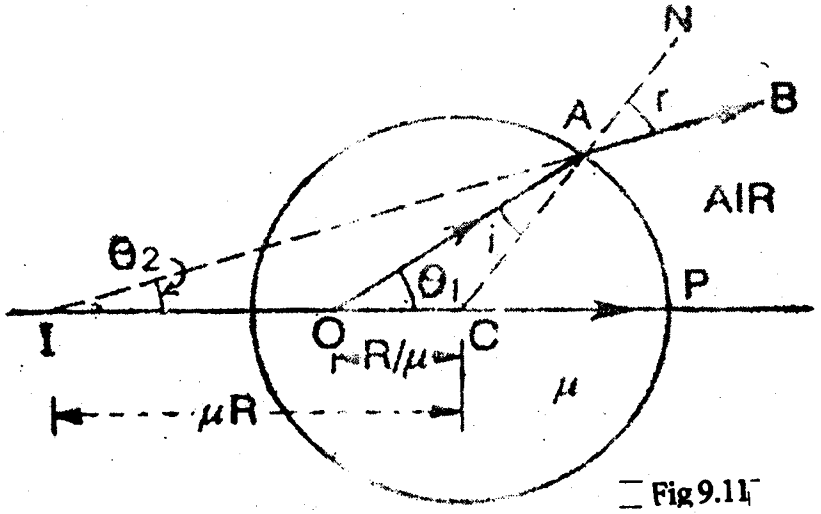

Fig 9.11

Proof :-From the fig.9.11

C is the centre of spherical surface.

CP=CA=R=Radius of spherical surface,

μ - refractive index of the surface,

O-object , I-image , OA- incident ray , AB- refracted ray

CN-normal to the surface at incident point.

OP- Second incident ray which is not deviated.

When the deviated ray AB and OP are extended back, image is formed at I. Hence I is the virtual image of object 'O'.

From figure,

$$\angle AIC = \theta_2, \angle OAC = i, \angle NAB = R, \angle AOC = \theta_1$$

In triangle OAC,

$$\frac{AC}{\sin\theta_1} = \frac{CO}{\sin i} \Rightarrow \frac{R}{\sin\theta_1} = \frac{R/\mu}{\sin i}$$

$$\sin\theta_1 = \mu \sin i \text{ ---------------------(9.40)}$$

Refraction is taking from denser medium μ into rarer medium air

$$\therefore \frac{\sin i}{\sin r} = \frac{1}{\mu} \Rightarrow \mu \sin i = \sin r \text{ --------------------------(9.41)}$$

From equation(9.40) and equation(9.41),

$$\sin\theta_1 = \sin r$$

$$\Rightarrow \theta_1 = r \text{ ------------------------------------(9.42)}$$

In triangle OAI,

external angle = sum of internal angles

$$\theta_1 = \theta_2 + (r - i)$$

$$\Rightarrow r = \theta_2 + (r - i) \Rightarrow \theta_2 = i \text{ ----------------------(9.43)}$$

Now from similar triangles OCA and ICA, we have

$$\frac{CI}{CA} = \frac{CA}{CO} \Rightarrow CI = \frac{(CA)^2}{CO} = \frac{R^2}{R/\mu} = \mu R \text{ ----------------(9.44)}$$

9. Equation (44) is independent of $\theta_1$andθ_2.Hence all rays starting from 'O' and incident on refracting surfaces at **different angles θ_1 ,after refraction** appear to be coming from same point I. Hence we obtain a single point **image I for point object 'O'**.

9.5. COMA :-

When we **consider a point on the object** which is away from principal axis , the image of that single **point on the object will be in the form** of a comet or egg like shape.This defect of image is called 'Coma'.

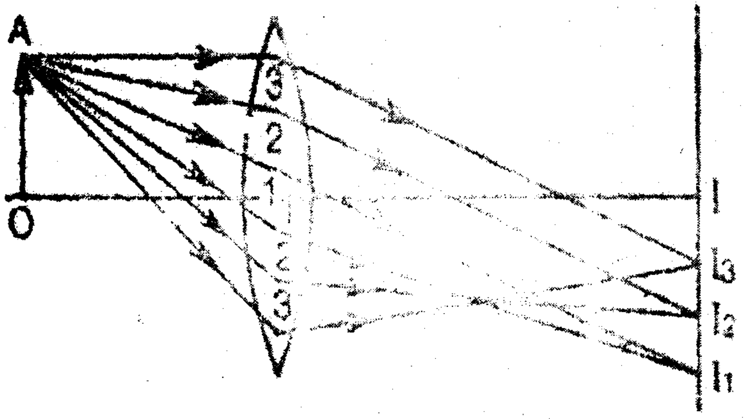

Fig 9.12

In above figure 9.12, the rays starting from single point A on the object (off the axis point) pass through different circular zones(1-1),(2-2),(3-3) etc of the lens.Each zone brings the rays into focus at different points I_1, I_2, I_3 etc.So smaller the radius of zone ,farther the image is formed and hence lateral magnification is more.

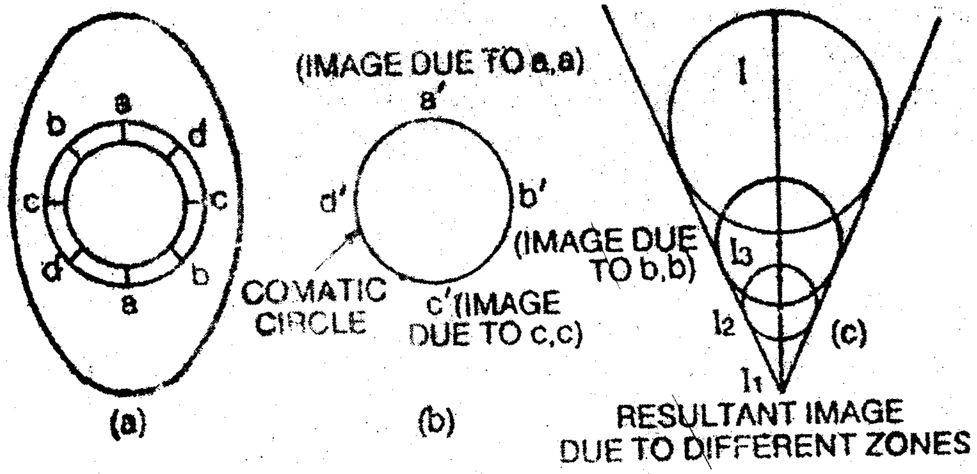

Fig 9.13

Now if we consider each circular zone of a lens as shown in the figure the points a-a, form image at a^1,b-b at b^1,c-c at c^1,d-d at d^1.Hence due to circular zone, we get a circular image

a^1 b^1 c^1 d^1 for a point object 'A'.As the radius of the zone increases, the image radius increases.So each zone gives one cicular image.So as we go from I_1 to I, the radius of circle increases and we get an image of comet shape as shown in figure for single point A.

This defect can be minimised by using

1.Stops

2.Crossed lenses of ratio of Radii curvature $\frac{R_1}{R_2} = \frac{-1}{9}$

3.Using lenses which follow Abbe's sine condition $\mu_1 y_1 \sin\theta_1 = \mu_2 y_2 \sin\theta_2$

where μ_1 and μ_2 are the refractive indices on two sides of spherical surfaces y_1, y_2 are heights of object and image respectively and θ_1 and θ_2 are the angles made by incident ray and emergent ray with the principal axis.

9.6 ASTIGMATISM :-

When a pont object is situated far off the axis of a lens, the image formed is a patch of light but not point image.The image consist of two mutually perpendicular lines separated by a finite distance and the lines in image are in perpendicular planes.This defect of image is called "Astigmatism".

Since lens is of finite thickness, we can imagine it consisting of different layers.These layers neednot be in same plane.In the figure 9.14 if we consider a point object 'O' far off from the axis the plane containing 'O',principal axis and plane M_1, M_2 of lens is called Meridian Plane.The plane perpendicular to meridian plane i.e. S_1S_2 and passing through principal axis is called Sagittal plane.These two planes of lens form the image of O at different points M and S in form of circles or patches perpendicular to each other.Hence we obtain a patch of light between M and S .This defect is termed as Astigmatism

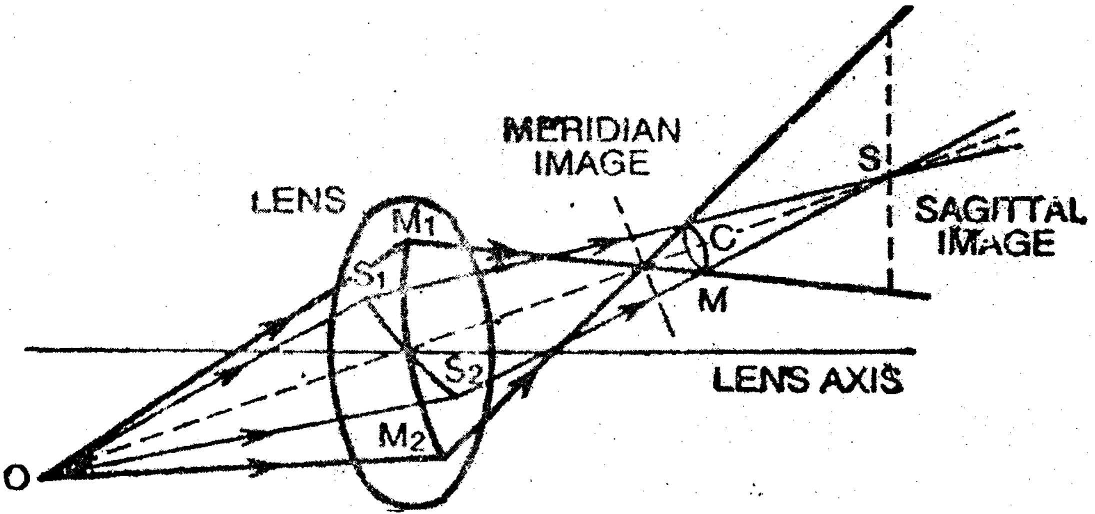

Fig 9.14

9.7 Solved Problems :-

1. **Calculate the focal length of a lens of** dispersive power 0.031 which should be placed in **contact with a convex lens of focal length 84cm** and dispersive power 0.021 to make the combination **achromatic?**

Solution :-

The condition for achromatism for lenses in contact is

$$\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}=0 \Rightarrow \frac{\omega_1}{\omega_2}=\frac{-f_1}{f_2}$$

$$\omega_1 = 0.021\ ,\ f_1 = 84\text{ cm}$$

$$\omega_2 = 0.031\ ,\ f_2 = ?$$

Substituting these values in above equation,

$$\frac{0.021}{84}+\frac{0.031}{f_2}=0 \Rightarrow \frac{0.021}{0.031}=\frac{-84}{f_2}$$

$$\Rightarrow f_2 = -84\times\frac{31}{21} = -124\text{cm}$$

So, the second lens in contact should be a concave lens of focal length 124 cm.

2. Find the focal lengths of two lenses of crown and flint glasses with dispersive powers 0.015 and 0.025 respectively inorder to make an achromatic converging lens of focal length 25 cm?

Solution :-

For an achromatic doublet

$$\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}=0$$

$\omega_1 = 0.015$ and $\omega_2 = 0.025$ ----------------(1)

The focal length of combination or achromatic doublet, F=25 cm.

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2} \Rightarrow \frac{1}{25}=\frac{1}{f_1}+\frac{1}{f_2} \text{ -----------------------------(2)}$$

$$\text{From (1), } \frac{0.015}{0.025}=\frac{-f_1}{f_2} \Rightarrow \frac{f_1}{f_2}=\frac{-3}{5} \Rightarrow \frac{1}{f_2}=\frac{-3}{5f_1}$$

Substituting in equation (2),

$$\frac{1}{25}=\frac{1}{f_1}-\frac{3}{5f_1}=\frac{5-3}{5f_1}=\frac{2}{5f_1}$$

$$\Rightarrow f_1 = \frac{2}{5}\times 25 = 10\text{cm}$$

$$f_2 = \frac{-5f_1}{3} = \frac{-5\times 10}{3} = -16.67\text{cm}$$

Hence focal length of two lenses are 10 cm and -16.67 cm respectively.

3. A flint glass biconvex lens has radii of curvature of +20 cm and -20 cm. If the refractive indices of flint glass for violet and red rays are given as 1.8 and 1.5 find the longitudinal chromatic aberration?

Solution :-

It is given that $\mu_v = 1.8, \mu_R = 1.5$

$$R_1 = +20 \text{ cm} = \frac{20}{100} \text{ m}, \ R_2 = -20 \text{ cm} = -\frac{20}{100} \text{ m}$$

The focal length for red rays is given by

$$\frac{1}{f_R} = (\mu_R - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.5-1)\left(\frac{100}{20} + \frac{100}{20}\right) = 5 \Rightarrow f_R = 0.2\text{m}.$$

Similarly the focal length f_v of violet rays is given by

$$\frac{1}{f_v} = (\mu_v - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.8-1)\left(\frac{100}{20} + \frac{100}{20}\right)$$

$$=8$$

$$\Rightarrow f_v = 0.125\text{m}$$

Longitudinal chromatic aberration is $f_R - f_v = 0.2 - 0.125 = 0.075$ m

4. A transparent sphere of radius 60 cm is made up of a material of refractive index of 1.5. Find the position of aplanatic points on the axis from the centre of lens?

Solution :-

$$R = 60 \text{ cm} = 0.6 \text{ m}, \quad \mu = 1.5$$

Aplanatic focii are given by R/μ and μR

$$R/\mu = 0.6/1.5 = 0.4 \text{ m},$$

$$\mu R = 0.6 \times 1.5 = 0.9 \text{ m}.$$

5. Two lenses of focal lengths 8 cm and 4 cm are placed at a certain distance apart. Calculate the distance between the lenses, if they form as achromatic combination?

Solution :-

For lenses seperated by a distance to be achromatic, the condition is

$$x = \frac{f_1 + f_2}{2}$$

$f_1 = 8$ cm $\quad f_2 = 4$ cm

Distance between the lenses, $x = \frac{8+4}{2} = 6$ cm $= 0.06$ m.

6. The focal lengths of a thin convex lens are 100 and 96.8 cm for red and blue rays respectively. Find the dispersive power of the glass?

Solution :-

Longitudinal chromatic aberration is given by

$$f_R - f_v = \omega f$$

$$\text{or} \quad \omega = \frac{f_R - f_v}{f}$$

where f is the mean focal length

$$f_R = 100\text{cm}, \; f_v = 96.8\text{cm}$$

$$\therefore f = \frac{100 + 96.8}{2} = 98.4 \text{ cm.}$$

$$f_R - f_v = 100 - 96.8 = 3.2 \text{ cm.}$$

$$\text{Dispersive power of the glass} = \frac{3.2}{98.4} = 0.033$$

9.8 SUMMARY :-

The defects produced by lens systems in forming images of the objects are explained. Methods to estimate the extent of aberrations produced by the optical systems is discussed. Methods to minimize the aberrations introduced by lens systems are explained. Chromatic and Spherical aberration are dealt in detail. Coma and Astigmatism are introduced.

9.9 KEYWORDS :-

Aberration, Deviation, Dispersion, Chromatic aberration, Monochromatic aberration, Refractive index, focal length, Marginal rays, Paraxial rays, Dispersive power, Achromatism, Spherical aberration, Stops, Crossed lens, Aplanatic surface, Coma, off the axis point, Circular zones, Astigmatism, meridian plane, saggital plane, circle of least confusion.

9.10 SELF ASSESSMENT QUESTIONS :-

9.10.1 Long Answer questions :-

1. What is chromatic aberration ? Obtain an expression for the chromatic aberration of a lens. Derive the condition for achromatism when two lenses are in contact ?
2. Derive an expression for the seperation between the two lenses if chromatic aberration is to be minimised in case of seperated doublet ?
3. What is Spherical aberration ? Distinguish between lateral and longitudinal spherical aberration. Mention two methods of minimising spherical aberration ?How?
5. Write a short notes on coma ?

9.10.2 Short Answer questions :-

1. Find an expression for longitudinal chromatic aberration for an object at infinity ?
2. Discuss different monochromatic aberrations ?
3. Deduce the condition for minimum spherical aberration in a combination of two lenses seperated by a distance ?How?
4. Can you eliminate chromatic aberration and spherical aberration in a lens system ? If so
5. Write a short notes on coma ?
6. What are aplanaic points ? Find the aplanatic focii for a spherical refracting surface ?
7. Write a note on astigmatism ?

9.10.3 EXERCISE :-

1. A double convex lens has radii of curvature of 40 cm and 10 cm . Find the longitudinal chromatic aberration for object at infinity .Given $\mu_v = 1.523, \mu_R = 1.5145$?

[ans : 0.253 cm]

2. Find the focal lengths of the two component lenses of an achromatic doublet of focal length 25 cm . The dispersive powers of the crown and flint glasses are 0.022 and 0.044 respectively.

[ans : 12.5cm , 25 cm]

3. The objective glass of a telescope is an achromat of focal length 90 cm . If the magnitude of the dispersive powers of the two lenses are 0.024 and 0.036 , calculate their focal lengths ?

[ans : 30 cm , -45 cm]

4. Two thin lenses of focal lengths f_1 and f_2 are seperated by a distance x have an equivalent focal length 50 cm . The combination satisfies the condition for no chromatic aberration and minimum spherical aberration. Find the values of the f_1 , f_2 and x . Assume that both are of the same material ?

[ans : 100 cm , 33.33 cm]

5. A convergent doublet of seperated lenses , corrected for spherical aberration has an equivalent focal length of 10 cm .The lenses of the doublet are seperated by 2 cm .What are the focal lengths of its component lenses ?

[ans : 18 cm , 20 cm]

6. An achromatic lens of focal length 30 cm is made of two lenses of materials A and B are in the ratio 1:2 , find the focal length of each lens ?

[ans :15 cm , -30 cm]

9.11 REFERENCE BOOKS :

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4. Optics	Ghatak
5. B.Sc Second year Physics	Dr.K. Ramakrishna D.V.Brahmaji A.Srinivasa Rao S.L.V. Mallikharjun

UNIT - 3
Lesson 10

INTERFERENCE

Objectives:-

1.Convey the impertance of interference phenomena in establishing the wave nature of light.
2.Introduce the principle of superposition in combining waves.
3.Obtain interference fringes practically using young's experment and determine the intensity at different points on interference pattern.
4.Importance of coherence in formation of interference fringe.
5.Explain formation of colours in thin filims through interference by division of wave front...
6.Calculation of wavelength of monochromatic source through fresnels biprism experment and formation of achromatic fringes through Lloyd's mirror experiment.
7.Derive the cosine law through division of amplitude
8.Understand the concept of non- reflecting films.
9.Learn wedge method experiment to check the optical plainess of glass surface.
10.learn the theory behind the formation of Newton's rings and its applications.
11.Importance of Michelson inter ferometer in determining the wavelenth of monocramatic source and the resolution of spectral lines.

Structure of the Lession:-

10.1.Introduction
10.2.Principle of superposition
10.3.Interference of light
10.4.Coherence and coherent sources
10.5.Condition for interference
10.6.Types of interference
10.7.Young's Experiment
10.8.Fresnal's Biprism
10.8.1.Adjustments to the Experimental setup
10.8.2. Theory of interference fringes
10.8.3Procedure
10.8.4.Determination of the thickness of a thin sheet of transport material
10.9.Change of phase on reflection
10.10.Interference by division of amplitude
10.11Cosine law
10.12Colours of thin flims
10.13.Non Reflecting flims
10.14.1.Interference by a plane parallel film illuminated by a point source
10.14.2.Interferene by a film with two non-parallel reflecting surfaces (wedge shaped film)
10.14.3.Determination of diameter of the wire-wedge method

10.1. Introduction:

Interference is a phenomena in which coherent light passing through closely placed pinholes and dark bands termed as interference fringes .It is a phenomena which proved Newtons corpuscular theory to be wrong and established the wave nature of light through Huygen's wave theory.It is a strong proof for the statement that "every point on the light wave acts as a secondary source and gives it secondary wavelets which propagates in forward direction".When secondary wavelets from each point on the light wave super- impose with other ,they produce interference fringes .The interference can be explained by two methodes .

1.Divison by wave front and

2.Divison by amplidude.

10.2 Principle of superposition:

" The resultant displacement of a particle of the medium acted upon by two or more waves simultaneously is the algebraic sum of the displacements of the same particle due to individual waves in the absence of others ".

Suppose the displacement produced in the particle of the medium acted upon by first wavetrain is y_1 ,in the absence of second wavetrain and y_2 is the displacement produced by second wavetrain in the particle in the absence of the first wavetrain.The resultant displacement produced in the particle when acted by the two wavetrain 'R' is given $R = y_1 + y_2$ according principle of superposition.if displacements are in opposite directions then $R = y_1 - y_2$

10.3 Interference of light:

When two light waves superimpose then the resultant intensity in the region of superposition is different than the amplitude or intensity of individual wave "this in the modification in the distribution of intensity in the region of superposition is called interference".

Constructive interference:

When the resultant amplitude is the sum of Individualwave amplitudes,then the interference is termed as constructive interference.

Destructive interference:

When the resultant amplitude is the difference of individual wave amplitudes ,then the interference is termed as "destructive interference".

10.4 Coherence and coherent sources:

When two light sources emit light waves of same wavelength and which maintain zero or constant phase difference between them are called coherent sources and the phenomenon is termed as coherence .The two coherent sources emit continuous waves of the same wavelength and time period.

10.5Condition for interference of light:

Stationary interfernce pattern can be obtained with a pair of light sources when following conditions are satisfied.

1 Conditions for sustained interference:

The two sources should be coherent.If a_1 and a_2 are the amplitudes of waves emitted by two coherent sources and δ is the phase difference between them ,then the resultant intensity at any point due to two sources is given by

$$I = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$$

If the sources are not coherent the phase difference δ continously changes and we obtain continuous illuminatation instead of interference bands .when $\delta = 0$ or constant or π

$$\cos\delta = 1 \text{ or } -1 \quad \text{and} \quad I = a_1^2 + a_2^2 + 2a_1a_2 = (a_1 + a_2)^2$$

$$I = a_1^2 + a_2^2 - 2a_1a_2 = (a_1 - a_2)^2$$

first is the continuous for constrnctive interference giving bright band and second is the condition for destructive interference giving dark band.

2.Condition for observation:

The band width of interference fringes is given by $\beta = \dfrac{\lambda D}{2d}$

so the seperation between two sources (2d) should be small and the distance between sources and the screen 'D' should be large,so that β is of considerable spread. Then only we can observe clear dark and bright fringes of considerable width. In other cases,although may form, we cannot observe interference pattern due to invisible widths and looks like uniform illumination.

3.condition for good contrast:

When interference fringes are formed,there should be good contrast between dark and bright bands so that we can see distinct interference pattern.as we observed in condition (1) that the intensity of bright band I=$(a_1+a_2)^2$ and dark band is I=$(a_1-a_2)^2$ .so for the dark band to be perfectly dark,"the amplitudes of the wave trains from two sources should be equal"

Also the sources should be narrow,or else each point in each sources behaves as a separate point source and the interference pattern due to them may overlap.

The sources should be monochromatic(sources of single wavelengh)if the source is white light,after passage through optical system it disperses into seven colours.Each colour forms a source.so the two coherent sources will emit light waves of seven different wavelengths and each wavelength produces irs own interference parttern.These patterns overlap each other and we may not observe distinct interference pattern.Similar thing may happen with other sources which are a combination of more than one wavelengh.

10.6 Types of interference:

The phenomenon of interference can be classified into two types 1.Divison of wavefront and 2.Divison of amplitude.depending on the method utilised to produce interference pattern.

1.Divison of wavefront:

When the light sources are point sources,the wavefronts from the source is divided into two parts by methods of reflection,refraction or diffraction.These wavefronts travel unequal distances before combining producing phase difference between two wavefronts.These wavefronts superimpose with each other to form interference friges.Experimental setups which utilise this principle are fresnel biprism,Lloyd's mirror.

2.Divison of amplitude:

Instead of point sources if we have extended sources like sodium vapour lamp behind a slit or sunrays from the sun on to a oil film,then the amplitude of these wavefronts are divided by methods of parallel reflection or refraction.these wave trains of divided amplitudes superimpose with each other to produce interference patterns.The experiments which utilise this principle are newton's rings,michelson interferometer etc.

10.7 Young's experiment and intensity at a point on the interference pattern:-

Young's experiment:

This experiment setup consists of a single source 'S'from which light rays travel and incident on two pinholes S_1 and S_2 as shown in figure. now S_1 and S_2 act as two coherent sources.the distance 2d between S_1 and S_2 is of order of the wave lengh of light.S_1 and S_2 emit spherical wavefronts.according huygen's wave theory,each point on these spherical wavefronts acts as secondary sources. they emit secondary wavelets emitted by two sources overlap each other forming inteference patterns.

When the crest of one wavefront overlaps with crest of another wavefront (or trough of one wave front with trough of another wavefront), Construtive interference takes place. Similarly when crest of one wavefront overlaps with trough of second wavefront, destructive interference pattern takes place. Hence all the points corresponding to constructive interference form bright bands and all points corresponding to destructive interference form darkbands in the interference pattern.

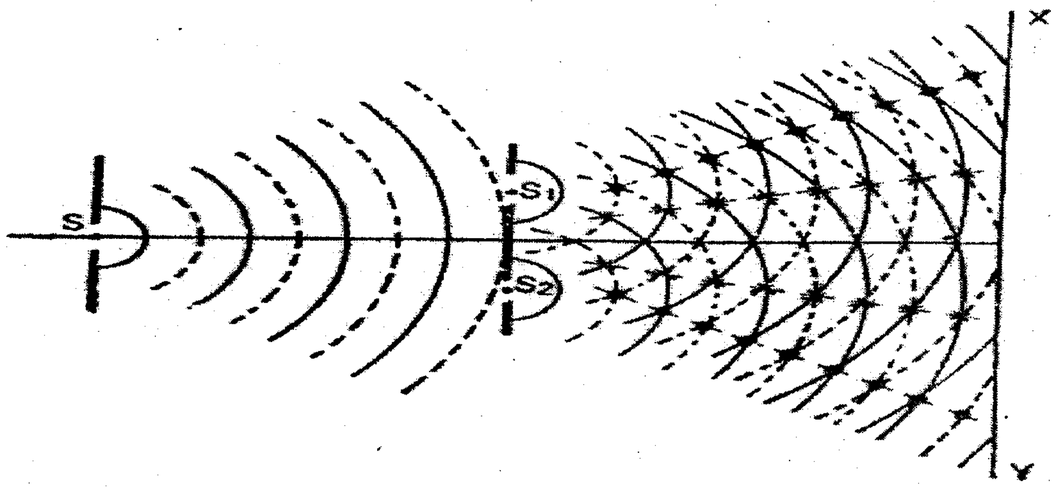


Fig. 10.1

Sources s_1 and s_2 are obtained from a single light source S, hence they form coherent sources. The two wave trains or light rays from two sources s_1 and s_2 travel different distances s_1p and s_2p (Fig. 10.2) and reach point p on the screen.

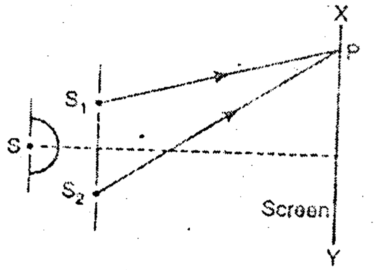


Fig.10.2

Path difference between two rays= $s_2p - s_1p$

This path difference introduces a phase differece δ between two rays s_1p and s_2p.

Phase difference = $\frac{2\Pi}{\lambda}$ x path difference

$$\delta = \frac{2\Pi}{\lambda}(s_2p - s_1p)$$

Let y_1 is the displacement produced at point p due to source s_1. It is given by $y_1 = a_1 \sin\omega t$ - (10.1) where a_1 is the amplitude of wavetrain given by s_1 and ω is its angular frequency. Similarly y_2 be the displacement produced by s_2 at point. It is given by $y_2 = a_2 \sin(\omega t + \delta)$ ---------(10.2)

where a_2 is the amplitude of the wave produced by s_2. δ is the phase difference introduced due to path difference between s_1p and s_2p. According to Principle of superposition, the resultant displacement at p is given by

$$y = y_1 + y_2 = a_1 \sin\omega t + a_2 \sin(\omega t + \delta)$$
$$= a_1 \sin\omega t + a_2 \sin\omega t \cos\delta + a_2 \cos\omega t \sin\delta$$
$$y = \sin\omega t(a_1 + a_2\cos\delta) + \cos\omega t(a_2 \sin\delta) \text{ - -----(10.3)}$$

Let $a_1 + a_2\cos\delta = R\cos\theta$ ---------(10.4), $a_2\sin\delta = R\sin\theta$ ----------(10.5)

Where R and θ are new constants.

Substituting in equatioin (10.3)

$$y = \sin\omega t R\cos\theta + \cos\omega t R\sin\theta$$
$$= R(\sin\omega t\cos\theta + \cos\omega t\sin\theta)$$
$$y = R\sin(\omega t + \theta) \text{ ---------(10.6)}$$

Squaring and adding equations (10.4) and (10.5), we get

$$R^2\sin^2\theta + R^2\cos^2\theta = (a_1 + a_2\cos\delta)^2 + (a_2\sin\delta)^2$$
$$R^2 = (a_1^2 + a_2^2 + 2a_1a_2\cos\delta)$$

Hence resultant intensity at point P is,

$$I = R^2 = (a_1^2 + a_2^2 + 2a_1a_2\cos\delta) \text{ ....(10.7)}$$

(i) Condition for maximum intensity:-

from equation (10.7) intensity at P is max,

$$I = I_{max} \text{ when } \cos\delta = 1 \Rightarrow \delta = 2n\lambda$$

$$\delta = 2n\pi \Rightarrow \frac{2\pi}{\lambda}(s_2p - s_1p) = 2n\pi$$

$$\Rightarrow (s_2p - s_1p) = n\lambda \text{ ....(10.8)}$$

$$\therefore I_{max} = a_1^2 + a_2^2 + 2a_1a_2 = (a_1 + a_2)^2 \text{ ....(10.9)}$$

So intensity at point P is maximum when path difference between two rays reaching point P is equal to 'n' times λ. where n= 0,1,2,3,.... etc.

(ii) Condition for minimum Intensity:-

Intensity at P is minimum I= ´I_{min} ,' where $\cos\delta = -1$

from equation (7)

$$\cos\delta = -1 \Rightarrow \delta = (2n+1)\pi \text{......(10.10)}$$

$$\Rightarrow \frac{2\pi}{\lambda}(s_2p - s_1p) = (2n+1)\lambda$$

$$\Rightarrow s_2p - s_1p = (2n+1)\frac{\lambda}{2} \text{..........(10.11)}$$

$$I_{min} = a_1^2 + a_2^2 - 2a_1a_2 = (a_1 - a_2)^2 \text{..(10.12)}$$

So intensity at point P is minimum when the path difference between waves reaching P is odd number times $\frac{\lambda}{2}$.

(iii) Special case :-When the amplitudes of wave fronts from two sources s_1, s_2 are equal then $a_1 = a_2$, Substituting in equation (9)

$$I_{max} = (a+a)^2 = 4a^2 = 4I \text{......(10.13)}$$

substituting in equation(10.12)

$$I_{min} = (a-a)^2 = 0 \text{....(10.14)}$$

In this case we obtain interference bands with perfectly bright bands and complete dark bands.

Energy distribution in interfernce fringes:-

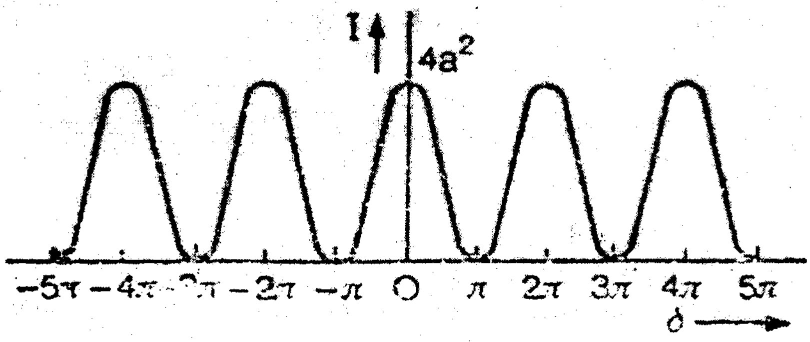


Fig. 10.3

Above figure 10.3 shows variation of intensity in interference fringes with phase. Law of conservation of energy is perfectly observed in the formation of interference fringes. The intensity lost in darkbands is gained in bright bands. Energy is redistributed between dark and bright bands.

Proof:- We will prove that average intensity is equal to the sum of the separate intensities.

Average intensity of interference fringes,

$$I_{average} = \frac{\int_0^{2\pi} I d\delta}{\int_0^{2\pi} d\delta} = \frac{\int_0^{2\pi} (a_1^2 + a_2^2 + 2a_1a_2 \cos\delta) d\delta}{\int_0^{2\pi} d\delta} \qquad (10.15)$$

$$= \frac{[a_1^2\delta + a_2^2\delta + 2a_1a_2 \sin\delta]_0^{2\pi}}{[\delta]_0^{2\pi}} = a_1^2 + a_2^2 ......(15)$$

Hence the result.

Nodal Lines:-

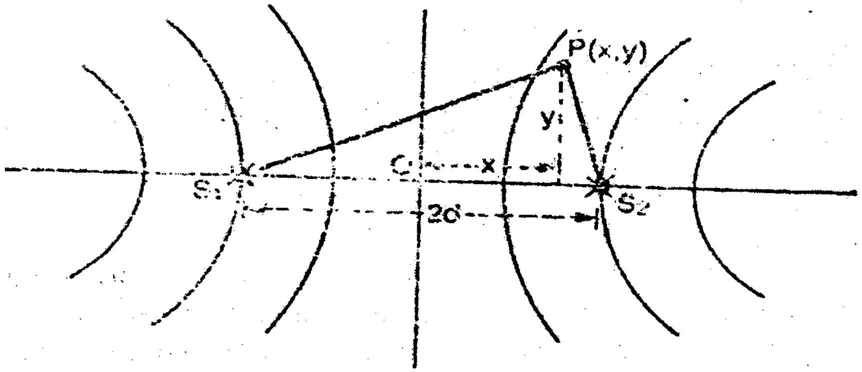


fig10.4

The secondary wavefronts from two sources and are overlap with each other to form interference fringes as shown in the figure 10.4. The line joining the points where the crest and trough meet is called nodal line. So the lines joining the points of minimum intensity are called nodal lines. Now we shall derive the equation of nodal line. Let us imagine the centre of co-ordinate axis O at the midpoint of two sources and . The co-ordinates of P is (x,y),of $s_1$is$(-d,0)$,of $s_2$is$(d,0)$.

From figure

$$(s_1p)^2 = (x+d)^2 + y^2$$

$$\Rightarrow s_1p = \sqrt{(x+d)^2 + y^{2p}} ....(16)$$

$$111^{ly} s_2p = \sqrt{(x-d)^2 + y^2} ......(17)$$

$$\text{pathdifference}\Delta = s_2p - s_1p = \sqrt{(x+d)^2 + y^2} - \sqrt{(d-x)^2 + y^2}$$

$$\Rightarrow \sqrt{(x+d)^2 + y^2} = \Delta + \sqrt{(d-x)^2 + y^2}$$

squaring on both sides we get

$$(x+d)^2 + y^2 = \Delta^2 + 2\Delta\left(\sqrt{(d-x)^2 + y^2}\right) + (d-x)^2 + y^2$$

$$\Rightarrow 4xd - \Delta^2 = 2\Delta\left(\sqrt{(d-x)^2 + y^2}\right)$$

Again squaring on both sides we get

$$16x^2d^2 + \Delta^4 = 4\Delta^2d^2 + 4\Delta^2x^2 + 4\Delta^2y^2$$

Dividing by $\Delta^2 d^2$ and rearranging we get,

$$\frac{x^2}{(\Delta^2/4)} - \frac{y^2}{(d^2 - \Delta^2/4)} = 1.........(18)\,(10.18)$$

This is an equation of hyperbola.

curves correspond to minima when $\Delta = (2n+1)\lambda/2)$

curves correspond to maxima when $\Delta = n\lambda$

10.8 Fresnel's Biprism:

Fresnel Biprism is used to produce interference fringes. Biprism produces two coherent images of a single source, which act as two coherent sources separated by a distance. So basically the action of Biprism is to produce two Pseudo coherent sources from a single source. It is formed by grinding a plane glass plate on two sides such that the obtuse angle of the prism is 179^0 and two acute angles are 30^1 each. The basic experimental arrangement is shown in the figure10.5.

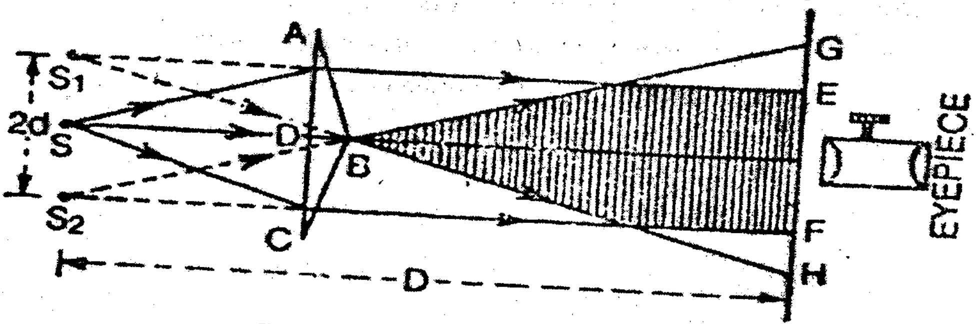


fig 10.5

The experimental set up consists of an optical bench with three stands of same height. The stands can move horizontally and also vertically to the optical bench. One stand is for slit S, the second stand is for biprism and third stand for micrometer eyepiece. The slit is illuminated with the monochromatic source. The biprism and the slit can be rotated in their own planes.

Action of Biprism:-The divergent light rays SA and SC emerge as parallel beam from the prism. Ray is divided by the edge of the prism B into two parts. One part which travels in downward direction appears to be emerging from a virtual source s_1 and the upperpart appears to be emerging from the virtual source s_2. So these divergent beams and the paralled beam overlap each other forming interference pattern in region EH. This interference pattern can be viewed through micrometer eyepiece.

10.8.1 Adjustments to the Experimental setup:

1. The optical bench is levelled with spirit level.
2. The three stands are adjusted to same height.
3. Eye piece is focussed on its cross-wires.
4. The slit is made narrow and vertical and illuminated with monochromatic light.
5. The biprism is moved at right angles to the optical bench till a bright patch of light is visible in micrometer eyepice. The slit is rotated with tangential of view of the eyepiece.
6. The fringes are made clear by rotating biprism in its own plane.
7. When we move the eyepiece, the fringes appear to be shifting. So the distance between biprism and slit is adjusted till there is no lateral shift in the fringe pattern.

10.8.2 Theory of interference fringes:-

Now let us derive the path differnce between two rays reaching a point P on the screen in the terms of the distance between two sources s_1 and s_2 and the distance between the source and the screen D(Fig. 10.6)..

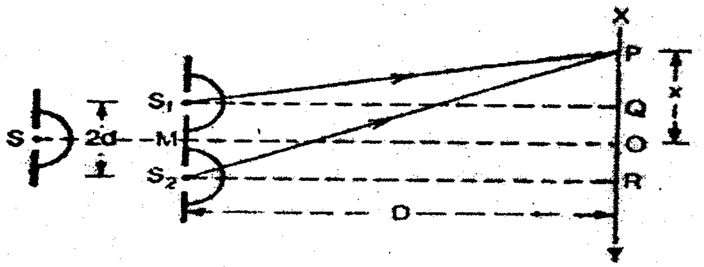


fig 10.6

Let us consider a point P on the screen at a distance 'X'from the centre point 'O'.From right angled triangle S_1PQ

$$(s_1p)^2 = (s_1Q)^2 + (QP)^2$$

$$s_1p^2 = D^2 + (x-d)^2$$

$|||^{ly}$ from rightangled triangle S_2PR

$$S_2P^2 = S_2R^2 + RP^2 = D^2 + (x+d)^2$$

$$\therefore S_2P^2 - S_1P^2 = (x+d)^2 - (x-d)^2 = 4xd$$

$$\Rightarrow \quad (S_2P - S_1P)(S_2P + S_1P) = 4xd$$

since the distance between slits s_1s_2 and screen is large, $s_2p - s_1p \approx D$

$$\therefore (s_2p - s_1p) = \frac{4xd}{2D} = \frac{2xd}{D} \text{------}(19)\ (10.19)$$

Bright fringes: For bright fringes the path difference $S_2P - S_1P = n\lambda$

$$\therefore \quad \frac{2xd}{D} = n\lambda \Rightarrow x = \frac{n\lambda D}{2d} \text{.................}(10.20)$$

The distance of first bright fringes from ' O ' is given by

$$\text{forn} = 1, x_1 = \frac{\lambda D}{2d}$$

$$\text{for} \quad n = 2, x_2 = \frac{2\lambda D}{2d},$$

$$\text{for} \quad n = 3, x_3 = \frac{3\lambda D}{2d}$$

The distance between the adjacent bright fringes is given by

$$x_2 - x_1 = \frac{2\lambda d}{2d} - \frac{\lambda D}{2d} = \frac{\lambda D}{2d} \text{ ....(10.21)}$$

Dark fringes:

For dark fringes the path difference

$$s_2p - s_1p = \frac{(2n+1)\lambda}{2}$$

First dark fringe $n = 1, x_1 = \frac{3\lambda D}{4d}$;

Second dark fringe $n = 2, x_2 = \frac{5\lambda D}{4d}$

nth dark fringe

$$x_n = \frac{(2n+1)\lambda D}{4d}.$$

The distance between the consecutive dark fringes

$$x_2 - x_1 = \frac{\lambda D}{2d}$$

Hence the spacing between consecutive bright fringes or dark fringes is same and is equal to $\frac{\lambda D}{2d}$

.Hence the fringe width, $x_2 - x_1 = \frac{\lambda D}{2d} = \beta$ ....(10.22)

Hence the wavelength of the source of light λ is given by $\lambda = \frac{\beta 2d}{D}$

10.8.3Procedure:

After the formation of interference fringes ,they are viewed through micrometer eye-piece .The vertical cross-wire of the eyepiece is adjusted on one bright fringe and reading of micrometer eyepiece is noted.The cross-wire is next adjusted to next bright fringe and reading of the micrometer is noted. The difference between these two readings gives the fringe width β . Distance D is measured from optical bench by measuring the distance between slit 'S' and the micrometer eyepiece .'2d'is measured with a small setup.A convex lens whose focal length<1\4(the distance between biprism and eye piece) is mounted on a stand in between biprism and eyepiece.

Then we can observe the image of slit s_1 and s_2 in the field of view of eye piece in two positions of lens.. If the lens is in position L_1 when first the slit images appears and L_2 the second position where slit image is again observed

The distance between real images of s_1 and s_2 viewed through eye piece in position $L_1 = d_1$

The distance between real images of s_1 and s_2 viewed through eye piece in position $L_2 = d_2$

From fig , $\frac{d_1}{2d} = \frac{v}{u}$ and $\frac{d_2}{2d} = \frac{u}{v}$

$\Rightarrow 2d = \sqrt{2d_1d_2}$ ....(10.23)

Thus we can determine the value 2d.

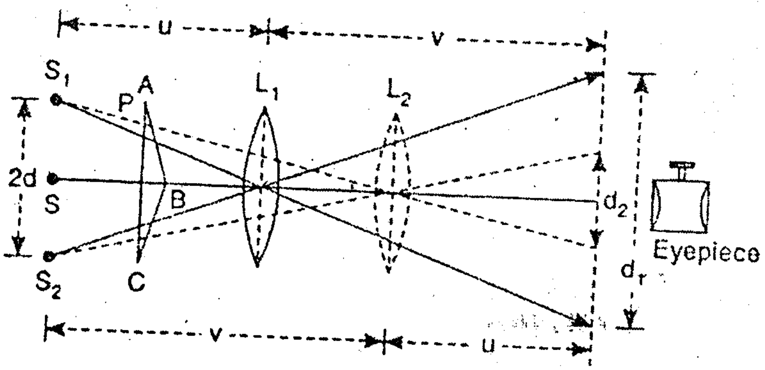


Fig. 10.7

Using the values of 2d ,D and β the wavelengths of monochromatic source is determined using the formula

$$\lambda = \frac{\beta.2d}{D}$$

10.8.4. Determination of the thickness of a thin sheet of transparent material:

When a thin sheet of glass of thickness 't' is introduced in the path of light rays between biprism and the screen.the fringe pattern shits or displaces through a small distance S. Using the shift 'S' we can find the thickness of the glass plate.

As shown in the figure light ray s_1p travels through a distance $(s_1p - t)$ through air and distance 't' through the glass plate .Let the velocityof light through air be 'C 'and through the glass or material be c_g .

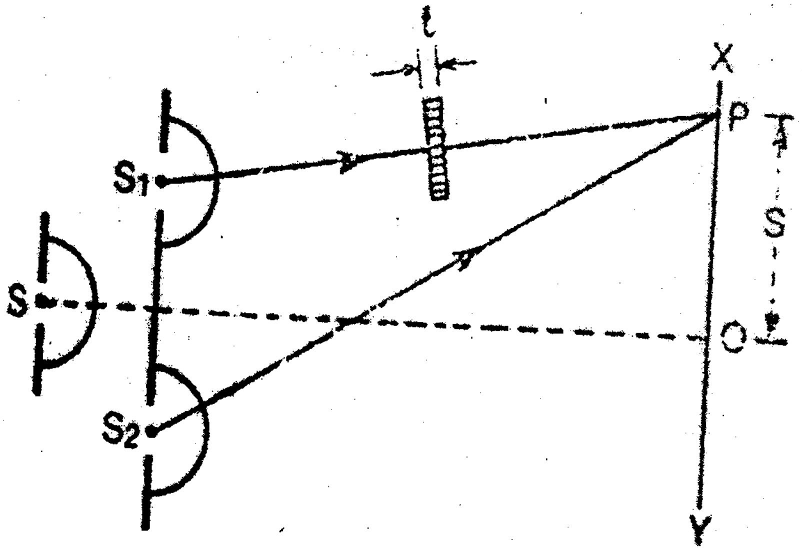


FIG 10.8

If μ is the refractive index of the material then $\mu=\frac{c}{c_g},\Rightarrow c_g=\frac{c}{\mu}$

The time taken by the light ray to travel the distance s_1p

$$=\frac{s_1p-t}{c}+\frac{t}{c_g}=\frac{s_1p-t}{c}+\frac{t\mu}{c}$$

$$=\frac{s_1p+(\mu-1)t}{c}....24\quad(10.24)$$

Thus the path or distance travelled by light ray s_1p is equivalent to

$$s_1p+(\mu-1)t=s_1p$$

Hence the path difference between the light ray s_1p and s_2p is given by

$$s_2p-s_1p=s_2p-(s_1p+(\mu-1)t)...25\quad(10.25)$$

But from eq:19, $s_2p-s_1p=\frac{2xd}{D}$, Path difference $= \Delta = \frac{2xd}{D} - (\mu - 1)t$

The n^{th} maxima the path difference $= n\lambda$ $\frac{2x_n d}{D} - (\mu - 1)t = n\lambda$ which implies

$$x_n = \frac{D}{2d}[n\lambda + (\mu - 1)t] \text{....26}$$

x_n denotes the position of n^{th} maxima.

When the glass plate is absent, t= o ,

$$x_n' = \frac{D}{2d} n\lambda$$

Hence the shift produced in n^{th} maxima due to the glass plate in path s_1p is given by

$$S = x_n - x_n' = \frac{D}{2d}(\mu - 1)t \text{....27} \quad (10.27)$$

EQ:27 is independent of 'n' hence the shift is same for all maxima.

Hence by measuring the displacements in fringe pattern by introduction of glassplate in path of light ray we can find the thicknes of the material using eq: 10.27

$$t = s.\frac{2d}{D(\mu - 1)} \text{....................}(10.28)$$

d,D values are measured as explained in biprism experiment of wave length. Then the monochromatc source is replaced by white source .We observe an interferene fringe pattern with central white band and remaining being coloured bands.The cross wire of micrometer is coincided with a white fringe Micrometer reading is noted Then the glass plate or mica sheet of thickness is inttroduced in the path of light ray The maxima or white band shifts. Again cross wire is coincided with white band The micrometer reading is noted .

The differeence between R_1andR_2 gives the displacement of fringes due to glassplate Using these values of d,Dand S ,the thickness of glass plate is deteemined using formula (10.28).

10.9 Change of phase on reflection:

Stokes law states that the light ray suffers a phase change of π or path change of $\frac{\lambda}{2}$ when it is reflected at the surface of optically denser medium. This does not happen , when it is reflected at the surface of optically rarer medium.

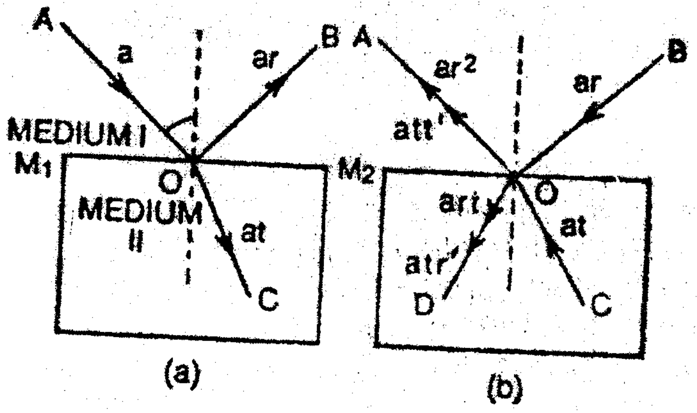


fig 10.9

proof : As shown in the figure10.9 let A be the incident ray on a glass slab .Let the refractive index of glass be μ_2 and of air is μ_1. $\mu_2 > \mu_1$ in this case hence , light is reflected at optically denser medium.OB isthe reflected ray and OC is refrated ray. Let ‘t’ and ‘r’ be the coefficients of transmission and reflection respectively .So if ‘a’ is the amplitude of incident ray , then the amplitude of reflected ray is a_r and the amplitude of transmitted ray is a_t

Suppose now if OC is taken as incident ray of amplitude a_t., the reflected ray will be OA. If the reflection and transmisson coefficient in denser medium are ‘r’and l, then the amplitude of OD is atr’ and amplitude of OA is att’.

On reversing OBor taking it as incident ray OB is ‘ar’ ,amplitude of reflected ray OA is ar^2. , amplitude of transmitted ray OD is art.

Reversing OB and OC we should obtain the original incident ray OA . Hence the ray OD should be absent.

$$art + art' = 0$$

Along OA we should have wave of amplitude ‘a’

$$ar^2 + att' = a.....30 \quad (10.30)$$

$$eq...29 \Rightarrow r' = -r.....31\,(10.31)$$

$$eq...30 \Rightarrow tt' = 1 - r^2.....32\,(10.32)$$

Negative sign in equation ... 10.31 shows that the ray reflected at denser medium suffers a phase change of π or path change of $\frac{\lambda}{2}$.

Example: Lloyd's mirror is an instrument to produce achromatic fringes with white light. This mirror utilises the above principle In this experiment direct rays from the source and light rays reflected from Lloyds mirror interferece with each other to produce interference pattern. When white light is used as a source, we obtain central dark band instead of white band. This fact proves that the reflection of light rays at mirror introduces a phase difference of π. So, the central band satisfies the condition of dark fringe at the centre of white light fringe pattern and coloured bands on either side..

10.10 Interference by divison of amplitude :

In some cases we may have extended light source .Interference fringes can be obtained from such source also using the method of division of amplitude of light rays. Interference fringes are obtained in such cases using the following procedure (Fig. 10.10).

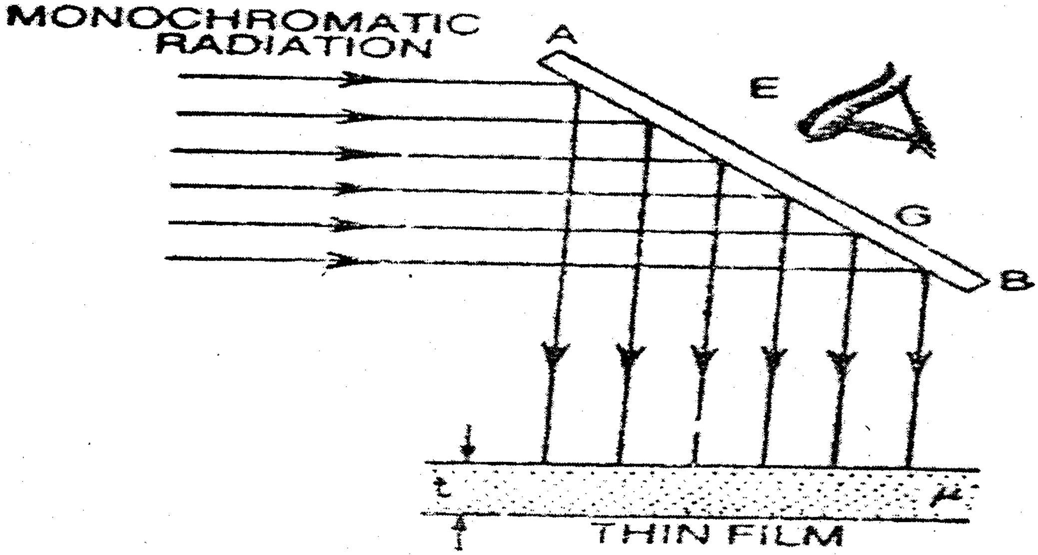


FIG 10.10

The divergent rays from extended source are made to be incident on a glass plate G placed at an angle of 45^0 to the incident rays. Then the light rays reflected at glass plate G get converted to a plane wavefront or parellel light beam. These parallel rays are made to incident normally on a thin film (or glass plate) of thickness 't' and refractive index μ. These rays get partially reflected at the upper surface of glass plate. 'So the incident amplitude a of light ray is divided into two rays of amplitudes ar and at .

This is division of amplitude the ray transmitted 'at' again gets reflected at the bottom surface of glassplate, transmitts through upper surface and comes out. So the two rays (1) reflected from upper surface (2) transmitted and then reflected ray at the second surface are obtained from same source but have a path differerence. Hence they interfere with each other to form interference fringes on the thin film itself (Fig. 10.10).

The extra path travelled by the transmitted ray or 2^{nd} ray $= 2\mu t$

Path difference between two rays $= 2\mu t$.

But first ray suffered reflection at denser medium. According to stokes law the path difference introduced due to reflection is $-\frac{\lambda}{2}$

Total path difference between two rays $\Delta = 2\mu t - \lambda/2$

For constructive interference the path difference $\Delta = n\lambda$

$$\therefore 2\mu t - \lambda/2 = n\lambda$$

$$\Rightarrow 2\mu t = (2n+1)\lambda/2 \quad \text{------}(10.33), n=0,1,2,3\text{--etc}$$

For destructive interference

$$2\mu t = n\lambda \text{-------}(10.34), n=1,2,3\text{-----etc}$$

10.11.Cosine law:Oblique incidence of plane wave on a thin flim

If the parallel rays from glass plate G are incident obliquely on a thin film(example wedge type film), then the procedure in finding the path difference is slightly different. That methodology is explained below (Fig. 10.11) which leads to cosine law. i.e., path difference in terms of cosine of the angle.

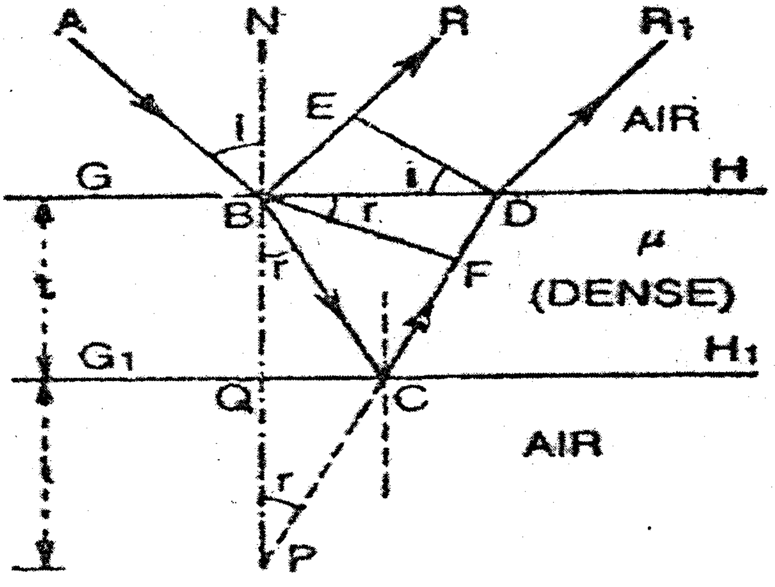


fig 10.11

AB is the incident ray incident obliquely on thin film at an angle of incidence 'i'.It is partly reflected along BR ,and partly transmitted along BC. Again it is reflected back along CD and transmits along DR_1 .BR and DR_1 are two parallel rays obtained from same ray AB. So they interfere with each other.The path difference between these two rays is BC+CD+BE.To determine this value normals BF and DE are drawn.

$\angle ABN = i, \angle QBC = r$

From geometry of figure $\angle BDE = i, \angle QPC = r$

DC is extended back to meet normal BQ at P

Path difference Δ = path(BC+CD)in thin film _path BE in air

$$\Rightarrow \Delta = \mu(BC + CD) - BE \text{ ---------(10.35)}$$

But from Snell's law $\mu = \frac{\sin i}{\sin r} = \frac{BE/BD}{FD/BD} = \frac{BE}{FD}$

$$BE = \mu FD \text{ -------(10.36)}$$

Substituting in above equation is

$$\Delta = \mu BC + \mu CD - \mu FD$$

$$\Delta = \mu BC + \mu(CF + FD) - \mu FD$$

$$\Delta = \mu(BC + CF) = \mu PF \quad ,(\because BC = PC) \text{ ------(10.37)}$$

In $\Delta^{le}BPF \rightarrow Cosr = PF/BP \Rightarrow PF = BPCosr = 2tCosr$ -------(10.38)

Substituting in equation (10.37)

$$\Delta = \mu \times 2tCosr = 2\mu tCosr \text{ -------(10.39)}$$

The optical path difference given by equation (10.39) is termed as "Cosine law". But the reflection at denser medium introduces a path difference of $\lambda/2$. Hence

Total path difference $= 2\mu tCosr \pm \lambda/2$ ---------(10.40)

condition for Constructive interference is

$$2\mu tCosr \pm \lambda/2 = n\lambda$$

$$\Rightarrow 2\mu tCosr = (2n \pm 1)\lambda/2 \text{ -----(10.41)}$$

Condition for destructive interference is

$$2\mu t\mathrm{Cos}r \pm \lambda/2 = (2n+1)\lambda/2$$

$$\Rightarrow 2\mu t\mathrm{Cos}r = n\lambda \quad \text{-------}(10.42)$$

10.12 Colours of thin films:

When white light is incident on a thin film (of water or glass or any transparent film), we observe beautiful colours on the film.The formation of these colours on films can be explained with the help of interference .

When white light is incident on thin film , the white light is reflected partly by the upper surface of the film and partly transmitted to second surface.It is then reflected at second surface and passes through first surface . The rays reflected at first surface and reflected by second surface will be able to interfere each other. Depending on the path difference between two rays, it will satisfy the condition of bright fringe for certain colours of light or wavelengths. so that colour will appear on the film. According to cosine law $2\mu t\mathrm{Cos}r = (2n+1)\lambda/2$ is condition for constructive interference.

Since the thickness t and refractive index μ of the film are constant,depending on the angle of the refraction above condition is satisfied for one colour.

When we change the position of the eye 'r' changes and we may observe different colour at the same position of the film. If we observe different parts of a film from same position also , may change and we observe another colour. Hence we can see different colours on same thin film depending on the position of the eye or the part of thin film. Other colours may be feeble. The colours for which minima condition is satisfied are absent.

If we observe the same film in transmitted light instead of reflected light,colours which are absent in reflected light will be visible in transmitted light.

10.13 Non Reflecting Films:

When a light ray is incident on transparent material like glass , a part of the light ray is reflected and remaining part is transmitted. According to Fresnel's equation , the intensity of the reflected light is given by .

$$I_r = \left(\frac{\mu_2-\mu_1}{\mu_2+\mu_1}\right)^2 . I \Rightarrow \frac{I_r}{I} = \left(\frac{\mu_2-\mu_1}{\mu_2+\mu_1}\right)^2 \quad \text{---------}(10.43)$$

Where μ_1 -refractive index of first medium(air)

μ_2 - refractive index of second medium(glass)

Generally μ_2 =1.5 and μ_1 =1

$$\therefore \left(\frac{\mu_2-\mu_1}{\mu_2+\mu_1}\right)^2 = \frac{I_r}{I} = 4\%$$

So when light is incident on any glass surface in an optical instrument, a part of light is lost at a rate of 4% at each surface.In certain objectives there will be about four lenses, hence a loss of 30% light .only 70% of light is transmitted. When the source of light is not intense, then image formation will be dull due to this loss. To overcome this loss by reflection of light, non reflecting films are used.

Non reflecting films are generally transparent materials like MgF_2 coated on glass as a thin layer of thickness $t = \lambda / 4\mu$,where λ is the wave length of the source, μ is the refractive index of coating material. Loss of light by reflection is overcome by destructive interference of reflected light rays. As shown in the figure10.12.

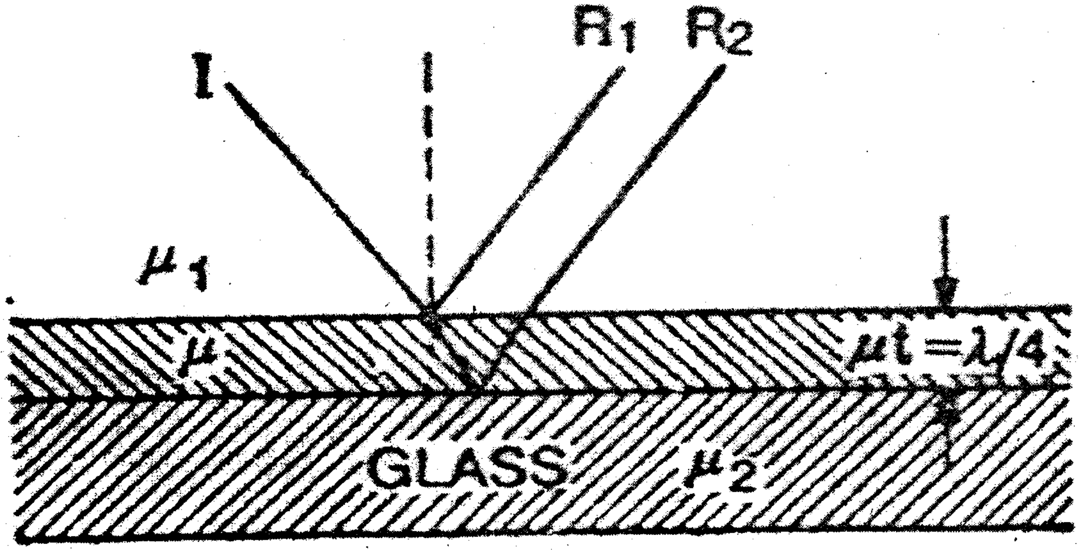


fig 10.12

The refractive index of air is μ_1 ,of the coating material is μ and of glass is μ_2 . The ray is incident on the coating material or non-reflecting film is reflected partly along R_1 and partly transmitted. Again it is reflected back along R_2 by the lower surface of the non-reflecting film. The refractive index of non-reflecting film is selected such a way that $\mu_1 < \mu < \mu_2$. So both the reflections are at surface of denser medium. So the path dfference introduced at both surfaces is $\lambda/2$ and $\lambda/2$ or phase difference π and π. These two rays R_1 and R_2 which differ by a path difference of $\lambda/2$ interfere destructively.

$$2\mu t = \lambda/2 \Rightarrow t = \lambda/4\mu$$

Hence the thickness of the film should be equal to $\lambda/4\mu$. The refractive index of MgF_2=1.38 in between that of air and glass. Phase difference between R_1 and R_2 $= \frac{2\pi}{\lambda} \times$ pathdifference

$$\frac{2\pi}{\lambda} \times 2\mu t = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi$$

For complete destructive interference of R_1 and R_2 their intensities should be equal.

$$\therefore \left(\frac{\mu-\mu_1}{\mu+\mu_1}\right)^2 = \left(\frac{\mu_2-\mu}{\mu_2+\mu}\right)^2 \quad \text{-------(10.44)}$$

If outside medium is air i;e $\mu_1 = 1$,then

$$\left(\frac{\mu-1}{\mu+1}\right)^2 = \left(\frac{\mu_2-\mu}{\mu_2+\mu}\right)^2, \quad \Rightarrow \frac{\mu-1}{\mu+1} = \frac{\mu_2-\mu}{\mu_2+\mu} \quad \text{-----(10.45)}$$

adding (1) on both sides of equation

we get
$$\frac{\mu}{\mu+1} = \frac{\mu_2}{\mu_2+\mu} \quad \text{--------(10.46)} \quad \text{and}$$

$$\frac{1}{\mu+1} = \frac{\mu}{\mu_2+\mu} \quad \text{---------(10.47)}$$

Dividing eq (45) by (47) we get $\mu^2 = \mu_2 \Rightarrow \mu = \sqrt{\mu_2}$ ------(10.48)

Hence the refractive index of the non-reflecting films should be equal to the square root of the glass material. The process of decreasing reflectivity of a surface using non-reflecting films is known as "Blooming".

10.14.1 Interference by a plane parallel film illuminated by a point source;

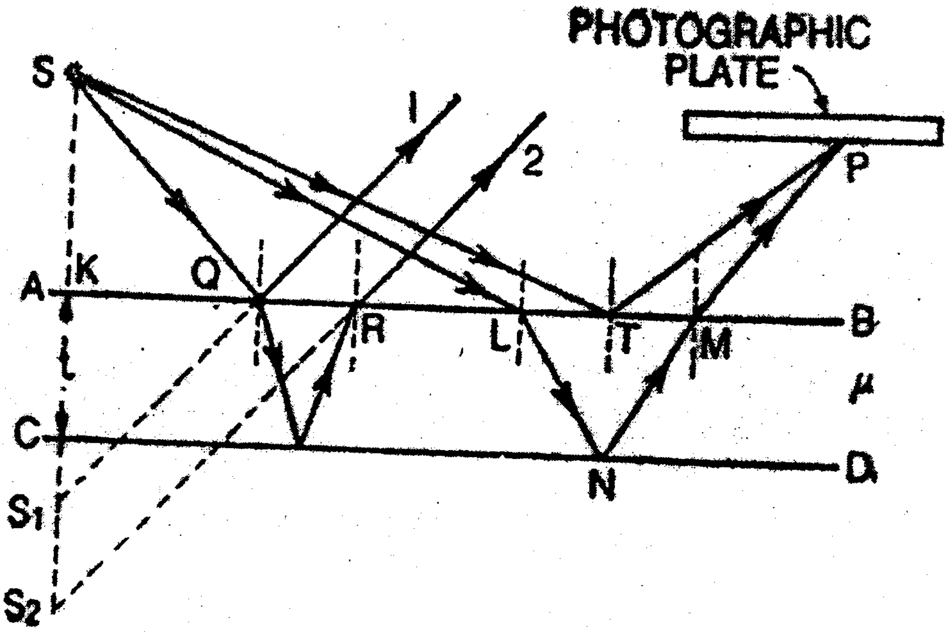


fig10.13

When light is incident from point source on a plane parallel film,a part of the ray gets reflected along 1.Remaining part transmits to second surface,again gets reflected and transmits along 2. the rays 1 and 2 appear to be coming from two point sources S_1 and S_2. These two rays interfere with each other .similarly the rays getting reflected along TP and rays transmitted and reflected along MP interfere each other. These two rays produce interference pattern (Fig. 10.13).

The fringes will be dark if path difference $\Delta = SL + \mu(LN + NM) + MP - (ST + TP)$ -(10.49) is equal to $n\lambda$,else bright fringe.

10.14.2 Interference by a film with two Non -parallel reflecting surfaces (Wedge shaped film):

Let us consider two plane surfaces GH and G_1H_1 inclined at an angle α and wedge shaped air film is enclosed between these surfaces as shown in the fig 10.14. The thickness of the film increases along GH.Interference takes place between the rays reflected from the first surface (BR) and the ray transmitted to second surface and transmitted along (DR_1). These two when extended back meet at S and appear to interfere at S.

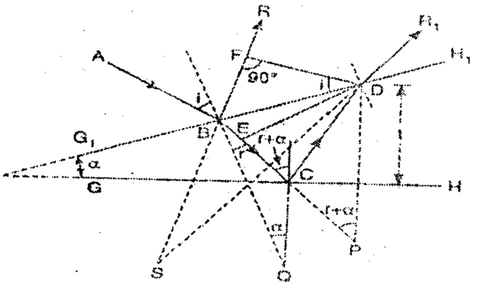


Fig. 10.14

The path difference between BR and DR_1

$$\Delta = \mu(BC + CD) - BF$$

$$\Delta = \mu(BE + EC + CD) - \mu BE$$

$$\Delta = \mu(EC + CD) = \mu EP = 2\mu t \cos(r + \alpha)$$

Due to reflection at the surface of denser medium additional path difference $\lambda/2$ is introduced.

Total path difference $= \Delta + \lambda/2 = 2\mu t \cos(r + \alpha) + \lambda/2$ -----(10.50)

For Constructive interference $\Delta = n\lambda$

$$\Rightarrow \Delta + \lambda/2 = n\lambda \Rightarrow 2\mu t \cos(r + \alpha) + \lambda/2 = n\lambda$$

$\Rightarrow 2\mu t \cos(r + \alpha) = (2n \pm \lambda/2)$ ----(10.51) Condition for constructive interference

Similarly condition for destructive interference is

$$2\mu t\mathrm{Cos}(r+\alpha) = n\lambda \quad \text{---(10.52)}$$

From equations 10.51 and 10.52 it is evident that when refracting angle 'r' and angle of wedge appearance of bright or dark fringe depends on the thickness of the airfilm in the wedge .Thickness of the air film in the wedge is equal along the direction parallel to the thin edge.Hence interference pattern is formed parallel to thin edge of the wedge

Spacing Between Fringes:

For normal incidence of light ray, r=0 and if thin film in the wedge is air, $\mu = 1$

$$\therefore 2\mu t\mathrm{Cos}(r+\alpha) = (2n+1)\lambda/2 \quad \text{------Bright fringe}$$

$$\Rightarrow 2t\mathrm{Cos}\alpha = (2n+1)\lambda/2 \quad \text{-------(10.53)}$$

If we consider n^{th} fringe formed at a distance of x_n from the end of wedge (Fig. 10.15), then

$$t = x_n \tan\alpha \text{ ---------------- (10.54)}$$

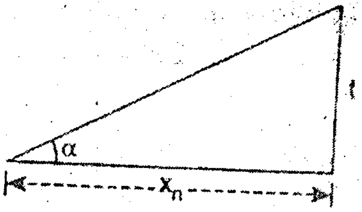


Fig. 10.15

Substituting in equation (10.53) we get

$$2x_n \tan\alpha\cos\alpha = (2n+1)\lambda/2$$

$$\Rightarrow 2x_n \sin\alpha = (2n+1)\lambda/2 \text{ ----------------------------(10.55)}$$

Similarly for $(n+1)^{th}$ fringe

$$2x_{n+1}\sin\alpha = [2(n+1)+1]\lambda/2 \text{ -----------------------(10.56)}$$

Spacing between two bright fringes is

Subtracting (10.55) from (10.56) $2x_{n+1}\sin\alpha - 2x_n\sin\alpha = \lambda$

$$2\sin\alpha(x_{n+1} - x_n) = \lambda \text{ ----------------}$$

This distance between two consecutive bright or dark fringes is termed as fringe width β.

$$\beta = x_{n+1} - x_n = \frac{\lambda}{2\sin\alpha}$$

When angle of the wedge 'α' is small, $\sin\alpha \approx \alpha$

$$\beta = \frac{\lambda}{2\alpha} \text{------------------------------} (10.58)$$

Since fringe width is independent of fringe number 'n',Hence the fringes are equally spaced.

10.14.3 Determination of diameter of the wire-wedge method :

Two optically plane glass plate are taken and a thin wire or paper is placed in between the plates at one end and tied up (see Fig 10.16).

Then air film in the form of wedge is formed between the plates. The thicknees of air film goes on increasing along the wedge as we move away from point of contact. This wedge is illuminated by a monochromatic source after making rays parallel with the glass plate G placed at an angle of 45^0. These parallel rays are incident normally on wedge of air film. Due to reflected rays from upper and lower surfaces of the film, interference takes place. Interference pattern or bands of equal thickness are formed along AB,CD,EF etc.

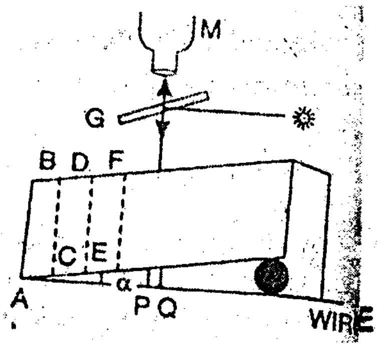
Since it is an air film $\mu = 1$, normal incident r=0.

Let the thickness of air film at CD is t_1. If dark fringe is formed at 'CD' then

$$2t_1 = n\lambda \text{------------------------} (10.59)$$

Similarlly another dark ring at EF where film thickness is t_2

$$2t_2 = (n+1)\lambda \text{------------------------} (10.60)$$



Fig,. 10.16.

From figure $\tan\alpha = \frac{t_1}{AP} = \frac{t_2}{AQ}$

$$\frac{t_2 - t_1}{AQ - AP} = \tan\alpha$$

But AQ-AP = fringe width β.

$$\therefore t_2 - t_1 = \beta\tan\alpha \text{ ----------------------(10.61)}$$

From equation (10.59) and (10.60) $\therefore t_2 - t_1 = \lambda/2$ ----------------(10.62)

Comparing (61) and (62) $\beta\tan\alpha = \lambda/2$

$$\Rightarrow \beta = \lambda/2\tan\alpha \approx \lambda/2\alpha \text{ ------------------(10.63)}$$ (When α is small.)

Let the diameter of wire placed between plates = d.

The distance between wire and line of contact of plates = x.

From the figure $\tan\alpha = \frac{d}{x} = \frac{\lambda}{2\beta}$

$$\therefore d = \frac{x\lambda}{2\beta} \text{ ----------------(10.64)}$$

Experiment :

Observe the fringes formed on wedge shaped film through microscope.Coincide the cross wire of microscope on n^{th} fringe. Get the microscope reading R_1. Count 5 fringes and coincide the cross wire with $(n+5)^{th}$ fringe. Note the reading as R_2. $\frac{R_2 - R_1}{5}$ gives the fringe width β .Measure the distance between the wire and the point of contact 'x' with a scale. Using the wavelength of the monochromatic source λ, the diameter of the wire 'd' can be determined uisng equation (10.64).

10.15 Newton's Rings :-

Newton's Rings are another practical demonstration of interference of light. When parallel light rays are incident normally on a convex lens placed on a glass plate alternate dark and bright circular fringes are formed round the contact point of lens and glass plate. These circular fringes are termed as "Newton Rings".

The reason for formation of newtons rings can be explained as follows in Fig. 10.17. When light ray AB is incident on the lens, it is partly reflected and partly transmitted along BC. This transmitted ray is reflected back by the lower surface of the lens and transmits into air along 1. A part of this ray BC gets transmitted along CD to the lower glass plate and gets reflected back and transmits through lens along '2'. These two rays 1 and 2 are obtained from same ray AB. Hence they interfere with each other forming Newtons Rings. The path difference between these two rays $=2\mu t\cos r + \lambda/2$

When film between the lens and glass plate is air, $\mu=1$ and for normal incidence of light ray, r=0.

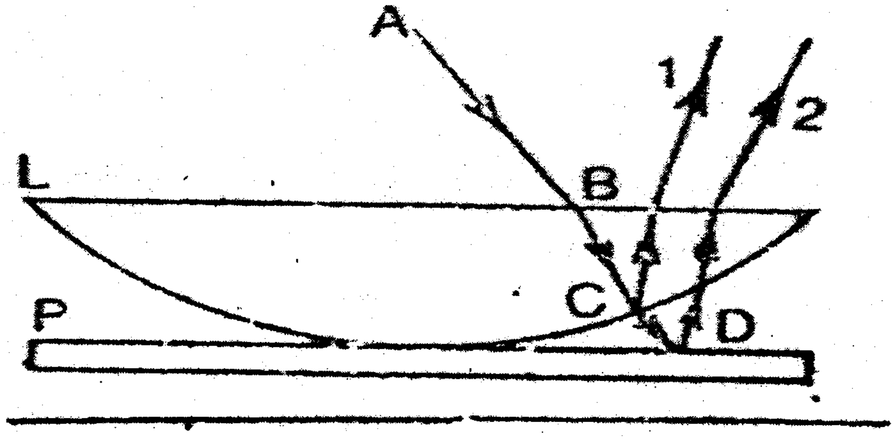


Fig. 10.17

Path difference $\Delta = 2t + \lambda/2$

For bright circular fringe $\Delta = 2t + \lambda/2 = n\lambda$

$$\Rightarrow 2t = (2n+1)\lambda/2 \text{ ---------------------- } (10.65)$$

For dark circular fringe $2t = n\lambda$ ----------------------(10.66)

From equations (10.65) and (10.66) it is evident that the formation of dark and bright fringe depends on the thickness of air film between the convex lens and the glass plate. 't' is equal along a circle with point of contact of lens and the glass plate. Hence we obtain a concentric circular fringes with point of contact as centre.

10.15.1 Newton's Rings in reflected light :-

In this section we try to findout the diameter of circular fringes in terms of λ and R is the radius of curvature of lens LOL^1 -Lens

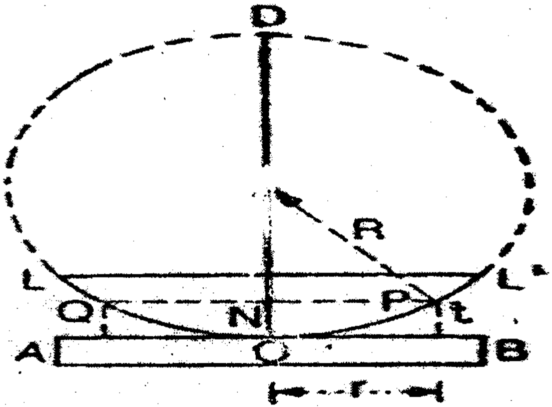


Fig10.17

. It is part of the circle D LOL^1 of radius R.

$\therefore$ OC= CD=R .Let us consider a Newton ring for[illegible]ed at QP. Let air film thickness at QP be 't'. The radius of this ring = r. PC is also equal to 'R' fr[illegible] geometry of this figure. The property of the circle gives the following equati[illegible]

$$NP \times NQ = NO \times ND$$

Substituting these values ,

$$r \times r = t \times (2R - t) = 2Rt - t^2$$

$$\because t << R, \quad r^2 \approx 2Rt \Rightarrow t = \frac{r^2}{2R} \quad \text{-------------------(10.67)}$$

Condition for bright fringe $2t = (2n-1)\lambda/2$

$$\Rightarrow 2.\frac{r^2}{2R} = (2n-1)\lambda/2$$

$$\Rightarrow r^2 = (2n-1)\lambda R/2 \text{ ---------------------(10.68)}$$

The diameter of the Newton's Ring $D = r/2$.

Substituting in above equation

$$\frac{D^2}{4} = \frac{(2n-1)\lambda R}{2} \Rightarrow D = \sqrt{2\lambda R(2n-1)}$$

$$\Rightarrow D \propto \sqrt{(2n-1)} \text{ ----------------------(10.69)}$$

For dark fringe $2.\frac{r^2}{2R} = n\lambda \Rightarrow r^2 = n\lambda R$

$$\Rightarrow D^2 = 4n\lambda R \Rightarrow D = 2\sqrt{n\lambda R} \text{ ----------------------(10.70)}$$

$$\text{or} \quad D \propto \sqrt{n} \text{ -------------------(10.71)}$$

Thus diameters of the newton's rings are proportional to square root of natural numbers. As the order of the fringe increases, width of the fringe decreases. In case of reflected light central ring is dark. Since the condition t=0 is satisfied by dark ring only.

10.15.2 Newton's rings in transmitted light :

In transmitted light,

$2t = n\lambda$ for bright fringe

$2t = (2n-1)\lambda/2$ for dark fringe.

Using equation (67)

$$2.\frac{r^2}{2R} = n\lambda \Rightarrow r^2 = n\lambda R$$

But $r = \frac{D}{2} \Rightarrow \frac{D^2}{4} = n\lambda R$

$$\Rightarrow D^2 = 4n\lambda R \Rightarrow D = 2\sqrt{n\lambda R} \text{ ----------------------(10.72)}$$

$$\Rightarrow D \propto \sqrt{n}$$

Similarly for dark ring $\Rightarrow D \propto \sqrt{(2n-1)}$

In transmitted light central ring is bright ring. The Newton's rings in reflected light and transmitted lights are complementary.

10.15.3 Determination of Wavelength of Sodium light using Newton's Rings :-

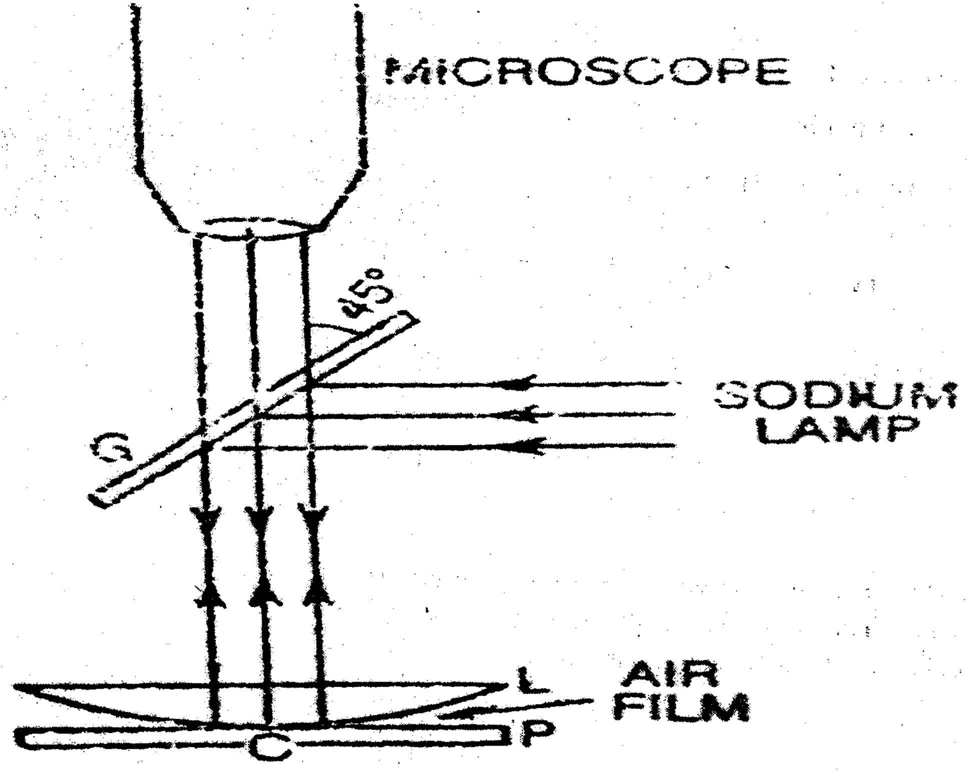


Fig10.19

Experimental Arrangement:-

A plane -convex lens of large radius of curvature is placed on a plane glass plate with its convex surface in contact with glass plate.The convex surface touches glass plate at C,and an air film of increasing thickness is formed between the convex surface and glass plate from C to L. Light rays from an executed monochromatic source [sodium vapour lamp placed behind a slit] are made to incident on a glass plate G,placed at an angle of 45° to the normal .This glass plate reflects the light rays and makes the light rays to be incident normally on the convex surface or lens. A part of these rays are reflected by convex surface and remaining part transmits to the plane glass plate. The plane glass plate reflects these rays and they pass through the lens and interfere with the light rays reflected by convex surface. As a result Newton's rings are formed. These are viewed using travelling microscope.

Theory:-

Let 'R' be the radius of curvature of the lens the wavelength of the monochromatic source, D_n -the diameter of the nth ring, D_{n+p} -the diameter of the (n+p) th ring. Then from equation-(10.71)

$$D_n^2 = 4n\lambda R \qquad \text{and} \qquad D_{n+p}^2 = 4(n+p)\lambda R$$

$$\therefore D_{n+p}^2 - D_n^2 = 4p\lambda R \Rightarrow \lambda = D_{n+p}^2 - D_n^2 \text{ ---------------------- } (10.73)$$

Using above formula wavelength of the monochromatic source can be determined.

Procedure:

Experimental arrangement is done as described above. After obtaining Newton's rings, the cross-wires of the travelling microscope are made to coincide with central dark spot .Then counting 20 rings travelling microscope is moved to 20 th ring. The cross wires are made to coincide with the end of the 20 th ring, such that the vertical wire looks like tangent to 20 th ring . Microscope reading is noted . Moving 5 rings at a time , the reading of 15 th,10 th ,5 th and 0 th ring are noted towrds right side . Similarly proceeding in same direction the readings of the same rings on left sides are also noted, The difference between right hand side of 5 th ring gives the diameter of 5 th ring.(Fig. 10.20) Similarly diameters of other rings are also determined . A graph is drawn with no. of rings on X-axis and its *diameter*^{2} on Y-axis.

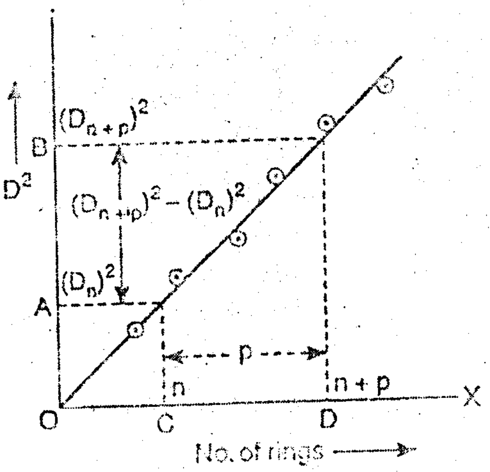


Fig. 10.20

It will be a straight line . Select a ring number n and ring number n+p and determine D_n^2 and D_{n+p}^2 from the graph

$$(D_{n+p}^2 - D_n^2) / p = AB/CD$$

The Radius of curvature of plano convex lens is determined using spherometer $R = \frac{l^2}{\Delta} + \frac{h}{2}$ or through Boy's method

Substituting these values in eq(10.73),we can find the wave length of the monochromatic source.

10.16 Michelson interferometer:-

It is a permanent instrument used to produce interference fringes without any further experimental arrangement as in Newton's rings or Wedge method. It can be used to find the wave length of the monochromatic source, resolute closely spaced spectral lines,to determine the refractive index and thickness of thin transparent materials and standardisation of meters in terms of the wave length of the light.

0.16.1 Construction:-

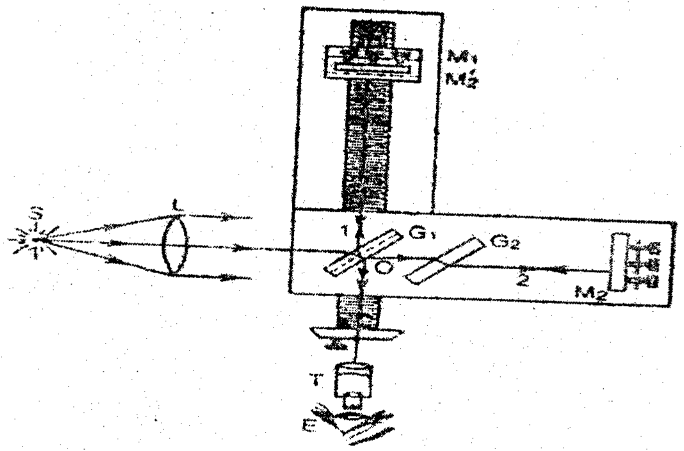


Fig 10.21

It consists of two optically plane,highly polished mirrors M_1 and M_2 at right angles to each other. It also consists of two semi-silvered glass plates G_1 and G_2 made of same material and same thickness at an angle of 45^0 to the plane of mirror M_2 .Mirror M_1 is mounted on a carriage which can be moved forward or backward with fine screw adjustment (See Fig. 10.21). The mirrors are mounted on three levelling screws with the help of which the mirrors can be titled (at an angle to) horizontal. The rays from a monochromatic source are made to be incident on a lens L , to attain a parallel beam of light.The interference fringes formed can be viewed through the telescope.

Working:-The parellel light beam from lens is incident on glass plate G_1 . A part of light ray '1' is reflected back to mirror M_1 and passes through G_1 . Another part of the light ray '2' is transmitted to M_2 .These two rays are incident on M_1 and M_2 normally. So they are reflected back normally and follow the same path and reach G_1 .The two rays coming out of G_1 after reflection from M_1 and M_2 are derived from same source, hence are in a position to interfere. Depending on the path difference between rays '1' and '2', interference fringes are formed. But Ray1 travels through G_1 twice ,while the ray2 passes through only G_1 once(see Fig. 10.22). To compensate this path difference, second glass plate G_2 of same thickness as G_1 is introduced.So this is termed as compensating glass plate.The interference fringes formed can be viewed through the telescope T.

When we view the mirror M_1 through telescope, we can also view the virtual image of M_2 , M_2^1 formed by the glass plate G_1 behind M_1 . So the two rays reflected from M_1 and M_2 can be viewed as reflected from M_1 and M_2^1 . So Michelson interferometer is equivalent to an air film formed between M_1 and M_2^1

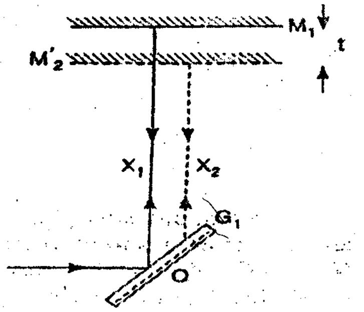


Fig. 10.22

From figure $OM_1 = x_1$, $OM_2 = x_2$

The path difference between interfering rays $\Delta = 2(x_1 - x_2)$

and additional path difference of $\lambda/2$ for reflection from M_1

$$\Delta = 2(x_1 - x_2) \pm \lambda/2 \text{------------------------} (10.74)$$

The interference fringes formed may be circular,straight or paraboic depending on path difference between M_1 and M_2^1.

10.16.2 Types of fringes :

The path diffrence between the light rays coming from M_1 and M_2^1 depending on

1.The seperation between M_1 and M_2^1

2.The angle subtended at an eye.

3.The inclination between M_1 and M_2^1.

Circular fringes :- When the two mirrors M_1 and M_2 are perfectly perpendicular to each other or M_1 and M_2^1 are perfectly paraellel and a uniform thin air film is formed, then circular fringes are formed. The path difference between the rays from M_1 and M_2^1 is given by

$$2t\cos r + \lambda/2 = n\lambda$$

Since 't' the thickness of air film and λ are constant, above equation is the locus of radius 'r'. Locus of radius 'r' are concentric circles. Hence we obtain circular fringes which can be viewed through the telescope T (Fig. 10.22).

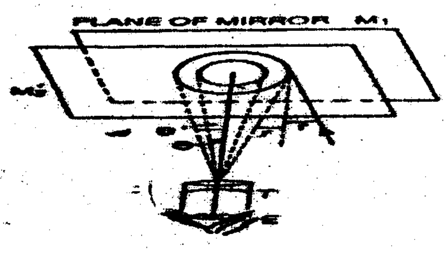
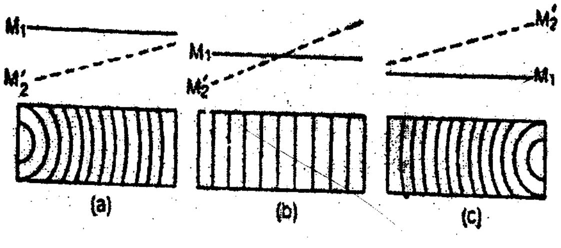


fig 10.23



(a) (b) (c)

fig10.24

Localised fringes:- When M_1 and M_2' are at all angle to each other or cross each other ,then the our film between the mirrors is wedge shaped. Depending on the thickness of the film and the angle of incidence shape of fringes changes. When the mirrors cross-each other, stright fringes are formed . When the mirror are wedge shaped ,then circular fringes with lourelity towards the thin edge of wedge are observed as shown in the figure.

Localised fringes with white light :-

When monochromatic source is replaced by white light,when the air film between M_1 and M_2 is thin coloured circular fringes with dark fringes in between are formed .

10.16.3 Mathematical Analysis :-

Let the light ray from M_1 produces a displacement of y_1 in the medium.

$$y_1=a\exp\{i\left[\omega t-k\left(2x_1+\frac{\lambda}{2}\right)\right]\}$$

Similarly M_2 produces a displacement

$$y_2=a\exp\{i\left[\omega t-k\left(2x_2+\frac{\lambda}{2}\right)\right]\}$$ The resultant displacement

$$y=y_1+y_2=2a\cos\left(\frac{\theta}{2}\right)\exp\left[i\left(\omega t-\alpha_1+\frac{\theta}{2}\right)\right]$$

Intensity $I=\left(\sqrt{yy^*}\right)^2=\left(2a\cos\frac{\theta}{2}\right)^2=4a^2\cos^2\frac{\theta}{2}$

Intensity is maximum when $\cos\frac{\theta}{2}=\pm\Rightarrow\frac{\theta}{2}=n\pi$

$$\Rightarrow\frac{1}{2}\left[k\left(2x_1+\frac{\lambda}{2}\right)-k\left(2x_2+\frac{\lambda}{2}\right)\right]=n\pi$$

Interms of path difference, $2(x_2-x_1)=n\lambda\pm\frac{\lambda}{2}$........(75)(10.76)

Condition for minimum intensity is, $\cos\frac{\theta}{2}=0=\frac{\theta}{2}=(2n+1)\frac{\pi}{2}$

$\Rightarrow\frac{1}{2}\left[k\left(2x_1+\frac{\lambda}{2}\right)-k\left(2x_2+\frac{\lambda}{2}\right)\right]=(2n+1)\frac{\pi}{2}$. Interms of path difference $2(x_2-x_2)=(2n+1)\frac{\lambda}{2}\pm\frac{\lambda}{2}$

10.16.4 :Uses of Michelson interferometer

1.Determination of difference of wavelength of monochromatic light:

The Michelson interferometer is arranged for circular fringes with central bright spot. The cross wires are coincided with centre of bright spot. Let the bright spot be n^{th} bright spot.Now if we move the distance between M_1 and M_2^1 by $\lambda/2$ [by adjusting the micrometer screw attached to M_1] a path difference of λ is introduced and $(n+1)^{th}$ bright spot appears at the crosswires. Like wise as we move M_2^1 away from M_1, for each distance $\lambda/2$ increase between them, one bright spot disappears and another one appears.Suppose M_1 is moved from distance x_1 to x_2 and N bright spots appear the centre then, $N\frac{\lambda}{2} = x_2 - x_1 \Rightarrow \lambda = \frac{2(x_2 - x_1)}{N}$ By counting no.of spots N an measuring the distance $(x_2 - x_1)$, we can determine the wave length of monochromatic source.

2.Determination of difference of wavelength of closely spaced spectral lines :

When we use monochromatic sources like Sodium also, they emit two wavelengths λ_1 and λ_2 (D_1 and D_2 lines) which have small difference. To find the difference $\lambda_1 - \lambda_2$ interferometer can be used.

Arrange interferometer for circular fringes. When the path difference $x_1 - x_2$ or the distance between M_1 and M_2^1 is such that the central bright spot of circular fringes of λ_1, exactly falls on the central bright spot of circular fringes of λ_2, then we can observe distinct circular fringe pattern in the field of view of the telescope.When we move M_2^1 away from M_1, the bright fringe of λ_1 may fall on dark fringe of λ_2. Then we observe that indistinctness in the field of view of telescope. So by moving M_1 through 'x' distance, suppose we find next indistinct position, then n^{th} fringes of λ_1 and (n+1) fringes of λ_2 appear in the field.

$$x = \frac{n\lambda_1}{2} \text{ and } x = \frac{(n+1)\lambda_2}{2} \qquad \Rightarrow n = \frac{2x}{\lambda_1}, \quad n+1 = \frac{2x}{\lambda_2}$$

Substracting above equation from each other

$$(n+1) - n = \frac{2x}{\lambda_2} - \frac{2x}{\lambda_1} = \frac{2x(\lambda_1 - \lambda_2)}{\lambda_1\lambda_2}$$

$$\Rightarrow \lambda_1 - \lambda_2 = \frac{\lambda_1\lambda_2}{2x} = \frac{\lambda_{av}^2}{2x} \text{ where } \Rightarrow \lambda_{av} = \frac{\lambda_1 + \lambda_2}{2}$$

Thus measuring 'x' we can find the difference of wavelengths $\lambda_1 - \lambda_2$

3. Determination of thickness of a thin plate :

Interferometer is illuminated with white light source and is adjusted for localised straight line fringes. Then the cross-wires of the telescope are coincided with the central fringe. Thin plate of thickness 't' and refractive index μ is introduced in one of the paths 1 or 2 . Then the central fringes shifts due to introduction of path difference = $2(\mu-1)t$ by the plate.

Now the mirror M_1 is moved such that again the cross wires are set on central fringe. If the distance moved by M_1 is x, then

$$2x = 2t(\mu-1) \quad \text{or} \quad t = \frac{x}{\mu-1}$$

Using above formula, we can find the thickness of the glass plate.

10.17. Interference Filter :

To choose few wavelengths around a particular wavelength for experimental purpose , Interference filters are used. When white light is incident on parallel plates seperated by a distance 't' then bright fringes are formed when

$$2\mu t = n\lambda \Rightarrow \lambda = \frac{2t}{n}$$

If the medium between plates in air , $\lambda = \frac{2t}{n}$. From above equation we can infer that if the thickness of air film between plates 't' is extremely small, condition for maximum is satisfied only for few wavelengths . Suppose $t = 500\times10^{-9}$ m and $\lambda = 600\times10^{-9}$ m then $n \approx 1$ there is only one maxima. Thus we can get wavelength 5000 A^0 and two more wavelengths 10000 A^0 and 3333 A^0 around it. This type of arrangement to filter few wavelengths from incident light is called "Interference Filter".

It consists of a glass plate coated with thin metallic reflecting film of dielectric (MgF_2 or quartz) and again coated with metallic reflecting film. Then it is closed with another glass plate. This interference filter transmits only a band of order 'n' at wavelength λ

10.18 Solved problems :

1. If a slit illuminated by sodium light (5893 A°) is placed 0.15 cm from the plane of Lloyd's mirror, what will be the distance between consecutive bands formed on a screen 1.0 m from the slit ?

Solution : Fringe width $\beta = \frac{\lambda D}{2d}$

λ=5893 A°, D=100 cm, 2d = 2×0.15=0.30cm

Distance between consecutive bands or fringe width

$$\beta = \frac{5893\times10^{-8}\times100}{0.3} = 0.02 \text{ cm}$$

2. Find the thickness of a soap film (μ=1.33) which gives constructive second order interference of reflected light of $\lambda = 700\text{m}\mu$ (millimicron). Assume normal incidence given that $1\text{m}\mu = 10^{-9}\text{m}$?

Solution: For constructive interference $2\mu t = (2n-1)\lambda/2$

For second order interference n = 2

$$2\mu t = 3\lambda/2 \quad \text{or} \quad t = \frac{3\lambda}{4\mu}$$

But $\lambda = 700\times10^{-9}$ m, μ=1.33

$$\therefore t = \frac{3\times700\times10^{-9}}{4\times1.33} = 3.95\times10^{-7} \text{ m.}$$

3. A broad source of light ($\lambda = 5890\text{A}^0$) illuminates normally two glass plates 10cm long which touch at one end and are seperated by a wire 0.05 mm in diameter at the other end. How many bright fringes appear over the 10cm distance.

Solution: Diameter of a wire $d = \frac{\lambda x}{2\beta}$ in wedge method, β - fringe width, x- distance between wire and contact end of glass plates.

$$\beta = \frac{\lambda x}{2d}$$

$\lambda = 5890\text{A}^0$, x=10cm, $d = 0.05\text{m} = 5\times10^{-2}\times10^{-1} = 5\times10^{-3}\text{cm}$

$$\beta = \frac{5890\times10^{-8}\times10}{2\times5\times10^{-3}} = \frac{1178\times10^{-4}}{2} \text{cm}$$

since length of glass plates= 10cm

No.of. bright fringes or dark fringes in this distance

$$n = \frac{10}{\beta} = \frac{10\times2}{1178\times10^{-4}} = 0.00085\times10^5 = 85\times2 = 170$$

4. In an experiment on Newton's rings, the diameter of the tenth dark ring formed by yellow sodium light (589nm) and seen in reflection is 3mm. What is the radius of curvature of the lens surface ?

Sol: Diameter square of nth ring

$$D_n^2 = 4n\lambda R, \quad \text{or} \quad R = \frac{D_n^2}{4n\lambda}$$

$D_n = 3\times10^{-3}$m, n=10, $\lambda = 5.89\times10^{-7}$m

$$\therefore \; R = \frac{\left(3\times10^{-3}\right)^2}{4\times10\times5.89\times10^{-7}} = 0.382\text{m}$$

5.In an experiment in the Michelson interferometer, it is found that 40 rings to merge the centre, the mirror had to be moved thriugh 0.01mm. Calculate the wavelength of the light used.

Sol: Wavelength of light used $\lambda = \frac{2x}{N}$

where 'x' is the distance moved by M_1 and N - no. of rings merge in the centre

N=40, x=0.01mm =0.001cm

$$\therefore \; \lambda = \frac{2x}{N} = \frac{2\times0.001}{40} = 5\times10^{-5}\text{ cm}$$

6. A thin film with refractive index μ =1.58 for light wavelength λ=5890 A° is placed in one arm of a Michelson interferometer. If there is a shift of 20 fringes, calculate the thickness 't' of the film.

Sol: In Michelson interferometer ,path difference $2(\mu-1)t = N\lambda$

where N- no. of fringes moved

$$t = \frac{N\lambda}{2(\mu-1)}, \quad N=20, \quad \lambda = 5890\times10^{-8}, \quad \mu = 1.58$$

$$\therefore t = \frac{20\times5890\times10^{-8}}{2(1.58-1)} = 0.001\text{cm}$$

10.19 Summary: Interference establishes the wave nature of the light. Principle of superposition leads to addition of waves. Thischapter explains the condition for interference, the position and intennsity of fringes, fringe width, conditions for the bright and dark fringes. It also explains how to produce interference by division of wavefront and division of amplitude. It explains concepts of colours in thin films, non reflecting films, change of phase on reflection and cosine law. We understand the methods of producing interference using Fresnels Biprism, Wedge method, Newtons rings and Michelson interferometer. Using these instruments we can find wave length of monochromatic source, thickness of a thin sheet of glass or mica, diameter of wire, and resolution of spectral lines.

10.20 Key words:- Interfernce, Interfernce bands, Princple of superposition, Wave front, amplitude, displacement,coherence, crest, trough, path difference, phase difference, Intensity, Normal lines, Fresnel Biprism, Bright fringe, dark fringe, fringe width, cosine law, constructive interfernce, destructive interference, thin film, Non-reflecting films, wedge, Newton's rings,Interferometer, circular fringes, Localised fringes, interference filters.

10.21 Self assesment questions:

Long Answer Questions:

1. Derive the equation for intensity at a point on the screen in Young's double slit experiment. Show that interference phenomenon obey's law of conservation of energy.
2. Derive the equation for fringe width in Young's double slit experiment.
3. What is Fresnel's Biprism ? Explain how to form interference fringes in Fresnel's biprism experiment. Show that how would you use this to determine the wavelength of monocromatic light.
4. Derive the expression for the displacement of fringes when a thin transparent sheet is introduced in the path of one of the interfering beams in biprism. How this method is used for finding the thickness of mica sheet?
5. What is wedge shaped film? With necessary theory explain the interference of reflected light from wedge film when illuminated by a plane monocromatic wave.
6. How do you determine the diameter of a thin wire by the wedge shaped air film? Discuss about fringe pattern.
7. Describe the theory and experiment of Newtons rings to determine the wanelength of monocromatic source of radiation.
8. Describe the principle, construction and working of a Michelson interferometer. Explain how the wavelength of light is determined with it.
9. How Michelson interferometer is used to determine the refractive index of a thin transparent shee?

Short Answer Questions:

1. What is the principle of superposition?
2. What are the conditions to produce sustained interference pattern?
3. Write a note on interference with white light.
4. What is nodal line? Derive equation for nodal line?
5. Discuss the phase change due to reflection of light from the surface of a denser medium.
6. Explain the occurence of the colours of thin films.

7.Explain cosine law.
8. What is the basic principle behind a non-reflecting film? And explain its needs.
9.What is Blooming?
10. Show that the refractive index of the coating material should be equal to square root of the refractive index of coated surface material.
11.Explain how you can check up optical planeness of a glass plate.
12. Explain the formation of different types of fringes in Michelson interferometer.
13.Describe the principle and application of non-reflecting films. How are they prepared.
14.Explain why the central spot in Newton's rings obtained with reflected light is perfectly black.
15. Discuss the phase change due to reflection of light from the surface of denser medium.
16. Explain the principle and action of an interference filter.

Exercise :-

1. In the young's double slit experiment, the distance between the two holes is 0.5mm, $\lambda = 5\times10^{-5}$ cm and D=50cm. Calculate the fringe width.(Ans 0.05m)

2. A thin sheet of transparent material (μ =1.6) is placed in the path of one of the interfering beams is biprism experiment. Using sodium light λ =5890 A^0 , the central fringe shifts to a position originally occupied by the 12th bright fringe. Calculate the thickness of the sheet.(Ans 1.778×10^{-3} cm)

3. In a biprism experiment the slit and eyepiece are at 1cm and 100cm marking on the optical bench. The seperation between two images of coherent sources were 1.2mm and 0.3mm for two positions of a convexlens between the biprism and eyepiece.If 10 fringes were 9.72mm apart,then find the wavelength of light used.(Ans λ =5890 A^0)

4. Find the thicknessof a soap film (μ =1.33) which gives constructive second order interference of reflected red light of λ =7000 A^0 under normal incidence.(Ans $t = 3.95\times10^{-5}$ cm)

5.Light of wavelength 6000 A^0 falls normally on a thin wegde shaped film of refractive index.

1.4 forming fringes that are 1.5mm apart .Find the angle of the wedge.(Ans $\alpha = 1.428\times10^{-4}$)

6. In a Newton's rings experiment the diameter of 5th ring was 0.336cm and the diameter of the 15th ring was 0.59cm .Find the radius of curvature of the plano convex lens if the wavelength of light used is 5890 A^0 (Ans R=99.82)

7. In Micheison interferometer 200 fringes cross the field of view when the movable mirror is displaced through 0.0589mm. Calculate the wavelength of the monocromatic light used
(λ =5890 A^0)

8. A transparent film of glass of refractive index 1.5 is introduced in the path of one of the interfering beams of Michelson's interferometer which is illuminated with light of wavelength 4800 A^0 . This causes 500 dark fringes to sweep across the field. Determine the thickness of the film(Ans t =0.024cm)

10.22 REFERENCE BOOKS :

1. Unified physics vol-2	Dr. S.L.Gupta Sanjeev Gupta Jai Prakash Nath & Co Meerut
2. B.Sc Optics	Telugu Academy
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4. Optics	Ghatak
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Unit IV

LESSON-11

DIFFRACTION

OBJECTIVES:

We will able to

1. Define diffraction and understand the phenomenon of bending of light waves around obstacles and formation of diffraction bands.
2. Theory behind the resultant of n-wavefronts.
3. Understand diffraction at single slit, experimental set up,theory behind the diffraction bands formed and intensity distribution in the fringes.
4. Understand Fraunhofer diffraction at circular aperture and applying to find the resolving power of the optical instruments.
5. Fraunhofer diffraction at double slit and its extension to N-slits.
6. Learn Fourier transform and its application to diffraction at single slit.
7. Construction and usage of diffraction grating and to determine the wave length of 'light with grating.
8. Explain Fresnel class of diffraction, half period zones and zone plates.
9. Concept of diffraction at st.edge using Fresnel class and diffraction of plane waves

STRUCTURE OF THE LESSON

11.1:INTRODUCTION:

Bending of the light waves around obstacles and small apertures is called as "diffraction". Diffraction effects are observable only when the size of the obstacle or aperture is of the order of wavelength of the light. Thus when light falls on obstacle or small aperture whose size is comparable with wavelength of light, there is a departure from straight line propagation, the light bends round the corners of the obstacles or apertures and enters in the geometrical shadow. This bending of light is called diffraction.

When an obstacle comes in the way of light, its geometrical shadow is cast on the screen. But we can observe a small amount of light intensity in shadow region also. This is due to diffraction. We can also observe bright and dark bands of varying widths and varying intensities in the exposed region (The region not stopped by the obstacle) due to interference of secondary wavelets originating from various points of wavefront which are not blocked by the obstacle. Diffraction effects are observed only when a portion of wavefront is cut off by some obstacle.

Diffraction phenomena can be classified in two types depending on the position of the source and the screen

1.Fraunhofer diffraction:

In this class of diffraction source and screen are effectively at infinity. In this case the incident wavefront on the aperture or obstacle is plane wavefront.

2.Fresnal's diffraction:

In this class, source and screen are placed at finte distances from the aperture or obstacle having sharp edges. The incident wavefront is either spherical or cylindrical.

11.2. Differences between Interference and Diffraction :

Interference	Diffraction
1.The interaction takes places between two separate wavefronts originating from the two coherent sources.	1.The interaction takes place between the secondary wavelets originating from different points of exposed parts of the same wavefront.
2.Regions of minimum intensity are perfectly dark.	2. Minimum intensity regions are not perfectly dark
3.Width of the fringes are generally equal.	3.Width of the fringes are never equal.
4.The maxima of interference pattern are of same intensity.	4.The diffraction maxima are of varying intensity

11.3 RESULTANT OF N-SIMPLE HARMONIC WAVES:-

Generally the diffraction phenomena or bands are observed when the secondary wavelets originating from different points of primary wavefront obstructed by an obstacle or aperture interfere or superimpose or overlap with each other. Hence we will try to find the resultant of n-wavefronts or n-simple harmonic waves. Let 'a' be the amplitude of the wave, 'd' be the phase

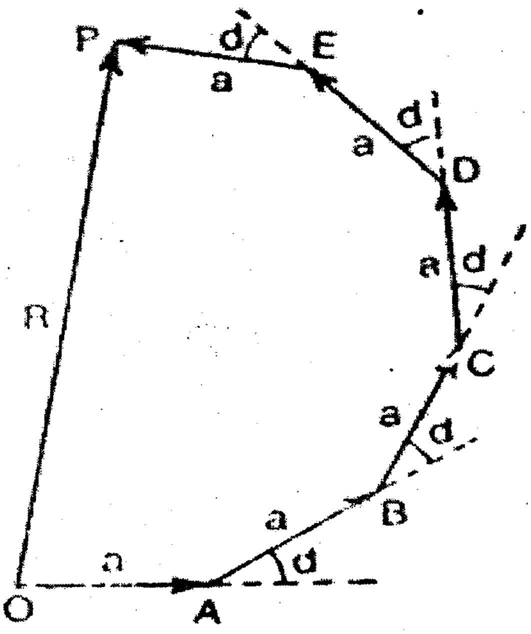


Fig:(11.1)

The resultant of these 'n' waves is given by wavefront wavefront OP,with resultant amplitude R and resultant phase θ .Using principle of superposition , we obtain

$$R = a\frac{\sin\alpha}{\left(\alpha/n\right)} = na\frac{\sin\alpha}{\alpha} = A\frac{\sin\alpha}{\alpha} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_(11.1)}$$

where na=A.

$$\theta = \frac{(n-1)d}{2} \approx \frac{nd}{2} = \alpha \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_(11.2)}$$

11.4 . Fraunhofer Diffraction due to a single slit -Normal Incidence :-

AB is a narrow slit of width 'e' perpendicular to plane of the paper .Let a plane wavefront ww' be incident on the slit . λ is the wavelength of the incident light .Then according to the Huygens-Fresnels theory , each point on the wavefront passing through slit act as secondary source and emit secondary wavelets in all directions .All these secondary wavelets interfere with other. The diffracted light is focussed by means of convex lens on a screen placed in the focal plane of the lens (Fig. 11.2). All wavefronts travelling norm the slit i.e. travelling along OP_0 are brought to focus at P_0 by the lens forming bright central image.

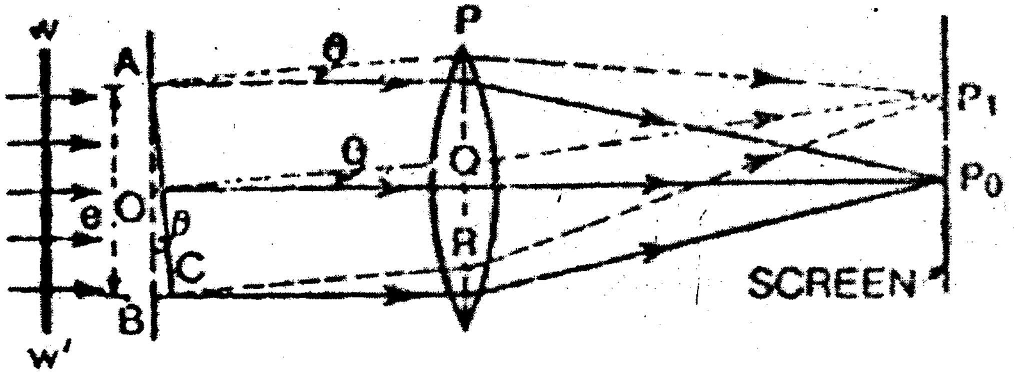


Fig:(11.2)

Secondary wavelets travelling along direction 'θ' with incident wavefront are brought to focus at P_1 on the screen .The point P_1 is of minimum or maximum intensity depending on the path difference between the secondary wavefronts.

THEORY :-

The path difference between secondary wavelets from A and B in direction

$$\theta = BC = AB\sin\theta = e\sin\theta$$

$$\text{Phase difference} = \frac{2\pi}{\lambda} e\sin\theta$$

Let the slit be divided into n- equal parts and amplitud eof the wave from each part is a The phase difference between consecutive wavefronts in these n- wavefronts =1/n(Total Phase)

$$= \frac{1}{n}\left(\frac{2\pi}{\lambda} e\sin\theta\right) = d(\text{say})$$

The resultant amplitude of 'n' wavefronts 'R' is given by

$$R = \frac{a\sin(nd/2)}{\sin(d/2)} = \frac{a\sin(\pi e\sin\theta/\lambda)}{\sin(\pi e\sin\theta/n\lambda)} \quad \{\text{From equation (11.1)}\}$$

$$=\frac{a\sin(\alpha)}{\sin(\alpha/n)} \quad \text{where} \quad \alpha=\frac{\pi e\sin\theta}{\lambda}$$

$$\Rightarrow R\approx\frac{a\sin\alpha}{\alpha/n}=\frac{na\sin\alpha}{\alpha}=\frac{A\sin\alpha}{\alpha}$$

Intensity at point $P_1=I=R^2=\dfrac{A^2\sin^2\alpha}{\alpha^2}$

'R' when expanded in terms of Sin α,

$$R=\frac{A}{\alpha}\left[\alpha-\frac{\alpha^3}{3!}+\frac{\alpha^5}{5!}-\frac{\alpha^7}{7!}+\dots\dots\dots\right]$$

$$=A\left[1-\frac{\alpha^2}{3!}+\frac{\alpha^4}{5!}-\frac{\alpha^6}{7!}+\dots\dots\dots\right].$$

when negative terms vanish ,R will be maximum.

Principal maximum:

When $\alpha=0$, we obtain principal maximum

$$\alpha=\frac{\pi e\sin\theta}{\lambda}=0 \quad \Rightarrow\sin\theta=0 \quad \Rightarrow\theta=0 \qquad (11.3)$$

Then $I_{max}=A^2$

Hence Central maximum or principal maximum is formed when secondary wavelets travel normal to the slit.

Minimum intensity positions:

Minimum intensity will be observed when $\sin\alpha=0\Rightarrow\alpha=\pm\pi,\pm2\pi,\pm3\pi,\dots\dots=\pm m\pi$

$$\Rightarrow\frac{\pi e\sin\theta}{\lambda}=\pm m\pi$$

$$\Rightarrow e\sin\theta=\pm m\lambda \qquad (4)(11.4)$$ where m=1,2,3,.........etc,,

Secondary maxima:-

In addition to principal maxima , there are secondary maxima between minima equally spaced. These are obtained by differentiating intensity 'I' according to calculus

$$\frac{dI}{d\alpha}=\frac{d}{d\alpha}\left[A\left(\frac{\sin\alpha}{\alpha}\right)^2\right]=0$$

$$\Rightarrow A^2\frac{2\sin\alpha}{\alpha}\frac{(\alpha\cos\alpha-\sin\alpha)}{\alpha^2}=0 \qquad (4) \quad (11.5)$$

$\therefore \sin\alpha = 0$ or $\therefore \alpha\cos\alpha - \sin\alpha = 0$

But $\sin\alpha = 0$ gives values of 'α' for minima

$\therefore$ Position of secondary maxima are given by

$$\alpha\cos\alpha - \sin\alpha = 0$$

$$\Rightarrow \quad \alpha = \text{Tan}\alpha$$

Consider the equations as two graphs, $y = \alpha$ and $y = \text{Tan}\alpha$ .Draw both the graphs and they look like the figure 11.3shown .

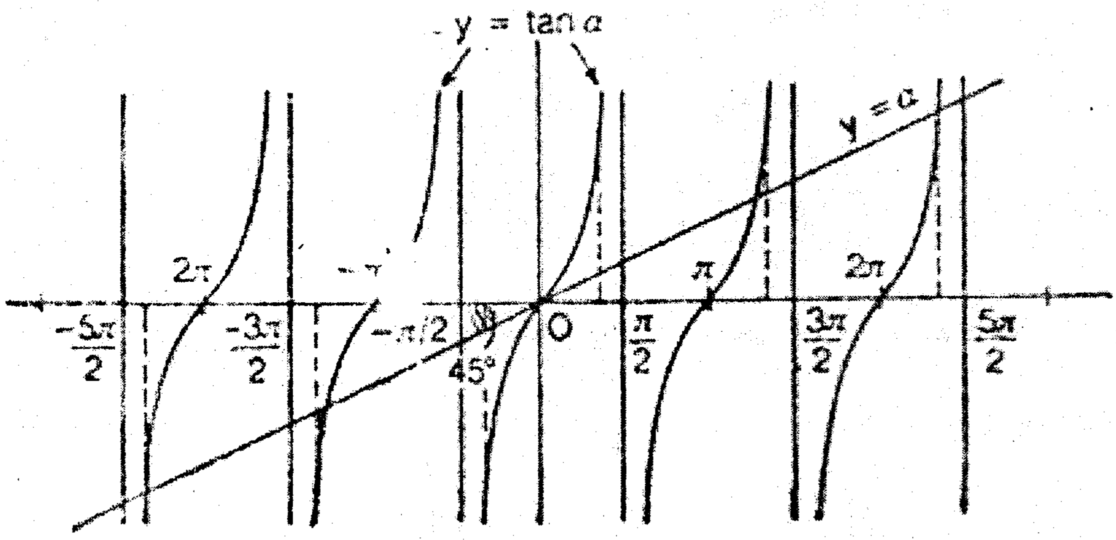


Fig 11.3

The points where $y = \alpha$ and $y = \text{Tan}\alpha$ intersect each other are points of secondary maxima.These are given by $\alpha = 0, \quad \pm 1.43\pi, \quad \pm 2.462\pi, \quad \pm 3.471\pi$ etc.,

Intensity of principal maxima $I_0 = A^2$

Intensity of first secondary maxima $I_1 = \frac{A^2}{22}$

Intensity of second secondary maxima $I_2 = \frac{A^2}{62}$ etc.

So, the intensity of maxima increase drastically.Due to $+1.430\pi$ [+or-signs], secondary maxima are situated on both sides of principal maxima .This intensity distribution is shown by following graph with on X-axis and intensity on Y-axis. (see Fig. 11.4)

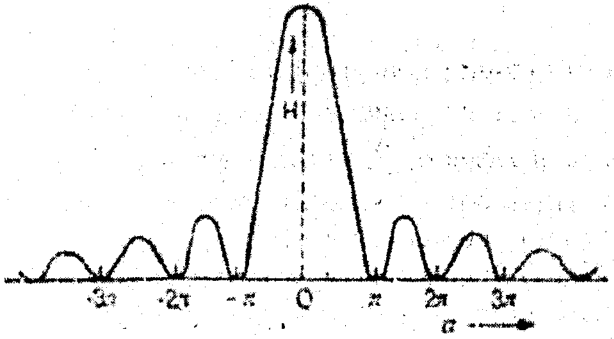


Fig. 11.4

The diffraction pattern consists of a central principal maximum in the direction of incident rays .The subsidiary maximum of decreasing intensity on either sides of it at positions

$\alpha = \frac{\pm 3\pi}{2}, \frac{\pm 5\pi}{2},$ etc.,. Between subsidiary maxima are secondary minima at

$\alpha = \pm\pi, \pm 2\pi, \pm 3\pi$, etc.,Secondary maxima are not exactly between secondary minima, but slightly displaced towards the centre of the pattern.

11.5 Single Slit Oblique Incidence :-

Let the wavefront WW^1 be incident on the slit at an angle 'i' i.e.oblique incidence. Now we consider the diffraction effects at angle 'θ' .Now again divide the slit into n-parts of equal length .Width of each part is Δx (Fig. 11.5)

The path difference between wavelets from A and B $= e(\sin i + \sin\theta)$

The total phase difference between wavelets from A and B $= 2\alpha$

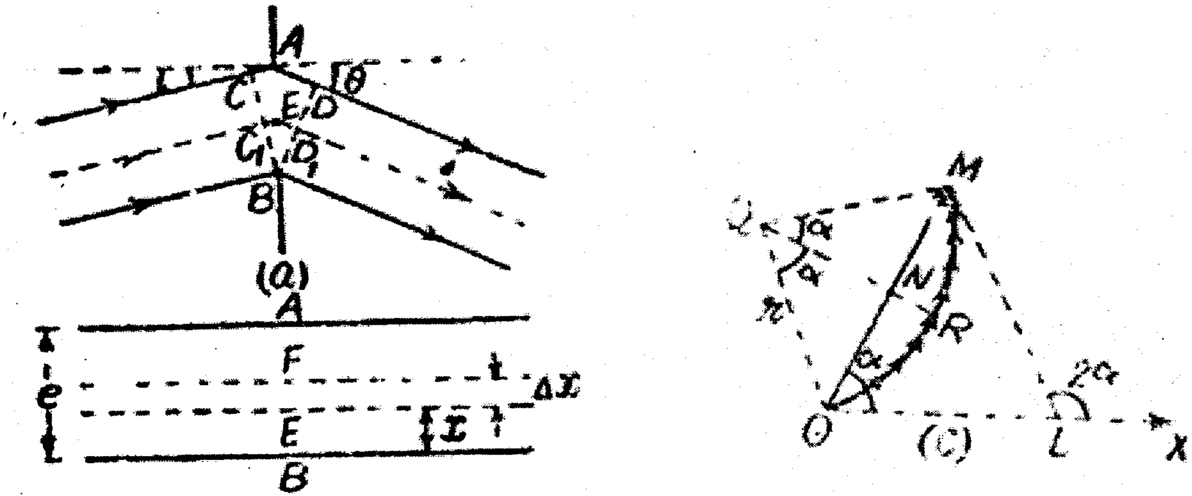


Fig. 11.5

$$= \frac{2\pi e}{\lambda}(\sin i + \sin\theta) \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_(11.6)}$$

The resultant amplitude is given by chord OM

R=chord OM=2ON=2r Sin α

Length of the arc $\propto$ Width of the slit

Length of the arc OM=ke=A

$$2\alpha = \frac{\text{ArcOM}}{\text{Radius}} = \frac{A}{r} \Rightarrow 2r = \frac{A}{\alpha} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_(11.7)}$$

From (6) and (7)

chord OM=A Sin α /α and

$$R = \frac{A\sin\alpha}{\alpha}$$

$$\text{Intensity } I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_(11.8)}$$

Minimum intensity when $\sin\alpha = 0 \Rightarrow \alpha = \pm\pi, \pm 2\pi, ...$

$$\Rightarrow \frac{\pi e}{\lambda}(\sin i + \sin\theta) = \pm m\pi, m = 1,2,3........$$

Since ‘i’ is constant , depends on ‘m’ only

$$\sin i + \sin \theta_m = \pm \frac{m\lambda}{e}$$

11.6 Fraunhofer Diffraction at a circular Aperture:-

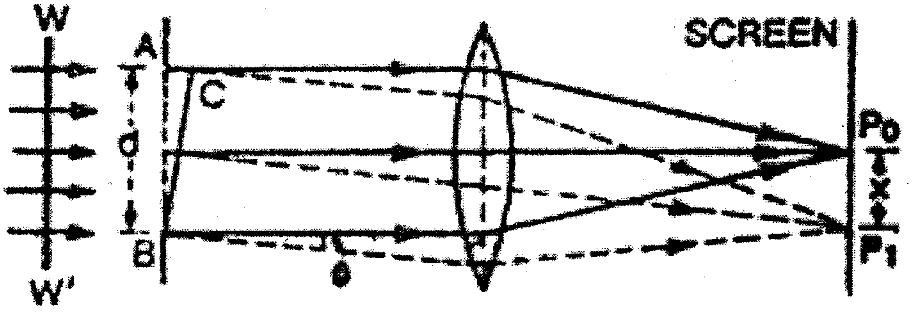


Fig 11.6

Let AB be a circular aperture of diameter ‘d’ .A plane wavefront ww is incident on circular aperture normally .The diffracted beam by the circular aperture is focussed on screen by lens .All the wavefronts(secondary) eliminating from circular aperture , normal to aperture, travel equal distances and reach P_o.So these wave fronts with zero path difference reinforce each other fprming central maximum at P_o.When we consider secondary wavefronts travellig along an angleto the slit, they get focussed at P_1 (see Fig. 11.6) .Depending on the phase difference between wanefronts from A to B, P_1 will be maxima or minima.

Let P_o, P_1 =x,the path difference between extreme waves from point A and B is

$$AC = AB\sin\theta = d\sin\theta$$

Point P_1 will be minimum , when path difference is multiple of λ and maximma when it is odd multiple of $\frac{\lambda}{2}$

$$\therefore \quad d\sin\theta = m\lambda \text{ manima} \_\_\_\_\_\_\_\_\_\_\_\_ (9)$$

$$d\sin\theta = (2m+1)\frac{\lambda}{2} \text{ maxima} \_\_\_\_\_\_\_\_\_\_\_\_ (10)$$

m=0 corresponds to central maxima at Po

If point P_1 is of minimum intensity then all points which are at a distance 'x' from Po are also minima .This diffraction may be considered by rotating intensity distribution graph of single slit about the central axis passing through Po about which the circular aperture is perfectly symmetrical .The point Po traces out a circular ring of uniform illumination .Thus the diffraction pattern consists of a central bright disc called Airy's disc surrounded by alternate dark and bright concentric rings called the Airy's rings .The intensity of dark rings is zero and that of the bright rings decreases gradually outwards from.The angular radius θ of the Airy's disc i.e. angular seperation between the centre of the bright disc and the first dark ring is given by

$$\theta = \frac{1.22\lambda}{d}$$

λ-wavelength of the incident light,

d-diameter of the circular aperture.

If the collecting lens is very near to the circular aperture and the screen is at very large distance 'x' from the lens then

$$\text{Sin}\,\theta = \theta = \frac{x}{f} \quad \text{-----(11.11)}$$

f-focal length of the lens.

For secondary minimum, $d\text{Sin}\,\theta = 1.\lambda$

$$\Rightarrow \quad \sin\theta = \theta = \frac{\lambda}{d} \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_(11.12)}$$

From (11.11) and (11.12),

$$\frac{x}{f} = \frac{\lambda}{d} \Rightarrow x = \frac{f\lambda}{d} \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_(11.13)}$$

x is the radius of the Airy's disc .

The exact value of 'x' is given by

$$x = \frac{1.22\lambda}{d} \text{\_\_\_\_\_\_\_\_\_\_(11.14)}$$

Thus if the diameter of the aperture is large, the radius of the central disc is small.

The diffraction pattern due to circular aperture is used in determining the resolving powers of telescope and microscopes.

11.7 Limit of resolution or Resolving power :-

When the two objects are very near to each other or when the two objects are far away from the eye ,we may not be able to see them as seperate objects. Similarly closely spaced spectral lines in spectra may not be distinguishable by naked eye. In such situations telescopes or microscopes are used to view those objects seperately.But these optical instruments have also limitations.The two objects will be resolved or shown seperately by the optical instrument,only when the central maxima of two diffraction pattern produced by two objects are distinguishable.

This is because the image of a point object or line produced by optical instrument is not a point or line, but a diffraction pattern with a bright central maxima and secondary maxima of rapidly decreasing intensity.

"The ability of the instrument to produce the separate diffraction patterns of closely placed objects or spectral lines is called as 'Resolving power' "

The limit of resolution of an optical instrument is the smallest angle subtended at its objective by two point objects which can just be distinguished as seperate.

Resolving Power=1/limit of resolution

According to Rayleigh's criterion, two sources are resolvable by an optical instrument when the central maximum in the diffraction pattern of one falls over the first minimum in the diffraction of the other and vice-versa.

$$\text{Limit of resolution of telescope} = \frac{1.22\lambda}{d} \qquad (11.15)$$

d-diameter of the objective of telescope.

11.8 Fraunhofer Diffraction by a double slit :-

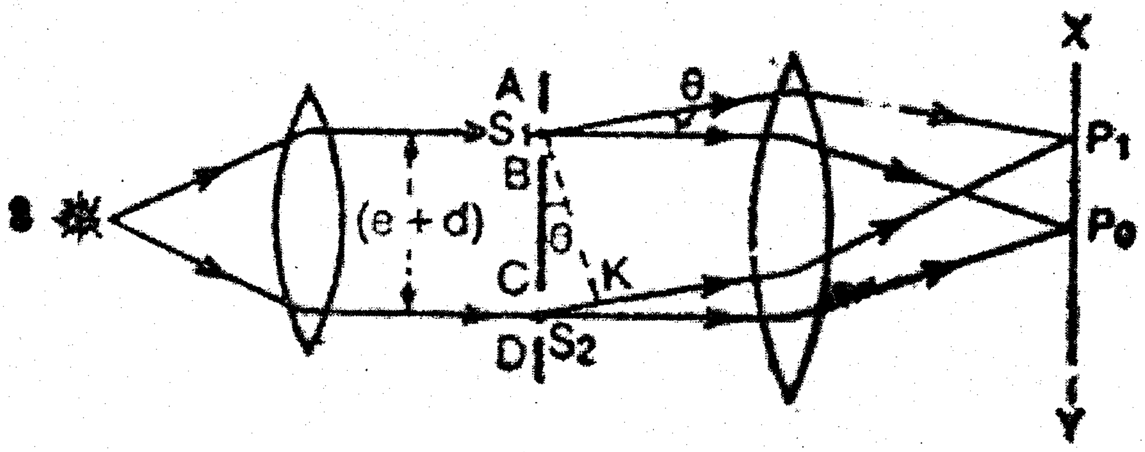


Fig 11.7

Let AB and CD be two parallel slits of equal width e and seperated by a distance 'd'.The distance between central point of two slits is (e+d). When plane wavefront from source 'S' through first lens is incident on the double slit ,each point in both slits act as source of second ary wavelets .Secondary wavelets from the slit travel forward in all directions.All the secondary waves from the two slits travelling normal to form central maxima at Po.Now the fringe pattern formed on screen XY is the superposition of diffraction pattern produced by a single slit ,with the interference pattern produced by two slits (se fig. 11.7).

That is to say, each slit produces its own diffraction pattern and secondary waves from two slits interferes with other forming interference fringes. So the resultant pattern is superposition of interference pattern on single slit diffraction pattern.

The secondary waves travelling from two slits at an angle 'θ' to the normal meet at point P_1 on the screen. Depending on the total phase difference between wavelets from the two slits, P_1 will be maxima or minima.

The path difference between wavelets travelling from middle points of the slits

Theory :- $=(e+d)\text{Sin}\,\theta$

We can consider each slit giving two waves of equal amplitude

$$R = \frac{A\text{Sin}\alpha}{\alpha} \qquad \text{Where, } \alpha = \frac{\pi e \text{Sin}\theta}{\lambda} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_(11.16)}$$

The phase difference between these two waves starting from centre of the slits

$$\text{Path difference} = S_2K = (e+d)\sin\theta$$

$$\text{Phase difference} = \frac{2\pi}{\lambda} = (e+d)\sin\theta \qquad \text{------------(11.17)}$$

The resultant amplitude 'R' at P_1 is given by

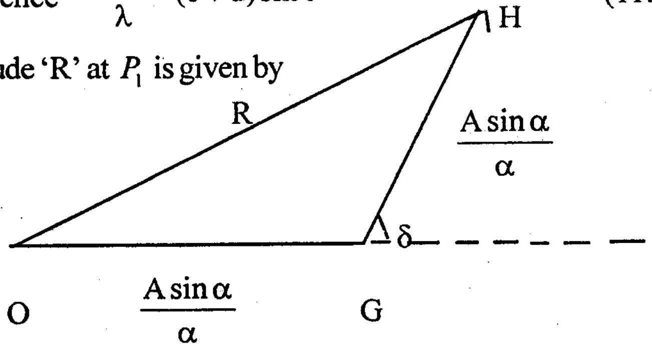


Fig 11.8

$$(OH)^2 = R^2 = OG^2 + GH^2 + 2OG.GH\text{COS}\delta$$

$$= \left(\frac{A\text{Sin}\alpha}{\alpha}\right)^2 [2 + 2\text{Cos}\delta]$$

$$= 4\left(\frac{A\text{Sin}\alpha}{\alpha}\right)^2 \text{Cos}^2\frac{\delta}{2}$$

$$R^2 = 4\left(\frac{A\text{Sin}\alpha}{\alpha}\right)^2 \text{Cos}^2\left[\pi/\lambda(e+d)\text{Sin}\theta\right]$$

$$\text{If} \quad \beta = \frac{\pi}{\lambda}(e+d)\text{Sin}\theta \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_}$$

$$I = R^2 = 4\frac{A^2\text{Sin}^2\alpha}{\alpha^2}\text{Cos}^2\beta \qquad (11.18)$$

Intensity distribution of double slit pattern :-

As evident from eq.(11.18) Intensity pattern is superposition of diffraction pattern by single slit $\frac{A^2 \mathrm{Sin}^2\alpha}{\alpha^2}$ and interference pattern by two parallel slit $\mathrm{Cos}^2\beta$

Diffraction maxima are obtained when $\mathrm{Sin}\alpha = 0$ and $\alpha \neq 0$

$\Rightarrow \alpha = \pm m\pi$, m=1,2,3-----,

$$\frac{\pi e \mathrm{Sin}\theta}{\lambda} = \pm m\pi, \Rightarrow e\mathrm{Sin}\theta = \pm m\lambda \text{ \_\_\_\_\_\_\_\_\_\_ (11.19)}$$

Central maxima is given when $\theta = 0$

Secondary maxima are given when $\alpha = \pm 3\pi/2, \pm 5\pi/2, \pm 7\pi/2 - - -$

In interference pattern, bright fringes or maxima are obtained when

$$\mathrm{Cos}^2\beta = 1, \Rightarrow \beta = \pm n\lambda$$

$$\Rightarrow \frac{\pi}{\lambda}(e+d)\mathrm{Sin}\theta = \pm n\lambda \ ,$$

$$\Rightarrow (e+d)\mathrm{Sin}\theta = \pm n\lambda \text{ ,n=0,1,2,3,------ \_\_\_\_\_\_\_\_\_\_ (11.20)}$$

Intensity distribution due to diffraction, interference and combined effect is shown in the following figure11.9.

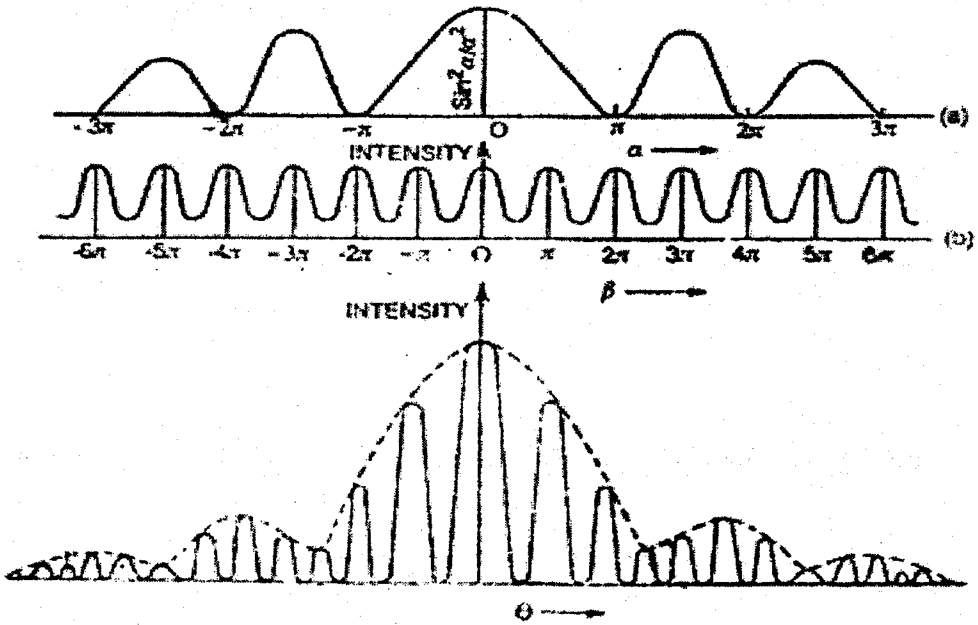


Fig.11.9

11.9 Oblique incidence on Double slit :-

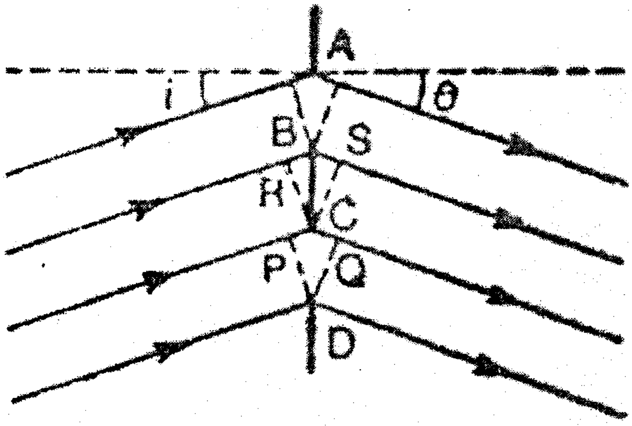


Fig .11.10

Let AB and CD be two slits separated by distance d and width e. Let i and θ be the angles of incidence and angle of diffraction respectively(Fig. 11.10).

The phase difference between the waves from C and D $= \delta_1 = \frac{2\pi}{\lambda} \times$ path difference,

$$\delta_1 = \frac{2\pi}{\lambda} e(\text{Sin}i + \text{Sin}\theta) \quad ,$$

The phase difference between the wavelets from B and C $\delta_2 = \frac{2\pi}{\lambda} d(\text{Sin}i + \text{Sin}\theta)$.

The amplitude of each resultant wave from each slit $= R = \frac{A\text{Sin}\alpha}{\alpha}$

The total phase difference between two waves $\delta = \delta_1 + \delta_2 = \frac{2\pi}{\lambda}(e+d)(\text{Sin}i + \text{Sin}\theta)$

The resultant amplitude of two wavefronts of equal amplitudes $\frac{A\text{Sin}\alpha}{\alpha}$. and phase difference δ is given by

$$R = 2\left(\frac{A\text{Sin}\alpha}{\alpha}\right)\text{Cos}\delta/2$$

$$\therefore \text{ Intensity } I = R^2 = 4\left(\frac{A\operatorname{Sin}\alpha}{\alpha}\right)^2 \operatorname{Cos}^2\delta/2$$

$$= 4\frac{A\operatorname{Sin}^2\alpha}{\alpha^2}\operatorname{Cos}^2\left[\pi/\lambda(e+d)(\operatorname{Sin}i+\operatorname{Sin}\theta)\right]$$

$$= 4\frac{A^2\operatorname{Sin}^2\alpha}{\alpha^2}\operatorname{Cos}^2\beta \qquad (11.21)$$

Where $\beta = \frac{\pi}{\lambda}(e+d)(\operatorname{Sin}i+\operatorname{Sin}\theta)$,

$$\alpha = \frac{\pi}{\lambda}e(\operatorname{Sin}i+\operatorname{Sin}\theta)$$

So, again the resultant diffraction pattern produced by a double slit in oblique incidence is combination of diffraction pattern of single slit and the interference pattern by double slit.

Condition for zero intensity is either $\left(\frac{\operatorname{Sin}\alpha}{\alpha}\right)^2 = 0$ or $\operatorname{Cos}^2\beta = 0$

$$\Rightarrow \alpha = \pi, 2\pi, 3\pi --- = m_1\pi, m_1 = \pm1, \pm2, \pm3 --- \_\_\_\_\_\_\_ (11.22)$$

and $\beta = (m_2 + 1/2)\pi, m_2 = 0, \pm1, \pm2, \pm3$ eq (23)

Let $\psi = \operatorname{Sin}i + \operatorname{Sin}\theta$

From eq (11.22), $\frac{\pi}{\lambda}e\psi = m_1\pi$

$$\Rightarrow \psi = \frac{m_1\lambda}{e}, m_1 = \pm1, \pm2, \pm3 \text{ ------}\_\_\_\_\_\_\_\_\_ (11.24)$$

From eq (24), $\frac{\pi}{\lambda}(e+d)\psi = (m_2 + 1/2)\pi$

$$\Rightarrow \psi = (m_2 + 1/2)\frac{\lambda}{e+d}, m_2 = 0, \pm1, \pm2 --- \_\_\_\_\_\_\_\_ (11.25)$$

So there is interference maxima under central diffraction maxima enclosed between the minima given by $m_1 = +1$ and $m = -1$

Width of central diffraction maximum $= \frac{\lambda}{e} - \left(\frac{-\lambda}{e}\right) = \frac{2\lambda}{e}$ ______ (11.26)

From eq (11.25),

The interval between two consecutive interference maxima $= \frac{\lambda}{(e+d)}$

$\therefore$ No of interference minima between two first diffraction minima is given by

$$\frac{2\lambda/e}{\lambda/(e+d)} = 2\left(\frac{e+d}{e}\right) \text{ ---------------}(11.27)$$

Interference maxima occur when

$$\cos^2\beta = 1 \Rightarrow \beta = \pm n\pi$$
$$\Rightarrow \frac{\pi}{\lambda}(e+d)\psi = \pm n\pi$$
$$\psi = \pm\frac{n\lambda}{(e+d)}, n = 0, \pm1, \pm2 --- \quad (11.28)$$

The Missing Order :-

Suppose a certain diffraction angle θ, simultaneously satisfies the conditions for Interference maxima and Diffraction minima ,that ordered interference maxima will be missing .This is known as missing order.

$$\psi = \sin i + \sin\theta = \pm\frac{n\lambda}{(e+d)} \text{ (Interference maxima)}$$

$$\psi = \sin i + \sin\theta = \pm\frac{m_1\lambda}{e} \text{ (Diffraction minima)}$$

Since both conditions are satisfied during missing order condition,

$$\frac{n\lambda}{e+d} = \frac{m_1\lambda}{e}$$

$$\Rightarrow \frac{n}{m_1} = \frac{e+d}{e} \text{ ------------------- } (11.29)$$

If d=e ,n=2m =2,4,6

Hence interference maxima of order 2nd, 4th, 6th etc will be missing.

If d=2e , $n = 3m_1 = 3, 6, 9$

The 3rd, 6th, 9th etc interference maxima will be missing.

11.10 PLANE DIFFRACTION GRATING :-(Normal Incidence)

Diffraction Grating :-

An arrangement consisting of large number of parallel slits of the same width and seperated by equal opaque spaces is known as diffraction grating.

Construction :-

Grating is constructed by ruling equidistant parallel lines on a transparent material like glass with a diamond point. The places drawn with diamond points act as opaque spaces and in between glass(transparent) parts act as transmission medium and act as slits.Hence these are known as 'Plane Transmission Grating'.When the spacing between the slits is of order of wavelength of light, then diffraction of waves takes place at each slit.

Theory :-

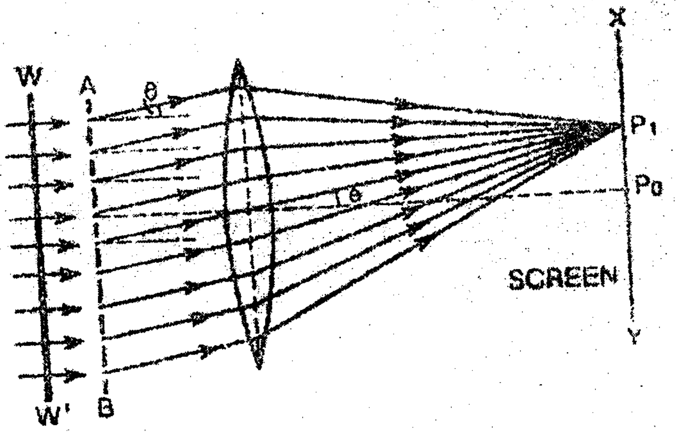


Fig. 11.11

Plane transmission grating is placed perpendicular to the plane of the paper. 'e' is the width of each slit and 'd' is the width of each opaque part (e+d) is known as grating element. XY is the screen placed perpendicular to the plane of the paper. Plane wavefront WW is incident on the grating. According to Huygen's wave theory each point in each slit acts as a secondary source and sends out secondary wavelets in all directions . The secondary wavelets travelling perpendicular to the slit interfere with each other constructively and form central maximum at Po on the screen .The secondary wavelets from all the slits travelling at an angle 'θ' to the normal come to focus at P_1 on the screen (see fig. 11.11). The waves reaching P_1 may be of different phases , so P_1 has maximum intensity or minimum intensity depending on overall phase.As a result dark and bright bands on both sides of central maximum are obtained .

The intensity at P_1 can be calculated by considering Fraunhofer diffraction at a single slit.The wavelets proceeding from all points in a slit along the direction θ are equivalent to a single wave of amplitude $\frac{A\sin\alpha}{\alpha}$ from the middle point of the slit where $\alpha = \frac{\pi e \sin\theta}{\lambda}$

Because there are N-slits in a diffraction grating , there will be N-diffracted waves from grating. The path difference between consecutive slits or waves is (e+d) Sin θ .Hence phase difference is $\frac{2\pi}{\lambda}(e+d)\sin\theta$ .This phase difference is constant and hence termed as 2β .

Hence the problem of determining the intensity in a direction 'θ' reduces to finding the resultant amplitude of N-vibrations each of amplitude $\frac{A\sin\alpha}{\alpha}$ and having common phase difference

$$\frac{2\pi}{\lambda}(e+d)\sin\theta = 2\beta$$

By method of vector addition of amplitudes, the resultant intensity in direction 'θ' is given by

$$I = R^2\left(\frac{A\sin\alpha}{\alpha}\right)\left(\frac{\sin N\beta}{\sin\beta}\right)^2 = I_0{}^2\left(\frac{A\sin\alpha}{\alpha}\right)\left(\frac{\sin^2 N\beta}{\sin^2\beta}\right) \quad \text{\_\_\_\_\_\_\_\_(11.29)}$$

$\left(\frac{A\text{Sin}\alpha}{\alpha}\right)^2$ gives the distribution of intensity due to a single slit while factor $\left(\frac{\text{Sin}^2 N\beta}{\text{Sin}^2\beta}\right)$ gives the distribution of intensity due to interference of waves from all the slits.

Intensity Distribution :-

Principal Maxima :-

Intensity will be maximum when $\text{Sin}\beta = 0$ but at the same time $\text{Sin}N\beta = 0$ then the quantity $\frac{\text{Sin}N\beta}{\text{Sin}\beta}$ becomes indeterminant .So by applying L.Hospital's rule , the intensity equation is differentiated seperately in numerator and denominator.

$$\text{Lim}_{\beta\to\pm n\pi}\frac{\text{Sin}N\beta}{\text{Sin}\beta} = \text{Lim}_{\beta\to\pm n\pi}\frac{d/d\beta(\text{Sin}N\beta)}{d/d\beta(\text{Sin}\beta)}$$

$$= \text{Lim}_{\beta\to\pm n\pi}\frac{N\text{Cos}N\beta}{\text{Cos}\beta} = \pm N$$

$$\therefore \text{Lt}_{\beta\to\pm n\pi}\left(\frac{\text{Sin}N\beta}{\text{Sin}\beta}\right)^2 = N^2$$

Resultant intensity of principal maxima= $\left(\frac{A\text{Sin}\alpha}{\alpha}\right)N^2$

These maxima are intense.The positions of principal maxima are obtained when

$$\beta = \pm n\pi$$

$$\Rightarrow \frac{\pi}{\lambda}(e+d)\text{Sin}\theta = \pm n\pi \quad \text{or}$$

$$(e+d)\text{Sin}\theta = \pm n\pi \text{ ,where n=0,1,2,3} \quad \text{\_\_\_\_\_\_\_\_(11.30)}$$

n=0 responds to zero order maxima.

Minima :-

Series of minima occur when $\mathrm{Sin}N\beta = 0$

but $\mathrm{Sin}\beta \neq 0$

$$\Rightarrow N\beta = \pm m\pi \Rightarrow N\frac{\pi}{\lambda}(e+d)\mathrm{Sin}\theta = \pm m\pi$$

$$N(e+d)\mathrm{Sin}\theta = \pm m\lambda \quad \underline{\hspace{6cm}} \quad (11.31)$$

where m has all values except m=0,N,2N,3N --------nN, because for these values the $\mathrm{Sin}\,\beta$ becomes zero and we get principal maxima .Then m=1,2,3,-------(N-1).Hence there are adjacent principal maxima.

Secondary maxima :-

Since there are (N-1) minima between two adjacent principal maxima, there must be (N-2) secondary maxima between two principal maxima.To find the position of secondary maxima , differentiate eq.(11.29) with respect to β and equate it to zero.

$$\frac{dI}{d\beta} = 0 \Rightarrow \left(\frac{A\sin\alpha}{\alpha}\right)^2 .2\left(\frac{\sin N\beta}{\sin\beta}\right) \times \left[\frac{N\cos N\beta \quad \sin\beta - \sin N\beta \quad \cos\beta}{\sin^2\beta}\right] = 0$$

$$\Rightarrow \quad N\cos N\beta \quad \sin\beta - \sin N\beta \quad \cos\beta = 0$$

$$\Rightarrow \quad N\tan\beta = \tan N\beta \quad \underline{\hspace{6cm}} (11.32)$$

Roots of this equation give the positions of secondary maxima.As N increases, the intensity of secondary maxima decreases .The intensity distribution with diffraction grating is shown in the figure 11.12.

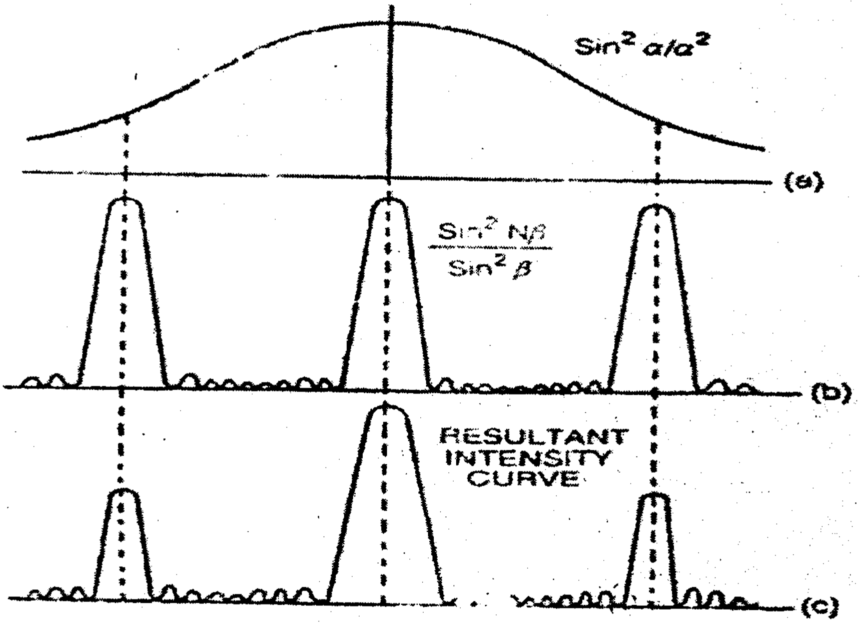


Fig .11.12

11.11 Multiple Spectra with grating :-

Condition for the formation of principal maxima in direction 'θ' is given by

$$(e+d)\sin\theta = \pm n\lambda$$

n-order of the maxima,
e+d-grating element,
λ-wavelength of incident light

1. From above equation, it is evident that for a particular wavelength (monochromatic source), we get different orders(n) of principal maxima for different diffraction angles.

2. For a particular order 'n', angle of diffraction θ changes with colour or wavelength. So when the source is white light, for n=0 and $\theta=0$ i.e. zero order wavelengths of all colours have same diffraction angle $\theta=0$. Hence we obtain central maxima or zero-order maxima as white coloured band. For other orders i.e. n=1 etc, θ varies with wavelength λ. Larger the wavelength, larger is the diffraction angle θ. So red colour diffracts more than violet colour. Similarly for n=2, we get second order maxima of different colours. Hence we obtain multiple spectra on either side of central white band with white light source in a grating. As no. of slits 'N' in grating becomes more, spectral lines will be sharper.

Spectra of different orders are situated symmetrically on both sides of zero-order image. Spectral lines are almost straight and sharp. Spectral colours are in the order from violet to red. Spectral lines are more dispersed, as we go to higher orders. Most of the incident light intensity is concentrated in zero-order.

11.12 Maximum no of orders available with a Grating :-

Condition for principal maxima is

$$(e+d)\sin\theta = n\lambda \Rightarrow \quad n = \frac{(e+d)\sin\theta}{\lambda}$$

Maximum angle of diffraction $\theta = 90^0$.

Hence maximum possible order

$$n_{(max)} = \frac{(e+d)\sin 90^0}{\lambda} = \frac{e+d}{\lambda} \quad \text{\_\_\_\_ eq(33)}$$

Ex : Suppose grating element $(e+d) < 2\lambda$, then $n_{max} \le \frac{2\lambda}{\lambda} < 2$

Hence we get only first order spectrum.

11.13 Absent Spectra with Diffraction Grating :-

Sometimes certain orders of grating spectra are found to be absent. This happens when the path difference between the waves from extreme ends of the slit in grating is equal to an integral multiple of λ

Condition for Principal maxima $(e+d)\sin\theta = n\lambda$ ______________eq(11.34)

Condition for minima in interference pattern with a single slit is given by

$(e+d)\sin\theta = m\lambda$ ______________eq(11.35)

m=1, 2, 3 ,............

If both the conditions (11.34) and (11.35) satisfied at same time for a certain angle θ, then

$$\frac{(e+d)\sin\theta}{e\sin\theta}=\frac{n}{m}\Rightarrow\frac{e+d}{e}=\frac{n}{m} \qquad (11.36)$$

then the order 'n' maxima (principal) will be missing from grating spectra.Eq (11.36) is called condition of absent spectra.

Ex : When e=d , n=2m

$$\frac{e+d}{e}=\frac{n}{m}\Rightarrow\frac{e+d}{e}=\frac{n}{m}$$

$$\Rightarrow n=2m \quad \text{when } m=1, n=2.$$

Hence 2^{nd} order spectra will be absent.

11.14 Determination of wavelength using Diffraction Grating :-

The diffraction grating is used to determine the wavelength of light in laboratories using spectrometers .The principal maxima of grating are obtained when

$$(e+d)\sin\theta=n\lambda$$

where (e+d) is grating element ,
n-order of the principal maxima and
θ is the diffraction angle for wavelength λ .

If N=no of lines /inch ruled on grating then grating element is given by

$$(e+d)=\left(\frac{2.54}{N}\right)\text{cm}$$

In particular order 'n' [Example first order],by measuring the diffraction angle 'θ' ,wavelength of monochromatic source or wavelengths of polychromatic sources can be determined.

Adjustments :-

1. The spectrometer is adjusted for parallel rays.
2. The grating is adjusted for normal incidence .For this purpose, the slit of the collimator is illuminated with given source of light.The position of telescope is adjusted such that the cross-wires of telescope eyepiece coincide with slit image.In this position collimator and telescope are in same line . The position of telescope is noted on circular scale .From this position , it is moved through 90° , so that the axis of collimator and telescope axis are perpendicular to each other. It is clamped in that position.The transmission grating is mounted at the centre of prism table such that the grating surface is perpendicular to the prism table.The prism table is now rotated such that the reflected image of the slit at grating appears at the cross-wire of the telescope.The prism table is fixed in this position. Then the vernier table of spectrometer is turned through 45° from that position,such that the grating comes exactly faces collimator .Then telescope is released and brought to original position.This adjustment is known as Normal Incidence , since with above adjustments the rays from collimator incident normally on grating .

The telescope is now turned to get the first order spectrum. The cross-wire coincide with the line whose wavelength is to be determined(say violet).The readings of the two verniers of telescope position are recorded. The telescope is turned to same colour of same order on the other side of central white band . Again the position of telescope is read on two verniers .The difference between readings of the same vernier on two sides gives the value of 2θ i.e., twice the angle of diffraction . By substituting the value of 'θ' in above equation , we can calculate the wavelength . The process is repeated for second order to obtain wavelength of same colour . Following same procedure, we can find the other wavelengths also.

11.15 Dispersive Power of grating :-

"Dispersive power of grating is defined as the rate of variation of angle of diffraction with wavelength."

$$\text{Dispersive power} = \frac{d\theta}{d\lambda}$$

Condition for principal maxima is $(e+d)\sin\theta = n\lambda$

Differentiating the above equation $(e+d)\cos\theta d\theta = nd\lambda$

$$\Rightarrow \quad \frac{d\theta}{d\lambda} = \frac{n}{(e+d)\cos\theta} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_}(11.37)$$

1. Dispersive power is directly proportional to order of the spectrum .Higher the order, larger is the dispersion.
2. It is inversely proportional to grating element . Smaller is the grating element, spreadly the spectrum wider.
3. It is inversely proportional to Cosθ i.e. larger the θ value,smaller is the $\cos\theta$ value and higher the dispersive power.

11.16 Resolving power of grating :-

Diffraction grating has the ability to seperate closely spaced spectral lines."The resolving power of diffraction grating is defined as the capacity to form seperate diffraction maxima of two wavelengths which are close to each other".

$$\text{Resolving power} = \frac{\lambda}{d\lambda}$$

Let λ_1 and λ_2 be the two closely spaced spectral lines and their angles of diffraction θ_n and $\theta_n + d\theta$ .According to Rayleigh's criterion,minimum of λ should overlap on the maxima of $\lambda + d\lambda$.

Condition for maxima with wavelength λ is given by

$$N(e+d)\sin\vartheta = m\lambda \quad [m \neq 0, N, 2N, \ldots\ldots\ldots, nN]$$

The first minima adjacent to n^{th} principal maxima is given by

$$m = nN + 1 \quad \text{for } \lambda$$

$$\therefore N(e+d)\sin(\theta_n+d\theta)=(nN+1)\lambda \quad \text{\_\_\_\_\_\_\_\_\_\_(11.38)}$$

Condition for maxima with wavelength $\lambda+d\lambda$ is

$$(e+d)\sin(\theta_n+d\theta)=n(\lambda+d\lambda)$$

Multiplying above equation with N,

$$N(e+d)\sin(\theta_n+d\theta)=nN(\lambda+d\lambda) \quad \text{\_\_\_\_\_\_\_\_(11.39)}$$

From equation (11.38) and (11.39),

$$(nN+1)\lambda = nN(\lambda+d\lambda)$$

$$\lambda = nNd\lambda$$

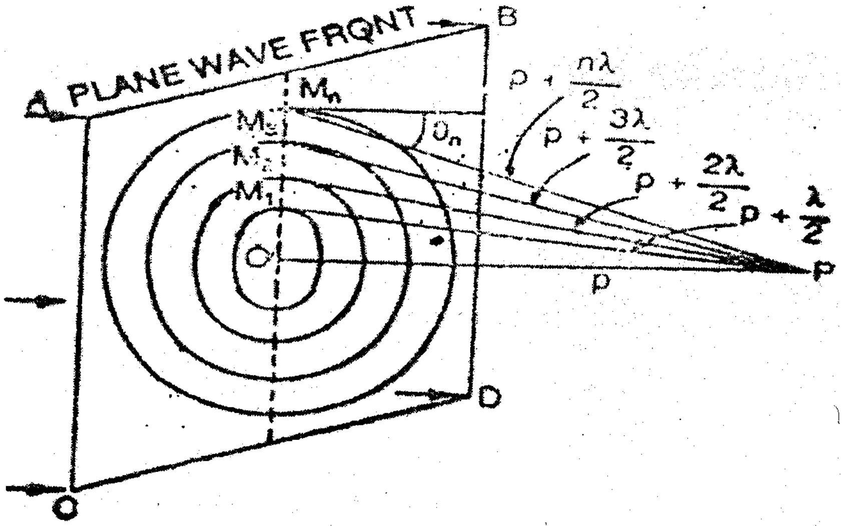
$$\text{(or)} \quad \frac{\lambda}{d\lambda} = nN = \text{Resolving Power}$$

So larger the number of lines on grating , larger is its resolving power,and resolving power is more in higher orders.

11.17 Huygen's-Fresnel theory of light propagation :-

According wavetheory, every point in light source sends out waves in all directions.These waves vibrate the particles in either medium.The locus of vibrations of either particles is called as wavefront.Now each point on wavefront acts as a secondary source and sends out wavelets.Fresnel proposed these wavelets(secondary) sent by each print on wave front interfere with each other to produce intensity at any point on its way.To determine the intensity at a point due to secondary wavelets . Fresnel introduced the concept of "Half-period zones".

11.18 FRESNEL'S HALF PERIOD ZONES :-



21)

Fig 11.13

As shown in the fig 11.13, ABCD is a wavefront moving in forward direction. We want to calculate the intensity at point 'P' due to interference of secondary wavelets. Although it is difficult to find the intensity, Fresnel solved the problem with half-period zones. Draw a normal from P to the wavefront-OP . length of OP=p. Now P as the centre,draw spheres of radii p, $p+\lambda/2$, $p+2\lambda/2$,---- ------------- $p+n\lambda/2$. These spheres cut the wavefront ABCD in circles OM_1, OM_2 ------------------- OM_n. The area in circle OM_1 is called first half-period zone, the area between OM_1 and OM_2 is second half-period zone and so on. It is assumed a resultant wave starts from each zone.

$M_1P = OP + \lambda/2 = p + \lambda/2$ .So the ray reaching P from 'O' differs in phase from M_1 by π or path difference of $\lambda/2$. The phase difference between the points lying between O and M_1 varies from 0 to π . Hence the overall phase or mean phase of secondary wavelets from first zone = $=(0+\pi)/2 = \pi/2$. Similarly the phase difference between points M_1 and M_2 is π and mean phase of second half period zone $=((\pi+2\pi)/2 = 3\pi/2$ .Mean phase of third half period zone is $5\pi/2$ etc. The successive half period zones differ by phase π or time period $T/2$. Hence they are called as 'Half period zones'.

The amplitude of the disturbance at P due to the wave from the zone depends on area of the zone, average distance from the zone and obliquity of the zone.

Area of first half period zone $= \pi(OM_1^2) = \pi$(M1p2-Op2) $= \pi[(p+\lambda/2)^2 - p^2] = \pi[p\lambda + \lambda^2/4] \approx \pi p\lambda$

Radius of first half period zone $= \sqrt{p\lambda}$

Similarly the radius of second half period zone $= \sqrt{2p\lambda}$

Area of second half period zone $= \pi[OM_2^2 - OM_1^2]$

$$= \pi[2p\lambda - p\lambda]$$

$$= \pi p\lambda$$

Radius of nth half period zone $= \sqrt{np\lambda}$

Average distance of n th zone from P

$$= \frac{(p + n\lambda/2) + (p + (n-1)\lambda/2)}{2}$$

$$= p + (2n-1)\lambda/4 \text{ ----------------(11.41)}$$

Obliquity factor θ_n is the angle between normal to the zone and line joining the zone to P

Amplitude of nth zone

$$\propto \frac{\pi\left[p+(2n-1)\lambda/4\right]\lambda}{p+(2n-1)\lambda/4} . f(\theta_n) \propto \pi\lambda f(\theta_n) \text{ ----------------(11.42)}$$

As order of zone increases $f(\theta_n)$ decreases, the amplitude of wave from that zone at P decreases. Resultant amplitude at P due to all zones=R

$$R = R_1 - R_2 + R_3 - - - - - - - - + R_n \quad \text{if n is odd.}$$

$$R = R_1 - R_2 + R_3 - - - - - - - - - R_n \quad \text{if n is even.}$$

The amplitudes due to zones go on decreasing i.e. $R_1 > R_2 > R_3 - - - - - - - > R_n$

when 'n' is odd $R = \frac{R_1}{2} + \frac{R_n}{2}$ since R2 = (R1+R3) / 2

when 'n' is even $R = \frac{R_1}{2} + \frac{R_{n-1}}{2} - R_n$

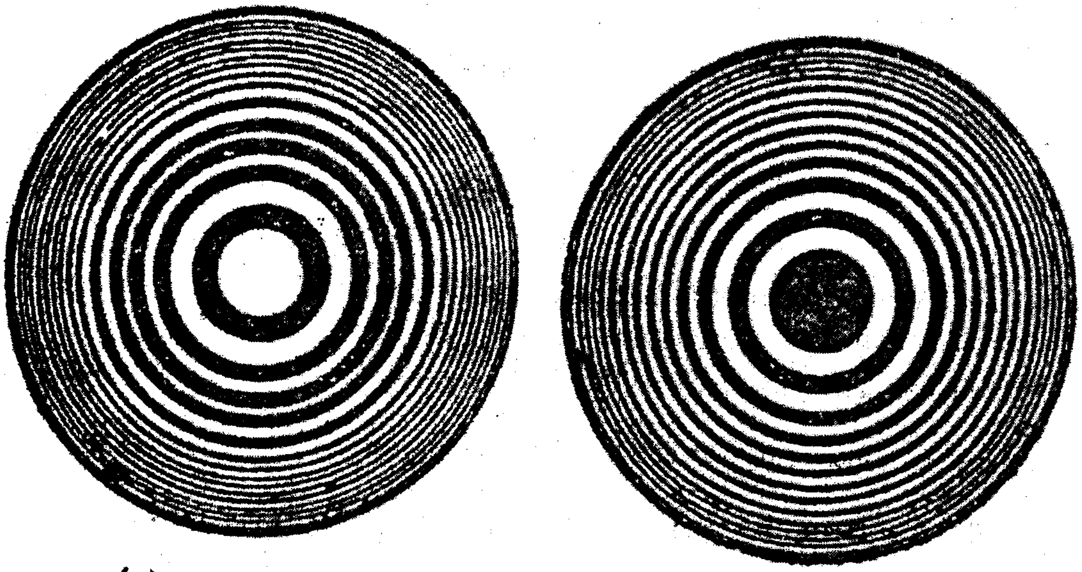
As $n \to \propto$, Rn and Rn-1 $\to 0$

Hence resultant amplitude at P due to whole wavefront $R \approx \frac{R_1}{2}$

Intensity at P due to wavefront ABCD = $I \propto \frac{R_1^2}{4}$ ---------------(11.43)

11.19 ZONE PLATE :-

Fresnel theory says that the radii of Fresnel half period zone is $\tau = \sqrt{np\lambda}$ .So to test the correctness of Fresnel theory and also to use zone plate(see Fig. 11.14) for focussing action just like convex lens ,zone plate is constructed . From above equation it is evident that radii of half period zones are proportional to square root of natural numbers.



(a) Positive (b) Negative

Fig 11.14

Construction of zone plate :-

A White paper is taken and concentric circles of radii equal to square root of natural numbers are drawn. They are painted black alternately. The reduced size photograph of this paper is taken on a glass plate. The resulting glass negative is called 'Zone Plate'. If the central zone is transparent and odd zones are transparent and even zones are opaque that zone plate is called as 'Positive zone plate'. If the central zone is dark, all odd numbered zones are black and even number zones are transparent, the zone plate is called as 'Negative zone plate'.

Action of zone plate :-

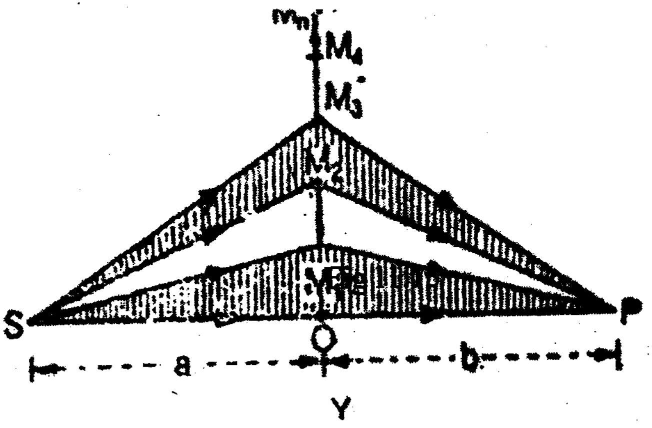


Fig 11.15

Let XY be the zone plate placed perpendicular to the plane of paper. 'S' is the light source at a distance 'a' from zone plate. P is a point on the screen at a distance 'b' from zone plate (Fig. 11.15). The radii of half period zones is $OM_1, OM_2,$ ---------- OM_n The position of the screen is such that there is increasing path difference of $\lambda/2$ from one zone to the next. Hence

$$SM_1 + M_1P = a + b + \lambda/2$$

$$SM_2 + M_2P = a + b + 2\lambda/2$$

$$SM_n + M_nP = a + b + n\lambda/2 \quad \text{-----(44)}$$

But from figure $SM_n^2 = SO + OM_n^2$,$SM_n^2 = a^2 + r_n^2$

$$SM_n = [a^2 + r_n^2]^{1/2} \quad ,SM_n \approx a[1 + \frac{r_n^2}{2a^2}] = a + \frac{r_n^2}{2a}$$

Similarly $M_nP = b[1 + \frac{r_n^2}{2b^2}] = b + \frac{r_n^2}{2b}$

Substituting these values in eq. (11.44)

$$\left(a + \frac{r_n^2}{2a}\right) + \left(b + \frac{r_n^2}{2b}\right) = a + b + n\lambda/2$$

$$\Rightarrow r_n^2\left(\frac{1}{a} + \frac{1}{b}\right) = n\lambda$$

With sign convention $r_n^2\left(\frac{1}{b} - \frac{1}{a}\right) = n\lambda$,$\Rightarrow \frac{1}{b} - \frac{1}{a} = \frac{n\lambda}{r_n^2}$ ______________(11.45)

Eq.11.45 is similar to lens equation

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \qquad , \therefore f_n = \frac{r_n^2}{n\lambda} \text{ ______________(11.46)}$$

This is the equation for the focal length of zone plate. Hence we conclude that "Zone plate behaves as a converging lens".

Focussing action :-

In zone plate alternate zones are transparent. So the amplitudes or waves that reach P from alternate zones differ in phase by π and hence reinforce each other with constructive interference. $R_1 + R_3 + R_5 + -----$,Hence the resulatant intensity $>> \frac{R_1^2}{u}$ and this shows that zone plate focusses the light from source S, to point P.

CONVEX LENS	ZONE PLATE
1. Forms real images $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$	1. Forms real images. $\frac{1}{b}-\frac{1}{a}=\frac{n\lambda}{r_n^2}$
2. Focal length depends on λ hence shows chromatic aberration.	2. Focal length depends on λ hence shows chromatic aberration.
3. Convex lens brings the rays to focus by refraction.	3. Zone plate focusses light by diffraction
4. All rays reaching image have same phase.	4. Rays reaching image from different zones differ by in phasenby 2π
5. Image is more intense and $f_v < f_r$	5. Image is less intense $f_v > f_r$
6. Convex lens has only one focii.	6. Zone plate has more than one focii $\frac{r_n^2}{n\lambda}, \frac{r_n^2}{3n\lambda}, \frac{r_n^2}{5n\lambda} --- \frac{r_n^2}{(2m+1)n\lambda}$ Intensity decreases as focal length decreases.

11.20 DIFFRACTION AT STRAIGHT EDGE :-

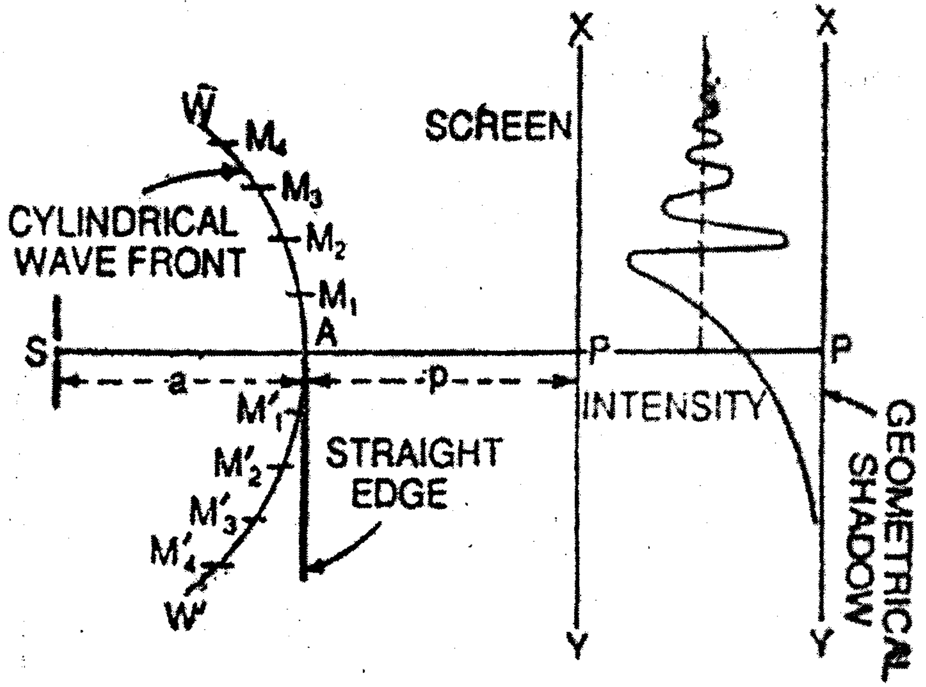


Fig 11.16

As shown in the fig 11.16, light from source 'S' is made to pass through small slit. It gives a sylindrical wavefront W W' .A straight edge (like edge of a razor blade or knife) is placed at a distance 'a' from slit .A screen XY is placed at a distance 'p' from the straight edge. A normal SP is drawn to screen XY which meets knife edge at A. Let us divide cylindrical wavefront W W' into half period zones A M_1 or M_0M_1, M_1M_2, M_2M_3 -----etc and $M_0M'_1, M'_1M'_2$ ----- etc in shadow region. According to geometric optics, the region below P on screen XY should be dark and above 'P' should be bright. But we can observe some light inside the geometrical shadow PY and alternate dark and bright bands of varying width and intensities i.e. diffraction bands above P upto certain point above P and after that equal illumination.

Intensity Distribution :-

1. Intensity at point P : For this point A is the pole .So it is completely exposed to upper half of the wavefront. The amplitudes sent by half period strips AM_1, M_1M_2, M_2M_3 ---- at P are R_1, R_2, R_3 ----

$$\therefore \text{Resultant amplitude at P is } R' = R_1 - R_2 + R_3 - R_4 + ---- = \frac{R_1}{2\pi}$$

$\because$ Alternate strips differ in phase by π

So intensity at P $\propto \frac{R_1^2}{4}$

2. Intensity at point P_1 :- Suppose M_0 is the pole for point P_1. (see Fig. 11.17). Now it is exposed to M_0M_n and M_0A portions of wavefront.

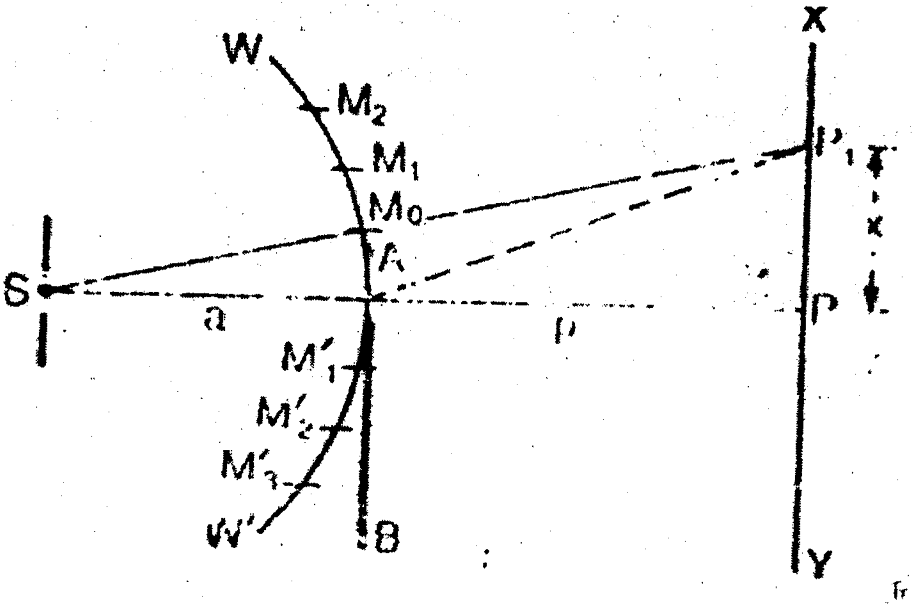


Fig 11.17

Suppose there is only one half strip between Mo and A.

Then the resultant amplitude at P_1 is

$$R' = \frac{R_1}{2} + R_1 = \frac{3R_1}{2} \quad [\frac{R_1}{2} \text{ is due to upper half of wavefront}]$$

Intensity at $P_1 \propto \frac{9R_1^2}{4}$, 9 times largerthan the intensity than at P. Hence bright band at P_1.

If there are two half zone strips between A and M_0

Resultant amplitude $R' = \frac{R_1}{2} + R_1 - R_2 \approx \frac{R_1}{2}$,

$$\text{Intensity} \propto \frac{R_1^2}{4}$$

Hence dark band at P_1.

Similarly if there are three strips ,

$$R' = \frac{R_1}{2} + (R_1 - R_2' + R_3)'$$

$$= \frac{R_1}{2} + \left(\frac{R_1}{2} + \frac{R_3}{2}\right) \qquad \because \left(R_2 = \frac{R_1 + R_3'}{2}\right)$$

$$= \frac{R_1}{2} + R_1 \approx \frac{3R_1}{2}$$

$$I \propto \frac{9R_1^2}{4}.$$

This intensity is slightly less than that of the first bright band due to R_3

In general "the intensity at P_1 will ne maximum if the number of half period strips enclosed between A and Mo is odd and minimum if no of half period zones enclosed is even.

3. Intensity at P_2 [in shadow region] :-

As shown in the fig 11.18 If the position of P_2 is such that the first half strip M_0 M_1 in upper half of the wavefront is obstructed and lower half of wavefront is completely obstructed, then the resultant amplitude

$$R' = -R_2 + R_3 - R_4 + R_5 \text{-------------} = -R_2/2$$

$$\text{Intensity} \propto \frac{R_2^2}{4} << \frac{R_1^2}{4}$$

If the position of P_2 is such that it obstructs two half period strips from upper wavefront and lower half of wavefront, then the resultant amplitude is given by

$$R^1 = R_3 - R_4 + R_5 - R_{6+} \text{-----------} \approx \frac{R_3}{2}$$

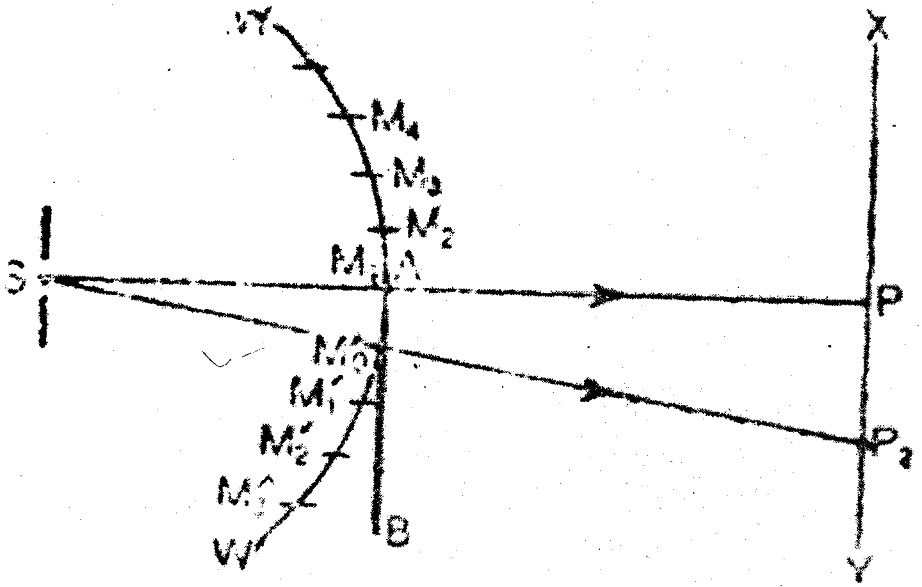


Fig 11.18

$$\therefore I \propto \frac{R_3^2}{4}$$

As we know from Fresnel's theory that $R_1 > R_2 > R_3$ ----------etc,so the intensity at P_2 decreases as we go deep into the shadow region.

Positions of maximum and minimum intensity :-

Point P_1 in the figure is maximum or minimum according as AM_0 contains an odd or even numbere of half period zones.

$P_1A - P_1M_0 = (2n-1)\lambda/2$ for maxima --------------(11.47)

$P_1A - P_1M_0 = 2n\lambda/2$ for minima ---------------(11.48) n= 1,2,3.....

But from above figure $P_1A^2 = PA^2 + PP_1^2$

$$= p^2 + x^2$$

$$P_1A = \sqrt{p^2 + x^2}$$

$$= p\left(1 + \frac{x^2}{2p^2}\right)^{\frac{1}{2}} \approx p\left(1 + \frac{x^2}{2p}\right) \text{------------------}(11.49)$$

Similarly $P_1S=(a+p)+\frac{x^2}{2(a+p)}$

$$P_1M_0=P_1S-M_0S=[(a+p)+\frac{x^2}{2(a+p)}-a]$$

$$=p+\frac{x^2}{2(a+p)} \text{-------------------(11.50)}$$

From (11.49) and (11.50), $P_1A-P_1M_0=\frac{ax^2}{2p(a+p)}$

Substituting these in eq.(47)

$$\frac{ax_n^2}{2p(a+p)}=(2n-1)\lambda/2$$

$$\Rightarrow x_n=\left[\frac{(2n-1)(a+p)p\lambda}{a}\right]^{1/2} \text{-------------(11.51)}$$

From eq (48)

$$\frac{ax_n^2}{2p(a+p)}=2n\lambda/2$$

$$\Rightarrow x_n=\left[\frac{2n(a+p)p\lambda}{a}\right]^{1/2} \text{-------(11.52)}$$

Thus equations (11.51) and (11.52) indicate the position of maxima and minima of diffraction bands with a straight edge.

11.21 Diffraction of a plane wave by a straight edge :-

Now we consider a plane wavefront incident on straight edge placed perpendicular to plane of the paper as shown in the figure11.19. The straight edge and screen are parallel to each other. P is a point on the screen where we want to calculate the intensity due to a plane wavefront striking the straight edge. Let P be a point on the screen with co-ordinates (0,y) with respect to origin taken on the edge of geometrical shadow. Consider an arbitrary point M on the plane of straight edge whose co-ordinates are (X,Y). Distance between straight edge and screen is 'd'. Let us consider an infinitesimal area dXdY around M on the plane of straight edge .The field is proportional to $\frac{Ae^{ikr}}{r}dxdy$

A-amplitude of plane wave in the plane of straight edge

k-propagation constant= $\frac{2\pi}{\lambda}$

The resultant field at P is given by integration over the whole area $u(p)=c\iint\frac{Ae^{ikr}}{r}dxdy$

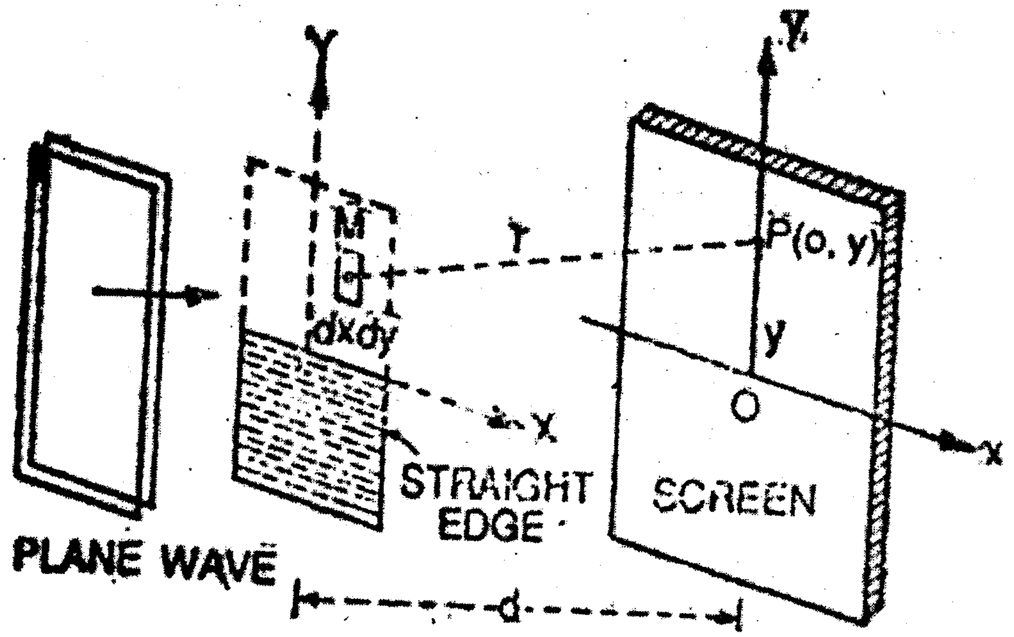


Fig 11.19

c- constant of proportionality $= -\frac{ik}{2\pi} = -\frac{i}{\lambda}$

$$\therefore u(p) = -\frac{i}{\lambda} \iint \frac{Ae^{ikr}}{r} dxdy \text{ -- } (11.53)$$

From figure 11.19, $r = MP\left[x^2 + (Y-y)^2 + d^2\right]^{1/2}$

$$= d\left[1 + \frac{X^2 + (Y-y)^2}{d^2}\right]^{1/2}$$

$$= d\left[1 + \frac{1}{2}\frac{X^2 + (Y-y)^2}{d^2}\right]$$

$$= d + \frac{X^2 + (Y-y)^2}{2d} \text{ -------------} (11.54)$$

Substituting the value of 'r' in (53) where $r \approx d$ in denominator.

$$u(P) \frac{-i}{\lambda}\frac{A}{d} \int_{-\infty}^{+\infty} dX \int_{0}^{\infty} \exp\left[ik\left\{d + \frac{X^2+(Y-y)^2}{2d}\right\}\right] dY$$

The above equation gives the intensity distribution due to straight edge, when a plane wavefront is incident on it.

11.22 SOLVED EXAMPLES :-

1. A Screen is placed 2 m away from a narrow slit. Find the slit width if the first minima lies 5 mm on either side of the central maximum when plane waves of $\lambda = 5\times10^{-5}$ cm are incident on the slit?

Solution :-

Minimum intensity positions are given by $e\sin\theta = \pm m\lambda$

For first minima, m=1

$$e\sin\theta = \lambda$$

When is small $\sin\theta \approx \theta$

$$\therefore \theta = \frac{\lambda}{e} = \frac{5\times10^{-5}}{e}$$

But θ is given as $\theta = \frac{0.5}{200}$ radians

$$\therefore \frac{5\times10^{-5}}{e} = \frac{0.5}{200}$$

$e = 0.02$ cm.

2. In a double slit Fraunhöfer diffraction pattern, the screen is 160 cm away from the slits. The slit width is 0.08 mm and they are 0.4 mm apart. Calculate the wavelength of light if the fringe spacing is 0.25 cm. Also find the missimg orders?

Solution :-

Fringe spacing 'β' is given by

$$\beta = \frac{\lambda D}{2d} \quad \text{or}$$

$$\lambda = \frac{\beta\times 2d}{D}$$

β=0.25 cm, 2d=0.04 cm, D = 160 cm $\lambda = 6250\times10^{-8}$ cm

Missing orders are given by equation

$$\frac{e+d}{e} = \frac{n}{m}$$

d=0.04 cm, e=0.008 cm

n= 6m where m=1,2,3, -------etc.

3. A ground control radar generates a beam of 4 cm microwaves. How large should be the parabolic antenna dish, if the radar is to resolve two aeroplanes seperated by 20 of arc?

Solution :-

Limit of resolution of any instrument is given by $\theta = \frac{1.22\lambda}{d}$

θ-minimum angle of resolution $\therefore d = \frac{1.22\lambda}{\theta}$

$\lambda = 4\times10^{-2}$ m ,

$\theta = 20^{1} = 0.00582$ radians

$$d = \frac{1.22\times4\times10^{-2}}{0.00582} = 6.29 \text{ metres}$$

4. Light is incident normally on a grating with 250 lines per mm. A second order spectral line makes an angle of 18^0 with the central zero order image. Calculate the wavelength of spectral line ?

Solution :-

Condition for principal maxima is $(e+d)\sin\theta = \pm n\lambda$

Grating element is $(e+d) = \frac{10^{-3}}{250}\text{m} = 4\times10^{-6}\text{m}$

Order of the maxima n=2 ,

Angle of diffraction= 18^0

$$\lambda = \frac{(e+d)\sin\theta}{n} = \frac{4\times10^{-6}\times\sin18^0}{2},$$

$$= 6.18\times10^{-7} \text{ m}$$

5. What should be the maximum number of lines in a grating which will fully resolve in the second order the lines whose wavelengths are 5890 A^0 and 5896 A^0 ?

Solution :-

The resolving power of grating is $R = \frac{\lambda}{d\lambda}$

But $\frac{\lambda}{d\lambda} = Nn$

where N = no of lines in grating

n = order of the spectrum

$$N = \frac{1}{n}\left(\frac{\lambda}{d\lambda}\right)$$

$\lambda_1 = 5896\ A^0$

$\lambda_2 = 5890\ A^0$,

$d\lambda = \lambda_1 - \lambda_2 = 6\ A^0$

$$\lambda = \frac{5896+5890}{2} = 5893\ A^0$$

n= 2 (order of the spectrum)

$$N = \frac{1}{2}\left(\frac{5893\times10^{-8}}{6\times10^{-8}}\right) = 492$$

6. How many half period elements are contained in a circle of radius 1 cm for a wavelength 5.56×10^{-5} cm with respect to a point 60 cm away ?

Solution :-

Radius r_n if n th half period zone is given by

$$r_n^2 = pn\lambda \Rightarrow n = \frac{r_n^2}{p\lambda}$$

r_n =1 cm, p=60 cm ,

$\lambda=5.56\times10^{-5}$ cm

$$n = \frac{1}{60\times5.56\times10^{-5}} \approx 300$$

7. Find the radii of the first and hundreth circles on a zone plate behaving like a convex lens of focal length 50 cm ?

Solution :-

Focal length of zone plate is given by

$$f = \frac{r_n^2}{n\lambda} \Rightarrow r_n^2 = n\lambda f$$

For first circle $n=1, f=50$ cm.

$$r_1 = \sqrt{1\times5000\times10^{-8}\times50} \approx 0.05 \text{ cm}$$

For 100 th circle $n=100, f=50$ cm

$$r_{100} = \sqrt{100\times5000\times10^{-8}\times50} = 0.5 \text{ cm}$$

8. A grating has slits that are each 0.1 mm wide the distance between the centres of any two adjacent slits is 0.3 mm. Find the higher order maxima that will be absent in the resulting spectrum ?

Solution :-

The absent Spectra with grating are given by the equation

$$\frac{e+d}{e} = \frac{n}{m} \quad , \quad m=1,2,3,\text{-----------------}$$

e=0.1 mm ,
d=0.3 mm

$$\frac{0.1+0.3}{0.1} = \frac{n}{m}$$

$$\Rightarrow \frac{0.4}{0.1} = \frac{n}{m}$$

$$\Rightarrow \frac{n}{m} = 4 \text{ or } n=4m$$

The missing order of maxima are 4 th, 8th, 12th -----------------etc.

10.23 SUMMARY :-

The bending of light waves at the obstacles and apertures is termed as diffraction. The differences between Fraunhofer and Fresnel diffraction, Interference and diffraction are highlighted. The intensity distribution in diffraction bands with single slit, circular aperture and double slit are derived. The position of principal maxima, secondary maxima and minima are also derived. Intensity distribution patterns are explained. Resolving power of telescope is defined using diffraction at circular aperture. Construction,working, theory of plane diffraction grating is dealt in detail . Formation of multiple spectra with grating . Absent spectra, missing orders, maximum no of orders possible are defined.

Fresnel's class of diffraction is explained with the help of half period zones. Construction and working of zone plate is detailed. Diffraction at straight edges is explained with the help of half-period zones.

10.24 KEY WORDS :-

Diffraction, Obstacle, Geometrical shadow, Intensity distribution, Principal maxima, Secondary maxima, minima, Oblique incidence, Wavefront, Airy disc, Resloving power, Double slit, missing order, Diffraction grating, Multiple spectra, Absent spectra, Grating element, Dispersive power, Resolving power, Half period zones, Zone plate, Focussing action, Straight edge.

10.25 SELF ASSESSMENT QUESTIONS :-

10.25.1 <u>LONG ANSWER QUESTIONS</u> :-

1. Describe Fraunhofer diffraction due to a single slit and deduce the positions of maxima and minima . Draw the representative graphs of the intensity distribution . What is the effect of change in a. slit width and b. wavelength of the incident light ?

2. Distinguish between Fresnel's and Fraunhofer diffraction. Discuss the Fraunhofer diffraction at a double slit . What is the effect on diffraction pattern by changing
 a. slit width keeping the slit seperation constant
 b. slit seperation keeping the slit width constant ?

3. Give the construction and theory of a plane diffraction grating of the transmission type and explain the formation of spectra by it ?

4. Explain how a plane transmission grating can be used to determine the wavelength of the spectrum of mercury. Discuss the formula used ?

5. Discuss the Fraunhofer's diffraction due to N-slits and obtain the intensity distribution and positions of maxima and minima ?

6. What is zone plate ? How is it constructed ? Show that a zone plate has multiple focii. Compare the zone plate with a convex lens ?

7. Describe with necessary theory, the Fresnel type of diffraction due to a straight edge . How would you use it to determine the wavelength of light ?

10.25.2 <u>SHORT ANSWER QUESTIONS</u> :-

1. Explain what is meant by diffraction of light .Distinguish between Fraunhofer and Fresnel classes of diffraction ?

2. Distinguish between single slit and double slit diffraction patterns ?

3. Distinguish between resolving power and dispersive power of a plane transmission grating ?

4. What do you mean by limit of resolution ?Discuss the Rayleigh's criterion ?

5. Find the maximum no of orders available with a grating ?

6. What are missing (or absent) spectra in a diffraction grating ? Deduce the condition for the n th order spectra to be absent ?

7. Define dispersive power of a grating and obtain an expression for it ?

8. Obtain the formula for the resolving power of an optical grating ?

9. What are Fresnel's half period zones ? Show that the intensity due to a large wavefront at a point is one-fourth that due to the first half-period zone acting alone ?

10. Explain the nature of diffraction at a straight edge ?

11.25.3 EXERCISES :-

1. A Slit of width e is illuminated by white light. The first minimum for red colour is observed to fall at θ =15.Given λ for red colour is 6500. Find the value of slit width e?

2. A light of wavelenth 600nm is incident on a slit of width 1 μm . Find the angular separation between the first order minima on either side of the central maximum.

3.A convex lens of focal lenth 24cm has a diameter of 32mm. Find out the angular separation between two points such that they will be just resolved as per Rayleigh's criterian. Also find out how far a part are the centers of the diffraction patterns formed in the focal plane of the lens. Assume λ =5500 A^0.

4. A plane transmission grating having 4250 lines per cm has got sodium light incident on it normally. In the second order spectrum the spectral lines is observed to be deviated through 30^0 What is the wavelenth of the spectral line consemed?.

5. A grating has got 5000 lines/cm on its surface. The opaque space are 2 times the transparent spaces Find the order of the spectrum that will be the absent in this grating.

6. A grating has a got a length of 15cm .The lines that are ruled on the grating surface are 6000 lines/cm. Find the resolving power of the grating in the first order.

7. The pupil of the eye is 3mm in diameter with the wavelength 5500 A^0 , find the resolution of the eye.

8. Find the radius of the first zone in a zone plate of focal length 20 cm for a light of wavelength 500 nm ?

9. With respect to point 50 cm distance for a wavelength of 6000 A^0 ,calculate the no of half period zones in circular hole of radius 1 cm?

11.26 REFERENCE BOOKS :

1. Unified physics vol-2	Dr. S.L.Gupta Sanjeev Gupta Jai Prakash Nath & Co Meerut
2. B.Sc Optics	Telugu Academy
3. Introduction to modern Optics	G.K.Fowles(Holt-Rinchert)
4. Optics	Ghatak
5. B.Sc Second year Physics	K. Ramakrishna D.V.Brahmaji A.Srinivasa Rao S.L.V. Mallikharjun

Unit IV
LESSON-12

POLARIZATION

OBJECTIVES:-

1. Prove that the light waves are transverse in nature through polarization of light waves.
2. Explain the concept behind the polarization of light waves and define plane of polarization and plane of vibration.
3. Prove that the angle between reflected and refracted light is 90^0 when the reflected light is completely polarized through Brewster's law.
4. Estimate the intensity of polarized light through Malus law.
5. Explain the concept of double refraction and practically show it through Nicol prism.
6. Give a brief account of Huygens wave theory in the explanation of double refraction.
7. Usage of quarter wave plate and half wave plate in introducing a phase difference between ordinary and extraordinary rays.
8. Selective absorption and polaroids.
9. Process of producing plane, circular, elliptical and partially polarized light using a quarter wave plate.
10. Importance of Babinet's compensator in introducing variable phase difference between ordinary and extraordinary light.
11. Define optical activity, specific rotation and determine their values through Laurent's half-shade polarimeter

STRUCTURE OF THE LESSON:-

12.1 INTRODUCTION :

Interference and diffraction phenomenon proved beyond doubt that light exhibits wave nature. But it was not confirmed whether it is a longitudinal wave or transverse wave. But the phenomenon of polarisation proved that the light wave has transverse nature.

12.2 POLARISATION OF WAVES:

Light energy is transmitted from one place to the other in the form of waves. Wave propagation is nothing but the propagation of energy from one place to the other by vibrating the particles of the medium in all three directions. Then the vibrations caused by light wave are said to be symmetrical about direction and the light wave is called unpolarised wave. If this light wave is passed through a tourmaline crystal, the vibration which are parallel to its optic axis are only 'transimitted and other vibrations are stopped. So the resultant wave coming out of the crystal is called "Polarised wave" and the wave is unsymmetrical about a direction.

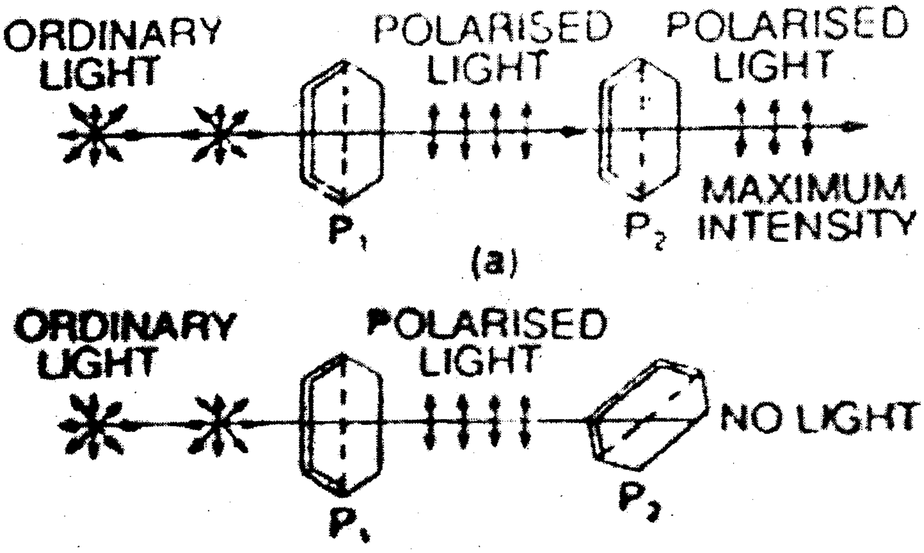


Fig.12.1

As shown in above figure 12.1, when light passes through a tourmaline crystal it attains one-sidedness parallel to optic axis of the crystal (optical axis in crystal just behaves like a slit and passes the vibrations parallel to it and stops other directional vibrations). The light wave which has acquired this one-sidedness is called "Polarised light". If these vibrations are confined to a single plane it is called as "Plane polarised light".

As shown in the figure 12.5 the incident ray AB splits into ordinary ray travelling along BD and extra-ordinary ray along BC .But these two rays emerge parallel to each other and also to incident ray . So the refractive index of ordinary ray $\mu_0 = \text{Sin}i / \text{Sin}r_1$ and of extra-ordinary ray $\mu_e = \text{Sin}i / \text{Sin}r_2$.$\because r_1 < r_2, \mu_0 > \mu_e$ .The velocity of ordinary remains same in all directions, but the velocity of extra-ordinary ray is different in different directions .Hence μ_e varies with angles of incidence

12.8 NICOL PRISM :-

Nicol's prism is a device for producing and analysing plane polarised light.

Principle :-When unpolarised ordinary light is incident on a crystal like Calcite, it is split up into two refracted rays, ordinary and extra-ordinary rays. These rays are completely plane polarised with plane of vibrations perpendicular to each other.One of these rays is eliminated by total internal reflection at the surface of canadabalsam layer placed between two calcite crystals.

As a result either ordinary or extra-ordinary ray is only emitted from Nicol prism.

Construction :-

A Calcite crystal whose length is three times as its width is taken . The end faces of the crystal are grounded in such a way that the angles in the principal section become 68^0 and 112^0 .This is done to increase the field of view .The crystal is cut into two pieces by a plane perpendicular to the principal section as well as end faces PR and QS (Fig. 12.6). The two cut surfaces are grounded and polished optically flat and then connected together by canadabalsam .The refractive index of canadabalsam lies between the refractive indices for the ordinary and extra-ordinary ray for calcite. $\mu_0 = 1.658$,Canadabalsam $\mu = 1.55, \mu_e = 1.486$.

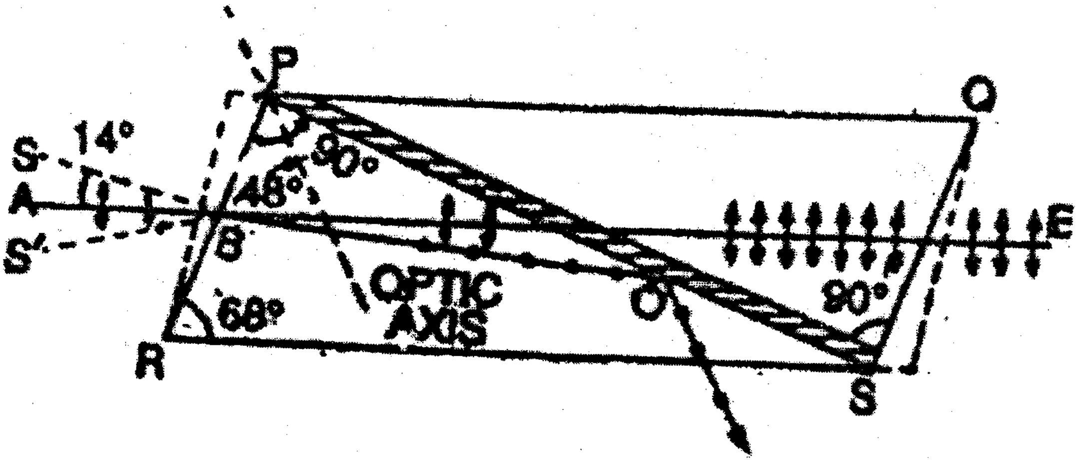


Fig 12.6

Action :-

The incident light AB is splitup into ordinary ray BO and extra-ordinary ray BE in the crystal.Since $\mu_0 > \mu$,it gets totally reflected at the canadabalsam layer. Similarly $\mu_e > \mu$,so extra-ordinary ray passes through and emerges out of the crystal parallel to incident ray .Since the extra-ordinary ray is plane polarised, we obtain plane polarised light from Nicol prism.

Limitations :-

The incident beam should have an angle of $\angle SAB < 15^0$ .Else ordinary ray hits PS at an angle < critical angle and it will not be totally reflected. So we obtain mixed light of o-rays and e-rays.

Uses:-

Nicol prism is used as a polariser and analyser.

Polariser :-

It acts as a polariser i.e the instrument which gives only plane polarised light as explained in the previous section .

Analyser :-

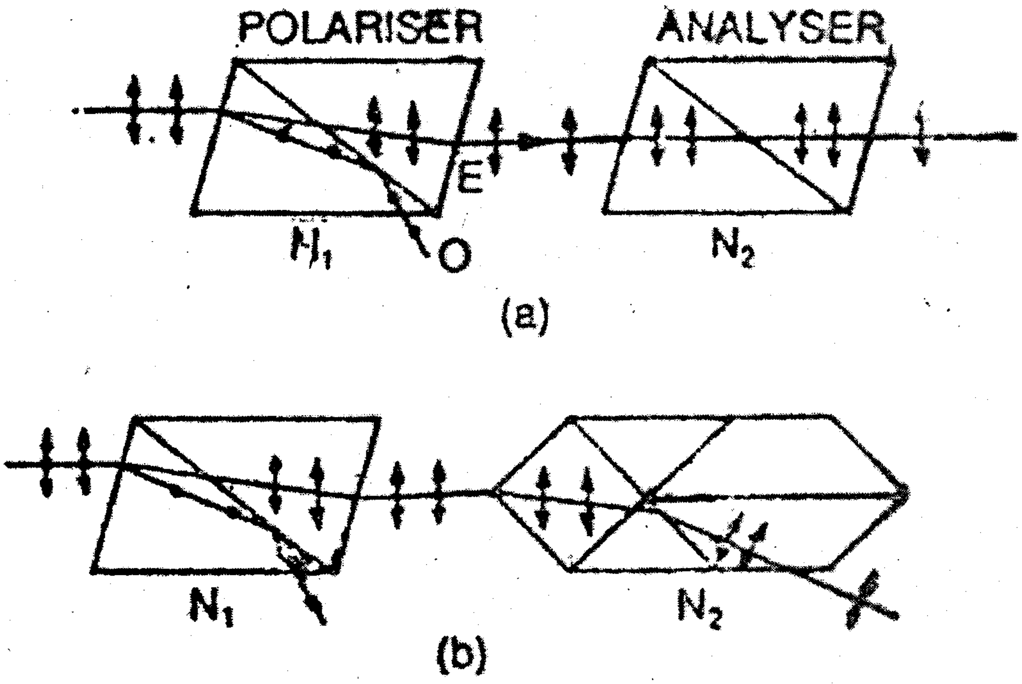


Fig 12.7

As shown in the figure 12.7, two Nicol prisms are taken and arranged co-axially .One of them acts as polariser. The second one is the analyser .when the principal sections of the two prisms are parallel to each other, the plane polarised light emitted by polariser passes through analyser without obstructions and we can observe intense light at the end. Now if we start rotating analyser, the intensity of light slowly decreases . At one position when the two principal sections are perpendicular to each other, light is completely shut off .This proves that the light from polariser is plane polarised light. If the light from polariser is not completely polarised, we never attain above condition of zero intensity. Then we can infer that the light from polariser is not completely plane polarised . Since the position and action of second Nicol prism analyses whether the light from polariser is polarised or not , it is called as analyser.

12.9 Huygen's Theory of Double Refraction :-

According to Huygen's theory, each point on the crystal through which light ray is passing emits two wavefronts instead of one wavefront , in double refracting crystals . One wavefront corresponds to ordinary ray and the second wavefront corresponds to extra-ordinary ray wavefront.The ordinary wavefront has same velocity in all directions . So there envelope will be sphere or spherical wavefront. But the velocity of extra-ordinary wavefronts is different in different directions. Hence its envelope is ellipsoid of revolution. In certain crystals, velocity of extra-ordinary ray > velocity of ordinary rays. Then sphere of ordinary ray lies within the ellipsoid of extra-ordinary ray as shown in the figure 12.8(a)

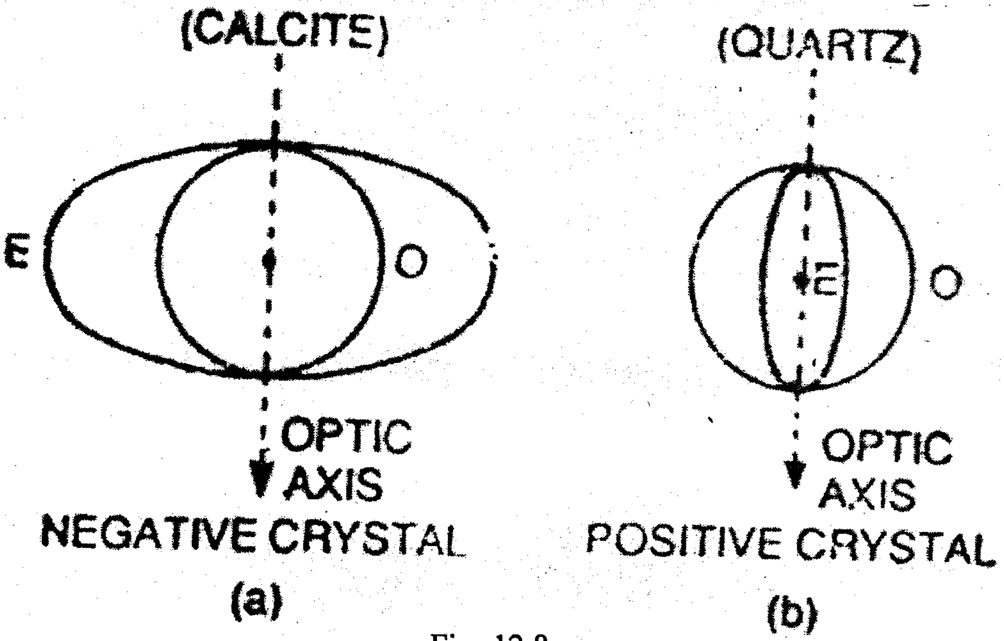


Fig 12.8

Such crystals are called as 'Negative crystals' (Ex : Calcite,Tourmaline). In certain other crystals ,velocity of ordinary ray > velocity of extra-ordinary ray. Such crystals have ellipsoid within the sphere of revolution and they are called as "Positive crystals" (Ex : Quartz , Ice)(12.8b)

But in both the cases, the sphere and ellipsoid of revolution touch each other along the optic axis. Since these velocities are same along the optic axis.

12.10 Polarisation by Selective Absorption or Dichorism :-

Dichorism is the practical application of double refraction. Certain types of crystals like tourmaline exhibit a special type of action. When unpolarised light is incident on the crystal, it is split up into ordinary and extra-ordinary rays. This crystal absorbs the ordinary ray and emits extra-ordinary ray which is plane polarised. "This type of selective absorption one refracted ray by the crystals is termed as Dichorism" and the crystals are called Dichoric Crystals.

12.11 POLAROIDS :-

This Dichorism principle is used in the construction of polaroids. Heraphite is a synthetic dichoric small crystal of Iodosulphate of quinone. It is suspended between two thin glass sheets in series in nitrocellulose. They are arranged in such a way that all their optical axis are aligned. The whole arrangement between the glass plates is termed as a polaroid and exhibits dichorism. It can give polarised light of all colours or wavelengths. Other type of polaroids are H-Polaroids and K-Polaroids. Poly Vinyl Alcohol film is taken and is stretched 3 times to its width. Then it starts behaving like dichoric crystal under stress. It is impregnated with Iodine. These are called as H-Polaroids. If the Poly Vinyl Alcohol is heated in the presence of strong dehydrating agent, instead of impregnating with Iodine, they are termed as K-Polaroids.

When two polaroids are uncrossed (optical axis parallel to each other), as shown in the fig 12.9(a) the emergent beam is plane polarised. When the two polaroids are crossed (optical axis are perpendicular to each other), as shown in the fig 12.9(b) there is perfect extinction.

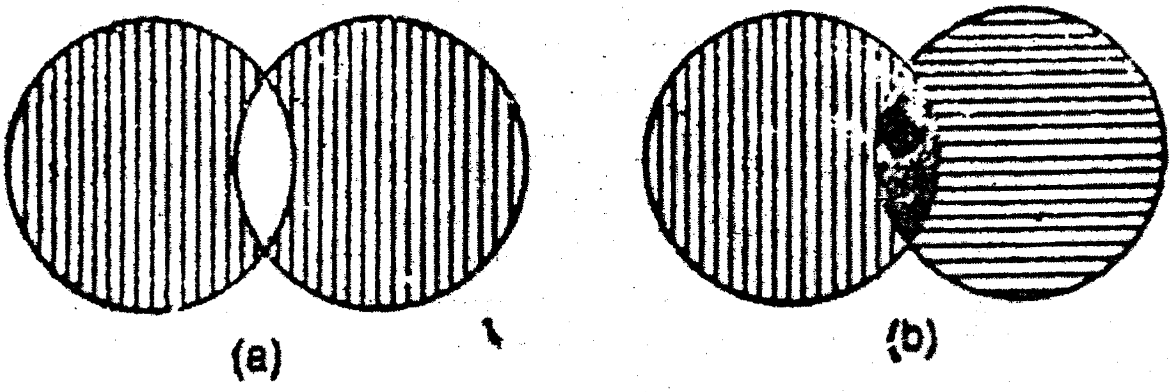


Fig 12.9

USES :-

1. They are used as polarisers and analysers in modern polarising instruments.
2. They are used in sunglasses to cut-off the glair produced by light reflected from polished and semi smooth surfaces.
3. They are used to control intensity of light in trains and aeroplanes.
4. Polaroid glasses are used to produce three dimensional moving pictures.
5. They are used to improve colour contrast in old oil paintings.
6. They are used to eliminate the head light glare in motor cars.

12.12 ANALYSIS OF POLARISED LIGHT :-

Quarter Wave Plate :-

"It is a crystal plate which introduces a path difference of $\lambda/4$ or phase difference of $\pi/2$ between ordinary ray and extra-ordinary ray produced by doubly refracting crystals". When we take a calcite plate cut with optic axis parallel to the surface,as shown in the fig 12.10 the unpolarised light incident on it splits into ordinary and extra-ordinary rays. These two rays travel along the optic axis with different velocities ,hence travel different distance.Hence path difference is introduced between the ordinary and extra-ordinary rays. If the thickness of the Calcite plate is selected appropriatly, we can obtain a path difference of $\lambda/4$ between the two . Hence it is termed as Quarterwave plate.

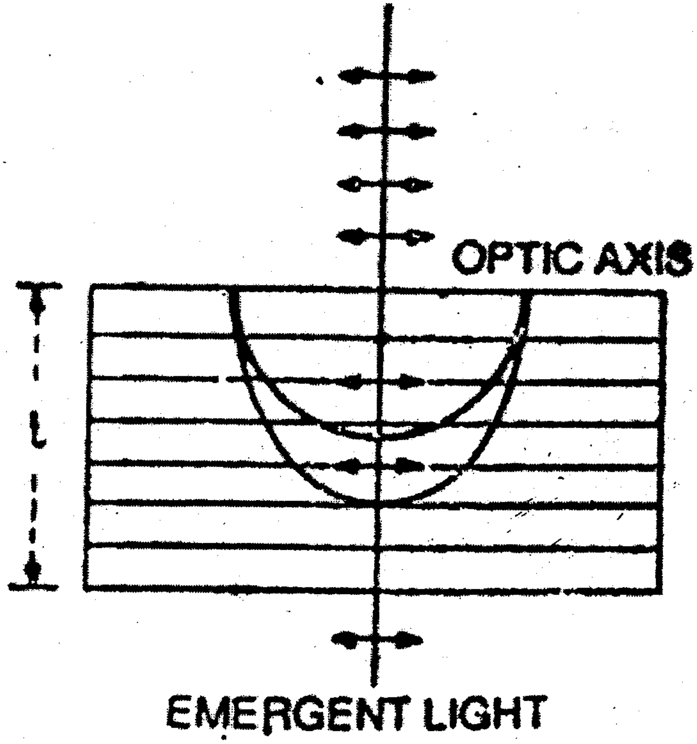


Fig 12.10

Let 't' be the thickness of the plate, μ_0, μ_e the refractive indices of ordinary and extra-ordinary rays. Path difference between o- and e- waves $= \mu_0 t - \mu_e t = (\mu_0 - \mu_e)t$ $[\because \mu_0 > \mu_e$ in calcite$]$.

For quarter wave plate, path difference $= \lambda / 4$

$$\therefore (\mu_0 - \mu_e)t = \lambda / 4$$

or Thickness of the plate $t = \dfrac{\lambda}{4(\mu_0 - \mu_e)}$

Similarly if the calcite plate introduces a path difference of $\dfrac{\lambda}{2}$ or phase difference of then it is called as "Half-wave plate". Thickness of Halfwave plate $t = \dfrac{\lambda}{2(\mu_0 - \mu_e)}$

12.13 Production of Circularly and Elliptically Polarised light :-

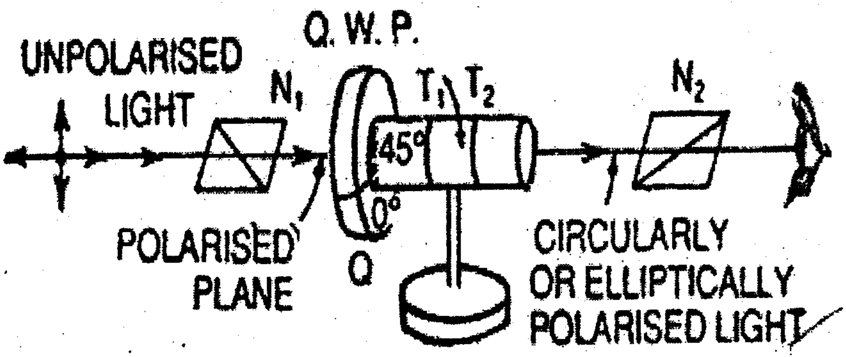


Fig 12.11

Circularly Polarised Light :-

Circularly polarised light is the resultant of two waves of equal amplitudes ,vibrating at right angles to each other and having a phase difference of $\pi/2$

As shown in the figure12.11, unpolarised light is incident on Nicol prism N_1. It gives out plane polarised light .Another Nicol prism N_2 is placed at a distance from N_1 in crossed position. So there will be darkness behind N_2. Now a quarter wave plate is mounted on a tube T_1 between two nicols. The tube T_1 can rotate about the outer fixed tube T_2. Thus the $\lambda/4$ quarter wave plate can be rotated about a horizontal axis through any desired angle . Because of quarterwave plate now some light appears after N_2 . Now quarterwave plate is rotated till the field of view is again dark. This happens when vibrations of light incident on quarterwave plate are along the optic axis and so perpendicular to N_2. Now the quarterwave plate is rotated through 45^0 so that the vibrations of light incident make an angle 45^0 with its optic axis . At this position the amplitude of o-ray and e-ray becomes equal. According to property of quarterwave plate, it introduces a phase difference of $\pi/2$ between o-ray and e-ray so that the resultant beam after quarterwave plate will be circularly polarised light.

Elliptically Polarised Light :-

Elliptically Polarised Light is the resultant of two waves of unequal amplitudes vibrating at right angles of each other and having a phase difference of $\pi/2$.

Same arrangement is repeated as in circularly polarised light .After introducing quarterwave plate , the field of view behind N_2 again becomes bright. The quarterwave plate is rotated in such a way that the field of view is again dark. Again the quarterwave plate is rotated such that the vibrations of light incident on it makes any angle other than 45^0.This makes the amplitudes of ordinary and extra-ordinary rays unequal and so the resulting light from quarterwave plate is elliptically polarised.

12.14 Analysis of Polarised light of different kinds using Quarterwave plate :-

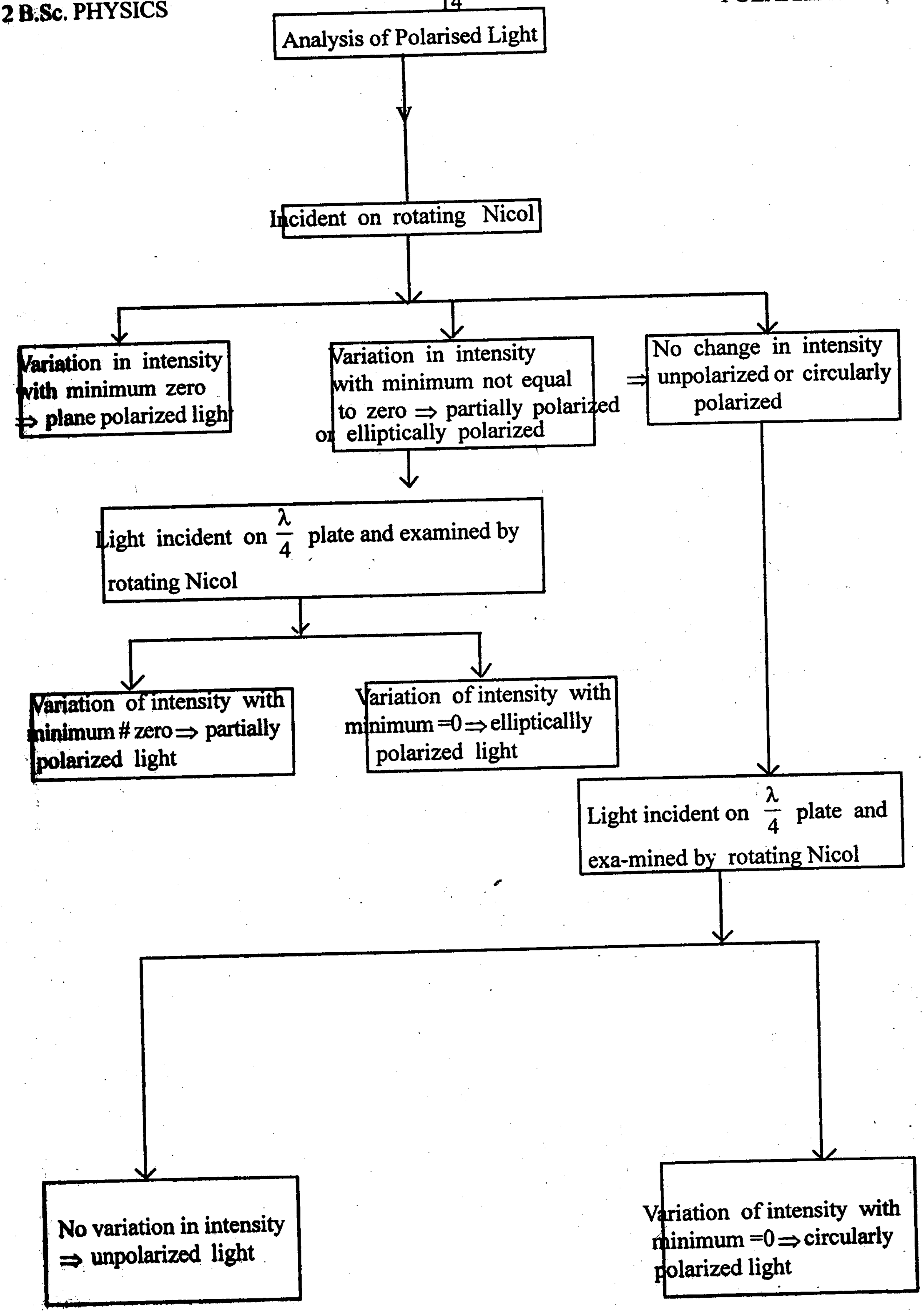
When Quarterwave plate is placed between Nicols, it introduces a phase change of $\pi/2$ between two vibrations. This helps us to differentiate between

1. Partially Polarised Light and Elliptically Polarised Light
2. Unpolarised Light and Circularly Polarised Light

1. Elliptically Polarised Light after passing through a quarterwave plate becomes plane polarised and after analysis by the rotating Nicol show variations in intensity with minimum as zero.

Patially Polarised Light after passing through a quarterwave plate remains practically unaffected and hence shows variation in intensity i.e. maximum and minimum (not zero) when passed through rotating Nicol.

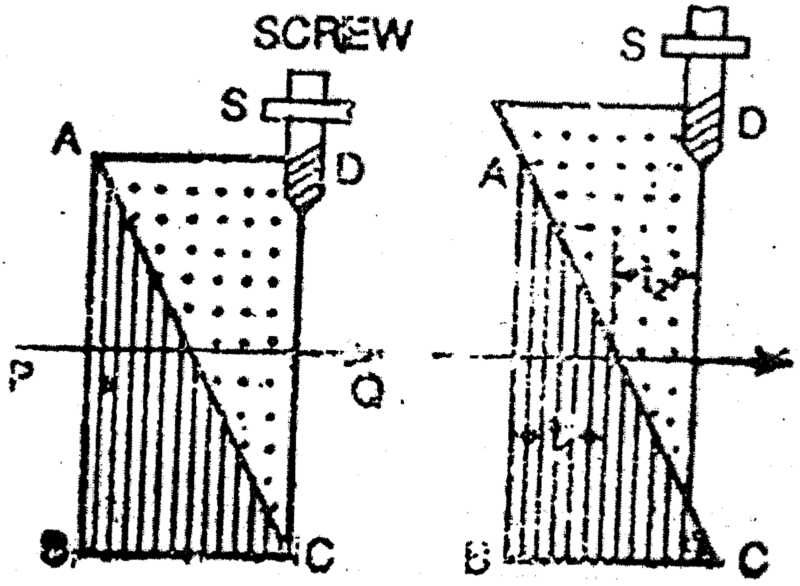
2. Circularly Polarised Light after passing through a quarterwave plate becomes plane polarised and hence the rotating Nicol shows variation in intensity with minimum as zero. Unpolarised light after passing through a quarterwave plate remain unaffected and hence shows no variation in intensity when passed through a rotating Nicol.


Analysis of Polarised Light
Incident on rotating Nicol
Variation in intensity with minimum zero ⇒ plane polarized ligh
Variation in intensity with minimum not equal to zero ⇒ partially polarized or elliptically polarized
No change in intensity ⇒ unpolarized or circularly polarized
Light incident on λ/4 plate and examined by rotating Nicol
Variation of intensity with minimum # zero ⇒ partially polarized light
Variation of intensity with minimum =0 ⇒ elliptically polarized light
Light incident on λ/4 plate and exa-mined by rotating Nicol
No variation in intensity ⇒ unpolarized light
Variation of intensity with minimum =0 ⇒ circularly polarized light

12.15 BABINET'S COMPENSATOR :-

It is an improvement over the quarterwave plate to produce variable phase difference between ordinary and extra-ordinary wavefronts of different wavelengths. It is used to produce and analyse the elliptically polarised light.

Construction :-



(a) Fig 12.12 (b)

The Babinet's Compensator consists of two small angled wedge shaped pieces of quartz placed with their hypotenuse in contact so as to form a rectangular block as shown in fig 12.12. The optic axis of the left wedge is parallel to its refracting edge AB while that of right wedge is perpendicular to the optic axis of wedge. Thus the optic axes of two wedges are perpendicular to one another and also perpendicular to the incident beam. One of the edges is fixed while the other can be moved relative to fixed edge in its own plane by a micrometer screw.

Theory:-

When Plane polarised light is incident on the compensator normally, it makes an angle θ with optic axis. After entering the first wedge, it splits up into ordinary and extra-ordinary components. As quartz is a positive crystal hence velocity of ordinary component > velocity of extra-ordinary components. When it enters second wedge, since the optic axis is perpendicular, ordinary component becomes extra-ordinary and vice-versa. Hence both the components exchange their velocity while passing from one edge to other edge. Hence the two wedges tend to cancel each other's effect

Let μ_0, μ_e be the refractive indices of quartz for ordinary and extra-ordinary ray. Let t_1, t_2 be the thickness of the two wedges respectively traversed by a particular ray.
The path difference introduced by first wedge between ordinary and extra-ordinary rays

$$= \Delta_1 = \frac{2\pi}{\lambda}(\mu_e - \mu_0)t_1.$$

Similarly phase difference created by second wedge

$$= \Delta_2 = \frac{2\pi}{\lambda}(\mu_0 - \mu_e)t_2$$

Total phase difference created by Babinet's Compensator between ordinary and extra-ordinary rays

$$\delta = \Delta_1 + \Delta_2 = \frac{2\pi}{\lambda}(\mu_e - \mu_0)(t_1 - t_2).$$

At centre of the compensator, $t_1 = t_2 \Rightarrow \delta = 0$ .So emergent light is plane polarised . By moving the screw, variable thickness ($t_1 - t_2$) can be introduced and can introduce desired phase difference. So by introducing $\pi/2$ phase difference between ordinary and extra-ordinary rays of variable amplitudes, we can produce elliptically polarised light . Hence compensator is similar to a wave plate of varying thickness.

12.16 Analysis of Elliptically Polarised Light :-

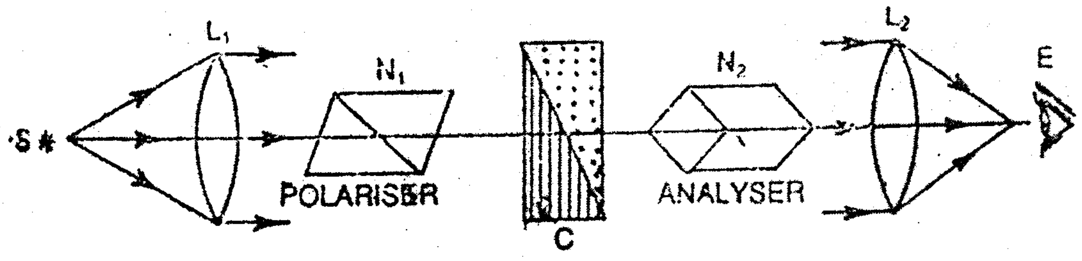


Fig 12.13

We can use to determine the characteristics of elliptically polarised light i.e.

1. Phase difference between the components
2. Position of the axes
3. Ratio of the axes.

Calibration :-

Babinet's Compensator is placed between two crossed Nicols N_1 and N_2 which act as polariser and analyser. The plane polarised light from N_1 falls on the compensator normally and makes an angle θ with the optic axis of the first wedge. Due to variation of effective thickness along the compensator from the midpoint, it behaves as a wave plate. For those points of compensator with phase difference $n\lambda$, the emergent light is plane polarised only. As N_2 is in crossed position, this plane polarised light is stopped and we get a set of 'dark bands' in the field of view. At the intermediate positions in the compensator, the phase difference will be $(2n+1)\lambda/2$ and the emergent light is plane polarised is inclined at an angle of 2θ with the incident plane polarised light. So N_2 will not stop these rays . When $\theta = 45^0$, the light from these places will be completely transmitted and we obtain bright bands in field of view. Hence we obtain dark and bright bands in field of view (Fig. 12.13). Positions of the elliptically polarised light will be inbetween these bright and dark band.

For caliberation, the micrometer screw of wedge is rotated, so that the dark band coincides with the cross-wire of micrometer piece through which we are observing. Then the reading of micrometer screw θ_1, is noted. The screw is again turned to get the next dark band at the cross-wires and reading θ_2 is noted .The difference between $\theta_1 - \theta_2 = \alpha$ gives the angle of rotation α for a path difference of λ or phase difference of 2π.Using α we can determine the constants of elliptically polarised light.

1. Phase difference :-

If an elliptically polarised light is incident on Babinet's Compensator, it can be resolved into two components ,one parallel to the axis and other perpendicular to the axis . The two components are

$$x = A\sin(\omega t + \alpha);$$
$$y = B\sin(\omega t + \beta)$$

So, phase difference between two components is $\alpha - \beta$. Now the Babinet's compensator introduces a phase difference of δ given by

$$\delta = \frac{2\pi}{\lambda}(\mu_e - \mu_0)(t_1 - t_2) \text{ at a given point on the compensator.}$$

So the total phase difference between o-ray and e-ray is $\alpha - \beta + \delta$ .If phase difference= $2n\pi$,we get a dark band and if phase difference $= (2n+1)\pi$,we get bright band.

The monochromatic source is replaced by white light source which gives central band and coloured side bands. The micrometer cross-wire is coincided with central dark band. There the white light source is replaced with monochromatic light source. Then it is made to pass through Nicol N_1 and quarterwave plate so that elliptically polarised light emerges from quarterwave plate. This elliptically polarised light is made to incident on the Babinet's Compensator.Now when we view through mocrometer eyepiece, we observe a shift in the dark band. The screw of Babinet's compensator is moved through a distance 'x', so that the dark band again coincides with the cross-wires. Now the phase difference between two components of elliptically polarised light is given by $\Phi = \frac{2\pi}{\alpha} \times x$.

2. Position of axes :-

White light source is used and plane polarised light is obtained from Nicol N_1. Then after passing through Babinet's compensator ,it forms dark and bright bands in the field of view. Micrometer cross-wire is coincided with central dark band.Now the screw of Babinet's Compensator is rotated through $\alpha/4$ such that it introduces $\lambda/4$ path difference or $\pi/2$ phase difference. Now white light source is replaced with monochromatic source and light incident on compensator is elliptically polarised. Now the central dark band is displaced. By rotating compensator screw,it is again brought back to cross-wires. The analyser N_2 is also rotated such that the central dark band is perfectly dark. Then "the axis of elliptically polarised light are exactly parallel to the optic axes of the wedges of the compensator".

3. Ratio of the axes :-

If the compensator screw is rotated through an angle ϕ ,to bring back the central dark band to cross-wires then the ratio of axes of the ellipse if given by

$$\text{Tan}\phi = b/a.$$

12.17 OPTICAL ACTIVITY :-

The property of rotating the plane of vibration of plane polarised light about its direction of travel by some crystal or by some solution is known as Optical Activity. The phenomenon is known as optical rotation and angle through which the plane of vibration is rotated is called as angle of rotation (Fig. 12.14).

There are two types of optically active substances . Substances which rotate the plane of vibration in clockwise direction are called 'dextro-rotatory' or 'right handed ' substances . The substances which rotate plane of vibration in anti-clockwise direction are 'laevo-rotatory' or 'left handed' substances.

The angle of rotation 'θ' is directly proportional to the length of the substance through which polarised light travels directly proportional to the concentration c of the solution and inversely proportional to square of the wavelength of plane polarised light.

$$\theta \propto l$$

$$\theta \propto c$$

$$\theta \propto /\lambda^2$$

$$\Rightarrow \theta \propto lc/\lambda^2$$

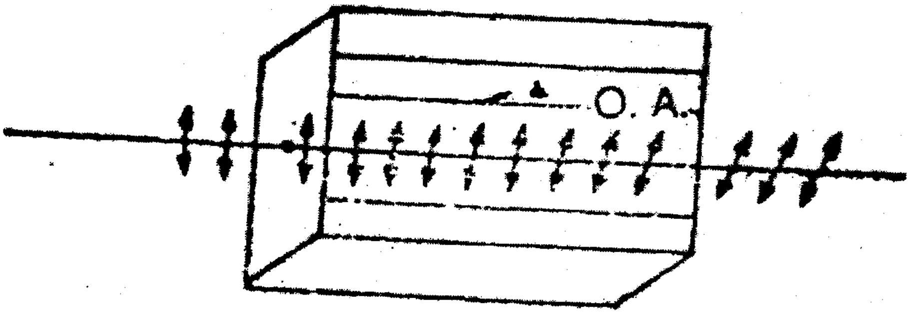


Fig 12.14

Specific Rotation 'S':-

The Specific rotation of a substance at a particular temperature and for a given wavelength 'λ' of light ray may be defined as the rotation (of plane of vibration of polarised light) produced by one decimeter length of its solution when the concentration is 1gm/cc.

$$S = \frac{\theta}{l \times c}$$

12.18 LAURENT'S HALF SHADE POLARIMETER :-

Polarimeters are the instruments designed to measure the angle of rotation produced by a substance. They can be used to find the specific rotation of sugar solution when concentration is known and vice-versa. If they are caliberated directly to find the percentage of cane-sugar in a sugar solution, they are termed as Saccharimeters.

Construction :-

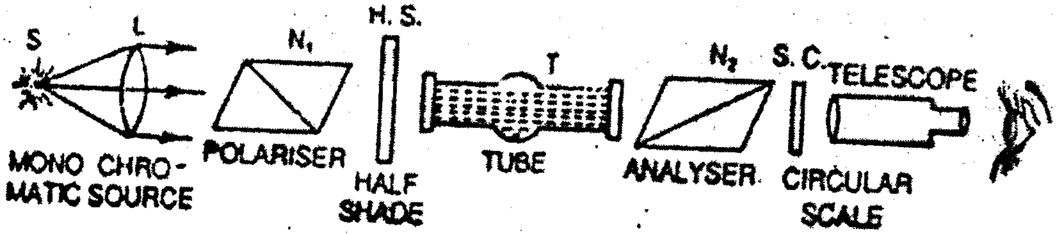


Fig 12.15

S' is a monochromatic source of light. Light from 'S' is rendered into parallel beam by lens L. The parallel beam of light is incident on the Nicol prism N_1 which acts as a polariser. The plane polarised light from N_1 is made to incident on a half shade device and then enter into a hollow tube 'T' filled with sugar solution (or any other solution). The diameter of the central part of tube T is large to avoid air bubbles in the solution. The polarised light is rotated by this solution (optical activity). This rotated light is incident on second crossed Nicol N_2 which acts as a analyser. The rotation of analyser N_2 can be measured using the circular scale attached to it. It consists of vernier scale, which can read upto a fraction of degree. The emergent light from N_2 is viewed through a telescope (see Fig.12.15).

Action of half shade

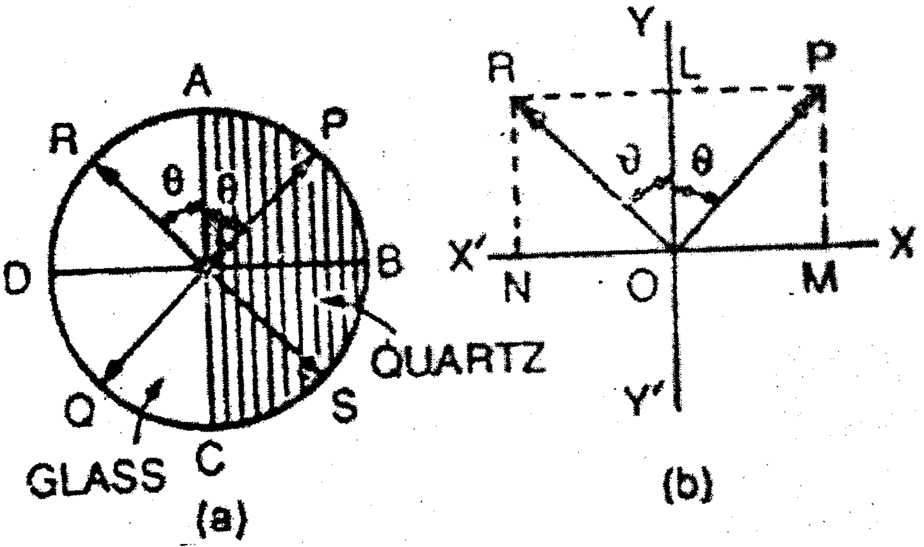


Fig 12.16

When an optically active substance is introduced between two crossed Nicols, field of view in telescope is not dark, as expected. This is because the plane of vibration of polarised light is rotated by optically active substance. Now if we rotate Nicol N_2 also, no where the field of view is perfectly dark. To overcome this difficulty, half-shade plate is used.It consists of semi-circular halfwave plate. ABC made of quartz. So it introduces a phase change of π between ordinary and extra-ordinary rays coming out from Nicol N_1. Another semi-circular glass plate ADC is cemented to ABC along diameter AC. The thickness of glass plate is such that it absorbs same amount of light as quartz plate ABC.

Let 'θ' be the angle between plane of vibration of polarised light and AC. It comes out of glass plate without any change along PQ (see Fig. 12.16). But when they enter quartz plate, the polarised light is split into ordinary ray along XX^1 direction and extra-ordinary along YY^1. Although both the components travel along same direction, they travel with different speeds.

velocity of 0-ray > velocity of e-ray and the phase difference between them is π. Hence the direction of o- ray is reversed and emerges along ON. [It is travelling initially along OM]. Hence the resultant of ON and OL will be along OR making an angle θ with Y-axis. "Thus the vibration of the beam emerging out of quartz will be along RS.

If the plane of analyser N_2 is parallel to PQ, glass plate looks bright . If it is parallel to RS, quartz half appears bright. If it is parallel to AC or Y-axis, both the portions appear equally bright.If Nicol N_2 is slightly rotated from this position also, we observed drastic intensity change in field of view. This particular aspect of equal brightness position of quartz and glass is used in finding the specific rotation of the solution.

Determination of Specific Rotation :-

$$s = \frac{\theta}{l \times c}$$

Solution of known concentration 'c' is prepared. The length of the solution is measured in the tube T directly. The tube is initially filled with water. Viewing through telescope, second Nicol N_2 is rotated till we observe half-shade equally bright in field of view. Then the circular scale attached to N_2 is read. Then the tube is filled with solution of known concentration.Now if we observe through the telescope the field of view will be partially bright.Now N_2 is again rotated till we observe equal brightness of glass and quartz parts. The reading of N_2 is noted. The difference between the previous reading of Nicol N_2 and present reading gives the angle of rotation θ.

Experiment is repeated for different concentrations of the solutions and a graph is drawn between C and θ. We obtain a straight line graph from which we can obtain θ/c .Using 'l' and θ/c values, we can determine the specific rotation 's' using above formula.

12.19 SOLVED PROBLEMS :-

1.A ray of light is incident on the surface of a glass plate of refractive index 1.55 at the polarising angle. Calculate the angle of refraction ?

Solution :-

$\mu = \tan p$ where μ is the refractive index of glass

p is the polarising angle.

$\mu = 1.55$

$\therefore 1.55 = \tan p \Rightarrow p = 57^0 10^1$

Angle of refraction = r

But from Brewster's Law

$$r + p = 90^0 \Rightarrow r = 90^0 - p$$

$$= 90^0 - 57^0 10^1 = 32^0 50^1.$$

2. The critical angle of incidence of water for total reflection is 45^0. What is the polarising angle and angle of refraction of light incident on water at an angle that gives maximum polarisation of reflected light ?

Solution :-

Refractive index of water $\mu = \dfrac{1}{\sin c}$

where c is the critical angle .c= 45^0

$$\mu = \frac{1}{\sin 45^0} = 1.345$$

If 'p' is the polarising angle , then according to Brewster's Law

$$\mu = \tan p$$

$$1.345 = \tan p$$

$$p = 53^0 22^1$$

Angle of refraction r

$$= 90^0 - p$$

$$= 90^0 - 53^0 22^1$$

$$= 35^0 28^1$$

3. Calculate the thickness of a quarter wave plate of quartz for light of wavelength of wavelength 5893 A^0 assuming the principle refractive indices as 1.5442 and 1.5553 ?

Solution :-

Thickness of quarter wave plate 't' for a given wavelength 'λ'is given by

$$t = \frac{\lambda}{4(\mu_e - \mu_0)}$$

μ_0, μ_e be the refractive indices of ordinary and extra- ordinary rays

$$\mu_0 = 1.5442, \mu_e = 1.5553, \lambda = 5893A^0$$

Substituting in above equations

$$t = \frac{5893 \times 10^{-8}}{4(1.5553 - 1.5442)} = 0.001327 \text{ cm}$$

4. A half wave plate is constructed for a wavelength of 6000 A^0 . For what wavelength does it work as a quarterwave plate ?

Solution :-For quarter wave plate $t = \dfrac{\lambda_1}{4(\mu_e - \mu_0)}$

For half wave plate $t = \dfrac{\lambda_2}{2(\mu_e - \mu_0)}$

So according to the problem $\lambda_2 = 6000A^0$

From the above two equations, when the same plate acts as quarter wave plate and half wave plate 't' is the source in two equations

$$\lambda_1 / 4 = \lambda_2 / 2 \Rightarrow \lambda_1 = 2\lambda_2$$

$$\therefore \lambda_1 = 2 \times 6000A^0 = 12,000A^0$$

So it acts as a quarterwave plate for wavelength 12000 A^0

5. A tube of sugar solution 2 cm long is placed between crossed Nicols and illuminated with light of wavelength 6×10^{-5} cm. If the optical rotation produced is 13^0 and specific rotation is 65^0, determine the strength of the solution ?

Solution :-

Concentration of the solution 'C' is $C = \theta / l \times s$

where θ =optical rotation produced in plane of vibration $=13^0$

s= specific rotation of the solution $=65^0$

l=length of the sugar solution =2 cm

Concentration=$\frac{13}{2 \times 65} = \frac{1}{10}$, So percentage of solution strength = 10%

12.20 SUMMARY :-

Polarisation is the phenomenon exhibited by light when they pass through crystals like Calcite, Tourmaline, Quartz etc which proves that light rays are transverse in nature. Plane of polarisation and vibration are defined. Brewster's Law proves that the light can be polarised by reflection completely when $\mu = \tan p$. Law of Malus gives the intensity of transmitted light through analyser. Concept of double refraction, ordinary and extra-ordinary rays is explained. Construction and working of Nicol prism on basis of double refraction to produce and analyse polarised light is dealt in detail. Quarter waveplate, half wave plate positive and negative crystals, dichorism, polaroids are briefed. The production and analysis of circular, elliptical and plane polarised light is detailed. Importance of Babinet's Compensator to produce variable phase difference between o-ray and e-ray are explained and also the production and determination of constants for elliptically polarised light is highlighted. Optical activity and specific rotation are defined. Laurent's half shade polarimeter in determination of concentration of sugar solution is taken in detail.

12.21 KEY WORDS :-

Longitudinal wave, Transverse wave, Polarisation, Unpolarised and Polarised lights, Plane of polarisation, Plane of vibration, Polarising angle, Polariser, Analyser, Optic axis, Double Refraction, Ordinary ray, Extra-ordinary ray, Negative crystal, Positive crystal, Quarter wave plate, Half wave plate, Dichorism, Polaroid, Elliptically polarised light, Circularly polarised light, Compensator, Phase difference, Path difference, Optical activity, Specific rotation, Polarimeter, Half-shade device, Crossed Nicols.

12.22 SELF ASSESSMENT QUESTIONS :-

12.22.1 LONG ANSWER QUESTIONS :-

1. Describe the construction and working of Nicol prism. Explain how it is used as polariser and an analyser ?

2. Write an essay on the prodution and analysis of different types of polarised light ?

3. Describe the phenomenon of double refraction in uniaxial crystals. Distinguish between negative and positive crystals. How can it be explained with Huygen's theory ?

4. Give the construction of Babinet's Compensator and explain how you would use it to analyse elliptically polarised light ?

5. Define Specific rotation. Describe the construction and working of Laurent's half shade polarimeter . Explain how you would use it to determine the specific rotation of sugar solution ?

12.22.2 SHORT ANSWER QUESTIONS :-

1. Explain the terms 'Plane of vibration' and 'Plane of polarisation' ?

2. Explain the method of producing polarised light by reflection. Explain Brewster's Law?

3. Write a short note on Law of Malus ?

4. Describe the phenomenon of double refraction in uniaxial crystals ?

5. What is a quarterwave plate ? What is its use ?

6. What is meant by a. Plane polarised light
   b. Circularly polarised light
   c. Elliptically polarised light

7. What are negative and positive crystals ?

8. What is meant by optical activity ? Define Specific Rotation ?

9. Explain the action of half-shade device ?

10. What is meant by crossed Nicols ?

12.22.3 EXERCISE :-

1. Calculate the Brewster's angles for the following liquids : Ethyl Alcohol for which $\mu = 1.361$ and Carbon tetrachloride for which $\mu = 1.461$

[Ans: $53^0 42^1, 55^0 36^1$]

2. The critical angle in a certain substance is 45^0. What is the polarising angle ?

[Ans : 35.26^0]

3. A ray of light is incident on the surface of benzene of refractive index 1.5. If the reflected light is linearly polarised, calculate the angle of reflection ?

[Ans : $33^0 42^1$]

4. An analysing nicol examines two adjacent plane polarised beams A and B whose planes are mutually perpendicular. In one position of analyser beam B shows zero intensity .From this position, a rotation of 30^0 shows the two beams matched . Deduce I_A / I_B of two beams ?

[Ans :1/3]

5. Calculate the thickness of a mica sheet required for making a quarter wave plate $\lambda = 5460A^0$. The indices of refraction for ordinary and extra-ordinary rays in mica are 1.586 and 1.592.

[Ans: 2.275×10^{-3} cm]

6. Calculate the thickness of halfwave plate made of quartz to be used with sodium light $\lambda = 5893A^0, \mu = 1.544, \mu_e = 1.553$

[Ans : 3.273×10^{-3} cm]

7. A sugar solution of specific rotation 52^0/decimeter/gm/cc causes a rotation of 12^0 in a column of 10 cm long. What is the concentration of the solution ?

[Ans : 0.23 gm/cc]

8. Calculate the specific rotation if plane of polarisation is turned through 26.4^0 ,traversing 20 cm length of 20 % sugar solution ?

[Ans : 66^0]

12.23 REFERENCE BOOKS :

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2. B.Sc Optics	Telugu Academy
3. Introduction to modern Optics	G.K.Fowles(Holt-Rinchert)
4. Optics	Ghatak
5. B.Sc Second year Physics	Dr.K. Ramakrishna D.V.Brahmaji A.Srinivasa Rao S.L.V. Mallikharjun

Unit IV
LESSON-13

LASERS, FIBER OPTICS AND HOLOGRAPHY

Objectives: We are able to
1.To understand the laser principle, working and its appliations.
2.Obtain the knowledge of parts of an optical fiber, types of fibers and modes of propagation.
3.To understand the prinicple behind the fiber communication technology.
4.Understand the basic principle of Holography a three dimensional imaging technique. Get an overview of construction of hologram and reproduction of three dimensional image.
5.Know about few appliations of holography.

Structure of the lesson

13. 1. LASERS :-

13.1.1. Introduction :

Laser is a device which produces intense, monochromatic, coherent light in a narrow parallel beam. Laser is the short form of "Light Amplification by Stimulated Emission of Radiation". The basic principle behind laser is Einstein's stimulated emission of radiation.

An atom consists of electrons in different energy states called ground states. When the atom collides with another energy particle, it atains some kinetic energy from colliding particle. This energy is utilized in exciting the electron from ground energy state to upper energy state. The other method of excitation of electrons is through <u>irradiation</u>. It means, when optical radiation of energy $h\upsilon$ is incident on the atom, the electron in the ground state absorbs the energy $h\upsilon$ and gets excited to higher energy level. Number of electrons excited to higher energy levels through absorption N_{Ab} is given by

$$N_{Ab} = B_{12}N_1\rho(\upsilon)\Delta t$$

N_1 - number of atoms in ground state

$\rho(\upsilon)$ - energy per unit volume in frequency range υ and $\upsilon + d\upsilon$

B_{12} -proportionaly constant,

Δt -time

So absorption of light energy by the atoms in the material leads to attenuation of light travelling through the medium

These electrons in the excited state will not be there for infinite time. They get deexcited to ground state in time of order 10^{-8} sec, known as 'life time' of the exicited state. So the deexcitation process is of two types

1. Spontaneous emission and
2. Stimulated emission.

13.1.2. Spontaneous emission :-

As shown in fig.13.1 the process of deexcitation of electrons excited to higher energy levels after the life time of the energy level, by themselves to ground level is termed as "spontaneous emission".

No. of spontaneous emission taking place in time t is given by

$$N_{sp} = A_{21}N_2\Delta t$$

where A_{21} -Proportionality constant,

N_2 - No. of electrons in excited state

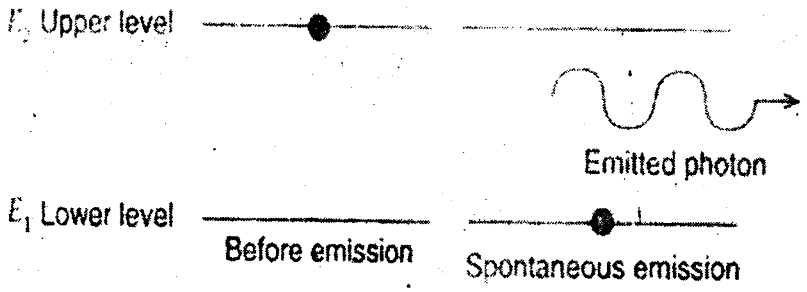


Fig . 13.1

Probability of transition occuring per second P_{sp} is given by

$$P_{sp} = \frac{N_{sp} / \Delta t}{N_2} = A_{21}$$

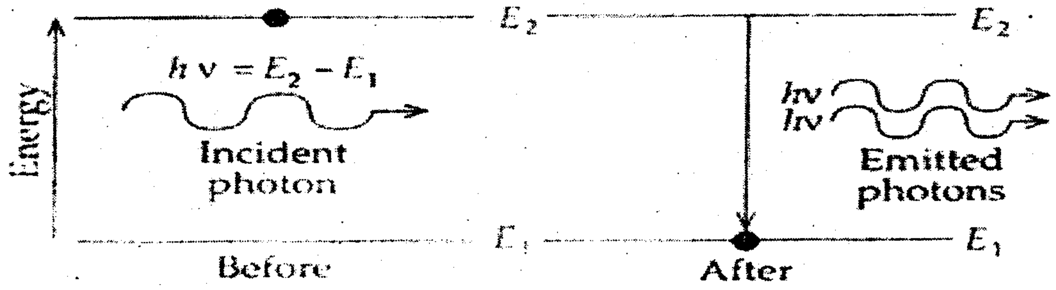
The photon emitted during spontaneous emission of the material **can be** any wave length or frequency $h\upsilon$ and may be emitted at any time (No phase relationship) and in **any** direction (non-directionality). So the light emitted during spontaneous emission is incoherent, non-monochromatic and without directionality.

Ex : Incandescent lamp emission, Emissions or light from monochromatic sodium vapour **lamp.**

13.1.3. Stimulated emission :-

Suppose the energy of ground level is E_1 and of the excited level in which e^θ us present in E_2 as shown in fig. 13.2. The energy difference between two levels **is** $E_2 - E_1$. Suppose a photon of energy $h\upsilon = E_2 - E_1$ hits the atom, it deexcites the electon from **excited** energy state to ground state. This process is termed as " stimulated emission"

During stimulated emission when one photon of energy $h\upsilon$ hits the atom, it gives **out** two photons of energy $h\upsilon$ i.e. $2h\upsilon$ in same direction, in same phase and same state of **polarization.** These two photons releasing from one atom, deexcite two more electrons,in excited



Fig,. 13.2

state in the material and give out four photons of same energy $h\nu$. In this way, a single photon $h\nu$ stimulating one deexcitaion gets multiplied by chain reaction as shown in the figure 13.3..

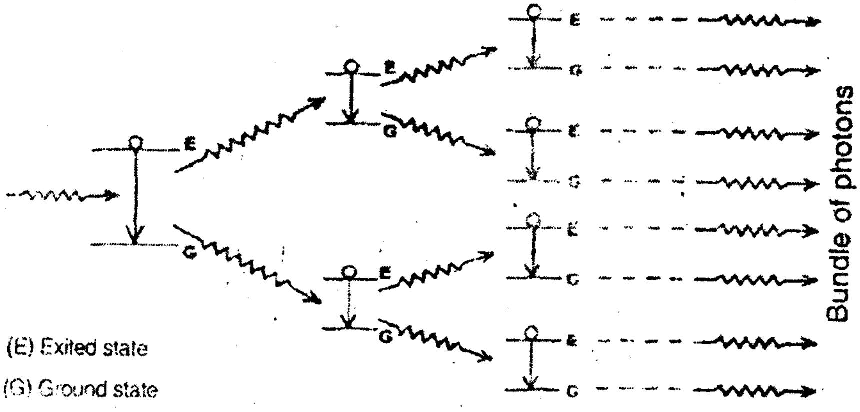


Fig. 13.3

Hence due to stimulated emission, we obtain an intense beam of light [Due to multiplication of photons with chain reaction] of same wave length or frequency ν, in same direction and at same time (same phase) and in same state of polarisation.The resultant wave which is highly monchromatic,coherent and intense is called 'LASER'.

Number of atoms deexcited per second by stimulated emission process N_{st} is given by

$$N_{st} = N_2 B_{21} \rho(\nu)$$

N_2 -density of atoms in exicited state

B_{21} -proportionality constant

$\rho(\nu)$ -spectral energy density

Under thermal equilibrium the rate of upward transitions is equal to the rate of downward transitions. Thisis known as 'Principle of detailed balancing."

$$N_{ab} = N_{sp} + N_{st}$$

The equation proposed by Einstein indicating the stimulated emission is

$$A^* + h\nu \rightarrow A + 2h\nu$$

where A^* is atom in excited state, $h\nu$- stimulating photon, A-atom in ground state, 2 $h\nu$- photons emitted from deexcited atom.

13.1.4. Condition for stimulated emission :-

Starting stimulated emission in ordinary excited atoms of the materials is not an easy process. According to Boltzmann, in thermal equilibrium, the population of energy levels is fixed by the equation

$$\frac{N_2}{N_1} = \exp\left[\frac{-(E_2 - E_1)}{kT}\right]$$

Equation indicates that the number of electrons in ground stae are always more than in excited state ($N_1 > N_2$). But stimulated emission needs reverse condition i.e. ($N_2 > N_1$). This condition is termed as "Population inversion". Also the population of electrons decreases exponentially as one goes to higher energy states.

To obtain the condition of population inversion i.e. number of atoms in excited state $N_2 >$ number of atoms in ground state N_1. The life time of excited state should be more i.e. of the order of 10^{-3} sec. Such energy state with large life time is termed as "metastable state". Generally the life time of ordinary excited states in the materials are 10^{-8} secs. So if we continuously pump energy to the material and take the electrons to the excited state, they come to ground level is 10^{-8} sec and we cannot obtain population inversion.

But crystals containing impurity atoms are taken. The energy levels of impurity atoms will be in between the energy levels of crystal is forbidden energy gap. These have lifetime oforder 10^{-3} sec. So when the electrons in ground state of crystal (E_1) are excited to next energy level

(E_2) they say there for 10^{-8} sec. Then they are made to get deexcited to E_3 level of impurity atom, in between E_1 and E_2. Since the lifetime of $E \approx 10^{-3}$ sec, more electrons get accumulated in E_3 and a state is reached when $N_3 > N_1$ where N_3 no. of electrons in E_3 state and N_1 number of electrons in E_1 state. Hence state of "population inversion" is achieved and E_3 is the "metastable state". Hence we can achieve the condition of stimulated emission by taking crystals with certain impurity atoms.

13.1.5. Laser principle:-

To produce lasers, we should follow the following steps.

1. <u>Induced absorption</u> :-

Exciting electrons in ground state E_1 to excited state E_2 by irradiating with light or photons of energy $h\upsilon$ (see fig. 13.4).

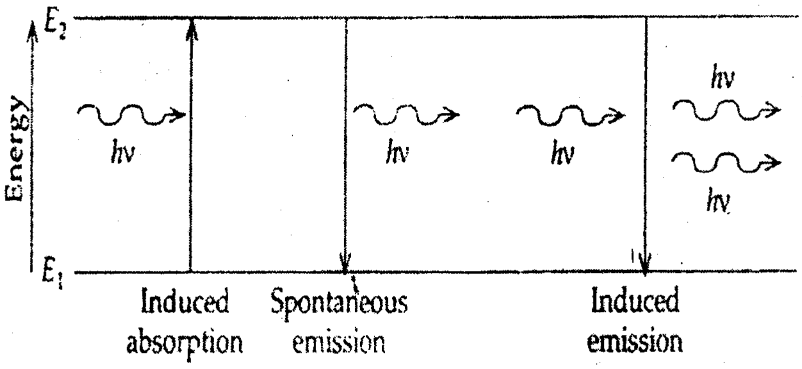


Fig. 13.4

2.Create a meta stable state in the material which has longer life time. Due to meta stable state conditions of population inversion i.e. more number of atoms in excited state than in the ground state is achieved. Population inversion is achieved by "pumping".

3.Once population is achieved - emission is induced. Implies the material is deexcited by hitting it with a photon $h\upsilon = E_2 - E_1$. This photon deexcites the atom and we obtain two photons of energy $h\upsilon$ in same direction and in same phase. These two photons deexcite two more atoms. This chain action continues giving out intense radiation which is monochromatic and unidirectional.This is nothing but the "Laser".

13.1.6. Pumping :-

As we have mentioned above, population inversion condition is achieved by a process called as "pumping". Now we will consider different types of pumping techniques and pumping schemes. The meaning of pumping is to transfer an atom from the ground state to an excited state to achieve population inversion in the medium.

Generally three types of pumping schemes are used.

1 .Optical pumping :-

Optical energy (light energy) is used to excite the atom from lower energy level E_1 to higher energy leverl E_2. High power flash lamps are used for this excitation . Ruby lasers and liquid dye laser which abosrb incident light considerably use these flash lamps.

2. Electrical Excitation :-

An electric field of several kV/m is applied between electrodes to produce electrons from cathode. These accelerated electrons while travelling towards anode, hit the atoms of laser species. The laser species receive the kinetic energy of electrons and go to excited states.

Ex: Used in He-Ne lasers and Argon ion lasers.

Sometimes instead of excited electron in cathode, high current is injected into barrier region in semi-conductor lasers, to excite the electrons to higher energy levels.

3. Chemical excitation :-

When two reacting species form the basic material for laser, the thermal energy release in exothermic chemical reactions is used to excite the atoms to higher energy levels or for population inversion. This priciple is used in CO_2 laser.

13.1.7. Pumping schemes :-

Before understanding three level and four level pumping schemes, let us define certain terms. The transition between the two levels that generate stimulated emission is called a 'lasing transition'.The lower level in lasing transition is called lower lasing level and upper level is called as upper lasing level. the upper level to which the electrons in lower enrgy state are excited is called as pumped level.

1. Three level pumping scheme :-

E_1, E_2, E_3 are three energy states of an atomic system E_1 - ground state. E_2, E_3 excited states. When a photon of frequency $\upsilon_1 = \frac{E_3 - E_1}{h}$ is incident on the system, the electrons in ground state get excited to E_3 level (Fig. 13.5). Since its life time is only 10^{-8} sec, they fall from E_3 to E_2 [a small part to E_1 also] by spontaneous emission. The difference in the energies ($E_3 - E_2$) is released as thermal energy and is given to the crystal. After some time more than half of the ground state atoms are acumulated at E_2 creating population inversion between E_1 and E_2. One spontaneous emission from E_2 to E_1 emit a photon $h\upsilon = E_2 - E_1$. This photon starts stimulated emission between E_2 and E_1 emitting laser.

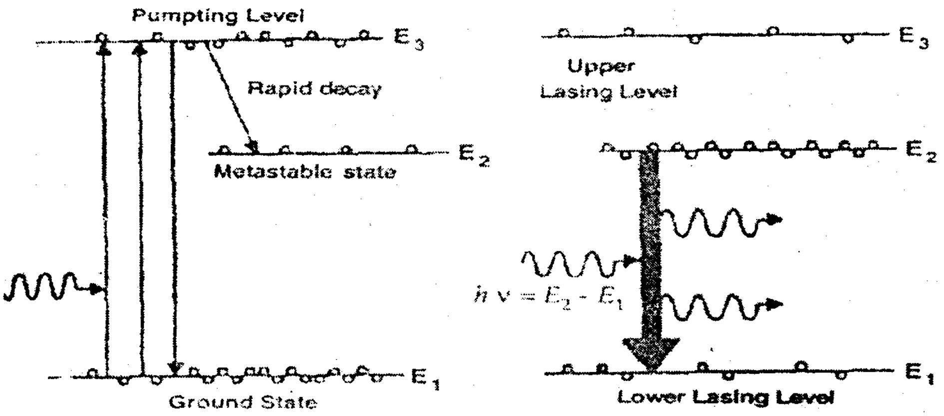


Fig. 13.5

Quantum efficiency of this laser $\eta = \frac{h\upsilon_{out}}{h\upsilon_{in}} = \frac{E_2 - E_1}{E_3 - E_1}$

2. Four level pumping schemes :-

E_1 is the ground state. E_4 is the pumping level, E_3 is the meta stable state and upperlasing level E_2 is the lower lasing level. The atoms or electrons in the ground state E_1 are excited to E_4 or pumped to E_4 (see fig. 13.6). The electrons in E_4 spontaneously dexcite to E_3 level. It is a level of high life time.So more number of electrons remain in this level. Generally E_2 is empty. So population inversion is obtained betwen E_3 and E_2.The life time of E_2 should be very small, so that electrons will not accumulate in E_2 against population inversion.

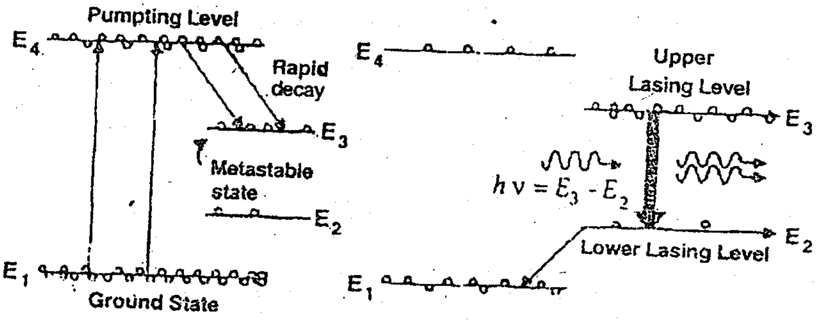


Fig. 13.6

A single spontaneous emission between E_3 and E_2 gives as shown in Fig. 13.6 a photon $h\upsilon = E_3 - E_2$. This photon starts lasing action between E_3 and E_2 givng out laser .Since E_2 has short life time, they fall back to E_1 .So always population inversion exists between E_3 and E_2.This scheme is used in He-Ne laser.

Quantum efficiency of this laser is

$$\eta = \frac{h\upsilon_{out}}{h\upsilon_{in}} = \frac{E_3 - E_2}{E_4 - E_1}$$

TYPES OF LASERS :-

13.1.8 Ruby laser :-

Ruby laser was first developed by Theodre Maimann. It is prepared as optically homogeneous, highly pure, insulator crystalline solid.Ruby is basically Al_2O_3 crystal with chromium ions as impurities. The red colour of Ruby is mainly due to Cr^{+3} ions concentration.

Construction :-

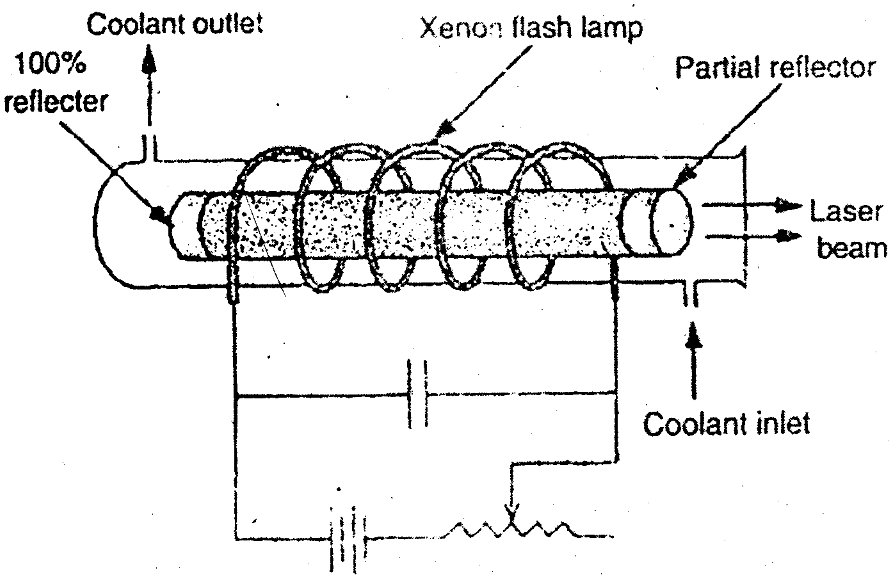


Fig. 13.7

As shown inabove fig. 13.7 Ruby rod is taken in the form of cylindrical rod of about 10 cm in length and 1 cm in diameter. Its ends are grounded and optically polished such taht the end faces are optically plane and exactly parallel and also perpendicular to the axis of rod. One face is silvered totally to produce complete reflection and other face is semi-silvered for partial reflecton .These two silvered faces act as optical resonationg cavity. The rod is surrounded by helical photographic xenon flash lamp to produce high intense flashes of white light whenever activated by a power supply. The system which gets heated due to pumping radiation is cooled with the help of coolant, circulated around the rod.

Working :-

E_1 is the ground state [belongs to Al_2O_3] as shown in fig. 13.8, E_3 and E_2 levels correspond to the chromium ions.These are not single levels, but band of energy levels. E_2 and E_3 are pumping levels.When xenon flash falls on the ruby rod, the green and blue components of the white light are absorbed by the chormium ions and raise to one of the energy levels E_2 or E_3.

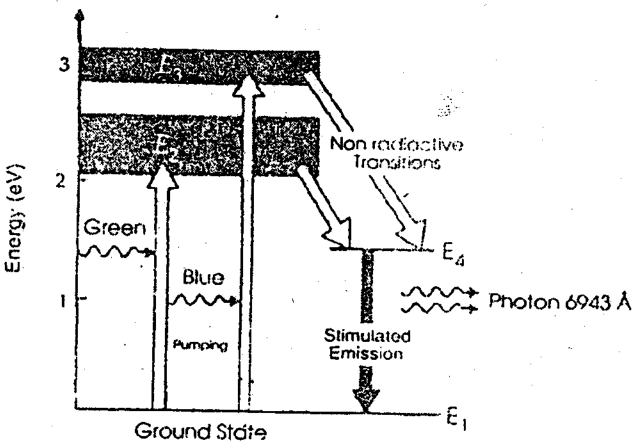


Fig.13.8

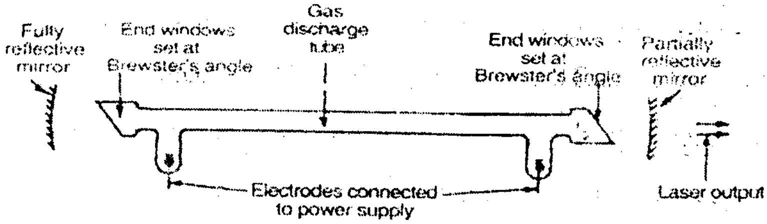
Due to their short life period in these level, they get deexcited to energy level E_4 through non-radiative transitions. E_4 level is the upper lasing level as well as metastable state. When population inversion. occurs between E_4 and E_1, some of electrons from E_4 fall to E_1 giving spontaneous emission. This spontaneous emission photons start stimulated emission between E_4 and E_1 and produces Laser light. When these photons are reflected by the two polished faces of ruby rod to and fro, they increase the stimulated emission. When the total emergent laser light reaches sufficient intensity, it emerges out of the partially silvered mirror.

The Xenon flash remains only for few seconds. During that period lasing action will not take place. Lasing action starts when population inversion occurs between E_4 and E_1. When E_4 is completely deleted of the electrons, lasing action stops. So laser light comes in the form of short pulses in Ruby laser but not continuous laser. Again the whole process should repeat for next pulse. Also the green and blue light energies are only used in flash light. Remaining light energy and the non-radiative emissions try to heat up the Ruby rod. Hence coolant is circulated around the Rubyrod.

13.1.9 He- Ne LASER :-

He-Ne laser is a gas laser. It is a mixture of Helium and Neon gases and is a "Continuous Laser"

Construction:



As shown in fig. 13.9, it consists of a long discharge tube of length about 50cm and diameter 1cm filled with a mixture of He- Ne gas in the ratio 10:1. Discharge is produced in the gas by arranging electrodes at the end and applying high voltage. The tube is sealed with two inclined windows at the end. Two reflections,one fully reflective and other partially reflective are arranged at the two ends of the tube.

Working :-

In He-Ne laser, Neon energy levels provide lasing action.Helium atoms provide efficient mechanism for excitation of neon atom. When power is switched on ,the electrons are released from cathode by gas discharge and travel towards the anode. In between they collide with light Helium atoms which are 10 parts of whole gas and transfer energy to them. So the helium atoms get excited to He_2 or He_3 levels with this energy. He_2 and He_3 are metastable states (fig. 13.10), hence the atoms remain in this state for longer time. They cannot return to ground state by spontaneous emission .So they transfer this excited energies to the Neon atom and Neon atoms excite to Ne_4, Ne_6 states of same energy. After transfer of energy He atoms return to ground and are again axcited by the field.

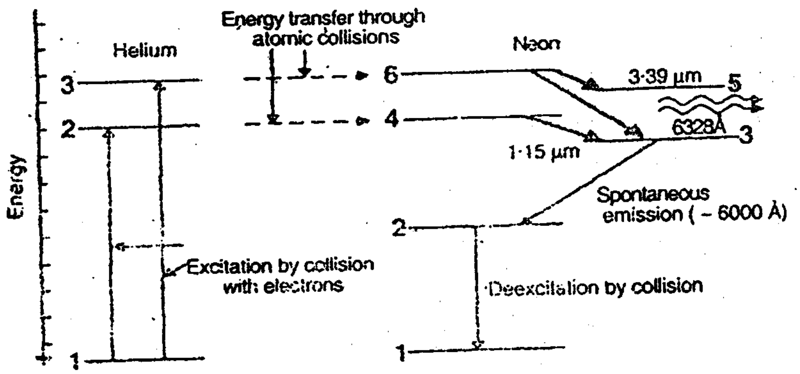


Fig 13.10

Now as more and more collisions occur between Helium and Neon atoms, Ne_6 and Ne_4 levels get more populated. Hence population inversion takes place. Now the transitions possible by stimulated emission are

$Ne_6 \rightarrow Ne_5 \rightarrow$ giving light of wavelength of 3.39 μm

$Ne_6 \rightarrow Ne_3 \rightarrow$ giving light of wavelength of 6328 A^0

$Ne_4 \rightarrow Ne_3 \rightarrow$ giving light of wave length of 1.15 μm

But we intend to obtain laser light of wavelenght 6328 A^0 {red colour} only. So to avoid other two wavelengths, the end windows of the tube are made of glass or quartz which absorb 1.15 μm and 3.39 μm radiations. The reflectors are also coated with thick dielectic material which reflect 6328 A^0 units only continuously. From lower lasing levels Ne_3 and Ne_5, atoms get deexcited to Ne_2 level. So Ne_2 may become more populated. However they (Neon atoms) give up their energy by collisions with the walls of the tube. Hence the tube is made narrow, so that the collisions are more and Neon atoms come to ground state. It is a lowpower device.

13.1.10. Einstein coeffeicients :-

Let us consider an assembly of atoms in thermal equilibrium at a temparatureT having two states. Let N_1 and N_2 be the no of atoms /unit volume in E_1 and E_2 energy levels ($E_2 > E_1$). When an electromagnetic radiation of energy $h\upsilon = E_2 - E_1$ fall on this

system, there are three possibilities

1. spontaneous emmision
2. Induced absorption
3. Stimulated emission.

1. No of atoms jumped / second from E_1 to E_2 i.e induced absorbtion rate N_{ab} is given by

$$N_{ab} = B_{12} N_1 \rho(\upsilon)$$

N_1 - number of atoms in E_1 state per unit volume

$\rho(\upsilon)$ - spectral energy density

B_{12} - Einstein coefficient of induced absorption

2. Number of atoms emitted by excited atoms per unit volume N_{sp} by spontaneous emission is given by

$$N_{sp} = A_{21} N_2$$

where N_2 is the number of atoms in the excited state

3. The number of atoms deexcited from E_2 to E_1 by stimulated emission is N_{st} per unit volume

$$N_{st} = B_{21} N_2 \rho(\upsilon)$$

B_{21} = proportionality constant

$\rho(\upsilon)$ = energy density in the range υ and $\upsilon + d\upsilon$

Energy of stimulating photon $= h\upsilon = E_2 - E_1$

The stimulated emission rate $N_{st} = B_{21} N_2 \rho(\upsilon)$

B_{21} - Einstein coefficient of stimulated emission

Fig,. 13.9

For system in equilibrium, upward and downward transition rates must be equal according to Einstein

$$B_{12}N_1\rho(\upsilon) = B_{21}N_2\rho(\upsilon) + A_{21}N_2$$

$$\rho(\upsilon) = \frac{A_{21}N_2}{B_{12}N_1 - B_{21}N_2}$$

Divide numerator and denominator by N_2B_{21}

$$\rho(\upsilon = \frac{\left(A_{21}/B_{21}\right)}{\left(\frac{B_{12}N_1}{B_{21}N_2}\right)-1}$$

After simplifying equation we obtain $B_{12} = B_{21}$

and $\frac{A_{21}}{B_{21}} = \frac{8\pi h\upsilon^3}{c^3}$

The above two equation are referred to as 'Einstein relations'. B_{12}, B_{21}, A_{21} are termed as Einstein coefficients.

13.1.11 Properties of Laser light :-

1.The line width of the laser light is $\Delta\lambda = 0.01$ nm i.e. it is highly monochromatic.

2.The maximum length of the resulting wave from a source on which any two points can be correlated is called coherence length and corresponding time is called coherence time. This tells us about the coherence of the light. Lasers are highly monochromatic with large coherence length 600km and largest coherence time of 2×10^{-3} sec. So it is highly coherent.

3.Directionality :-

Lasers emit light in one direction without any deviation upto to very large distances. The directionality is denoted by the angular spread $\Delta\theta$ given by $\Delta\theta = 1.22\lambda/d$ This is as small as 10^{-5} to 10^{-6} radians for lasers.

4. Lasers are bright and intense light source. An one milliwatt He-Ne laser is 10,000 times brighter than light from sun at earth surface.This is due to high coherence and directionality.

5. Lasers can be 'tuned' to emit radiations over a range of wavelengths.

13.1.12. Applications of lasers :-

1.Lasers in metrology :-

Lidar (Light detection and ranging) is an optical radar used to measure the distances between moon and earth upto an accuracy of centimeter. This gives information about drift of continents of earth, moons spiralling away from earth and so on.

2.Industry :-

Laser can be focussed into a fine beam and increase the temperature at focussed point upto to 1000k. So they are used to drill holes, welding and cutting of metals.

3.Communication :-

The amount of information that can be sent by an electromagnetic wave or carrier wave depends on its band width. The bandwidth of lasers is very large, hence it is used in optical and space communications.

4. Medicine :-

Lasers are used for welding retinal detachment by opthalmologists and also for cataract removal. Lasers are used in destroying kidney stones and gall stones.They are used in cancer diagnosis and therapy. Lasers are used as scalpels in bloodless surgery.

5.Defence :-

Lasers are used ot guide missiles and satellites for detecting and destroying enemy planes or tanks.

6.Nuclear fusion :-

Lasers can be used to induce the nuclear fusion.

7.Holography :-

Lasers are used in recording and reproduction of holograms

8.Electronic industry :-

Lasers are used as supermarket scanners to detect the bar codes and display the item code and price. Diode lasers are used to record and repoduce data from compact discs.

13.2. Optical Fibers :-

13.2.1. Introduction :-

Optical fibre (Fig. 13.11) is a thin glass of strand or plastic strand that serves as a medium in optical communcation. The communication is done by light through optical fibres.Communication through optial fibres is used in telephone, computer communications and also in the automation of factories. Due to large band width of laser light used in optical fibre communication, large infomation can be transferred than in radio waves or microwaves.

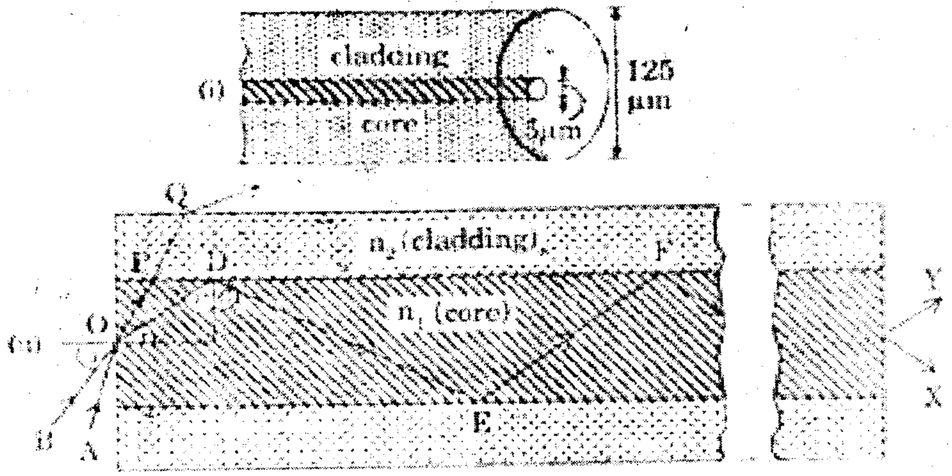


Fig .13.11

An optical fibre is a hair thin structure having a core diameter of 100 µm to 1000 µm and extremely light an can replace many copper cables because of its large information carrying capacity. The optical fibre has electromagnetic immunity and has very law losses of the signal during communication.Importanct appliation of optical fibres are in medicine. A bundle of fibres is used to illuminate the operated area in the body and other bundle of fibres are used to catch reflected light .This is the principle of endoscope . In retinopathy and angioplasty optical fibres are used as wave guides.

13.2.2. Modes of propagation :-

Mode is simply a path that a light ray follows in travelling through an optical fibre .When the light waves enter into a optical fibre, they get totally internally reflected when angle of incidence is greater than the critical angle. But only those waves which undergo constructive interference pass through optical fibre. Only the modes that fibre will support depends on $\frac{d}{\lambda}$ where d is the diameter of the core and λ the wavelength of the wave propagating.The number of modes supported by an optical fibre varies from one to one hundred thousand in a multimode fibre.

13.2.3 Types of optical fibres :-

1.Clasification based on materials :-

a) Glass fibres :-

These optical fibres are made of glass . Both core and cladding are glass. The glass used is ultra pure and ultra transparent silicon dioxide or fused quartz . Impurities are added to them to change the refractive index inside core or cladding .Germanium or phosporous impurities are added to increase the refractive index . Boron and fluorine impurities decrease the refractive index.

b) <u>Plastic-clad silica fibers</u> :-

This optical fibre has glass core and a plastic cladding.

c) <u>Plastic fibres</u> :-

These optical fibres will having plastic cladding and plastic core. These are comparitively limited in losses and bandwidth. They have got the advantage of security and electromagnetic immunity.

2.<u>Classification based on the refractive index of the core and the modes the fibres propagates:-</u>

Depending on mode of propagation, they are divided into single mode fibre and multimode fibre.

a) <u>Single mode fibres</u>:-

As shown in the Fig 13.12 these will be having a very narrow core of diameter 5 µm or less. Comparatively the cladding is big. Due to narrow core, single mode only can propagate parallel to the central axis through this type of fibre.

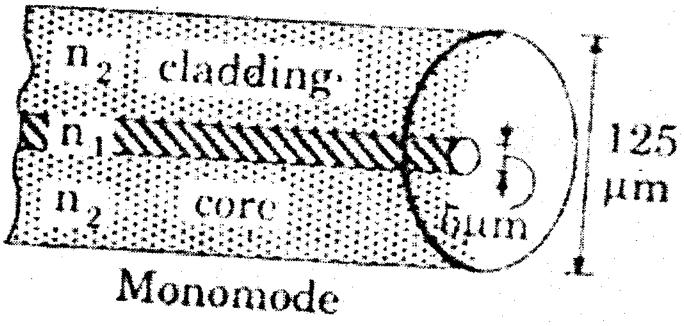


Fig. 13.12

Because there is a single mode through fibre, noise is eliminated These are used in long distance transimissions and laser beams are used. They are expensive **but best** in performance.

b) <u>Multimode fibres</u> :-

Core has a diameter of about 50 µm . So it can propagate upto 1,00,000 signals at a time. Depending on the nature of refractive index distribution of the material of the core, these are again divided into two types.

1.<u>Step index multimode fibres</u> :-

The refractive index of core is n_1 which remains constant through out the core. The refractive index of cladding which starts from cladding is n_2. So the refractive index varies from n_1 to n_2 at the boundary of core **and cladding**. It is termed as step index multimode fibre, as shown in fig 13.13.

But this type of optical fibres are not suitable for long distance propagation, but are relatively less expensive. Also $n_1 > n_2$

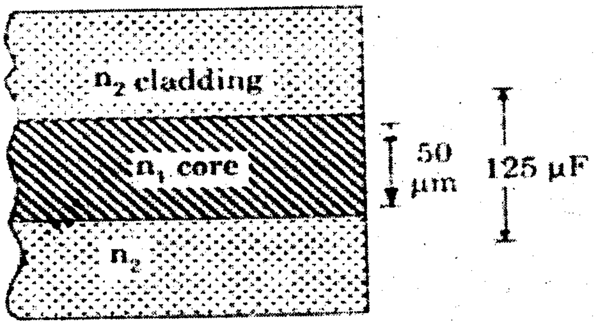


Fig. 13.13

2. Graded-index multimode fibers :-

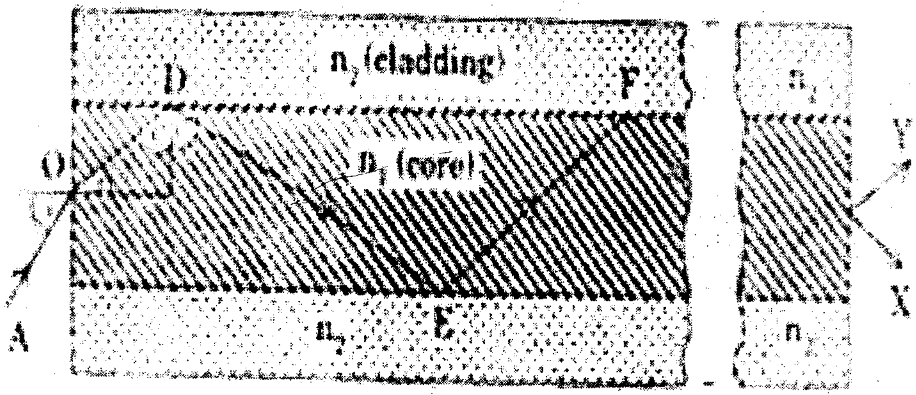


Fig. 13.14

As shown in above figure 13.14, the refractive index varies continuously in a parabolic manner from the centre of the core upto the boundary of the cladding. The refractive index of cladding < the refractive index of the fibre at the boundary of the core. Light signals are transmitted more efficiently in graded index fiber in smooth spiral.paths around the central axis There are of intermediate cost and used for intercity communication.

13.2.4. Rays and Modes in an optical fibers :- (Fig. 13.15 given below)

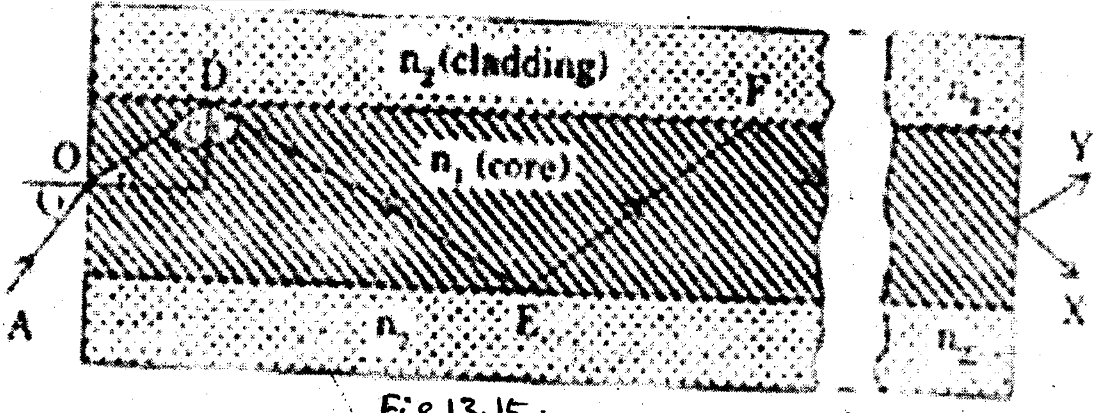


Fig 13.15.

The light travels through optical fibres on the principle of total internal reflection with out loss of intensity. As shown in the above figure 13.15, the light ray enters into the core along AO. After travelling through core of refractive index n_1 along OD (slighly refracted), it touches the boundary of core and cladding at an angle 'C' which is the critical angle for core-cladding boundary. Refractive index of core $n_1 >$ refractive of cladding n_2. Hence the ray is entering from denser to rarer medium at critical angle, hence gets total internally reflected along DE. Again the same total internal reflection takes place at E. Thus the light travels in Zig-Zag paths inside the core [For step-index fibre] and comes out of the fibre along X- or Y- direction.

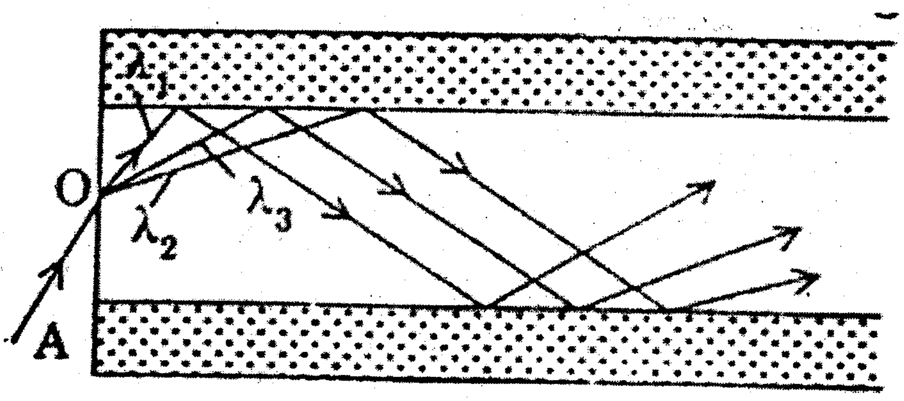


Fig. 13.16

If instead of a single wave or signal, three signals of wavelengths $\lambda_1, \lambda_2, \lambda_3$ enter the core of step index at critical angle and more than critical angle respectively. All the three undergo total internal reflections (Fig. 13.16).

But due to different angles of incidence on the boundary, they trace different paths and distances and come out of the optical fibre at different times. Hence although all the three signals entered at the same time, they come out at different times. This defect is termed as distortion in transmission of signal in step-index fibre.

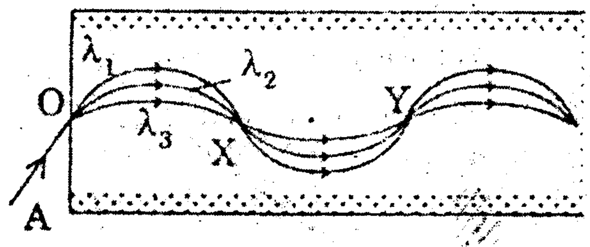


Fig. 13.17

But if we consider a graded index fibre, the refractive index of core changes continuously from the centre to the boundary, So the three signals are continuously bend towards central axis of the fibre, tracing semi-circular paths. Although λ_3 traces more path than λ_2 and λ_2 more than λ_1, the signal with larger paths enter into regions of lower refractive index, hence travel at high speeds. The signals with lesser paths,enter higher refractive index areas in core and travel with lower speed.Finally all the signals come out of the fibre at same time without any signal distortion (Fig. 13.17).

13.2.5.Principles of fibre communication :-

Optical fibres are mainly used in high speed telecommunications. The main advantage is that light is used as signal carrying wave and there is no loss of signal due to total internal reflecton propagation inside the optical cable. We can also send more messages per cable length due to their band width and also clearer sound. Higher the frequencyof the signal, higher would be the information carrying capacity (Fig. 13.18).

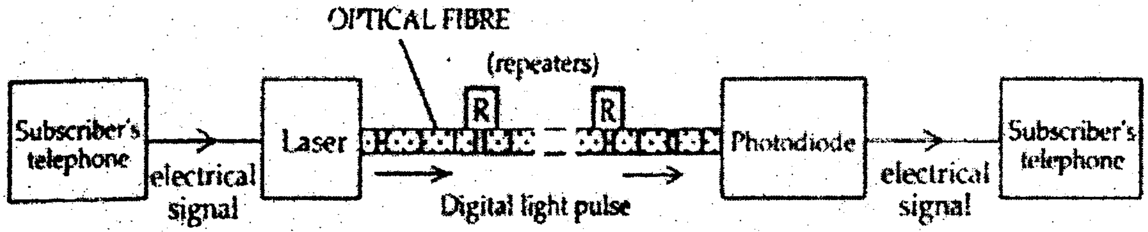


Fig. 13.18

1.Subscriber telephone converts sound signals into continuous electrical signals.

2. The analog to digital converter converts continuous electrical signals into digital pulses.

3.These digital electrical pulses are sent into optical transmitters [Lasers in the block diagram]. There they are amplified and semi-conductor laser is used to convert electrical signals or pulses into optical pulses.

4. The optical pulses are transimitted through optical fibers. But there may be loss in the intensity of signals due to absorption and scattering. Hence repeaters (R,R) are used to amplify the optical signals during transmission through fibres.

5. Then on reaching the destination, optical pulses are again converted into electrical pulses by photodiode. These are again converted into continuous electrical signals by digital to analog converters.

6 The receiver's telephone converts the electrical signals into sound signals.

7. Time - division multiplexing is used to transmit several thousands of telephone calls along a single optical fibre by using light pulses in digital form.

13.2.6. Advantages of Fibre Optic Communicatons :-

1. Optical fibres are cheaper, small in size. Light weight, flexible but mechanically strong.
2. Optical fibres are insulators, so there will be no electric shocks and signals are also immune to surrounding electromagnetic or radio frequency radiations.
3. They have very wide band width and low losses per unit length.

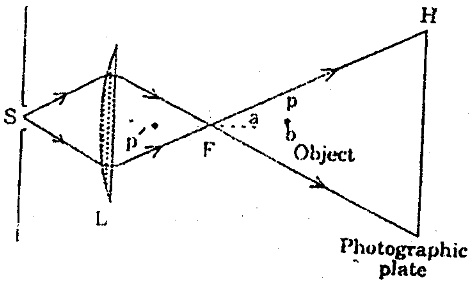
13.3. HOLOGRAPHY

13.3.1. Inroduction :-

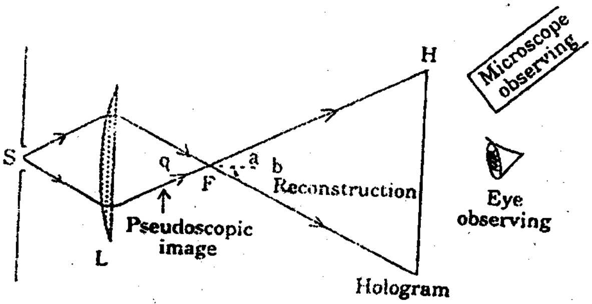
Any real life object is a three dimensional object. So it has different perspectives, when viewed through different angles. But when we try to take the photograph of the object, the intensity of the light reflected or scattered by the object is only recorded and the phase information of the light is lost. So when we develop the photograph we can view only two dimensional image with single perspective. But 'Holography' is a photography which records both the intensity and phase of the light scattered by the object and produces a three dimensional image of the object. This technique needs highly coherent light beam, which was available after the advent of lasers. So the coherent light is made to incident on the object and the reflected or scattered beam is made to interfere with a reference beam to produce interference pattern corresponding to the object. This interference pattern is recorded on a photographic plate and is termed as 'Hologram'. When this hologarm is illuminated with coherent light, it produces the three dimensional image of the object in front of us, as good as the object. This technique is termed as Holography.

13.3.2 Gabor Hologram :-

The actual arrngements to produce and reproduce Gabor holograms are shown in the figures (Fig. 13.19 a and b)



(a) Making hologram



(b) Reconstructing the im.

Fig. 13.19

The coherent light from source 'S' is incident on a lens 'L' with focus at F. The beam gets focussed at F, and diverges from that point. The object whose whole hologram has to be recorded is b .It should be transparent and smaller in size than the divergent beam. It is placed at a small distance from F. Now the light obstructed by the object b creates a diffracted wave. The undisturbed light out side the object acts as a carrier wave. Care is taken that the intensity of carrierwave is more than the diffracted wave. These two waves which are coherent interfere with each other forming interference pattern resembling the intensity of light and phone of the wave from the object. Due to strong carrier wave, the regions of constructive in terference, are more bright and dark regions perfectly dark. A photographic plate H is placed at a short distance from 'b' and interference pattern is recorded. So the negative consists of high intense interference spots as dark regions and viceversa. The positive developed from this negative is the 'Hologram'. Now if we place the hologram is same arrangement without the object as shown in set up (b), we can see the pseudoscopic image of 'b' at 'q' with naked eye placed before hologram or viewing through hologram.

`13.3.3. Limitations of Gaborhologram :-

1. During the advent of Gabor holograms strong coherent lights, which remain coherent for long distances like lasers were not available.

2. Gabor couldnot fully record the phase information as the intensity information on photographic plates available during those days.

3. Reconstruction has to be done by pseudoscopic image and hologam has to be illuminated with point source from rear side of hologram.

13.3.4. Applications of holography :-

1. Holographic interferometry :-

The deformations in surfaces of different industrial machines or scientific materials produced by stress can be detected and measured using "double exposure technique" of Holography. First the interference pattern of the surface with out stress is recorded on the photographic plate.Then the surface is exposed to stress and again its interference pattern is recorded on same photographic plate. When this hologram is developed or produced, the view of hologram shows the regions under stress and their extent.

2. Holographic microsocopy :-

With ordinary micorscopes we can record only still photograph of a biological event. But we cannot record continuous bilogical process happening three dimensionally . But using holography technique, we can record time varying biological or any other phenomena. Then afterwards viewing through the hologram, we can observe the depths of scene or three dimensional view of the scene at leisure, in detail .Useful in scientific research.

3. Acoustic holography :- Images formed by soundwaves can be recorded and reproduced by hologaphy.This technique is used mainly to view the under water objects and internal organs of our body three dimensionally. Coherent ultrasound wave is produced and sent into water towards the object (object beam). The object beam reflected from the object (water object or internal organs of our body) and the reference ultrasound wave are made to interfere with each other. This interference pattern is recorded on photographic plate to produce hologram .Viewing through hologram, we obtain the three dimensional information of the object or the organ.

4.Holographic cinema :-

Projection screen itself will be a large hologram which behaves like a multiple elliptical mirror Real images can be projected on to the viewer in cinema hall from series of hologams.

5. Hologaphy is used in spatial filtration, character recognition , long distance hologaphy with microwaves, rainbow holograms, integral holograms, holographic optical elements etc.

13.4. Solved problems :-

1. In an optical fibre the refractive index of core is 1.52 and that of cladding is 1.48. Calcualte the critical angle related to the system ?

Solution

$$\text{Sin C} = {}^{n_2}\!/_{n_1} \; ; \qquad n_1=1.52 \;, \; n_2=1.48$$

$$\therefore \sin C = \frac{1.48}{1.52} = 0.974$$

2. In an optical fibre, the refractive index of the core is 1.52 and that of cladding is 1.48 vhat is the maximum launching angle such that all the light gets totally reflected inside the fibre?

Solution :-

$$\frac{\sin i}{\sin r} = n_1 \Rightarrow \sin i = n_1 \sin r,$$

$$\sin C = \frac{n_2}{n_1},$$

$$\text{But } r = 90^0 - C$$

$$\Rightarrow \sin r = \sin(90^0 - C) = \cos C$$

$$\therefore \cos C = \frac{\sin i}{n}$$

$$\text{But } \sin^2 C + \cos^2 C = 1$$

$$\Rightarrow \frac{n_2^2}{n_1^2} + \frac{\sin^2 i}{n_1^2} = 1$$

$$\Rightarrow \sin i = \sqrt{\left(n_1^2 - n_2^2\right)}$$

$$\text{But } n_1 = 1.5 \text{ and } n_2 = 1.48$$

$$\text{Sin i} = \sqrt{\left(1.52^2 - 1.48^2\right)}$$

$$= 0.2828,$$

$$i \approx 20^0 \text{ (approximately)}$$

3. Consider a laser that emits a wavelength of λ=550 nm. If population inversion had not been generated , what if the ratio of the population of the atoms in state E_x to that in the ground state E_0?

Solution :-

$$\frac{N_x}{N_0} = \exp\left[\frac{E_x - E_0}{kT}\right]$$

Separation between energy levels

$$E_x - E_0 = h\upsilon$$

$$= \frac{hc}{\lambda}$$

$$= \frac{(6.63\times10^{-34})(3\times10^{8})}{(550\times10^{-9})(1.6\times10^{-19})}$$

$$= 2.26 \text{ eV}$$

The mean energy of thermal agitation kT of an atom at room temperature (300K) is

$$kT = (8.62\times10^{-5})(300)$$

$$= 0.0259 \text{ eV}$$

$$\therefore \frac{N_x}{N_0} = e^{-87.26} \approx 1.3\times10^{-38}$$

4. If a light ray is incident perpendicularly to one face of a trianglualr prism, it is totally reflected internally at glass - air surface. If angle of incidence between normal at the slant surface and incident ray , $\theta_1 = 45^0$ what will be the angle of refraction of glass ?

Solution :-

Refractive index of air $n_2 = 1$

Refractive index of glass $n_1 = n$

critical angle $C = \sin^{-1}\left(n_2/n_1\right) = \sin^{-1}\left(1/n\right)$

Since total internal reflection occurs , $\theta_c < \theta_1$ then

$$\sin^{-1}\left(1/n\right) < 45^0$$

$$\Rightarrow \frac{1}{n} < \sin 45^0$$

$$\Rightarrow n > \frac{1}{\sin 45^0}$$

$$n = 1.4$$

13.5. Summary :-

Laser is a device which produces intense, highly coherent and highly directional monochromatic beam of light. The process of spontaneous emission, stimulated emission, population inversion and pumping schemes are explained. Laser principles and lasing actions are explained. Different types of lasers are introduced. Applications and properties of lasers are briefed.

Optical fibres are the wave guides for optical communication. Construction of optical fibres, types of optical fibers and modes of transmission in optical fibres are briefed. Principle of optical communication through optical fibres are introduced.

Holography is a photographic technique which produces the three dimensional image of the object by recording and reproduction of hologram. Construction of gabor hologram and reconstruction of the image, its limitations are explained. Few applications of holography are explained.

13.6. Keywords :-

Lasers, Stimulated emission, Spontaneous emission, Population inversion, Metastable state, Pumping optical resonator, Lasing levels, Lasing transitions, Radio active and non radiative transitions, Line width, Coherence, Directionality, LIDAR, Laser welding, Laser scalpels, Scanners, Compact disc, Optical fibres, Core, Cladding, Mode of propagation, Band width, Single mode, Multimode, Step index fibre, Graded index fibre, Signal distortion, Optical amplifier, Repeater, Photo diode, Decoder, Time division multiplexing

13.7. Self assessment questions :-

13.7.1 Long Answer questions :-

1. Explain the principle and working of Ruby laser ?
2. What is "Laser" ? Explain the population inversion and metastable state and their role in laser action.
3. What is pumping ? Explain various pumping techniques. Explain three level pumping schemes ?
4. Explain the construction and working of He-Ne laser.
5. Explain the propagation of light rays and modes of propagation.
6. Explain the advantages of optical fibres in optical communication with a neat block diagram, explain qualitatively th principles of fibre communication ?
7. Explain the principles and applications of holography. How is the Gabor hologram prepared and reconstructed?.

13.7.2 Short Answer questions :-

1. Distinguish between Spontaneous and Stimulated Emissions ?
2. Explain Einstein Coefficients.
3. Explain the important characteristics of a Laser.

4. Explain the terms (a) Population Inversion and (b). Meta stable state ?
5. Mention various applications of Lasers .
6. What is a Laser and what is the principle of Laser ?
7. Explain (a). Step index fibre (b). Graded index fibre
8. What are modes of propagation? How are Optical fibres classified on the basis of modes of propagation ?
9. Classify the optical fibres on basis of materials used and their merits.
10. Compare holography and photography.
11. Explain how a hologram is prepared and viewed.
12. Explain the basic principles of holography.

13.8 REFERENCE BOOKS :

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2. B.Sc Optics	Telugu Academy
3. Introduction to modern Optics	G.K.Fowles(Holt-Rinchert)
4. Optics	Ghatak
5. B.Sc Second year Physics	Dr.K. Ramakrishna D.V.Brahmaji A.Srinivasa Rao S.L.V. Mallikharjun

B.Sc. Physics Second year

Practical Manual

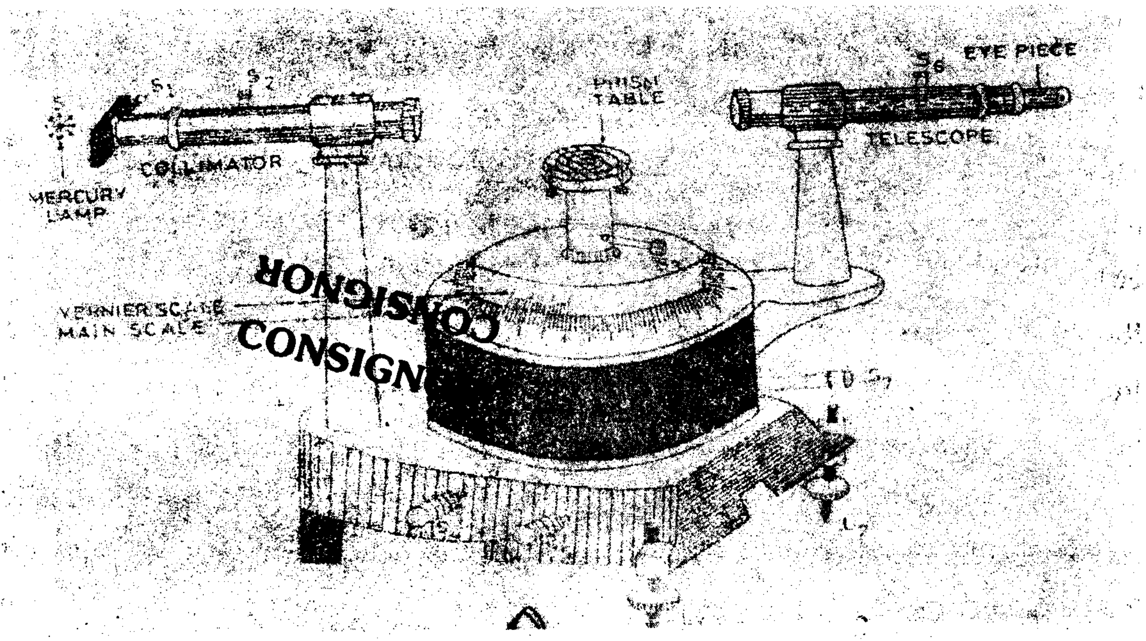
CONTENTS

SPECTROMETER DESCRIPTION AND ADJUSTMENTS

Aim: -To give spectrometer description and to explain the initial adjustments of the spectrometer before using it.

Apparatus: - Spectrometer

Diagram of Spectrometer:



Description:

The spectrometer mainly consists of

a) Collimeter,

b) Telescope,

c) Prism table and

d) Circular scale and the vernier scale

a). Collimator:

It consists of a convergent lens fitted to the inner end of a hallow tube, fixed to the instrument. Another hallow tube which exactly fits into the fixed tube and can be moved in or out by working a Rock and pinion arrangement caries at its outer end a slit of adjustable width. The axis of the collimator is set perpendicular to the axis of rotation of the prism table. The collimator is fixed to the instrument and can not be rotated. The collimator is used to obtain a parallel beam of light from a given source.

b) Telescope:

This is an astronomical telescope whole objective is fitted to the inner end of a hallow tube. Exactly fitting into this tube there is another hallow tube which can be moved in or out by working a rock and pinion arrangement. At the outer end the tube carries Ramsden's eye piece with cross wires. The cross wires consists generally of the fibers from a spider's web, fixed across the tube one vertically and another horizontally in front of the eye piece. The axis of the telescope is set perpendicular to the axis of rotation of prism table and can be clamped on any position by the screw S_1. The angle of rotation can be measured on a circular scale which is fixed to the telescope and moved along with it. By means of the tangent screw the telescope after it is clamped can be turned through very small angles and thus fine adjustments can be made. The telescope is used to receive the parallel beam of light from the collimator.

c) Prism table:

It is a small circular table provided with three leveling screws and is used for keeping the prism on it. The prism table can be raised or lowered and clamped in any position by a screw. By means of another screw it can be fixed to the vernier table and the two will then turn together. The vernier is provided with a clamping screw and a tangent screw for fine adjustment. The prism table can be rotated about a vertical axis passing through its centre.

d) Circular scale and the vernier scale:
This is a circular metal plate attached to the telescope and rotates with it. Usually graduated in to half degrees and the reading can be noted on two verniers which are fixed diametrically opposite to each other.

Adjustments:

Before the instrument is used for measurement purpose the following adjustments are to be made.

1) Eye piece: The telescope is turned towards a which surface say a wall and the eye piece is moved in or out until the cross wires are seen clearly.

2) Telescope: The telescope is directed towards a distance object say a telegraph post or a tree and by working the pinion the telescope is adjusted until the image of the object is formed in the plane of the cross wires with no parallax between the image and the cross wires. Now the telescope is ready to receive a parallel beam of light.

3) Collimator: The slit of the collimator is illuminated with sodium light. The telescope is brought in line with the collimator and the distance of the slit from the collimating lens is adjusted by rock and pinion until a clear image of the slit with well defined edges is formed in the plane of cross wires without any parallax error and also the slit is adjusted to be vertical and narrow.

4) Prism table: A spirit level is kept on the prism table parallel to the line joining the two leveling screws. The two screws are adjusted until the air bubble of spirit level come to the certre. The n the spirit level is turned on the table perpendicular to this position and the third screw is adjusted until the air bubble comes to the centre. Now the surface of the prism tube will be horizontal.

5) Before taking the readings, least can be measured of the circular scales which are on either side of the table.

Least Count = Main scale division / No. of divisions on the vernier scale = S/N

Experiment No. 1

SPECTROMETER – I-D CURVE

Aim: To draw a graph between the angle of incidence (i) and the deviation (d) produced by a glass prism and hence to determine the angle of minimum deviation (D) and refractive index of material of prism.

Apparatus: Spectrometer, glass prism, monochromatic source of light (sodium vapour lamp) magnifying lens, spirit level

Formula:

$$\mu = \frac{\sin(\frac{A+D}{2})}{\sin(\frac{A}{2})}$$

where μ = refractive index of material of prism

A =angle of prism

D =angle of minimum deviation

Procedure:

The preliminary adjustments of the spectrometer must be done.

Determination of angle of prism: The prism is placed at the centre of prism table such that both refracting edges of prism are facing the collimator symmetrically as shown in figure 1.1. Then the prism is fixed. The telescope is released and rotated to observe reflected image of the slit from one face say AB. The tangent screw of the telescope is worked until the refracted image coincides with vertical cross wire. The readings of the two verniers are noted. The telescope is rotated such that the reflected image of the slit from second face AC is focused. Then the readings of both verniers are noted. Then the difference between the respective readings of vernier gives the value of 2A from which the refracting angle can be determined.

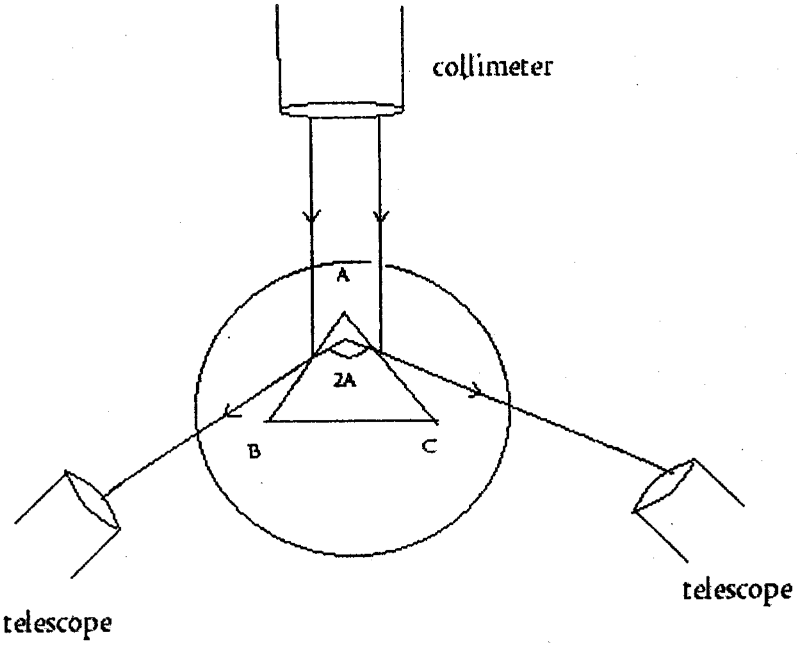


Fig 1.1

To measure angle of deviation: The telescope and the collimator are put in a straight line. The image of the slit is made to coincide with the vertical cross wire. The reading of the spectrometer is noted. From the position the telescope is rotated through an angle $\theta = 180 - 2i$ where 'i 'is the angle of deviation incidence for which the angle of deviation has to be found. The value of ' i' should be started from . The prism is mounted centrally on the table so that its ground surface is parallel to the axis of the collimator.

The prism table is rotated until the reflected image of the slit from one of the polished surfaces of the prism coincides with vertical cross wire. Then the prism table is fixed. The telescope is released and rotated to observe the refracted image of the slit. The refracted image of the slit is made to coincide with the vertical cross wire and the reading of the spectrometer is noted. The direct reading of the telescope is noted by removing prism. The difference between the two readings of spectrometer gives the deviation 'd' corresponding to the angle of incidence' i'. The experiment is repeated by increasing the values of 'i' by steps of and in each case the angle of deviation 'd' is obtained.

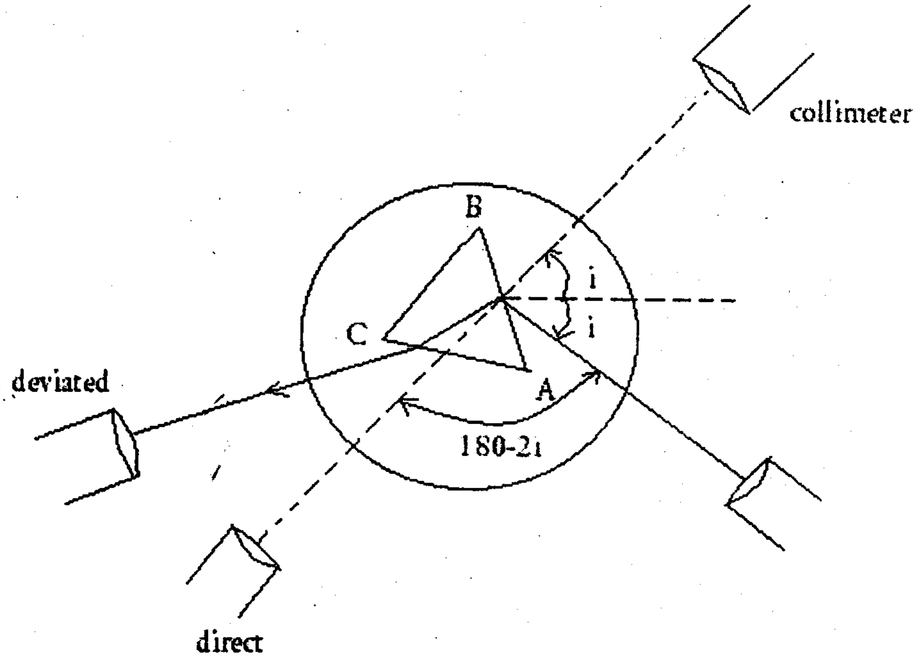
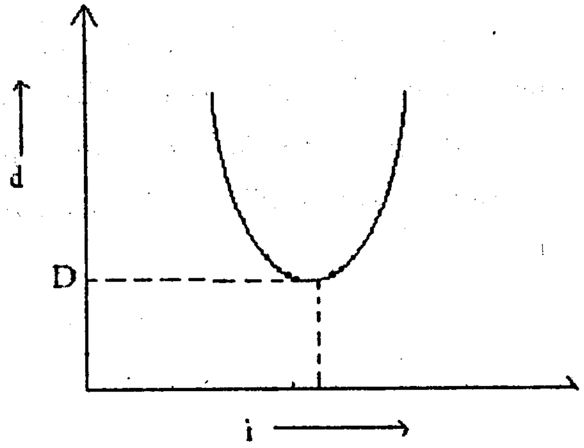


Fig.1. 2

Table:

Angle of incidence	θ= 180-2i	Readings of spectrometer		Direct reading		Angle of incidence		
		V-I	V-II	V-I	V-II	V-I	V-II	Mean
		X_1	Y_1	X_2	Y_2	$X_1 \sim X_2$	$Y_1 \sim Y_2$	

A graph is drawn between angel of incidence i on x -axis and angle of deviation on y-axis. The graph is a curve. The value of minimum deviation is found from graph and the refractive index of glass prism is calculated.



Figl. 3

Precautions:

1. The optical adjustments must be done carefully before starting the experiment.
2. The slit should be narrow as possible.
3. The prism must be set symmetrically on the prism table.
4. The polished surface should not be touched. It should be handled at edges.

Result: The refractive index of a given prism material is = -------------.

Viva Questions and Answers:

1. What is a spectrometer

A. Spectrometer is an optical instrument used for studying the nature of light given out by different sources.

2. What is meant by the angle of the prism?

A. The angle between the two refracting surfaces of the prism is called angle of prism.

3. What is meant by angle of deviation?

A. The angle between the ray emerging from and the ray incident on the prism is called angle of deviation.

4. Define refractive index.

when a ray of light passes from one medium to other medium it is bent towards the normal or away from it at the surface of separation of two media. The ratio of sine of angle of incidence (i) in the first medium to the sine of the angle of refraction (r) in the second medium is called refractive index of second medium with respect to first.

Experiment No.2

SPETROMETER- DISPERSIVE POWER OF PRISM

Aim: To determine the dispersive power of the material of the given prism using spectrometer.

Apparatus: Spectrometer, Prism, Mercury vapour lamp, spirit level

Formula: $\omega = \frac{\mu_2 - \mu_1}{\mu - 1}$ or $\omega = \frac{\mu_b - \mu_r}{\mu - 1}$

where ω = dispersive power of the material of the given prism

μ_b, μ_r = refractive indices of blue and red colours respectively

And $\mu = \frac{\mu_b + \mu_r}{2}$

Procedure: The usual initial adjustments of the spectrometer are to be made. The refracting angle of A of of the prism is found.

Then the prism is mounted on the prism table and the position of the prism is adjusted to observe the spectrum of the mercury vapour. Observing the blue line the spectrum through the telescope, the prisms adjusted for minimum deviation position. Working with the tangent screw of telescope the position of the prism is adjusted so that the blue line is just on point of retracing its path after coming to the point of intersection of the cross wires. The readings of the telescope for the minimum deviation of red line are noted. The telescope is brought in line with the collimator and removing the prism . The readings of direct image on the both verniers are noted. the respective differences give the minimum deviations for blue and red colours. the refractive indices are found by

$$\mu_b = \frac{\sin(\frac{A+D_b}{2})}{\sin(\frac{A}{2})} \quad \text{and} \quad \mu_r = \frac{\sin(\frac{A+D_r}{2})}{\sin(\frac{A}{2})}$$

The dispersive power of the material of the prism for blue and red colours is found by the relation $\omega = \frac{\mu_b - \mu_r}{\mu - 1}$

Observations

Least count (LC) = S/N

Angle of Prism A:

Spectrometer Reading				**Difference in readings**			**Angle A**
Left T_1		**Right T_2**		**$2A = T_1 \sim T_2$**			
V_1	**V_2**	**V_1^I**	**V_2^I**	**$V_1 \sim V_1^I$**	**$V_2 \sim V_2^I$**	**Mean**	

Determination of angle of prism:

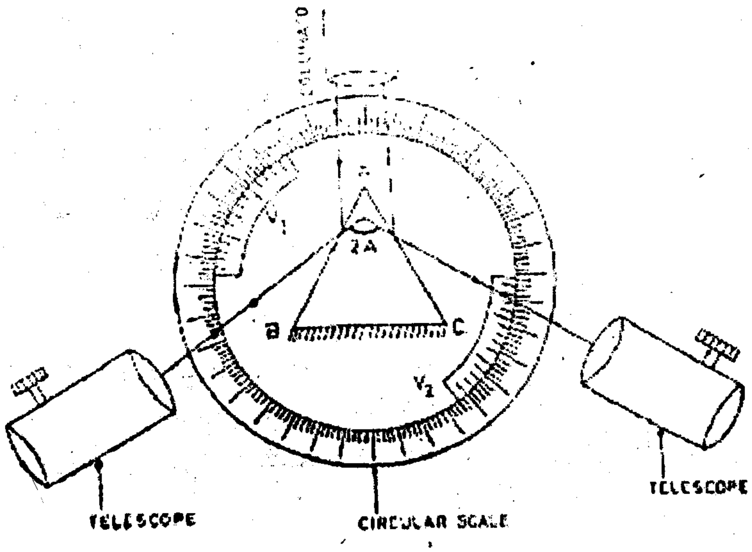


Fig. 2.1

Determination of angle of minimum deviation:

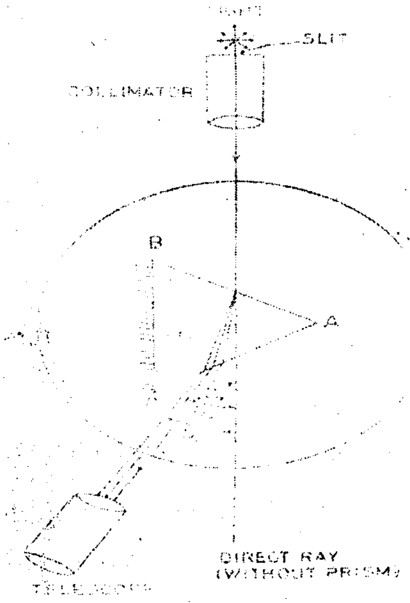


Fig. 2.2

Angle of minimum deviation

S.No.	Colour Spectral lines	Spectrometer Readings		Angle of minimum deviation(a~b)	$\mu = \frac{\sin(\frac{A+D}{2})}{\sin(\frac{A}{2})}$
		Minimum Deviation(a)	Direct Reading(b)		

Precautions:

1. The prism should be adjusted for each colour separately,
2. Readings should be taken without parallax error.

Result: Dispersive power of material of prism (Crown glass) is ω = …………

Experiment No. 3

DIFFRACTION GRATING - NORMAL INCIDENCE

Aim: To determine the wavelength of given source of light using plane transmission grating in the normal incidence position.

Apparatus: Spectrometer, plane transmission grating, magnifying lens, spirit level, mercury vapour lamp

Formula:

$$\lambda = \frac{\sin\theta}{Nn}$$

Where λ = Wavelength of light

θ = Angle of diffraction

N = Number of lines per centimeter on the grating (15,000 /2.54)

n = Order of the spectrum

Description: A plane diffraction grating consists of a parallel sided glass plate with equidistant fine parallel lines drawn very closely on it by means of a diamond point. The number of lines drawn is usually 15000 lines per inch.

Procedure: The initial adjustments of the spectrometer are done. The least count of the vernier of the spectrometer is found

Least Count = Value of one main scale division / total number of vernier scale divisions

a) **Normal incidence:** The slit of the spectrometer is illuminated with mercury vapour lamp. The telescope is placed in line with axis of the collimator and the direct image of the slit is observed. The slit is narrowed and the vertical cross wire is made to coincide with the centre of the image of the slit. The reading of one of the verniers is noted. The prism table is clamped firmly and the telescope is turned through and fixed in position.

The grating is held with the rulings vertical and mounted in its holder on the prisms table such that plane of grating possess through the centre of table and the ruled surface towards the collimator. The prism table is released and rotated until the image of the slit is seen in the telescope by reflection on the ruled side of grating. The prism table is fixed after adjusting the point of intersection of the cross wires is on the image of the slit. Then the vernier table is released and rotated through exactly from this position so that the ruled side of the grating faces the collimeter . The vernier table is fixed in this position and the telescope is brought back to the direct reading position. Now the light from the collimeter incidents normally on the grating.

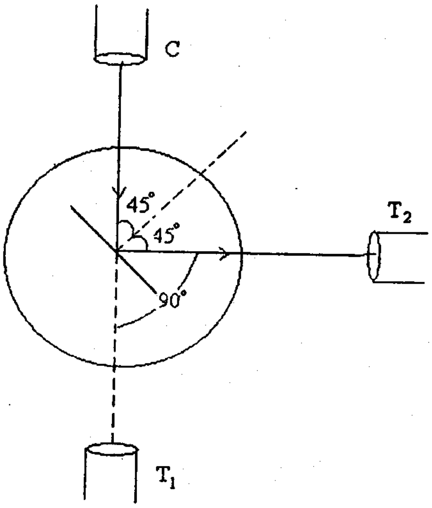


Fig. 3.1

b)Measurement of wavelength:

The telescope is rotated so as to catch the first order diffracted lines on one side say on left (see fig.3.2)

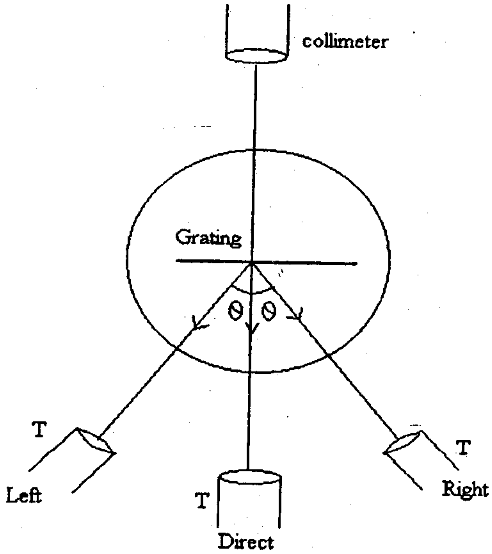


Fig. 3.2

With sodium light two lines very close to each other can be seen. They are called and lines. The point of intersection of cross wires is set on the line and its reading is noted on both the verniers. Similarly the reading corresponding to the line is noted. Then telescope is turned to the other side i.e. right side and similarly the readings corresponding to and lines of first order spectrum are noted. Half the difference in the readings corresponds to any one of the line gives the angle of diffraction. The experiment is repeated for 2nd order spectrum. The number of lines per centimeter of the granting (N) is noted and the wavelength of spectral line is found by the formula

$$\lambda = \frac{\sin\theta}{Nn}$$

Observations:

Number of lines per centimeter (N)

Order of Spectrum	Line	Reading of Spectrometer				2θ			$\lambda = \frac{\sin\theta}{Nn}$
		Vernier 1		Vernier 2					
		Left	Right	Left	Right	Vernier 1	Vernier 2	Mean	
First order n = 1	D_1								
	D_2								
First order n =2	D_1								
	D_2								

Precautions: 1. Always grating should be held by the edges the ruled surface should not be touched.

2. Light from the collimator should be uniformly incident on the entire surface of grating.

Result: Mean value of for D_1 lines = ...cm
D_2 lines = ...cm

Viva questions and answers:

1. What is meant by wave front?

A. When a wave passes through a medium the particles of the medium vibrate. The continuous locus of all the particles vibrating in the same phase is called wave front.

2. Are the spectra of different orders of the same intensity?

A. No. The intensity is maximum in the zero order and decreases with the increase of order of spectra.

3. How many order of spectral lines are formed when a plane transmission grating is used.

A. It depends on the number of rulings on the grating.

4. What will happen if the width of clear space and ruled space is made equal?

A. Even order of spectra 2,4,6... will be absent.

Experiment No. 4

DIFFRACTION GRATING - RESOLVING POWER

Aim: To determine the resolving power of the given grating.

Apparatus: Spectrometer, Plane diffraction grating, Sodium vapour lamp, Magnifying lens, Spirit level, Rectangular slit of variable width and Traveling microscope.

Formula:
The resolving power (R.P.) of grating is given by

theoretical, Resolving power $= \frac{\lambda}{d\lambda} \times \frac{\cos\theta}{a}$

Experimental, Resolving power = Nn

where λ = Mean wavelength of D_1 and D_2 lines (yellow)

$$= \frac{\lambda_1 + \lambda_2}{2} A^o$$

$d\lambda$ = difference in the wavelengths of two close lines D_1 and D_2= $(\lambda_1 \sim \lambda_2) A^o$

θ = angle of diffraction

a = mean width of slit = $\frac{a_1 + a_2}{2}$

where a_2 = width of the slit determined when the telescope is on left side of the direct image

W= width of slit determined when the telescope is on right side of the direct image

N= number of lines per centimeter on the grating (15000/2.54)

n = order of the spectrum

Theory: The resolving power of a grating is its ability to show or just resolve as separate and distinguishable images of two spectral lines whose wavelengths are close together. If and are wavelengths of two spectral lines just resolved by the grating, then the resolving power of grating is measured by where is mean wavelength of the two spectral lines and is the difference between the wavelengths. If a the width of adjustable slit which resolves the and lines of sodium light hen the width of the rating passing through the slit is for normal incidence, where is angle of diffraction. the resolving power for the entire length l of grating is given by $\frac{\lambda}{d\lambda} \times \frac{l\cos\theta}{a}$

this is equal to Nn l where N is the number of lines of the grating per centimeter and N is the order of spectrum i.e. $\frac{\lambda}{d\lambda} \times \frac{l\cos\theta}{a} = Nnl$

Procedure:

The preliminary adjustments to the spectrometer are to be done. Arrange the grating in normal incidence position. Turn the telescope to left side of direct image. First order spectrum consisting of $D_1$and D_2 lines appears. Turn the telescope to observe D_1 and D_2 lines and adjust its position until the vertical cross wire coincides with D_1 and D_2 then fix the telescope such that the slit is vertcial. First open the slit fully to see the first order spectrum fully and clearly. Now slowly reduce the width of the slit till the two lines just merge into a single line. Note the readinss of vernier V_1 and vernier V_2. Remove the width of slit using the micrometer attached to it. Release the telescope and turn to right side of direct image and follow the procedure that is done on left side. Measure the width of slit a_2. The angle of diffraction θ is measured.

Repeat the experiment for second order spectrum also. Find mean of and That gives average or mean width of slit 'a'. Find the length l of the ruled surface with microscope.

note : 1. If sodium light is used then

$\lambda_1 = 5890\ A^o$, $\lambda_2 = 5896\ A^o$

2. If mercury vapour lamp is used

$\lambda_1 = 5770\ A^o, \lambda_2 = 5790\ A^o$

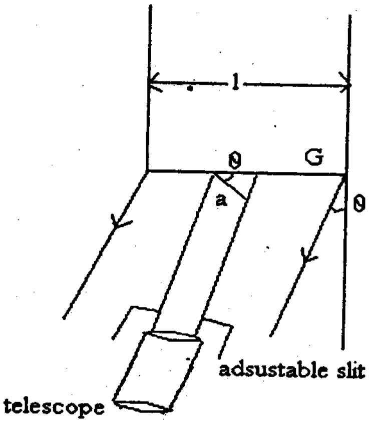


fig.4

To determine Angle of Diffraction θ

Order of the spectrum	Colour of the line	vernier	Telescope Readings						Diff. in vernier Readings 2θ L~R	Mean angle of diffraction θ	Resolving power of grating	
			Lert			Right					$\frac{\lambda}{d\lambda}\times\frac{l\cos\theta}{a}$	Nn Theoretical
			MSR a	VC n	TR (L) A+n.LC	MSR a	VC n	TR (R) A+n.LC				
		V_1										
		V_2										
		V_1										
		V_2										
		V_1										
		V_2										
		V_1										
		V_2										

To determine the width of adjustable slit using micrometer screw gauge

No.of head scale divisions =

Pitch of the screw made by head (n) = $\frac{\text{Distance moved on the pitch scale for n rotations}}{\text{No. of rotations}}$ =.......mm

Least Count = $\frac{\text{Pitch of the srew}}{\text{Total No. of head scale divisions}}$ =..... mm

Zero error = ...divisions

Zero correction = ... divisions

Order of spectrum	Position of telescope	PSR a in mm	Head Scale Reading		width of the slit A+n LC in mm
			Observed	Corrected n	
First	Left				a_1
	Right				a_2
Second	Left				a_1
	Right				a_2

Average width of slit of first order a $= \frac{a_1 + a_2}{2}$ mm

Average width of slit of second order a $= \frac{a_1 + a_2}{2}$ mm

Precautions:

1. While taking the observations the micrometer screw should be turned in one direction to avoid back-lash error.
2. The resolution of D_1 an dD_2 lines should be carefully observed to avoid personal errors which occur due to visual judgment.
3. the width of slit should be narrow.
4. The two lines D_1 and D_2 should merge exactly.

Calculations:

$\lambda =$ $\lambda + d\lambda$ N=

$\theta =$ n=

$\frac{\lambda}{d\lambda} \times \frac{l \cos\theta}{a}$

Nnl. Theoretical R.P.

Result: The theoretical and experimental values of resolving power of grating are calculated and compared.

Theoretical R.P.=

Experimental R.P.=

Viva questions and answers

1. What is resolving power of an optical instrument?

A. The ability to resolve the images of two nearby point is termed as resolving power.

2. what is resolving power of grating?

A. It is the ability to separate two spectral lines whose wavelengths are very close to each other.

3. On what factors resolving power of grating depends?

a. It depends on i) order of spectrum (n) ii) total no. of lines (N) ruled on grating

Experiment: 5

DIFFRACTION GRATING - MINIMUM DEVIATION

Aim: To determine the wavelength of spectral using a plane diffraction grating by measuring the angle of minimum deviation.

Apparatus: Plane diffraction grating, Spectrometer, Reading lens, Spirit level, Sodium vapour lamp.

Formula:

$$\lambda = \frac{2\sin(\frac{D}{2})}{Nn}$$

where λ= Wavelength of light.

D = Angle of minimum deviation.

N= Number f lines per centimeter.

n = Order of the spectrum.

Procedure:

The preliminary adjustments of the spectrometer are done. The slit is illuminated with sodium lamp. The grating is clamped to the prism table. The plane of the grating is adjusted to be vertical and perpendicular to the axis of the collimator.

The direct image of the slit is observed through telescope and by working the tangent screw of the telescope. The point of intersection of the cross wires is set on the image of the slit. The readings of the two verniers are noted. The telescope is moved to the left to observe the diffracted image of the first order. The prism table is released and it is rotated to the left .The first order image moves to the right reaches a limiting position and then tries to retrace its path . The telescope is fixed in this liming position such that the point of intersection of the cross wires is on the D_1line. The readings on both the verniers are noted.

The respective difference in the vernier readings gives the minimum deviation for the D_1line. The same procedure is repeated for the order line D_1 on the right hand side. The experiment is repeated for second order spectrum on both sides.

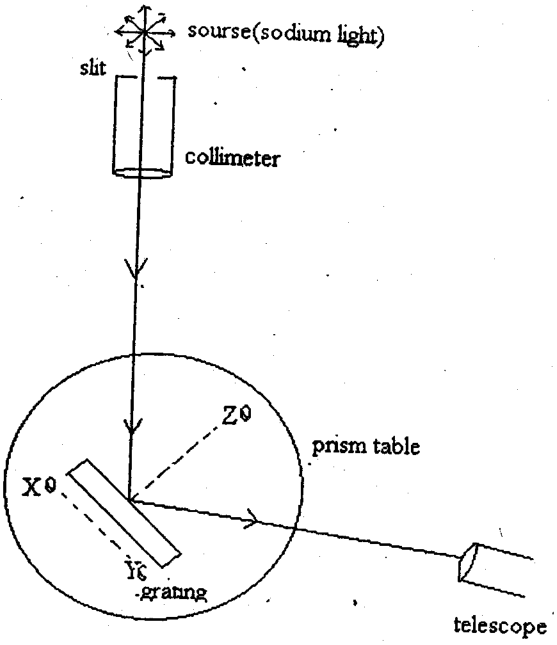


Fig.5

Observations:

No,o of lines per centimeter on the grating = ----------------

Value of the main scale division (1MSD) S =----------------

Number of divisions on vernier scale N = ------------

Least count of spectrometer =-S/N = --------------

Direct Reading of the image of the slit:

Vernier	Telescope Readings		
	M S R (a)	V C	Total Reading d = a + VCx LC
V_1			
V_2			

To measure angle of minimum deviation 'd' and λ

spectral line	Spectrometer Reading			Angle of minimum deviation		average D = $R_1 + R_2 / 2$	$\lambda = \frac{2\sin(\frac{D}{2})}{Nn}$
	Vernier	Left (d_2)	Right (d_3)	$d_1 \sim d_2 = R_1$	$d_1 \sim d_3 = R_2$		
D_1	V_1						
	V_2						
D_2	V_1						
	V_2						

Precautions:

1. After noting the readings on each side the direct reading is again checked to make sure that it has not altered.
2. For remaining refer to experiment No.

Result: The wavelengths of sodium D_1 line =
D_2 line=

Viva questions and answers

1. What is a diffraction grating?

A) An arrangement consisting of a large number of parallel slits of equal width and separated from one another by equal opaque spaces is called diffraction grating.

2. What is grating element?

A) The distance between the centres of two successive slits is called grating element. If a = Width of slit and b = Distance between two slits then (a+b) is known as grating element.

3. What are the requirements of a good grating?

A) The lines should be exactly uniform, equidistant, parallel and of equal width through the ruled space.

Experiment No.6.

THICKNESS OF A WIRE -WEDGE METHOD

Aim: To determine the thickness or diameter of a given wire by forming interference fringes due to a wedge shaped air film.

Apparatus: Two optically plane glass plates of same size, wooden frame, a piece of black paper, reading lens, traveling microscope, reflecting glass plate, wire of small thickness and a retort stand.

Formula:

$$d = \frac{\lambda d}{2\beta} \text{ cm}$$

where d = Thickness or diameter of the wire to be determined

λ =Wavelength of light used (sodium) 5893 A°

l= Distance between the point of contact of the glass plates and the axis of the thin wire fixed between them

β = Fringe width of one fringe.

Description: The experimental arrangement is shown in the fig. 1. Take two optically plane glass plates and clean them with a piece of cloth. Fix the wire whose thickness is to be determined between the two glass plates in such a way that the two glass plates touch at one end and separated at the other end. A thin film of air or wedge (shaped film) will be formed between the two glass plates. The thickness of the air film or wedge gradually increases from the point of contact of the two glass plates towards the other end. Now place this set on the base of traveling microscope. Now the lower surface is covered with a black paper. Light from a monochromatic source S (sodium) is allowed to incident normally on the combination of glass plates after partially reflected from a glass plate G which is inclined at angle of with the horizontal. The rays reflected at the lower surface of the upper plate and partly from the top surface of lower plate interfere and produce interference fringes. As these two rays are derived from the same incident ray and traveled over different paths. They are in a condition to produced interference fringes. The locus of all the points having the same thickness of air film is a straight line for different thicknesses. Thus, alternate dark and bright parallel straight lines will be observed, which can be viewed through a microscope held vertically above the centre of fringe system.

Procedure: The wedge shaped illuminated by sodium vapour .Adjust the glass plate G, until the inclination angle with the horizontal such that the light from the source S, after the refection from G, incident normally on the air film . Focus the microscope vertically above the fringe system and adjust the cross wires so that the fringes are clearly observed .Move the microscope horizontal and adjust it's position. So that the cross wires coincides with one of the fringes say first fringe near the point of contact. Note the main Scale reading and vernier coincidence of that fringe. Let the total reading be
For our convenience treat the first fringe as the 0'th fringe. By counting the number of fringes move the microscope so that vertical cross wire coincides with the 5th fringe. Then, note the M.S.R and V.C. Let the total reading be R . Repeat the experiment and note down the observation for the 10th, 15th, 20th, 25th and 30th fringes. While taking the readings the microscope should be moved always in one direction to avoid back-lash error. Then tabulate the readings. The difference between the two readings R_o, R_1 gives the width of 5 fringes. From this find the width of one fringe β known as fringe width. The distance between the point of contact of the two glass plates and the wire ?. The thickness d of the given wire can be calculated by substituting the values of λ, l, and β in equation of d.

Observations:

To determine the fringe width β using traveling microscope.

Value of one main Scale division S =

Number of divisions on the vernier N =

Least count of the vernier of microscope LC = S/N =

S.No.	No. of the Fringe	Main Scale Reading a in cm	Vernier coincidence n	b= n x LC in cm	Total Reading a+b in cm	Width of 5 fringes in cm
1.	0				R_0	x_1
2.	5				R_1	x_2
3.	10				R_2	x_3
4.	15				R_3	x_4
5.	20				R_4	x_5
6.	25				R_5	x_6
7.	30				R_6	x_7
8.	35				R_7	x_8
9.	40				R_8	x_9
						Total

Averae width of 5 fringes x = Total /9 = cm

Average width of one fringe β = x/5 = cm

Precautions.

1. The glass plate should be thin and optically plane.
2. The glass plate should be cleaned with spirit and to be wiped with clean cloth.
3. The wire should be uniform and thin. It should be without shrinks.
4. The vertical cross wire should be made to coincide with bright gringe.
5. The travelling microscope should be moved only in one direction to avoid back lash error.

Results:

Thickness of given wire d = cm

Calculations:

Values: l =

$\lambda = 5893 \times 10^{-8} cm$

β = cm

$$d = \frac{\lambda l}{2\beta} \text{ cm}$$

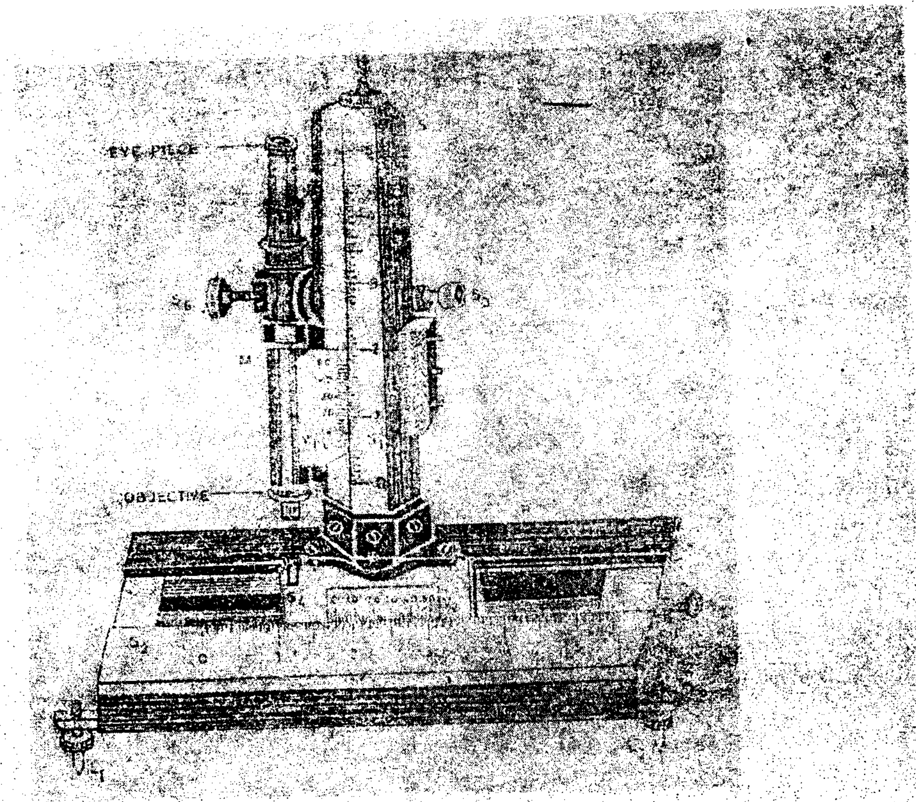


Fig6:

Viva Questions and Answers:

1. What is the principle of this experiment?

A). Interference phenomena.

2. Why the fringes are straight and parallel?

A). As the thickness of film is constant only in a direction parallel to the thin edge of the wedge straight and parallel fringes are obtained.

3. Why the fringes are of equal thickness?

A) Because, each fringe is the locus of the points where the thickness of the film is constant.

4 When the two glass plates are not perfectly plane, what is the nature of the fringes?

A) Fringes of unequal thickness will be formed.

Experiment No.7.

NEWTON'S RINGS

Aim: To determine the wavelength of sodium light by forming newton's rings.

Apparatus: convex lens f=100cm, two optically plane glass plates, microscope, condensing lens and a sodium vapour lamp

Formula:

$$\lambda = \frac{D_m^2 - D_n^2}{4R(m-n)} cm$$

where λ = wavelength of sodium source 5893 A (to be find out)

D_m = Diameter of mth dark fringe

D_n = Diameter of nth dark fringe

R = Radius of curvature of the surface of the lens in contact with the glass plate

m, n are the numbers of the rings chosen

Description:

The convex lens is placed on the optically plane glass plate P which is on the plat form of the traveling microscope. A black paper is placed under the glass plate. The condensing lens c is placed at a distance equal to the focal length of the lens from the sodium vapour lamp. The emergent parallel beam of light is directed towards the glass plate G kept directly above the centre of the lens and inclined at an angle of 45 to the vertical. The beam of light is reflected on the lens L. As a result of interference between the light reflected from the lower surface of the lens and the top surface of the glass plate P, Newton's rings with alternate bright and dark rings are formed having a black centre. These rings can be focused by the microscope. It may happen that the centre of the fringe system may be white. This is due to the dust particle between the lens and the thick glass plate. In such a case the surface of the lens and the glass plate have to be cleaned.

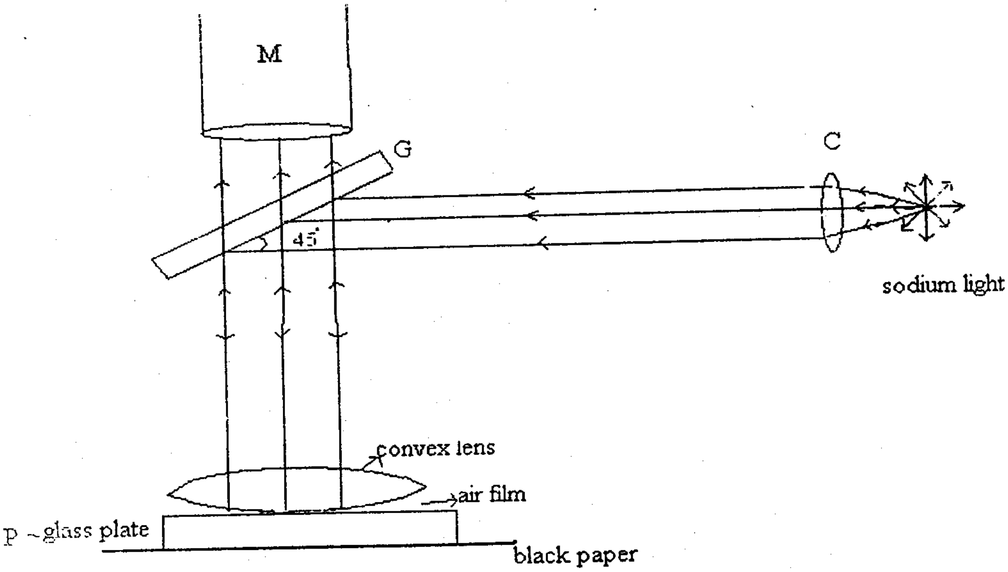
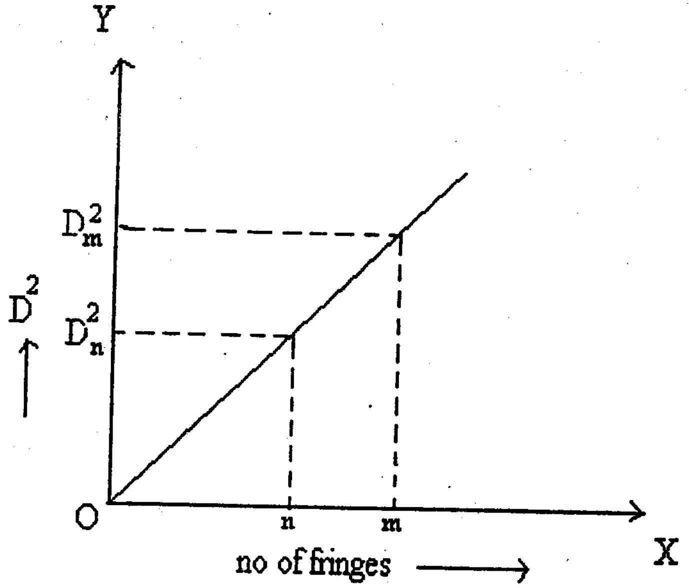


Fig.7

Procedure:

The microscope is focused at the centre of the ring system. The microscope is moved so that the cross wires pass over dark rigs. Then microscope is moved back until the vertical cross wire is set at the middle of the 15th dark ring. the reading of the main scale reading of microscope is noted, and vernier coincidence is also noted. From this microscope reading is obtained. The microscope reading $MSC + V \times LC$ where LC is the least count of microscope. The microscope is set at the middle of t14th dark ring. The reading of microscope is noted. Similarly the readings of the microscope with cross wire set successively at the middle of etc 5th dark ring. The microscope corresponding to 5,6,7 ...15th dark ring on the other side of the centre are noted. From these observations the diameters of 5,6...15 th dark rings can be found. The convex lens is removed and its radius of curvature R is determined by a spherometer. A graph is drawn between number of dark rings on x-axis and square of diameter on y-axis. The graph is a straight line passing through origin. From the graph the values of D_n^2 and D_m^2 corresponding to mth and nth rings are found. The values are substituted in the above formula and is calculated. It is compared with the standard wavelength of light.



Observations:

No. of divisions on micrometer screw

Pitch of the screw = distance moved by the micrometer screw/ number of rotations made

$$\text{Least count} = \frac{\text{pitch of the screw}}{\text{no. of divisions on micrometer screw}}$$

Microscope Reading = Main scale reading VC × LC cm

S.No.	No. of dart ring	Microscope Reading		Diameter D= b-a cm	D^2 cm^2	$\frac{D_m^2 - D_n^2}{m-n}$
		Left a cm	Rb cmight			

$$\text{Average} = \frac{D_m^2 - D_n^2}{m-n}$$

Radius of curvature of convex lens using spherometer:

$$\text{Pitch of screw} = \frac{\text{Distance moved by the screw}}{\text{number of rotations made}} = \text{-----mm.}$$

$$= \text{----- cm.}$$

$$\text{Least count} = \frac{\text{pitch of screw}}{\text{total number of head scale divisions}} = \text{------- mm}$$

$$= \text{-------- cm}$$

S.No.	P.S.R. a mm	Hed Scale Reading n	Extrafraction b= n xLC mm	Total a+b mm
	Reading on plane glass plate R_1			
1. 2. 3.				
	Reading on plane glass plate R_2			
1. 2. 3.				

Height of convex lens $h = R_2 - R_1\ mm$ =----- mm

Radius of curvature of convex lens

$$R = \frac{l^2}{6h} + \frac{h}{2}\,mm = \text{----- cm}$$

where l = length between any two legs of spherometer in cm

h = Height of convex lens in cm

Precautions:

1. While taking the readings the micrometer screw should be moved only in one direction to avoid back lash error
2. The lens L and glass plate P should be perfectly clean
3. The slow motion tangent screw one should be used in taking observations.
4. The central spot should be dark.

Result: Wavelength of the sodium light = ...

Viva questions:

1) What is interference of light?

A) When two light waves superimpose on each other then the resultant amplitude in the other region of supper position is different than the amplitude of individual waves. This modification in intensity distribution is known as interference.

2) Where the interference fringes formed?

A) The fringes are formed in the air film enclosed between the lens and glass plate.

3) Why Newton's rings are circular?

A) Path difference of a light ray along a circle is constant.

4) Are the rings equidistant?

A) No, spacing between the rings decreases with increasing of order.

5) Why do use sodium light?

A) In order to obtain the fringe pattern as alternate dark and bright fringes, monochromatic light should be used.

Experiment No. 8

THICKNESS OF A THIN WIRE USING OPTICAL BENCH

Aim: To determine the thickness of a thin wire using optical bench

Apparatus: Optical bench with uprights, thin wire, sodium vapour lamp, spirit level and micrometer eye piece

Formula:

$$t = \frac{\lambda(D_2 - D_1)}{(x_2 - x_1)} \text{ cm}$$

where t = thickness of thin wire

λ = wavelength of sodium light (5893 A)

x_1 = Average fringe width when the distance between the eyepiece and the wire is at D_1

x_2 = Average fringe width when the distance between the eyepiece and the wire is at D_2

Description:

The experiment al arrangement consists of an optical bench with uprights as shown in fig. S is a narrow vertical slit which is mounted vertically on one of the upright at one end of the optical bench. In order to illuminate ~~the~~ slit sodium vapour lamp is placed before it. The thin wire should be fixed in a rectangular metal frame by soldering its ends ad this frame is placed on another upright at a distance say 15 cm from the slit. The slit and the metal frame can be rotated in their own places about a horizontal axis with the help of tangential screws provided on the holders of the upright. The micrometer eyepiece is mounted on the third upright which is placed at the other end of the optical bench.

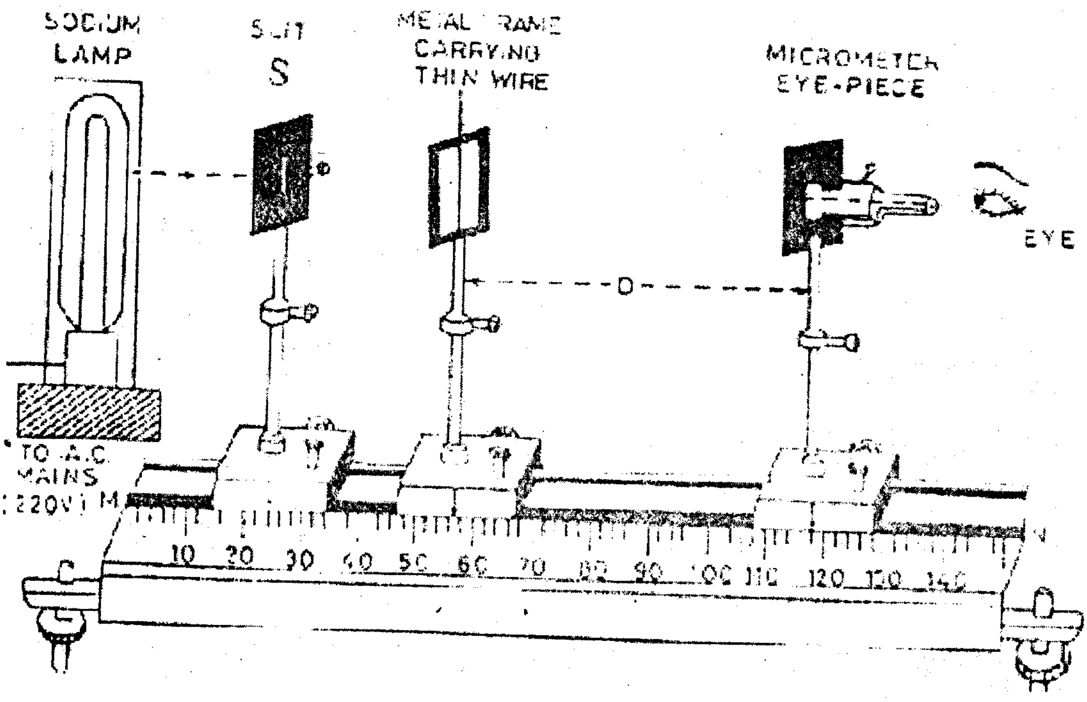


Fig 8.1

Theory: S is a narrow vertical slit and AB is a thin wire. XY is a screen placed behind the wire. The slit, the wire and the screen are arranged such that they are parallel to each other as well as perpendicular to plane of paper. When the slit s is illuminated by monochromatic light then the cylindrical wave front ww emanating from S strikes the wire AB and get diffracted at the edges A and B a\. The diffraction pattern can be observed on screen XY. Consider a point P on the screen such that OP is perpendicular to screen AB represents the region of geometrical shadow of the wire AB. So above a and below B the screen is illuminated.

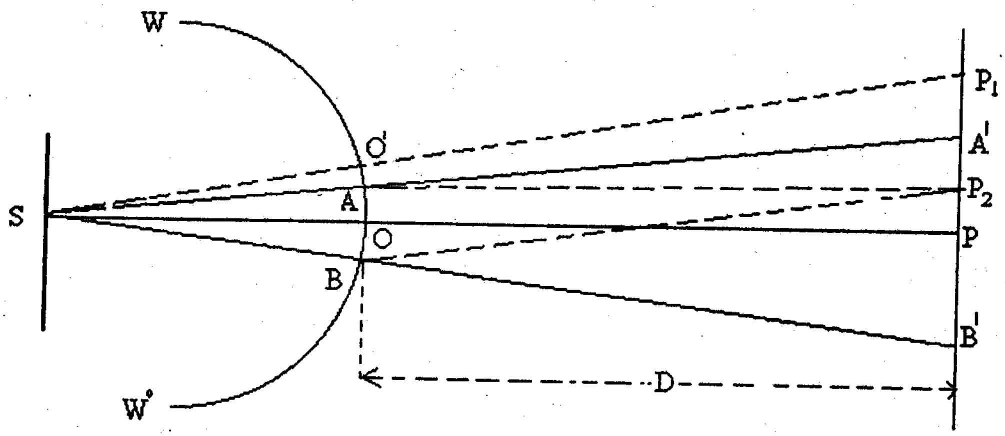


Fig.8.2

S = Slit
AB = Thin wire
WW^1 = Cylindrical wave front
$A^1 B^1$ = Geometrical shadow region
XY = Screen

Procedure:

a) To ~~obtain sharp~~ and distinct fringes

1. Level the optical bench so that it is perfectly horizontal by means of the leveling screws and spirit level.
2. Place the slit on one of the uprights at one end of optical bench. The slit is illuminated with sodium lamp. The width of the slit should be reduced and the slit is rotated in its own plane about an axis parallel to the axis of the bench with the help of a tangent screw until slit becomes vertical.
3. Mount the metal frame carrying the thin wire on the second upright and keep it at a distance of about 15 cm from the slit. Adjust the height of the slit and metal frame so that they are at same height and perpendicular to the length of the bench. By means of tangent screw adjust the position of the metal frame so that it is vertical and parallel to the slit.
4. Now mount the micrometer eyepiece on the third upright and keep it at a distance of 40 cm from the wire at the same height as that of the slit and the wire. Focus the eye piece on the cross wire and move the eyepiece on the eye piece on the cross wires and move the eyepieces in and out until the cross wires are clearly seen. Make one of the cross wires vertical by rotating the eyepiece. Looking through it adjust its distance from the wire until well defined equally spaced bright and dark interference fringes are formed in the foal plane of eyepiece.

b) 1) to determine the fringe width when the eyepiece is placed at a distance b1 from wire.

Find the least count of micrometer screw. Place the eyepiece at a convenient distance from the wire. viewing through the eye piece the interference fringes thus formed in the geometrical shadow of the wire, move the micrometer screw one side (say left side) of the interference pattern such that point of intersection of the cross wires coincide with centre of one of the bright fringes say first fringe (near the extreme left of fringe system). Adjust the position of micrometer screw so that the vertical cross wire exactly coincides with the centre of bright fringe. Note the pitch scale reading and head scale reading of that fringe. Let the total reading be R_0. For convenience treat that first bright fringe as 0th fringe. By counting the number of bright fringes, move the micrometer screw towards the right side and adjust the micrometer screw so that the vertical cross wire coincides with the 5th bright fringe. Then note the PSR and HSR. Let the reading be R_1. Repeat the experiment and note the readings for 10, 15, 20, 25 bright fringe. Tabulate the readings. the difference between two reading R_0 and R_1 gives width of 5 fringes. From this find the width of one fringe (x_1).

2) To determine the fringe width x_1 when the eyepiece is place at a distance d2 from the wire.

Place the eyepiece at a distance (>) from the slit . Find the fringe width x_2 by following same procedure as above.

Observations:

Number of divisions on head scale =

$$\text{Pith of the screw} = \frac{\text{Distance moved on the pitch scale for n rotations}}{\text{no. of rotations made by head}} = \ldots\ldots \text{ mm}$$

$$\text{Least count} = \frac{\text{Pitch of the screw}}{\text{number of head scale divisions}} = \ldots\ldots\text{mm}$$

To determine the fringe width x_1:

Distance between wire and eye piece, D_1 = cm

Zero error =........ divisions

Zero correction = ...divisions

S.No.	No.of the bright fringe	Micrometer Readings			total readings	width of 5 fringes x mm
		Pitch Scale Reading a in mm	Head Scale Readings		a+n x LC mm	
			Observed	Corrected n		
1. 2. 3. 4. 5.						

Average width of 5 fringes x =

Average fringe width $x_1 = x/5 =$ mm = cm

To determine the fringe width x_2 :
Distance between the wire and eye piece , D_2 = ...cm

S.No.	No.of the bright fringe	Micrometer Readings			total readings	width of 5 fringes x mm
		Pitch Scale Reading a in mm	Head Scale Readings		a+n x LC mm	
			Observed	Corrected n		
1. 2. 3. 4. 5.						

Average width of 5 fringes x =
Average fringe width $x_2 = x/5 =$ mm = cm

Result:
Thickness of thin wire t = ... cm

Precautions:

1. The optical bench should be made perfectly horizontal using leveling screws.
2. The slit should be vertical and narrow.
3. All the upright should be mounted at the same height.
4. The micrometer screw should always be moved in only one direction to avoid back lash error.
5. Lateral shift should be avoided.

Viva Questions& Answers:

1. When the film is illuminated with white light, which type of fringes are formed?

A: Coloured fringes are appearing near the edge of the wedge.

2. When the two glass plates are not perfectly plane, what is the nature of the fringes?

A: Fringes of unequal thickness will be formed.

3. What is the principle of this experiment?

A: Interference phenomena.

Experiment No. 9

STUDY OF OPTICL ROTATION - POLARIMETR

Aim: To determine the specific rotation of an optically active substance say sugar using Laurents sacchrrimeter.

Apparatus: Laurents saccharimeter, sodium lamp, cane sugar, distilled water, balance

Formula:

$$S = \frac{100}{Cl}$$

where S = Specific rotation of the optically active substance

C = concentration

l = length of the tube in centimeters

Descriptions:

The essential parts of Laurents saccharimeter are represented in Fig. Light from a source S is rendered parallel by means of a convex lens L. The emergent parallel beam passes through the polarizing nicol P.N. and then illuminates the half shade H.S The emergent beam is viewed through an analyzing nicol A.N. and a long tube with glass ends.

The analyzing nicol and the telescope are fixed in a tube capable of rotating on its axis. The position of the analyzing nicol may be observed by means of verniers attached to it moving on a fixed circular scale. After emergence due to the acquired phase difference π, they recombine to give plane polarized light with its plane of vibration OQ so that OP and OQ are equally inclined to the principal plane AB of the quartz plate. The rays transmitted through the right half are characterized by their vibrations along OP and the rays transmitted through the left half are characterized by their vibrations along OQ both being equally incident to AB Of these vibrations the analyzing nicol transmits only vibrations parallel to its principal plane. the two halve will be equally bright or equally dim when the principal plane of the analyzing nicol is either parallel or perpendicular to

AB. In any other position of the analyzing nicol one half appears much brighter compared to the other.

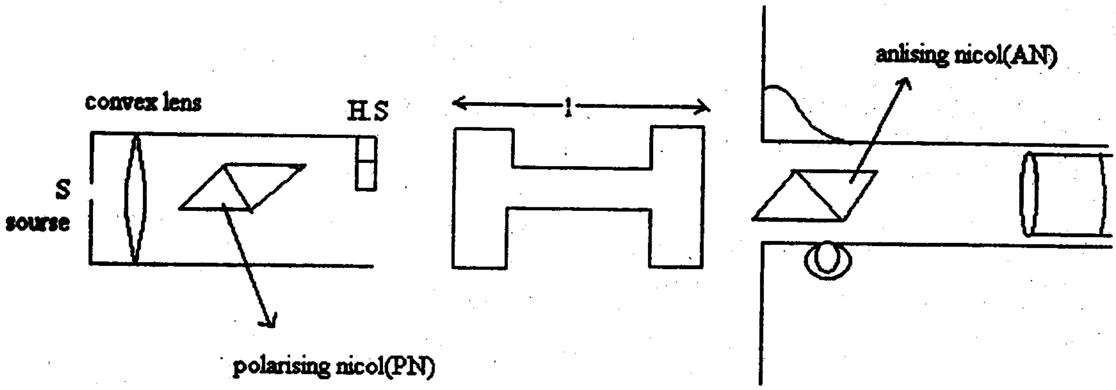


Fig.9.1

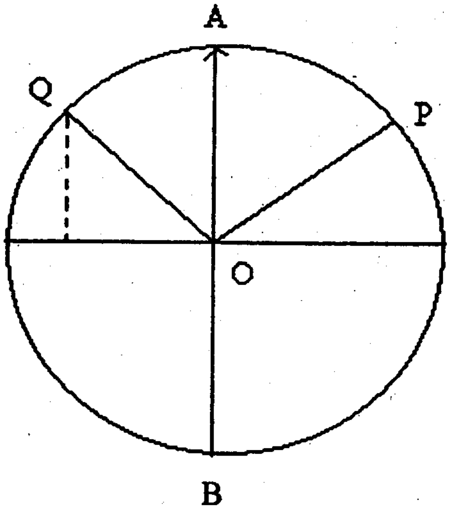


Fig.9.2

Principle:

The half shade consists of two semicircular placed side by side plates one of them being a quartz plate cut with its optic axes parallel to the refracting surfaces and the other a glass plate of such thickness has to absorb the same amount of light as absorbed by the quartz plate. The quartz plate is a half wave plate which introduces a phase difference π between the ordinary and extraordinary rays. The polarizing nicol transmits only extra ordinary ray with vibrations parallel to its principal plane. Suppose the plane of vibration of the light incident on the half shade is OP which is also the principal plane of the polarizing nicol. The plane of vibration of the incident light continues to the OP after passing through glass. But the plane polarized light incident on the quartz plate will be split up into components polarized in two mutually perpendicular directions. They travel in the same directions but with different velocities.

Procedure: The tube l is filled completely with distilled water leaving no air bubbles inside and it's introduced in the polarimeter between the half shade and he analyzing nicol. The analyzing nicol is rotated till the two halves appear equally dim. The vernier reading is taken. The position of analyzing nicol is disturbed and the point of equal dimness is obtained from the other side. The mean vernier reading is calculated. A weighted amount of sugar say 10 gms is dissolved in 100 cc of distilled water and the solution is filtered. The tube is taken out, empted, rinsed with sugar solution, and filled with the solution in place of water leaving no air bubbles inside. Th tube is replaced in the polarimeter and the analyzer is rotated to get the position of equal dimness. the vernier readings gives the angle of rotation θ of the plane of polarization produced by the solution. If M gms of the solute is dissolved in V cc of the solvent then the concentrations C = (M/V) gms/cc. Specific rotation S of the optically active substance is calculated.

From the formula

$$S = \frac{100}{cl}$$

where l is the length of tube in centimeters. The experiment is repeated for the concentrations C/2, C/4. and C/8 and the corresponding angles of rotation are obtained in each case A solution of concentrations C/2 is obtained by adding equal volumes of distilled water and the solution first prepared. In the same manner other concentrations are prepared. A graph is drawn between the angle of rotation and the concentration and it will be straight line.

Observations:

l = length of the tube = cm

Mean reading of the vernier with distilled water

Concentration(C)	Position of the analyzer with solution θ_s	angle of rotation $\theta_s - \theta_w$	$S = \frac{100}{cl}$

Precautions:

1. the end pieces of the tube must be screwed tightly.
2. The end pieces of the tube must be free from dust and grease.
3. Air bubbles must be avoided in the tube.

Results:

The specific rotation of an optically active substance is.....

Viva Question & Answers:

1. What is polarimeter?

A: It is an instrument used for measuring the angle through which the plane of Polarization of a plane polarized light is rotated by optically active substances.

2. What is a saccharimeter?

A: A polarimeter used exclusively to determine the quantity of sugar in a solution or the percentage of sugar in a solution or in sugar analysis is called a saccharimeter.

3. On what factors does specific rotation depend?

A: i) The nature of the solvent and solute.

ii) The temperature of the solution.

iii) The wavelength of light

Experiment No.10.

DIAMETER OF LYCOPODIUM PARTICLES

Aim: To determine the diameter of lycopodium particles forming diffraction fringes.

Apparatus: Optical bench with two uprights, a metal plate with a small hole (0.25 mm or 0.5 mm) in diameter at its centre, lycopodium powder, spirit level, glass plate, sodium vapour lamp and traveling microscope.

Formula:

$$d = \frac{1.22\lambda D}{r}$$

where d = diameter of the lycopodium particle

λ = wavelength of light used.

D= distance of the glass plate from the central hole of the metal plate

r = radius of the ring (dark or bright) or

distance of the pin hole from the central hole of the metal plate.

Description:

The metal plate is mounted on the upright of the optical bench and the central is illuminated with sodium vapour lamp.

The glass plate is dusted with lycopodium powder. The dusted plate is also mounted on one upright of the optical bench when viewed through the glass plate at the hole a series of circular fringes of yellow can be observed.

Procedure:

The distance of the glass plate from the metal plate is adjusted such that the first ring of diffraction pattern coincides with the first set of the pin holes. Let the distance of the glass plate from the metal plate be D. Let the distance of the first set of pin holes from the central hole be r.

The angle of diffraction of the first ring of the diffraction pattern due to the particle of the lycopodium powder is given by $\theta = \frac{r}{D}$. If d is the diameter of the lycopodium particle $\frac{1.22\lambda}{d} = \frac{r}{D}$.

$$d = \frac{1.22\lambda D}{r}$$

from which diameter of lycopodium powder can be found.

The distance of the glass plate is again altered so that the first dark ring falls on the second set of equidistant holes form the central hole. the experiment is repeated with other sets of equidistant pin holes. In each case the value of r/D is

Found and d can be calculated.

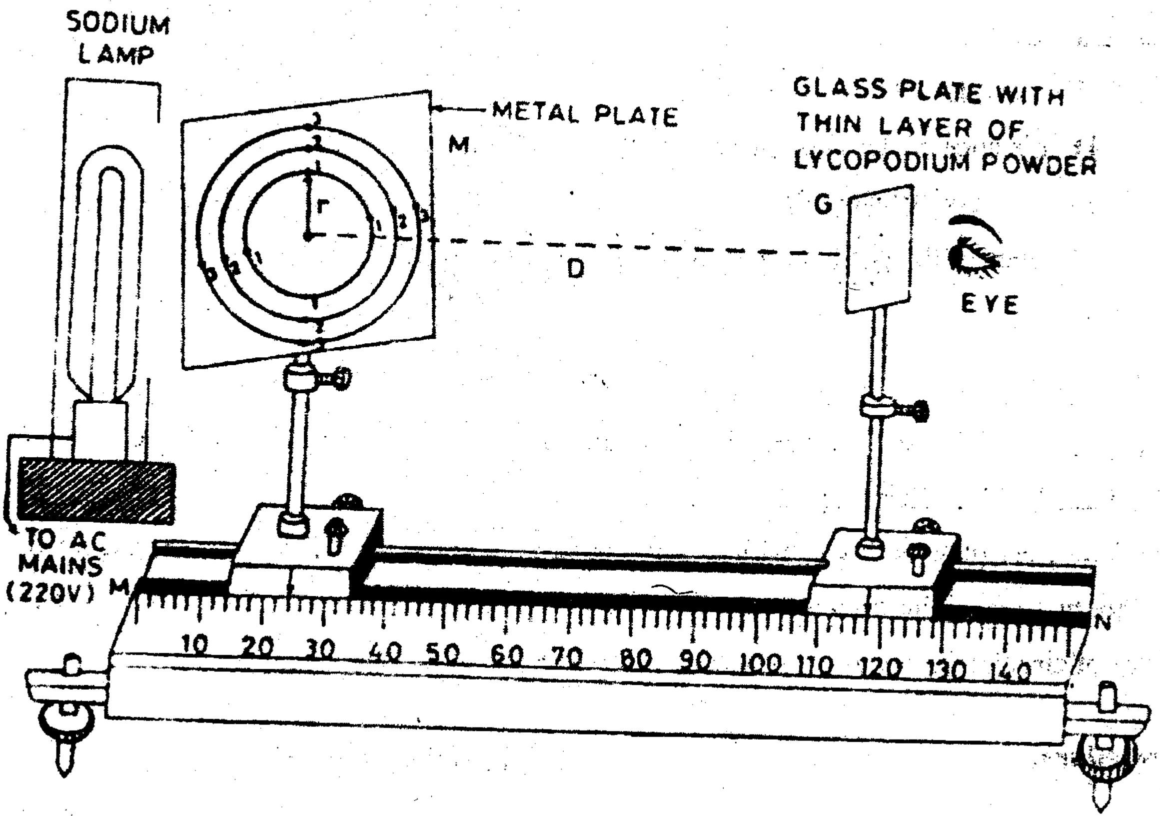


fig. 10

Observations:

Wave length of sodium light λ =5893 A^o

S.NO	D	r	$d = \frac{1.22\lambda D}{r}$

Precautions:

1. The central hole of the metal plate and the eye should be at the same level.
2. The glass plate should be parallel to the metal plate.

Result:

The mean diameter of lycopodium powder is = -------- cm

Viva question & Answers:

1. How does the intensity vary with the order of the ring?

A: The intensity decreases as the order of the ring increases.

2. Why pin holes are provided on the mental plate.

A: To have the same distance from the center of the central hole of the metal plate.

3. Why darks rings are preferred, while taking observations?

A: they are distinctly visible against the back ground of the metal plate.

Experiment No. 11.

HEATING EFFICIENCY OF AN ELETRICAL KETTLE WITH VARYING VOLTAGE

Aim: To determine the heating efficiency of an electrical kettle with varying voltage.

Apparatus: Electrical kettle, A.C. Voltmeter, Rheostat, A.C. Ammeter, Plug key, Connecting wires, Step down transformer and stop watch.

Formulae:

Efficiency of the kettle $= \dfrac{(\omega t + ms)\theta}{Eit} \times 100$

where w = water equivalent of the Kettle =

m_1 = mass of the kettle

s_1 = specific heat of the material of the kettle

m = mass of water taken in the Kettle -

m_2 = mass of the kettle + water

s = specific heat of water

θ = ride in temperature of water in t seconds -

θ_1 = initial temperature of water

θ_2 = final temperature of water

E = potential difference between the ends of conductor

i = current passing through the coil for t seconds

t = time of flow of current.

Description:

The apparatus consists of a step down transformer the primary of which is connected to AC mains. The secondary of the transformer is connected to rheostat by means of which the potential difference between the ends coil of the kettle can be varied. The kettle comprised a heating element of suitable resistance and a thermometer is inserted in the kettle to measure two temperature of water. An AC voltmeter and AC ammeter are connected to the kettle to measure the potential difference and current.

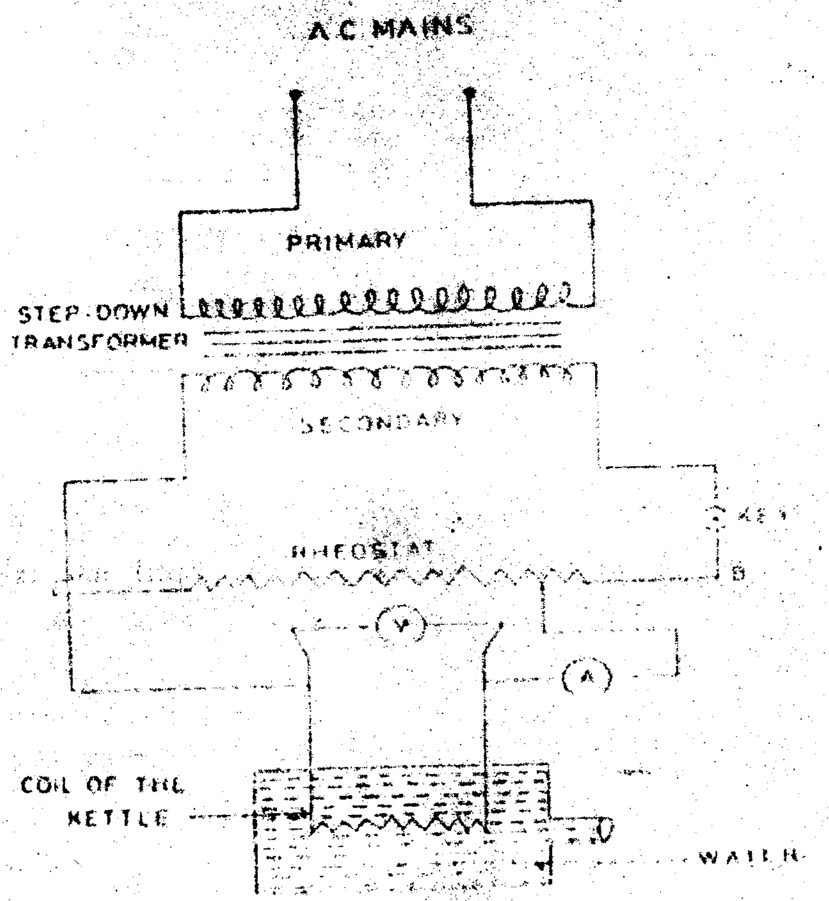


Fig. 11

Procedure:

The experimental arrangement is as shown in figure. take an empty kettle and clean it well. Determine the mass of the kettle using a rough balance. Take water in the kettle until the resistance coil is completely immersed in it. Find the mass of water and kettle. Note the initial temperature C of water using thermometer. Now switch on the current supply and simultaneously start a stop watch. the temperatue of water slowly rises. Adjust the rheostat so that the voltmeter reads 150 volts. Note the current i passing through the coil. After a few minutes Say 5minutes. Switch off the current and note the final temperature C of water. Now remove the water from the kettle and again first with fresh water. Repeat the experiment for various known voltages 180v, 200v and for each voltage not the corresponding current. The efficiency of the Kettle can be estimated for each voltage and the results are to be tabulated.

Observations:

S.No	Voltage applied to the kettle E in volts	Efficiency of kettle $\frac{(w+ms)\theta}{Ei\,t} \times 100$

Precautions:

1. The temperature of water should be noted carefully.
2. Current should be passed through the kettle only after the kettle is properly immersed in water.

Result: Heating efficiency of electrical kettle

Viva questions and answers:

1. What is water equivalent?

A. The water equivalent of a body is defined as the amount of water that has the same thermal capacity as that of the body. It is expressed in grams.

2. Distinguish between heat and temperature?

A. Heat is a form of energy where as the temperature is a measure of the degree of hotness of body.

Viva questions and answers:

1. What is water equivalent?

A. The water equivalent of a body is defined as the amount of water that has the same thermal capacity as that of the body. It is expressed in grams.

2. Distinguish between heat and temperature?

A. Heat is a form of energy where as the temperature is a measure of the degree of hotness of body.

Experiment No. 12

REFRCTIVE INDEX OF A LIQUID USING PULFRICH REFRACTOMETER

Aim: To determine the refractive index of a liquid by pulfrich refractometer.

Apparatus: Pulfrinch refractometer, Liquid, meter scale

Formula:

$${}_A\mu_L = \sqrt{({}_A\mu_G)^2 - \sin^2 i}$$

where ${}_A\mu_L$ = the refractive index of liquid
${}_A\mu_G$ = the refractive index of glass
i = angle of refraction at the boundary surface

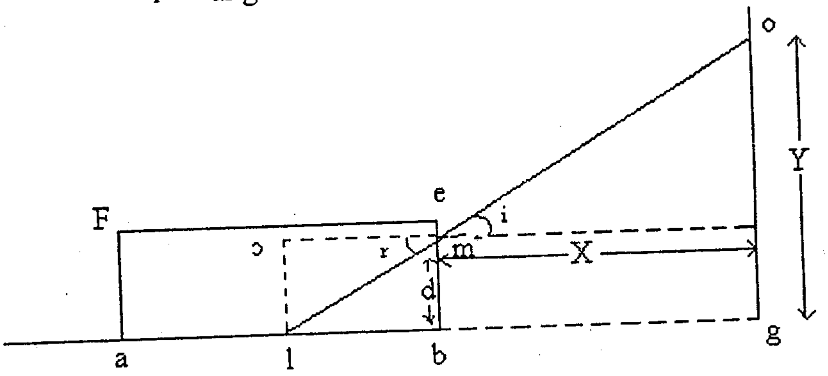


Fig.12

Description:

A metal sheet of dimension 20 cm x 4 cm is fixed perpendicularly to a wooden piece. AB of dimensions 30 cm x 3 cm. Two decolumn sheets M and N of sizes 10 cm and 4 cm and 5 cm x 2 cm are taken and cross shaped hole of size 1.5 cm x 0.4 cm is made on the sheet N. This is kept on the sheet M and passed with araldite and it is kept on AB. The given liquid is placed in the cross shaped space and a glass block is kept on it. A black paper is fastened to the upper surface of the block and sodium light is made to fall on it. If the cross is observed through a slit BC of size 2cm x 1cm appears faint.

The refractive index of the glass block is determined ( ) by real and apparent method by using a microscope. If R_1

is the cross reading on the paper kept on the horizontal board R_2 is the paper on the glass block. Then

$$ {}_A\mu_G = \frac{R_3 - R_1}{R_3 - R_2} = \text{ Real distance / Apparent distance} $$

If the sheet BC is kept at far convenient distance from the block and if the cross is observed at a certain position the cross will not appear and the face glows like silver. If the MN sheet system is moved towards EC at a certain position half of the cross is then the remaining half glows like silver. at this position the point m on BC is noted with a pen bd = x , bm = a , dn = y+a are measured. the refractive index of the liquid can be determined from the relation given in the theory by measuring 'Sin i' from the following relation

$$ \sin i = \frac{y}{\sqrt{x^2 + y^2}} $$

Reading R_2

The experiment is repeated by changing the heights of the aperture of the slit and every time the values Sin i are noted . The average reading is calculated and values are tabulated.

Precautions:

1. Care should be taken to see that there are no air bubbles in the cross.
2. The position of cross should be carefully observed.

Observations:

1.Reading R_1 =

2.Reading R_2 =

3.Reading R_3 =

S.No	x cm	(y+ a) cm	(y + ng)cm	$\sqrt{x^2 - y^2}$	Sin i	$_A\mu_L = \sqrt{(_A\mu_L)^2 - \sin^2 i}$

Result: The refractive index of the material =

Viva questions:
1. Define Refractive index of a material.
A. Refractive index is the ratio of velocities in air with respect to the material.

Experiment No. 13

REFRACTIVE INDEX OF THE MATERIAL OF A LENS AND OF A LIQUID - BOYS METHOD

Aim: To determine the refractive index of the material of a convex lens and that of a liquid.

Apparatus: Convex lens, needle, mercury, watch glass, retort stand

Formula:

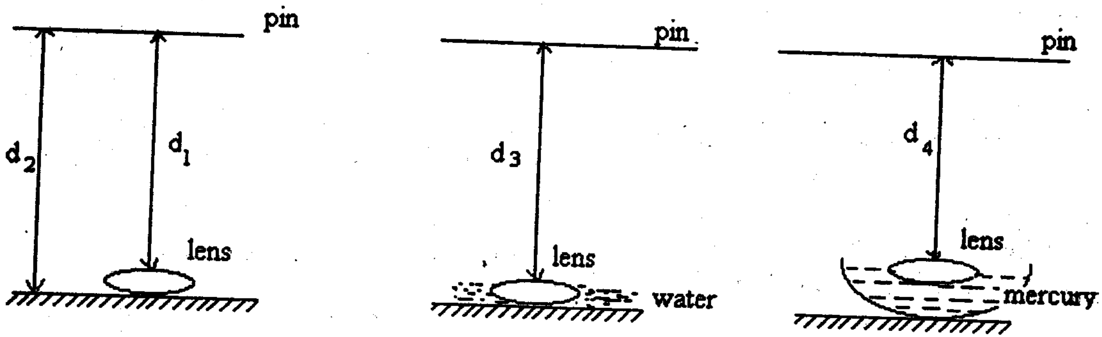


Fig.13

d_1 = distance of the pin from the top surface of the convex lens

d_2 = distance of the pin from plane mirror

d_3 = distance of the pin from top surface of convex lens kept on water

d_4 = distance of the pin from top surface of the convex lens floating in mercury

$t = d_2 - d_1$ =thickness of convex lens

$f_1 = d_1 + (t/2)$ Focal length of combination (convex lens and water lens)

$F = d_1 + (t/2)$ = focal length of liquid lens

$f_2 = \frac{Ff_1}{F - f_1}$ focal length of liquid lens.

Radius of curvature of convex lens $R = \frac{\mu f_1}{f_1 - \mu} + \frac{t}{2}$

where $\mu = d_4 + t/2$

Refractive index of material of convex lens

$$\mu_1 = 1 + \frac{R}{2f_1}$$

Refractive index of material of liquid (water)

$$\mu_2 = 1 + \frac{R}{f_2}$$

Procedure:

1. To determine 'f' of a convex lens.

The given convex lens is placed on a plane mirror which is placed on a table. A needle is clamped horizontally to a retort stand and the position of the needle is adjusted so that it is above the lens. Looking down from the needle the reflected image of the needle is observed through the lens and the height of the needle is adjusted such that the time of the needle and its reflected image coincide without any parallax where the eye is moved side ways. The distances of the needle from the top and bottom of the lens are measured. The average of the two gives the focal length f_2 of the convex lens.

2. To determine radii of curvature of the two surfaces of convex lens.

The lens is floated in a dish of mercury. the needle is held horizontally with the clamp of retort stand is kept such that its tip is vertically above the centre of the lens. Looking down from the top of the needle the height of the needle is adjusted such that its time coincides with reflected image in mercury. With out any parallax error, the distance between the tip of the needle and lens is noted. To this distance half the thickness of the lens is added.

Let it be u the radius of curvature of surface of lens in contact with mercury is calculate by formula

$$R = \frac{\mu f_1}{f_1 - \mu} + \frac{t}{2}$$

3. To calculate the refractive index of the material of the lens:

Knowing f_1, r_1 and r_2 of the lens its u is calculated by the formula

$$\mu_1 = 1 + \frac{R}{2f_1} \text{ where } f_1 = d_1 + t/2$$

4.To determine the focal length of liquid lens and hence the refractive index of the liquid (water):

Few drops as the given liquid are placed on plane mirror and the convex lens is placed above the liquid drops such that its surface is in contact with the liquid. the height of the needle is again adjusted such that there is no parallax error between the tip of needle and the reflected image of needle when seen vertically down from the top of needle. The distances of the needle from top to bottom surfaces of the lens are measure. The average of two readings gives the combined focal length (F) of the convex lens and f_2 of plano concave liquid lens

$$F = d_3 + (t/2)\,; \qquad f_2 = \frac{Ff_1}{F - f_1}$$

value of f2 will be negative since the liquid lens is plano concave. If μ_2 is refractive index of liquid then

$$\mu_2 = 1 + \frac{R}{f_2}$$

Precautions:

1. The needle is properly illuminated so that image is bright.
2. Mercury should be pure.

Result:

Refractive index of material of convex lens $\mu_1 =$

Refractive index of material of liquid (water) $\mu_2 =$

Viva questions and answers:

1. What is meant by reflection?

A: When a beam of light is incident on a plane surface separating two media, a part of the incident light is returned to the first medium .This phenomenon is called reflection.

2. Define a mirror?

A: It is a fine polished surface which reflects most of light (above 85%) that is incident on it.

3. What is meant by parallax?

A: The apparent separation of the object on moving the eye is called parallax.

4. What is spherical lens?

A: A piece of transparent material bounded by two spherical surfaces is called a spherical lens.

EXPT No:14

RESOLVING POWER OF TELESCOPE

Aim: To determine the resolving power of telescope

Apparatus:

A telescope wire, guage with fine uniform mesh, sodium vopour lamp, traveling microscope tape (or)a long thread, a rectangular slit of adjustable width attached to a micro meter, screw and reading lens.

Formulae: The resolving power of a telescope is given by

$$\text{Theoretical resolving power} = \frac{1.22\lambda}{a}$$

$$\text{Experimental resolving power} = \frac{d}{D}$$

Where λ = Wave length of sodium light

a = Width of rectangular slit .when the two point objects are just resolved.

d = Average distance between any two adjustment wires of the mesh.

D =Distance between wire gauge and the objective of the telescope.

Procedure:

To deter mine the width of the slit:

Mount the wire gauge vertically on a retort stand at a suitable and height place in front of a sodium light .Average the telescope at a distance from the wire gauge ,Adjust the position of the eye piece by moving it .In(or)out until the cross wire gauge clearly visible. Now illuminate the wire gauge with sodium light focus the telescope on the wire gauge. Do that the vertical and horizontal wires of the mesh are distinctly seen in the plane of the cross wires. Find the least count of the micrometer screw. Mount the adjustable slit fixed to the micrometer screw on the objective of the telescope. Adjust the position of micrometer screw .so that slit is vertical and parallel to the vertical wires of the mesh .Now open the slit and observe the horizontal ,vertical wires of the mesh. Then gradually reduce the width of the slit by turning the micrometer screw until the cross wires of the mesh .Just disappears on only horizontal wires are seen .Note the pitch scale reading, head scale coincidence and find the total reading .Which gives the width of the slit .Repeat the experiment by increasing the distance "D" between the wire gauge and objective of a telescope in steps of 50cm. i.e. at 150cm, 200cm and 250cm and in each case not corresponding width of a silt.

. To determine the distance‘d’ between the two adjacent wires of the mesh:

Remove the wire gauze and place it on traveling microscope. Find the least count of traveling microscope. Adjust the position of eye piece by moving it in (or) out unite the cross wires are clearly visible. Then focus the microscope on the mesh and observe the horizontal and vertical wires of the mesh. The point of intersection of the cross-wire is tangential to the center of vertical wire say first wire. Then after lightening the free motion screw with the help of tangential screw adjust the microscope such that vertical cross wire is tangential to left of the mesh. Note M.S.R and V.C. Find the reading R_0 for the vertical wire of the convenience treat the first wire as 0th wire. Now by containing the number of vertical wire move the microscope to wards the right side and adjust the microscope so that the vertical cross-wire tangential to the left edge of the 5th wire. Again note M.S.R. and V.C. find total reading R_1. Repeat the experiment. Note

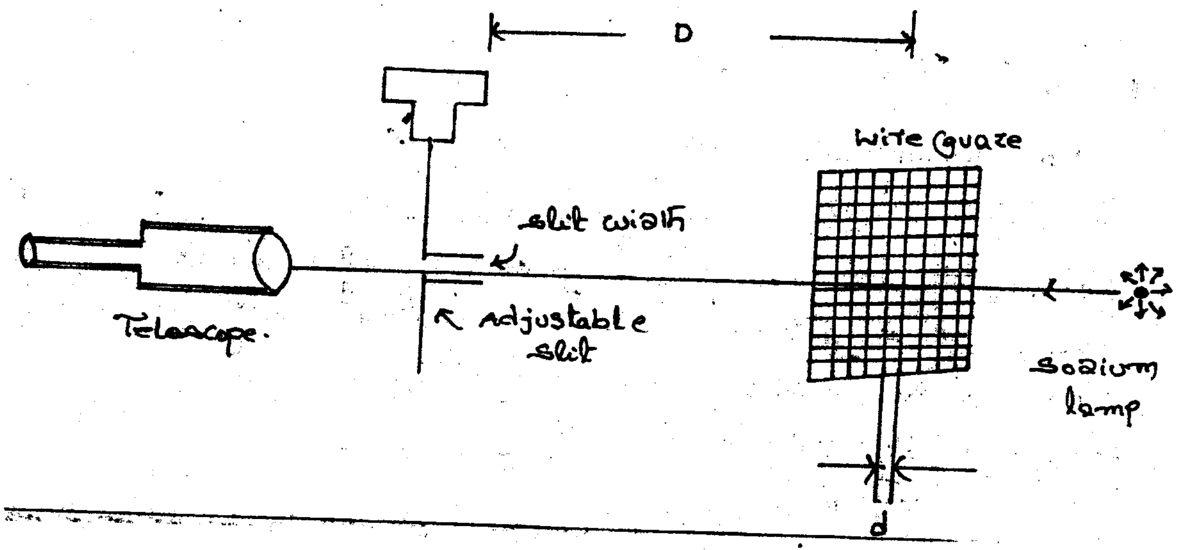


Fig.

observations for10th 15th, 20th, 25th and 30th vertical wires tabulate the observations in table. The difference between R_0 and R_1 gives the distance between five interspaces of mesh. Find average width of the 5 inter spaces. The theoretical and experimental values of resolving power of telescope can be calculated using formula.

Precautions:

1. The adjustable slit and wire gauze should be kept at the same height in the vertical position
2. The distance between wire gauze and objective of the telescope should be measured

   rope (or) long thread.
3. The plane of adjustable slit should be parallel to the vertical wires of the mesh.
4. While taking observations, the microscope should always be moved in one direction to avoid back-lash error.

Observations:

1. **To determine the width of the slit: micrometer screw gauge:**

Number of divisions on the head scale =

$$\text{Pitch of the screw} = \frac{\text{Distance moved on the scale for n rotations}}{\text{No of rotations made by head scale}}$$

$$\text{Least count (L.C)} = \frac{\text{pitch of the screw}}{\text{No of head scale divisions}}\, mm$$

S.No	Distance between wire and objective of the telescope	Micro meter screw reading			Width of the slit	Resolving power of the telescope		
		p.s.r x mm	Head scale coincidence			$\frac{1.22\lambda}{a}$ theor.	$\frac{d}{D}$ observ.	Differ
			observed	calculated				

2. **To determine the distance 'd' between two adjacent wires of mesh:- traveling microscope.**

$$\text{Least count}=\frac{\text{value of one division on main scale}}{\text{Total no of divisions on vernier}}\,cm$$

S.No	No of the vertical wire	Horizontsl scale reading on the microscope when v cross wires to left edge of verticle wire			Width of the 5 inter spaces cm
		m.s.r cm	v.c n	t.r x +(nxLC) cm	
1	0			R_0	X_0
2	5			R_1	X_1
3	10			R_2	X_2
4	15			R_3	X_3
5	20			R_4	X_4
6	25			R_5	X_5
7	30			R_6	X_6

average with 5 interspaces x = cm

Average distance between two adjacent wires $d=\frac{x}{5}=$

Calculations:

Wavelength of the sodium light $\lambda = 5893 \times 10^{-8}\,cm$

d= cm

1. When D = 100 cm, a = cm, $\frac{1.22\lambda}{a}$ = , $\frac{d}{D}$ =

2. When D = 150cm, a = cm, $\frac{1.22\lambda}{a}$ = , $\frac{d}{D}$ =

3. When D = 200 cm a = cm , $\frac{1.22\lambda}{a}$ = , $\frac{d}{D}$

=

4. When D = 250 cm a = cm, $\frac{1.22\lambda}{a}$ = , $\frac{d}{D}$ =

Result:

The theoretical and experimental values of the resolving power of telescope are calculated and compared. They are found to be equal.

Viva questions and answers

1. Define resolving power of on instrument?

A. The ability of an optical instrument to just resolve the images of two near by objects is called its resolving power.

2. What is resolving power of normal eye?

A. 1minute.

3. When will be resolving power of telescope be higher?

A. The telescope large diameter of objective has a higher resolving power. How ever with the increase in diameter of the objective the effect of spherical aberration becomes appreciable.

EXPT.NO:15

BIPRISM

Aim: To determine the wavelength of a given monochromatic light (sodium) by forming interference fringes using Fresnel's Biprism.

Apparatus:

Fresnel's Biprism, optical bench with four up rights, convex lens of short focal length (15 to 20 cm) sodium vapour lamp, low power micrometer. Eye piece, variable slit, spirit level and reading lens.

Formula:

$$\lambda = \frac{d(\beta_2 - \beta_1)}{(D_2 - D_1)}$$

λ = wave length of monochromatic light to be determined.

d = distance between two coherent sources

= $\sqrt{d_1 d_2}$

d_1 = distance between two magnified images of the coherent sources

d_2 = distance between the two diminished images of the coherent sources.

β_1 = fringe width when the distance between the slit and eye piece is

β_2 = fringe width when the distance between the slit and eye piece is

Procedure:

The following preliminary adjustments are to be made in order to obtain well screws defined interference fringes.

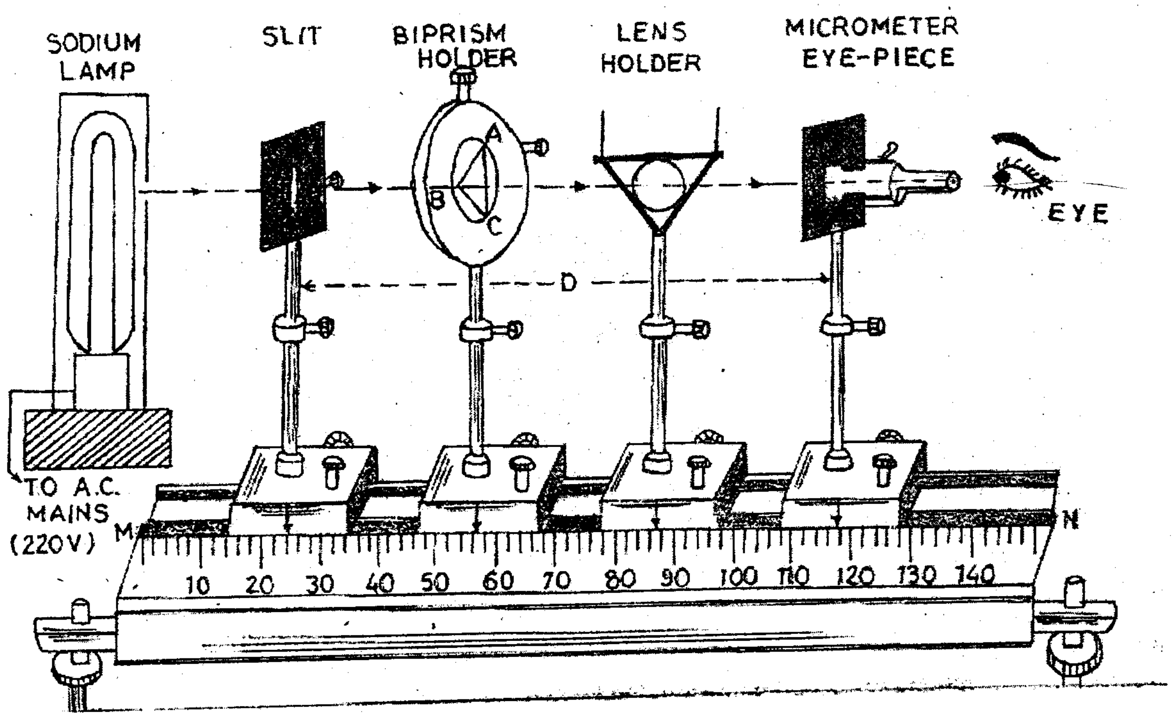


fig15.1

1. Priliminary adjustment to obtain interference fringes:

1. Level the optical bench so that it is perfectly horizontal by means of leveling screws and spirit level.

2. Place the slit of variable width on one of the up right at one end of the optical bench. Illuminate the slit with sodium vapour lamp. Reduce the width of the slit and rotate it in its own place, about an axis parallel to the axis of the bench with the help of a tangent screw until the slit becomes vertical.

3.Mount the Biprism on the second up right and keep it at a distance of about 10 cm from the slit w. Adjust the height of the slit and biprism so that their centers lie in a horizontal line parallel to the length of bench. Observe the slit through biprism with a naked eye. Then two virtual sources of slit will appear.

4. Now mount micrometer eye piece on third up right and keep it at a distance of 30 cm from the biprism at the same height as that of the slit and the biprism. Focus the eye piece on the cross-wires and move the eye piece in out until the cross wires of clearly seen. Make one of the cross wire vertical by rotating the eye piece. Looking through the eye

piece rotate the biprism slowly with the tangent screw unit closely spaced alternately dark and bright interference fringes of (bands) are obtained.

(2)To determine fringe width β_1 when eye piece is placed at a distance D_1 from the slit:

Find the least count of the micrometer screw. Place the eye piece at a convenient distance D_1 from slit. Viewing through the eye piece. The interference fringes thus formed move the micrometer screw to one side of the interference pattern such that the point of intersection of the cross-wires coincide with centre of one of the bright fringes say first fringe. Adjust the position of micrometer screw so that the vertical wire exactly coincides with the center of the bright fringe. Note the P.S.R and head scale coincidence of the fringe. Find the T.R R_0 as 0^{th} fringe by counting the no. of bright fringes move the micrometer screw towards right side and adjust the micrometer screw. So the cross wire coincides with the 5^{th} fringe. Then note P.S.R and H.S.C find total reading R_1 repeat experiment and note observations for $10^{th}, 15^{th}, 20^{th}, 25^{th}$ and 30^{th} bright fringe. Note readings in table. The diffraction between $R_0 and R_1$ gives the fringe width of 5 fringes. Find the average width of 5 bright fringes .From this find The width of the one fringe β_1

R₀ adopting the same procedure as above.

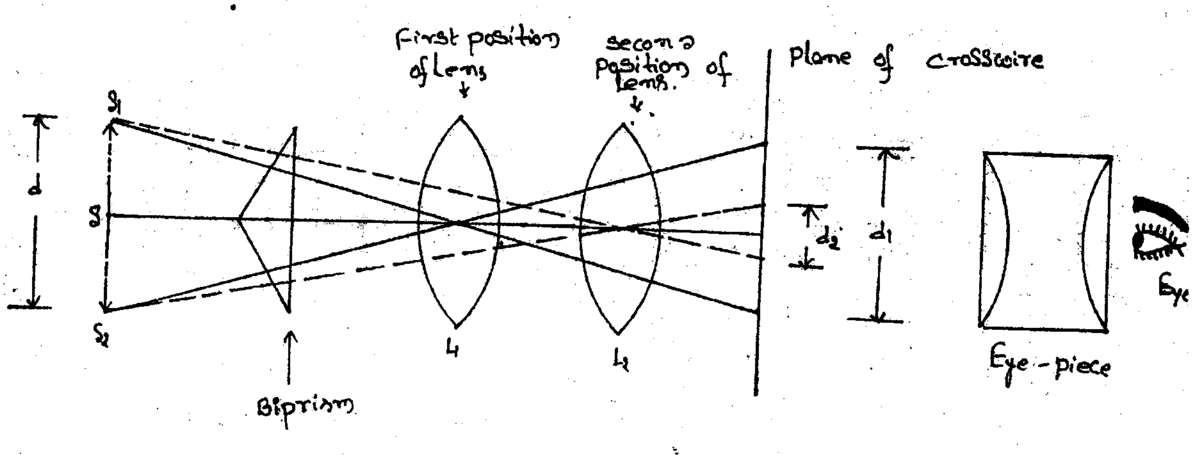


fig 15.2

(4).To determine distance 'd' between the virtual sources $S_1 and S_2$:Lens displacement method

Take a convex lens of focal length 15 cm and mount it on the fourth up right. interpose the up right carrying the convex lens between the biprism and the eye piece without disturbing the positions of the slit and biprism. Adjust adjust the position of the lens so that its center lies between along the line joining the center of the slit. The biprism and the point of intersection of cross-wires. In order to get two positions of the lens to form two distinct images of the coherent sources $S_1 and S_2$ .Move the eye piece to a suitable distance so that the distance between the slit and eye piece is greater than four times the focal length of the lens. move the lens slowly towards the biprism until two sharp and magnified images of virtual sources $S_1 and S_2$ are formed in the focal plane of. eye piece, adjust the position of micrometer screw such that the vertical cross wire coincides with left edge of one image. Note the P.S.R and H.S.R. Find total reading L_1 .Move the micrometer screw in one direction and adjust the position of micrometer screw so that the vertical cross wire coincides with left edge of other image.

then note P.S.R and H.S.C find T.R R_2 the diffraction between $L_1 and R_1$ gives the distance d_1 between the two magnified images of the coherent sources. Now move the lens towards eye piece. And biprism fixed. Repeat the same procedure as was adopted above and note readings. The difference between $L_2 and R_2$ gives the distance d_2 .Between diminished image of the coherent sources. The distance between two coherent sources can be found using the relation.

Observations:

No. of divisions on the head scale =

$$\text{Pitch of the screw} = \frac{\text{distance moved on pitch scale for n rotations}}{\text{No.of rotations made by Head}}$$

$$\text{Least count (L.C)} = \frac{\text{Pitch of the screw}}{\text{No of head scale divisions}}$$

1.To determine the fringe width β_1 :-

Distance between slit and eye piece D_1 = cm

zero error

divisions

correction

divisions

S No	No of bright fringe	Micro meter reading			Total reading $a+(n\times LC)$	Width of the 5 fringes
		P.S.R a mm	H. S coincidence			
			Observed	corrected		
1	0				R_0	x_0
2	5				R_1	x_1
3	10				R_2	x_2
4	15				R_3	x_3
5	20				R_4	x_4
6	25				R_5	x_5
7	30				R_6	x_6

Average width of five fringes x = …… mm

Average fringe width $\beta_1 = \frac{x}{5} =$…… mm

2.To determine the fringe width β_2 :

Distance between slit and eye piece D_2 = …….cm

Divisions

S No	No of bright fringe	Micro meter reading			Total reading $a+(n\times LC)$	Width of the 5 fringes
		P.S.R a mm	H. S coincidence			
			Observed	corrected		
1	0				R_0	x_0
2	5				R_1	x_1
3	10				R_2	x_2
4	15				R_3	x_3
5	20				R_4	x_4

6	25				R_5	x_5
7	30				R_6	x_6

Average width of five fringes x = ... mm

Average fringe width $\beta_2 = \frac{x}{5} =$...... mm

3. To determine the distance 'd' between two virtual sources $S_1 and S_2$:

L.C of the micrometer screw = ...mm.

Type of image formed	Micro meter readings when the vertical cross wires coincides with left edge of								Distance between two images
	First image				Second image				
	p.s.r a mm	Head scale coincidence		T.R L= $a+(n\times LC)$	p.s.r a mm	Head scale coincidence		T.R L= $a+(n\times LC)$	
		Observed	corrected			Observed	corrected		
Magni-fied									
Diminis-hed									

Distance between two virtual sources $d = \sqrt{d_1 d_2}$ =....... mm = ... cm.

Calculations:

$D_1 =$ $D_2 =$ $d_1 =$ $d_2 =$

$\beta_1 =$ $\beta_2 =$ $d = \sqrt{d_1 d_2}$

Precautions:

1. The optical bench should be made perfectly horizontal using leveling screw and the spirit level.
2. The slit should be vertical and narrow. The slit should be erected parallel to the reflecting edge of the biprism to get well defines fringes.
3. A large no. of observations should be taken to determine the mean fringe width.

4. The micrometer screw should always be moves only in one direction to avoid back lash error

8. The index error should be found and the correction has to be added to the distance D between the slit and the eye piece up right.

Results:

Wave length of the sodium light $(\lambda)A^0$	Exp.value	Std.value	Deviation
		5893 A	

VIVA QUESTIONS:

1. What is biprism ? How it is constructed?

A).A biprism is a combination of two acute angled thin prisms, each of refracting angle about joined base to base. It is made from a single glass plate by suitable grinding and polishing.

2. What is an optical bench?

A).A horizontal metal (or) wooden base of nearly two meters long with a millimeter scale fixed on one arm is called an optical bench.

3. What is function of biprism?

A) To produce two coherent images of the given slit which are separated by a small distance. The two images behave as coherent sources and when the wave trains from these two coherent sources superimpose on a screen then they produce equally spaced alternate bright and dark fringes.

EXPT: 16

LASER

Aim: To determine the wavelength of the laser source using a plane diffraction grating.

Apparatus::
A 0.5 mw He-Ne laser, a plane diffraction grating, a thin rope, meter scale and a screen.

Formula: $\lambda = \frac{\sin\theta}{Nm}$

Where λ =Wave length of the laser light

θ =angle of diffraction.

N=number of lines per cm on the grating

m=order of diffraction.

$$\sin\theta = \frac{Ym}{\sqrt{D^2 + ym^2}}$$

Where Ym =linear separation of different order diffraction maxima's from the central maxima.

D=distance between the grating and the screen.

Procedure:

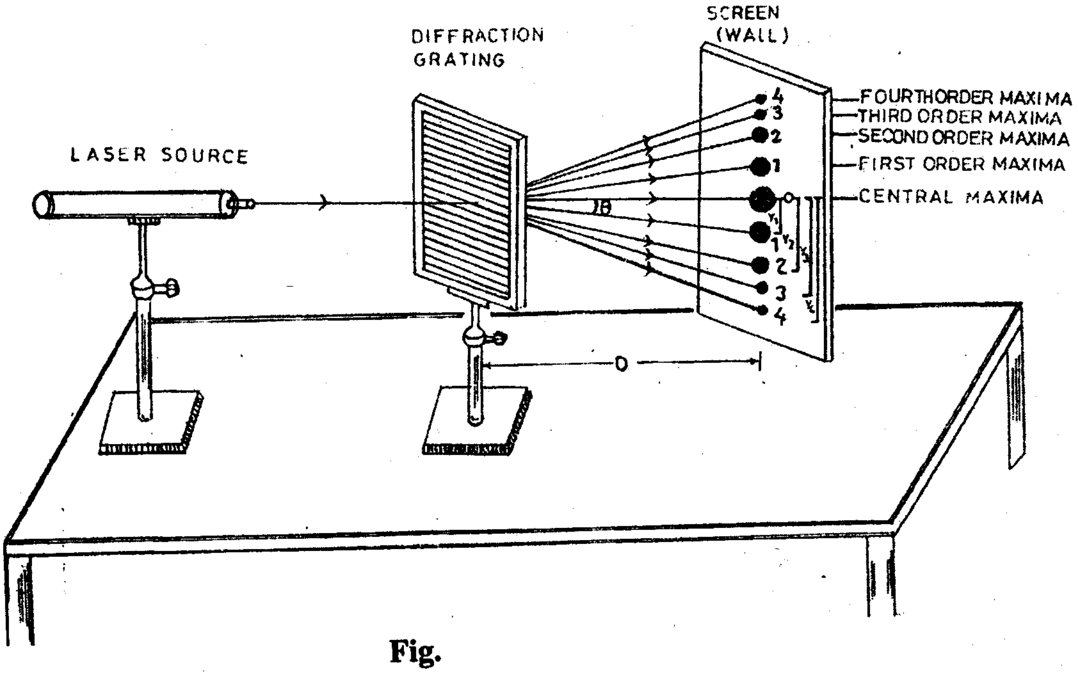


Fig.

Mount the given laser source whose wave length is to be determined, horizontally on a stand. Clamp diffraction grating vertically on a stand. Such that the light emitted from the laser source is incident \normally on the grating .Place the screen at a suitable distance from the grating in order to visualize the various orders of the diffracted images of the laser beam on it Now switch on the laser source, adjust the relative orientation of laser with position of the screen with respect to the diffraction till a central bright red spot with about (4)or(5)spots on either side of the central bright spot and measure the linear separation between screen and grating .Find the angle of diffraction θ and $\sin\theta$ .Substituting the values of N, m and $\sin\theta$ in the formula, the wave length λ of given laser source can be calculated.

Repeat the experiment with grating of different number of lines per cm and note the observations in table. In addition to by varying the distance between the screen and the grating measure the distance of first, second, third and fourth order maxima from the central maxima and find the values of $\sin\theta$ using the formula note the observations in table.

Result:

Wavelength of the given laser source is...... A^o

Observations:

1.To determine the linear separation Ym and the wave length .

No. of lines per cm on grating N =

Distance between screen and grating D = cm

Order of diffraction	Linear separation			$\sin\theta = \frac{Ym}{\sqrt{D^2 + ym^2}}$	$\lambda = \frac{\sin\theta}{Nm}$
	Left A cm	Right B cm	Mean (A+B)/2		

mean value of λ = ... cm

2. To determine the linear separation in Ym and the wave length for different values. of N:

Distance between the screen and grating D = cm.

N = 200/2.54, 500/2.54, 1000/2.54, 1500/2.54.

Order of diffra -ction	No of lines on grating per cm	Linear separation			$\sin\theta = \frac{Ym}{\sqrt{D^2 + ym^2}}$	$\lambda = \frac{\sin\theta}{Nm}$
		Left A cm	Right B cm	Mean (A+B)/2		
1	78.74					
2						
1	196.85					
2						
1	393.7					
2						
1	590.55					
2						

mean value of λ = ... cm

3. To determine the linear separation Ym for different values of N.

S. No	Distance between the grating and the screen D	Distance from the central maxima Ym	Order of diffraction m	$\sin\theta = \frac{Ym}{\sqrt{D^2 + y}}$	$\lambda = \frac{\sin\theta}{Nm}$
1	50		1		
2	45		1		
3	40		1		
4	35		1		
5	30		1		
6	25		1		
7	20		1		
8	15		1		
9	10		1		
10	5		1		

11	15		2		
12	10		2		
13	5		2		

mean value of λ = ... cm

Precautions:

1. The screen should be moved carefully till the spots are clearly visible on it.
2. Laser light source should be switched off after taking the observations.
3. The grating should be clamped on the stand so that light emitted bu laser should incident normally on it.
4. The grating and the screen should be at the same height.

Calculations:

1.For N = , D = cm, m =1, , Y_1 = cm

, $\sin\theta = \frac{Y_3}{\sqrt{D^2+Y_1^2}} =$ $\lambda = \frac{\sin\theta}{N} =$ cm=

2. For N , D = cm, m =1, Y_2 = cm

, $\sin\theta = \frac{Y_3}{\sqrt{D^2+Y_2^2}} =$ $\lambda = \frac{\sin\theta}{2N} =$

3. For N = , D = cm, m =1, Y_3 = cm

, $\sin\theta = \frac{Y_3}{\sqrt{D^2+Y_3^2}} =$ $\lambda = \frac{\sin\theta}{3N} =$

cm=

4. For N = , D = cm, m =1, Y_4 = cm

, $\sin\theta = \frac{Y_3}{\sqrt{D^2+Y_4^2}} =$ $\lambda = \frac{\sin\theta}{4N} =$

VIVA QUESTIONS AND ANSWERS:-

1. What is meant by laser ?

A.LASER means Light Amplification by Stimulated Emission of radiation.

2. What are the properties of laser ?

A.1.High monochromacity 2. High directionality

3.High degree of both spatial and temporal coherence.

4.High brightness. 5. Highly polarized 6. Highly energetic 7. High Intensity

EXPT.No.17

THERMO EMF

Aim:- To determine the thermo E.M.F generates in a thermocouple, using the potentiometer.

Apparatus:

Potentiometer, thermometer, rheostat, thermocouple, a storage cell, plug key, two resistance boxes, a standard cell of E.M.F (or) a Daniel cell with a fully saturated, copper sulphate solution, a sensitive galvanometer, a high resistance , jockey, connecting wires and water path with suitable heating arrangement.

Formula:

E=P.D. across 1 cm of the potentiometer wire

$\times$ mean balancing length.

$=5\times l$ micro volts.

where E= thermo E.M.F generated in the thermo couple.

l = mean balancing length in cm.

$$l=\frac{l_1+l_2}{2}$$

where l_1 = balancing length while heating part

l_2 = balancing length while cooling part.

Procedure:

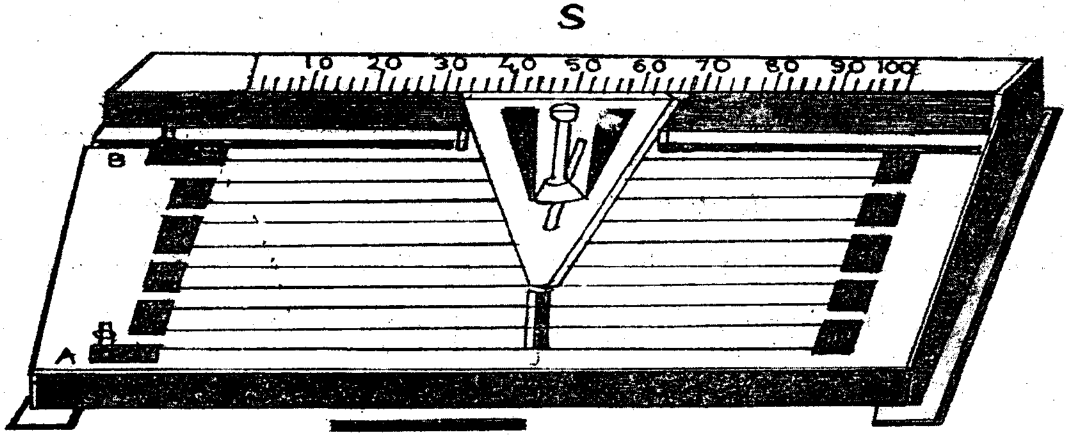


Fig. 1

AB is the potentiometer wire of length 'L' and resistance R_1 .Connect a storage cell of E.M.F volts. plug key K_1, Rh, and two resistance boxes R_1 and R_2 in series with the wire AB which forms a primary circuit as shown in diagram. Connect the + ve terminal of the standard cell Sc to the junction 'C' between the two resistance boxes R_1 and R_2 and connect its -ve terminal to the jockey (J) through a very sensitive galvanometer G and high resistance H.R before taking observations, close the key. Press the jockey on the potentiometer wire near the two ends A,B. then the deflection the galvanometer should be opposite.

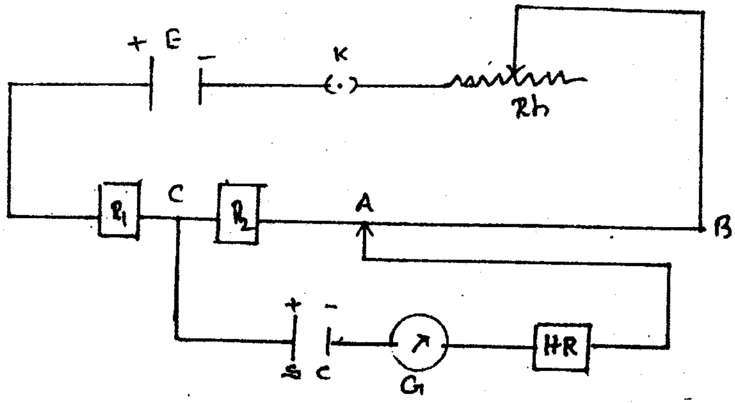


Fig. 2

a). keep the 'Rh' at zero resistance throughout the experiment put R_1 =0 and un plug a resistance of 5 ohm from box R_2. Close the primary circuit .press the jockey at different points on potentiometer wire and obtain the approximate balance point where the galvanometer shows no deflection. then short circuit the H.R. press the jockey on the wire and find exact balance unit point. Note the balancing length l_1 cm.

b). keep $R_1 = 5\Omega$ and $R_2 = 0$ repeat the same procedure as was adopts above and find the rough balance point. Again short circuit the H.R and locate the exact balance unit point. Then not the balancing l_2 cm. obviously, the resistance of $(l_2 - l_1)$ cm of the potentiometer wire =5 ohms.

The resistance 'r' of the entire length (1000cm) of the potentiometer is given by

$$r = \frac{5}{(l_2 - l_1)} \times 100 \text{ Ohm}$$

$$= \frac{5000}{l_2 - l_1} \text{ ohm}$$

Repeat the experiment varying resistances in R_1 and R_2 and in each case note the balancing length tabulate observations in table. Find mean value of 'r'

(II) CALIBRATION OF POTENTIOMETER WIRE:

(a) To calibrate the potentiometer wire for copper constantan couple:-

According to ohms law when a no. of resistances are joined in series the P.D across any resistance is proportion to its resistance, for the same current flowing through all the resistances

For a copper constantan couple the E.M.F developed is 40 micro-volts when the temp. Difference between the two junctions is

The maximum E.M.F developed in couple for $100^{\circ}c$ temp difference between the two junctions is given by

E.M.F=40100 micro volts

=40x10^3 micro-volts

= 4 milli volts

1 milli volt = 10^{-3} volts

1micro volt = 10^{-6} volts

according to equation $E = at + bt^2$ where a, b are constants and they vary from couple to couple.

The potentiometer wire should be sensitive enough to measure the voltage of micro-volts range. Therefore, the potentiometer is to be calibrated for fall of required potential drop per millimeter length of the wire and then the E.M.F of the thermo couple should be measured.

As the E.M.F developed in copper-iron couple is nearly 15 micro volts per °c . so the potentiometer wire should be calibrated such that the potential difference between its ends is 5 milli volts per cm. According to ohms law a P.D. of 5 milli volts develop between the ends of a potentiometer wire resistance 'r' Ohm. Then the resistance R in R_2 in series with the wire which develops a P.D of 1.08 volt. is given by

$$\frac{R}{R_L}=\frac{1.08}{0.005}$$

$$\therefore R = 216R_L \text{ ohm}$$

now connect the circuit shown in diagram.

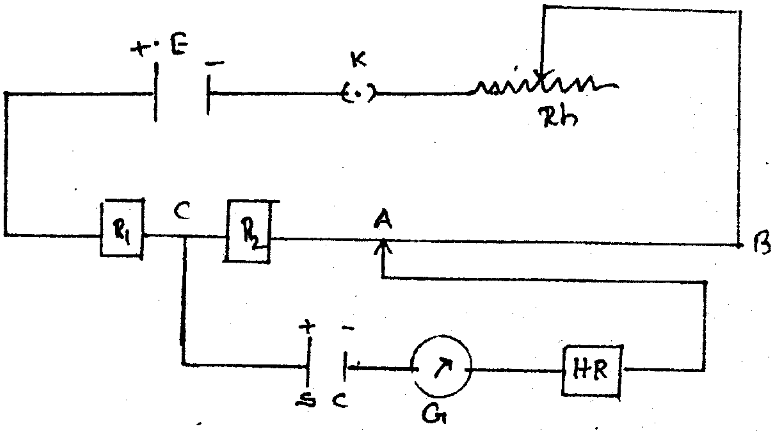


Fig. 3

Plug out a resistance of 216r ohm from resistance box R_2 .connect the +ve terminal of a Daniel cell of E.M.F 1.08volts to the junction 'c'of the two resistances boxes $R_1 \text{and} R_2$. Connect -ve terminal of the standard cell to the jockey through the galvanometer and a high resistance from R_1 nearly equal to the resistance R_2 in unit the galvanometer shows no deflection . then the circuit H.R adjust rheostat so that the galvanometer reads zero deflection now P.D across the resistance $R_2(1.08V)$ and P.D between the ends of the potentiometer wire will be 5-milli volts.

b).To calibrate the potentiometer wire for copper-iron couple:-

For copper-iron couple E.M.F developed is nearly 15 micro volts. When the temp. difference between junctions is

the max. E.M.F developed in couple for 100°c temperature. Difference between the two junctions is given by

$$\begin{aligned} \text{E.M.F} &= 15 \times 100 \text{ micro-volts} \\ &= 1500 \text{ micro-volts} \\ &= 1.5 \text{ milli volts} \end{aligned}$$

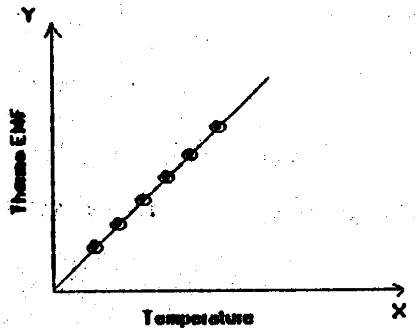
The potentiometer should be calibrated such that the P.D between its ends is 2 milli volts (or) 2 micro volts. According to ohms law 2 milli volts (or) 0.002 volts develop between ends of wire of resistance 'r' ohms, then the resistance R in R_2 in series with the wire which develops a P.D of 1.08 volts is given by

$$\frac{R}{R_L} = \frac{1.08}{0.002}$$

$$\therefore R = 540 R_L \text{ Ohm}$$

the Daniel cell of E.M.F=1.08 V is balanced with the E.M.F develops across the resistance and the P.D between the ends of the potentiometer wire will be two milli volts. The potentiometer is now ready for measuring themo E.M.F develops at different temperatures of the thermo couple.

Graph:



To measure the E.M.F of the copper constantan thermo couple:

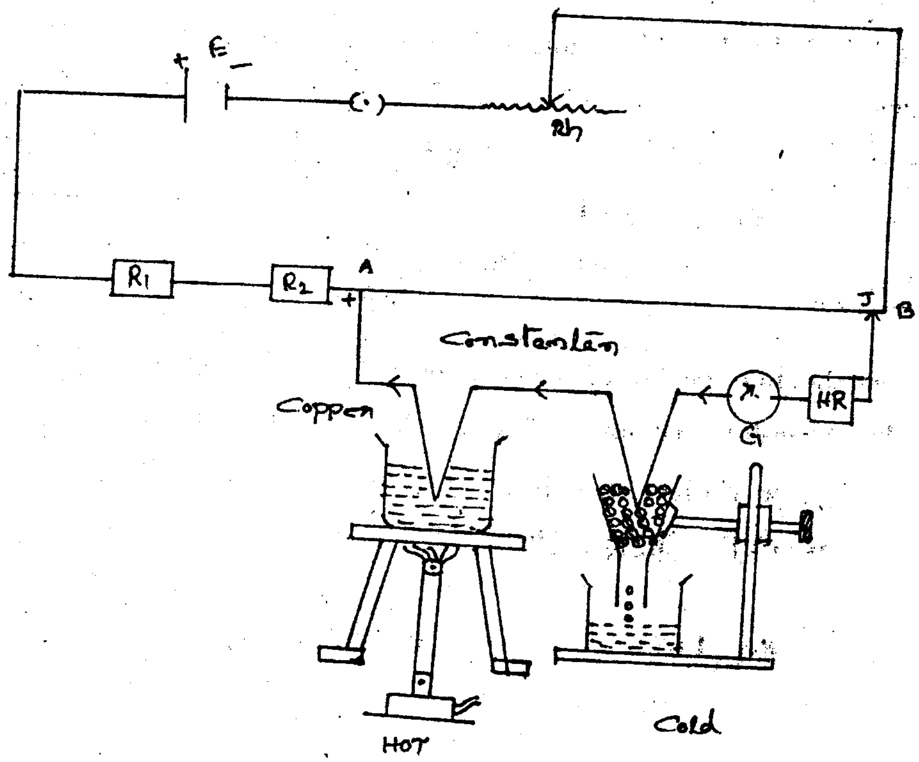


Fig 17.3

(III) To measure the E.M.F of the copper constantan thermo couple:

After calibrating the potentiometer wire for measuring the E.M.F of copper constantan couple disconnect the Daniel cell without disturbing the primary circuit. Connect the thermo couple between the terminals A and the jockey as shown in diagram. Arrange couple such that the hot junction in melting ice.

Graph:

Draw a graph with temp. of hot junction on the x-axes and thermo E.M.F developed in couple on the y-axes. A straight line passing the origin will be obtained.

Observations:

To determine the resistance ‘r’ of potentiometer wire:-

S.No	Resistance in R_1 ohm	R_2 cm	Balancing length	Difference $(l_2 - l_1)$	Resistance $R_L = \frac{5000}{l_2 - l_1}$
1	0	5	l_1		
	5	0	l_2		
2					

Mean of r = ……..ohm

To measure the thermo E.M.F of copper constantan couple:

S No	Temp of hot junction	Balancing length			Thermo EMF in micro-v E	Thermo EMF per degree difference of temp E/t
		While heating l_1	While cooling l_2	Mean l=$(l_2 - l_1)/2$		
1	10					
2	20					
3	30					
4	40					
5	50					
6	60					
7	70					
8	80					
9	90					
10	100					

Precautions:

1. The ends of the connecting wires should be clean and the light connections should be made.
2. The jockey should not be dragged on wires. t should be pressed gently.

3. The thermo couple should not touch the bottom (or) inside the vessel
4. The cold junction should be kept in convenient distance from burner.
5. While taking the observations, the temperature of hot junction should be maintained constant.
6. The junction of thermo couple should be properly welded.
7. The cell used in primary circuit must have a constant E.M.F.

VIVA QUESTIONS AND ANSWERS

1. What is a potentiometer?

A.It is a device used to measure the potential difference. it consists of a long, thin uniform wire of manganin (or) constantan fixed on a wooden board.

2. what are uses of potentiometer?

A.1. To measure the thermo E.M.F

2. To compare E.M.F of two cells.

3. To calibrate an ammeter and voltmeter.

4. To determine the internal resistance of cell.

3. What is potential gradient?

A.The change in potential per unit length of the potentiometer wire is called the potential gradient.

4. Why manganin is preferred for making potentiometer wire?

A.Manganin has low temperature coefficient of resistance and high resistivity.

5. What is meant by E.M.F of cell?

A. The E.M.F of cell is the potential difference across the terminals of the cell when the cell is in an open circuit i.e., when no current is drawn from the cell.

(కత్తిరించి పంపవలెను)

అధ్యాపకుల, విద్యార్థుల సలహాలు, సూచనలు :

అధ్యాపకులు, విద్యార్థులు ఈ స్టడీ మెటీరియల్‌కు సంబంధించిన సలహాలు, సూచనలు, ముద్రణ దోషాలు తెలియపరచినచో, పునర్ముద్రణలో తగు చర్యలు తీసుకొనగలము. తెలియపరచవలసిన చిరునామా : డిప్యూటీ డైరెక్టర్, దూరవిద్యా కేంద్రం, ఆచార్య నాగార్జున విశ్వవిద్యాలయం, నాగార్జున నగర్ - 522 510.

Course	Year	Paper No. & Title
B.Sc., Physics	2nd Year	Paper - I : Thermodynamics & Optics

(కత్తిరించి పంపవలెను)

(కత్తిరించి పంపవలెను)

(కత్తిరించి పంపవలెను)

How to prove the transverse nature of light ?

Now the light polarised by one tourmaline crystal is passed through other tourmaline crystal. When its optic axis is parallel to the former one, it passes through without obstruction .If the second crystal is rotated,then the intensity of light eminating from second crystal varies. Its intensity becomes zero when the optic axis of two crystals are perpendicular to each other .This means that the vibrations are perpendicular to the direction of propagation of light. If they are parallel to the propagation ,then intensity would have been the same in all cases. Hence we can infer that light has transverse nature. Generally the first crystal is termed as polariser and second crystal as analyser.

12.3 PLANE OF POLARISATION AND PLANE OF VIBRATION :-

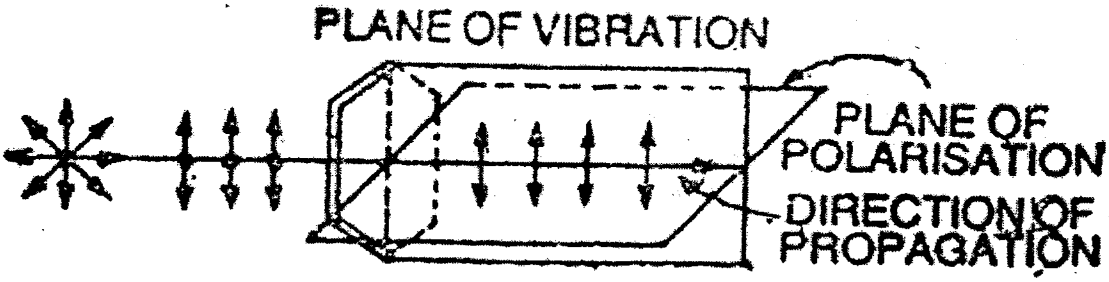


Fig 12.2

When ordinary light is passed through tourmaline crystal, the light is polarised . The vibrations of polarised light are confined to a single plane. The plane in which the light vibrations are confined and perpendicular to the direction of propagation is called "Plane of Vibration".The plane perpendicular to the plane of vibration and has no vibrations is called "Plane of Polarisation" (Fig. 12.2).

12.4 BREWSTER'S LAW :-

One simple method of producing polarised light is through reflection. The light ray reflected from a glass plate is partially polarised .The extent of polarisation changes with angle of incidence . This angle of incidence is termed as angle of polarisation .

LAW:-

Brewster conducted experiments with different reflecting surfaces and different angles of incidence. He proved that for each reflecting surface there is one angle of polarisation for which the reflected light is almost completely polarised. At this instance, the angle between reflected ray and refracted ray is 90^0 .

He also proved that the tangent of this particular angle of polarisation p(for which the reflected light is completely polarised) is equal to the refractive index of the material of the refracting surface $\mu = \text{Tan}p$.

Proof :-

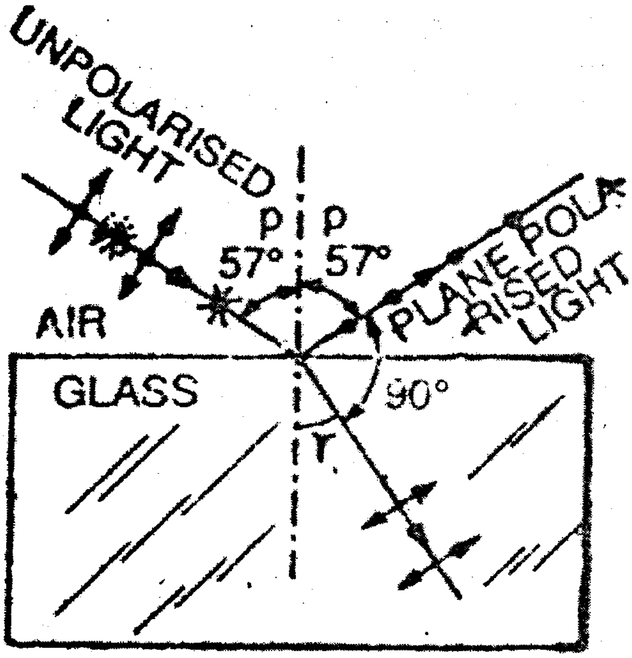


Fig. 12.3

From the fig 12.3 suppose the angle of incidence of light incident on a reflecting surface of refractive index μ is 'p'. This is also called as angle of polarisation. Let 'r' be the angle of refraction. By the principle of reflection, the angle of reflection = angle of incidence.

Angle of reflection = p , i = p

Brewster's law states that $\mu = \text{Tan}p$ --------- (12.1)

$$\text{But } \mu = \frac{\text{Sin}i}{\text{Sin}r} = \frac{\text{Sin}p}{\text{Sin}r} \text{ --------- (12.2)(from Snell's law), From (12.1) and (12.2)}$$

$$\text{Tan}p = \frac{\text{Sin}p}{\text{Sin}r} \Rightarrow \frac{\text{Sin}p}{\text{Cos}p} = \frac{\text{Sin}p}{\text{Sin}r}$$

$$\Rightarrow \text{Cos}p = \text{Sin}r$$

$$\Rightarrow \text{Sin}(90^0 - p) = \text{Sin}r$$

$$\therefore 90^0 - p = r \Rightarrow r + p = 90^0$$

Hence the angle between reflected and refracted rays is 90^0 when reflected light is completely polarised.

12.5 LAW OF MALUS :-

This law indicates the intensity of polarised light. When a completely plane polarised beam is incident on the analyser "the intensity of the polarised light transmitted through the analyser varies as the square of the cosine of the angle between the plane of transmission of the analyser and the plane of polariser".

Proof:-

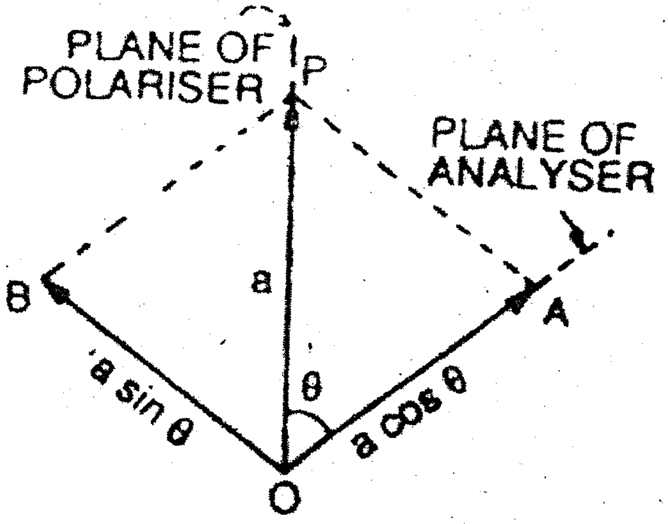


Fig 12.4

As shown in the fig 12.4 let 'a' be the amplitude of the plane polarised light incident on an analyser. Let 'θ' be the angle between the polariser and analyser. If we resolve the amplitude of the plane polarised into two perpendicular components, one parallel to the plane of analyser (a Cos θ) and the other perpendicular to the plane of analyser (a Sin θ). Then the component transmitted through the analyser = a Cos θ

∴ Intensity of light transmitted through analyser = $I_\theta = (a\cos\theta)^2$

$$= a^2\cos^2\theta = I_0\cos^2\theta$$

$$\therefore I_\theta = I_0\cos^2\theta \text{ or } I_\theta \propto \cos^2\theta - \text{MALUS LAW}$$

When $\theta = 0^0$ i.e. when analyser and polariser are parallel to each other

$$I_\theta = I_0$$

When $\theta = \pi/2$ i.e. when analyser and polariser are perpendicular to each other.

$$I_\theta = 0$$

This is what that is observed practically in the case of tourmaline crystals.

12.6 POLARISING CRYSTALS :-

Before going to the concept of double refraction .Let us define optic axis and principal section which will be an aid in understanding further concepts .Crystal is a regular structure .

Optic axis :-

A line passing through any one of the blunt corners(where the three obtuse angles meet in a crystal) and making equal angles with the three faces which meet at this corner , locate the direction of the optical axis of the crystal .

Principal section :-

Any plane which contains the optic axis and is perpendicular to the opposite faces of a crystal is called Principal Section .

The crystals having a single optic axis are called Uniaxial crystals and the crystals containing two optic axis are called Bi-axial crystals .

12.7 DOUBLE REFRACTION :-

Now since we understood a bit about the crystals . we consider anisotropic crystals [crystals which have different physical properties in different directions].

When unpolarised light travels through anisotropic crystals like Calcite , the refracted light is split up into two refracted rays . One which obeys ordinary laws of refraction called Ordinary refracted ray and the other which doesnot obey refraction laws called Extra-ordinary refracted ray .The ordinary ray has plane of vibrations perpendicular to the principal section of the crystal and extra-ordinary ray has plane of vibrations in the principal section.

In these doubly refracting crystals also , in uniaxial crystals the velocity of ordinary and extra-ordinary rays are equal along one single optic axis and in bi-axial crystals, there are two optic axis along which the velocities are different .

Hence the phenomenon of splitting up of refracted ray into ordinary and extra-ordinary rays in a double refracting crystals is called "Double Refraction ".

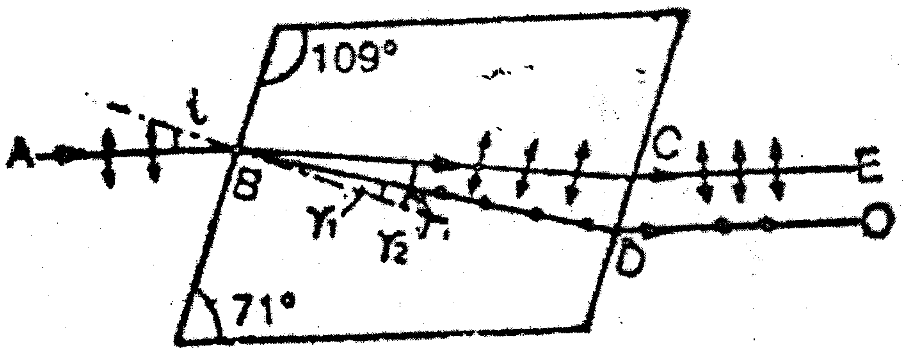


Fig 12.5